Disorder, Supersolidity, and Quantum Plasticity in Solid Helium 4

S. Balibar · A.D. Fefferman · A. Haziot · X. Rojas

Abstract Several years after Kim and Chan's discovery of an anomaly in the rotation properties of solid helium (Kim and Chan in Nature 427:225, 2004; Science 305:1941, 2004), the interpretation of the observed phenomena as a manifestation of supersolidity remains controversial. J. Beamish and his collaborators have shown that the rotation anomaly is accompanied by an elastic anomaly (Day and Beamish in Nature 450:853, 2007; Day et al. in Phys. Rev. Lett. 104:075302, 2010; Syshchenko et al. in Phys. Rev. Lett. 104:195301, 2010): when the rotational inertia apparently increases, the shear modulus decreases. This softening is due to the appearance, in the solid, of a large reversible plasticity that is a consequence of the evaporation of ^{3}He impurities from dislocations that become mobile. This plasticity is called "quantum plasticity" because the dislocations move by quantum tunneling in the low temperature limit.

Since the main evidence for supersolidity comes from torsional oscillator (TO) experiments, and since the TO period depends on both the inertia and the stiffness of solid ^{4}He, it is not totally clear if supersolidity really induces a change in inertia or if it is the disappearance of quantum plasticity that mimics supersolidity in TO experiments.

In order to distinguish between supersolidity and quantum plasticity, we have studied the rotational and the acoustic properties of solid ^{4}He samples with a variable amount of disorder and of ^{3}He impurities. Of particular interest is the comparison of single crystals to polycrystals but the whole problem is not yet solved.

This short review article is an opportunity to discuss several questions regarding the exact role of disorder in supersolidity and in quantum plasticity.

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1 Introduction

The successive studies of the stiffness of solid ^{4}He by J. Beamish and his collaborators [3–5] show that if supersolidity exists, it takes place in the stiff state of solid ^{4}He, not in its soft state. This is a little paradoxal because one might expect a supersolid state in which a fraction of the mass can flow through the rest to be softer than when no flow takes place. However, since supersolidity does not appear possible in bulk crystals without defects [6], and since the magnitude of the rotation anomaly appears to increase with disorder [7], various authors have assumed that supersolidity could occur in the core of either dislocations [8] or grain boundaries. This assumption has been supported by numerical studies [9, 10]. Given this, some authors tried to imagine how a solid with supersolid defects could be stiffer than with normal—i.e. non-supersolid—defects. One suggestion was that the core of a dislocation has a long range quantum coherence only if this dislocation is immobile because ^{3}He impurities kill its fluctuations [11]. After some preliminary studies of mass flow inside ^{4}He crystals, followed by studies of the properties of grain boundaries, we have studied the acoustic properties [12, 13] and the rotational properties [14] of solid ^{4}He samples having a variable amount of disorder and of ^{3}He impurities. However, at the present stage of our studies, the role of disorder is not totally clear in all these phenomena and the existence of supersolidity appears still somewhat questionable.

In the first section we consider why disorder is usually believed to be at the origin of both the rotational and the elastic anomaly of solid ^{4}He. In the next section, we discuss the respective role of dislocations and grain boundaries in the light of our experiments and of a few others. In the third section, we focus on dislocations and we discuss how they could be pinned by the adsorbtion of ^{3}He impurities or by other mechanisms. We then discuss various difficulties with the usual supersolid scenario. In our conclusion we propose a few more ideas for future experimental or theoretical work.

2 Disorder

Shortly after the first publications by Kim and Chan [1, 2], early models of supersolidity were revisited. The discussion focused on vacancies whose presence at low temperature was questionned. According to Fraass et al. [15], their activation energy was +8 K so that their density should be negligible at 0.1 K. Boninsegni et al. [16] used a "Path Integral Monte Carlo" method to calculate this energy and they found +13 K, that is a similar value. Despite the existence of various controversies about Fraass' data analysis and about the validity of Boninsegni's result (see Anderson [17] followed by Maris [18]; for a review, see Balibar and Caupin [19]), we believe that vacancies have an activation energy of order 10 K and that they should consequently be absent at low temperature. This value was confirmed by the recent experiments of Bossy et al. [20].

The next question was whether the presence of vacancies is necessary or not necessary for the existence of supersolidity. Prokof'ev et al. explained (see Prokofi'ev's review [6]) that, in the absence of vacancies, supersolidity is unlikely to occur. Their reasoning is twofold. If the wavefunction of the supersolid fraction is well defined, one needs to have fluctuations of the density because it is the conjugate variable of the wave function phase. For this one needs the energy of vacancies and interstitials to be zero, and this cannot be true in a continuous part of the phase diagram, only perhaps for particular values of the pressure P and temperature T, consequently of the density ρ. They added that, even if the vacancy energy is zero, their interaction is attractive so that an incommensurate solid with a finite density of vacancies should be unstable with respect to phase separation into a liquid in equilibrium with a solid.

To this reasoning one might object that the conjugate variable to the macroscopic phase should be the total number of atoms in the supersolid fraction, not the local density, and fluctuations of the total number of atoms contributing to supersolidity do not necessarily imply fluctuations of the local density thanks to existing vacancies or interstitials.

Furthermore, the attractive interaction between vacancies was found by Rossi et al. [21] to lead to the formation of dislocations, not to any instability into a liquid-solid phase separation (see also the review by Galli and Reatto [22]). According to the latter authors, the very concept of commensurability may be ill-defined because the number of lattice sites in not fixed in a crystal with a variable number of dislocations. In the absence of definite answer to such questions, the question whether supersolidity may exist in a real crystal without vacancies does not appear completely settled to us. But we will see below that experimental results vary from sample to sample, an observation which strongly supports the assumption that the rotation anomaly observed with solid ^{4}He samples is related to the existence of quenched disorder like dislocations or grain boundaries. If there were vacancies in the ground state at $T = 0$, it should be an equilibrium property of helium crystals and we do not see why their density should vary from sample to sample.

Some authors looked at the ^{4}He properties after annealing their samples. For example, Rittner and Reppy [23] found that the magnitude of the supersolid fraction vanished after annealing. This was a sample grown at constant volume by the "blocked capillary" method and such that its final pressure being 26 bar, its melting temperature was 1.5 K. The annealing was for 13 h at 1.4 K. It is likely that at 1.4 K no liquid appeared in their cell, even in tiny pores or slits where, due to capillary effects, the presence of liquid would be favored. This is important because if Rittner's cell had reached the liquid-solid equibrium, any temperature change along the melting curve would have induced pressure changes and consequently stressed the sample and created more defects. In some other experiments it might be the case, but in Rittner's case it is probably not the case. Their temperature cycle has not induced significant pressure changes. It is a real annealing in the usual sense meaning that some defects must have been removed.

Another indication that the amplitude of the rotation anomaly is associated to disorder is the work of Clark et al. [7] who compared samples grown at constant volume from normal liquid ^{4}He to samples grown at constant T and P on the melting line. It has been shown by Sasaki et al. [24] that the first method leads to polycrystals

and the second one to single crystals. This is probably due to multiple nucleation of crystallites when the temperature is not highly homogeneous—as when the liquid is normal—and to large stresses during growth along the high T part of the melting curve as necessarily happens when the growth takes place at constant volume from the normal liquid above 2 K. Clark et al. [7] have found that the rotation anomaly is much smaller in the case of single crystals. As we shall see in the next section, it does not mean that supersolidity occurs inside grain boundaries more than in the core of dislocations. It probably means that the dislocation density is larger in the highly disordered polycrystals than in the single crystals whose quality is better.

A further indication of the importance of disorder is the sensitivity to ^{3}He impurities which was first noticed in the first article by Kim and Chan [1], later developped by Kim et al. [25] and confirmed by Day and Beamish [3] who found exactly the same dependence to ^{3}He impurity concentration for the elastic anomaly. This dependence is large. Samples grown from natural purity ^{4}He show an anomaly around 100 mK (taken from the middle of the temperature variation) while, with 1 ppb, it occurs around 40 mK. It is very unlikely that a bulk property depends on impurities down to such a very low value of their concentration. But if the impurities condense on defects like dislocations whose density is also low, one understands that the impurity concentration on these defects may be large and substantially change the properties of these defects. As we shall see in Sect. 4, Rojas et al. [13] prepared single crystals with no impurity at all and found that they are anomalously soft down to 20 mK.

In summary, there are numerous observations supporting the assumption that both the rotational and the elastic anomaly are related to the existence of defects in the studied samples. However the amount of disorder, for example the dislocation density, was never measured quantitatively in parallel with acoustic or rotational properties so that the amount of disorder in each sample is usually only a guess based on what should be obtained as a consequence of one or another growth method. This is obviously something that needs to be measured more precisely in future experiments.

3 Dislocations or Grain Boundaries

Given the importance of disorder in the anomalies of solid ^{4}He, one needs to know more precisely what kind of disorder is relevant. Sasaki et al. [26] searched for superflow through solid samples with or without grain boundaries. For this, they started comparing polycrystals to single crystals. They prepared solid ^{4}He in equilibrium with liquid ^{4}He and they managed to obtain a liquid-solid interface at two different levels in two different parts of a cell. Optical access to the experimental cell allowed them to see if the two interfaces relaxed towards a single equilibrium level, indicating mass flow. With single crystals they did not observe relaxation down to 50 mK but with polycrystals they did. Their first interpretation was that there was no supersolidity in single crystals and that, in polycrystals, the supersolidity was confined inside the grain boundaries of the polycrystal. However, they later modified their interpretation [27]. Indeed they observed that the line of contact of a grain boundary with a solid wall (the glass wall of their cell) is in reality a liquid channel. This is a consequence of a local equilibrium between surface tensions. The width of these channels
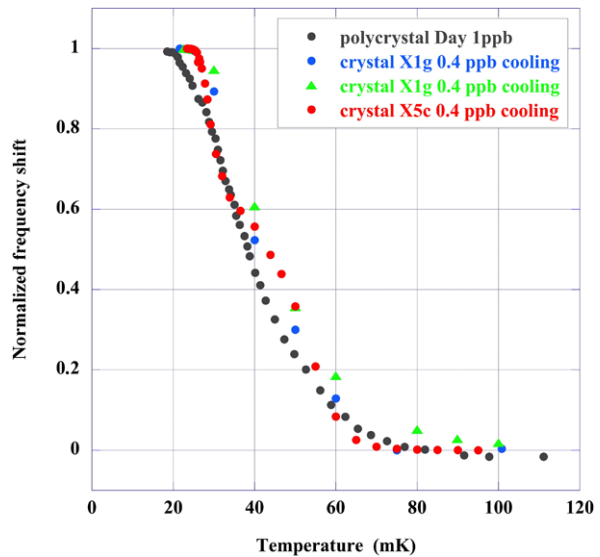
was predicted to vary from macroscopic (tens of micrometers) at the melting pressure $P_m = 25$ bar to an atomic size about 10 bars higher [24]. For the same reason there are also liquid channels at the junction of three grain boundaries [24, 28]. As a consequence it is likely that the superflow measured in 2006 by Sasaki et al. [26] in polycrystals was actually taking place along these liquid channels, not along the grain boundaries themselves, which are solid—not liquid—2D-defects. As for the absence of superflow through single crystals, it would need to be further investigated at lower temperature because, as Pantalei et al. [29] showed and as Rojas et al. [13] verified, the concentration of ^{3}He in the solid is much lower than in the liquid when the crystal is in equilibrium with a large volume of liquid at low temperature, so that the transition takes place at a temperature which could be much lower than 50 mK.

More recently, the study of mass flow through solid ^{4}He was improved by Ray and Hallock [30] using a clever technique for mass injection through porous Vycor pieces. They found superflow below 700 mK in samples that were grown at constant volume. They also found that the magnitude of this flow had a non-monotonic temperature variation with a minimum around 75 mK. These results are significantly different from what was found by other authors for either the rotation anomaly or the elastic anomaly of solid ^{4}He. It is obviously important to understand why.

Kim and Chan presented preliminary evidence for the existence of macroscopic mass superflow by comparing a TO where solid helium fills a continuous annular space with a similar TO where a metallic barrier blocked the annulus [2]. The presence of the barrier reduced the rotation anomaly by two orders of magnitude. This result was first understood as an evidence that the TO period shift was a consequence of a macroscopic mass superflow taking place along the whole annulus. In 2008, Rittner and Reppy confirmed that the presence of the barrier suppresses the rotation anomaly [31]. However, Reppy later found from comparisons of several annular cells that the anomalies were not consistent with the existence of a macroscopic superflow [32]. A more likely interpretation was found after understanding that inserting one or more solid barriers in the TO increased its rigidity and consequently decreased the period. The blocked annulus experiment would need to be repeated with a TO whose rigidity is very high even in the absence of a rigid barrier linking the inner cylinder and the outer one which define the annulus where helium is suposed to flow. Reppy has also invented a triple TO with three resonance frequencies [33] whose analysis should allow to distinguish between stiffening and superflow effects.

Paalanen et al. [34] and Ho et al. [35] had found evidence for elastic anomalies in solid ^{4}He, which they attributed to the adsorption of ^{3}He impurities. In 2007, Day and Beamish demonstrated that the elastic anomaly of polycrystals has exactly the same dependence on the temperature T and on the ^{3}He concentration X_3 as the rotation anomaly of TOs [3]. This striking similarity raised an important question. Since the resonance frequency of a TO is $f = (1/2\pi)(K/I)^{1/2}$ where I is the momentum of inertia of the oscillating mass and K is the torsion elastic constant of the TO, and since the helium inside has a contribution to both quantities, what is the origin of the frequency shift that is observed in TO experiments, a decrease of I or an increase of K? The contribution to the elasticity of the TO head could be calculated [36, 37] and it was shown to be small in many cases, except if the cell is not sufficiently rigid [32, 33]. But we realized in collaboration with J. Beamish that the contribution to

the torsion rod may explain the whole frequency shift when the cell is filled through a hole in the torsion rod, whose diameter is not small enough compared to the rod diameter itself [38]. In some other cases, the inertia effect and the elastic effect may add to each other to produce the observed frequency shift but the discovery by Day and Beamish requires carefull analysis of each particular TO experiment before considering its results as evidence for supersolidity.

In most experiments done by Kim and Chan, the elastic effect seems to be negligible so that the rotational anomaly and the elastic anomaly should be two different consequences of a single phenomenon. Day and Beamish proposed that the stiffening was due to the pinning of dislocations by ^{3}He impurities below a critical temperature that depends on the binding energy E_3 of ^{3}He impurities to these dislocations. Rojas et al. [13] further verified this interpretation by comparing the elastic anomaly of single crystals grown at constant T and P from the superfluid to that of polycrystals grown at constant V. Figure 1 shows that the stiffness change of a single crystal measured by Rojas et al. is the same as that of a polycrystal measured by Day and Beamish. Note that some of the preliminary results by Rojas et al. [12] were somewhat confusing because the samples in equilibrium with the liquid have a ^{3}He concentration which strongly depends on temperature if the volume of the liquid region is large. For such comparisons one also needs to be aware that there is some hysteresis between cooling and warming.

To find the same temperature dependence for polycrystals and for single crystals is a strong indication that the relevant defects are indeed dislocations, not grain boundaries. Of course, one could imagine that the binding energy of ^{3}He impurities is exactly the same on grain boundaries and on dislocations but it seems unlikely. Similarly, since the same temperature variation of the period shift has been measured in bulk samples and in samples confined inside a porous Vycor glass, one could imagine that this binding energy is also the same at the helium-glass interface but, once more, this is rather unlikely, so that the Vycor case is a standing puzzle.

In a polycrystal under stress, the grains do not slip against each other because the grain boundaries are solid, not liquid, as expected in any system with long range forces (here van der Waals) [24, 39]. This result could have been expected because, if the grain boundaries were liquid, they would be wet by the liquid phase at the liquid-solid equibrium and the grain boundary energy σ_{GB} would be just twice the liquid-solid interfacial energy. But this is not true. Sasaki et al. [24, 27] showed that σ_{GB} is strictly less than $2\sigma_{LS}$.

4 The Pinning of Dislocations: [3]He Impurities, Kinks and Jogs

John Beamish and his group have made a systematic study of the shear modulus μ of helium 4 solid samples as a function of T, X_3 and the frequency f. We assume that their samples were polycrystalline. At low temperature, they show a stiff state that is always the same but above a temperature T_c they show a soft state whose stiffness varies from sample to sample. What is thus " anomalous" is the shear modulus above a critical temperature of about 100 mK and that's why one should talk about an "anomalous softening" of the low temperature stiff state. The magnitude of this softening is between 7 and 14.5 % (note that these values are equivalent to the stiffening from the soft state by 7.4 to 17 % that Day and Beamish published [3–5]). The temperature T_c varies with the concentration X_3. It also increases with frequency [5] and it decreases with the stress amplitude above a critical threshold of about 5 microbars (4×10^{-8} strain). The amplitude dependance is likely to be a consequence of the existence of a threshold stress to unpin a dislocation from a [3]He impurity. This stress depends on the free length of the dislocation between two pinning sites [40, 41]. The whole set of data is convincingly explained by a model in which stiffness changes are a consequence of [3]He binding to dislocations with a distribution of binding energies $E_3 = 0.73 \pm 0.45$ K.

This interpretation of the anomalous softening of solid [4]He was clearly confirmed by Rojas' measurements [13, 42]. By growing crystals slowly (less than 1 μm/s) at low temperature (25 mK), they obtained very high quality crystals with a dislocation density Λ that should be less than 100 cm^{-2} according to Ruutu et al. [43] who estimated this number from a study of the growth dynamics. The latter study actually measured a density of screw dislocations but it looks reasonable to assume that the density of edge dislocations should be of the same order of magnitude. Other authors estimated the dislocation density in various crystals. For example, Rolley et al. [44] found 10^4 cm^{-2} screw dislocations in crystals grown at 100 mK. Lengua and Goodkind [45] found at least 1800 cm^{-2} screw dislocations in samples grown at 0.9 K. Syshchenko et al. [46] found 7500 cm^{-2} edge dislocations in single crystals grown between 0.87 and 1.4 K. In summary, our estimate of 100 cm^{-2} edge dislocations is an order of magnitude that would need to be more precisely measured.

Most importantly, Rojas et al. [13] obtained crystals with no [3]He impurity at all. This is because the potential energy of an [3]He atom in the liquid is 1.356 K lower in the liquid than in the solid [29, 47]. Even in the presence of dislocations where potential wells for [3]He atoms could be as deep as $0.73 + 0.45 = 1.18$ K with respect to the bulk solid and according to Syshchenko et al. [5], this is still higher than the
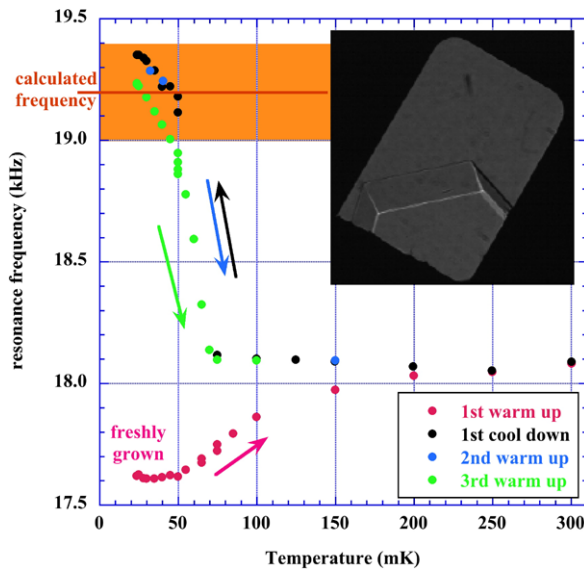
Fig. 2 Fresh from growth, the crystal studied by Rojas et al. [13] is soft so that its acoustical resonance frequency is low (17.6 kHz). They explained this observation by calculating the concentration of ^{3}He impurities in their crystal as a function of temperature from the thermodynamics of solid and liquid helium [29, 47]. At 25 mK, the crystal should contain no ^{4}He impurity at all, so that the dislocations are highly mobile. When warming this crystal for the first time, they observed an unusual stiffening that was understood as the consequence of some ^{3}He impurities entering the solid from the liquid phase nearby. A subsequent cooling produced a stiffening due to the pinning of dislocations by the ^{3}He impurities. Then the usual softening during warming corresponding to ^{3}He evaporating from the dislocations into the bulk solid. The theoretical value 19.2 ± 0.2 kHz is the result of a numerical calculation using Greywall's values [48] for the elastic coefficients (Color figure online)

potential energy in the liquid, so that the ^{3}He concentration on dislocations cannot be larger than in the liquid. Rojas et al. [13] starting with ultrapure ^{4}He containing 0.4 ppb of ^{3}He, the average distance L_i between two impurities on their dislocations could not be less than one meter, much larger than their cell size. As a consequence, when grown at low temperature, Rojas's crystals should be free of impurities. They found them anomalously soft (see Fig. 2) and they understood that it was simply because the dislocations could move freely in the absence of impurities.

When warming up their ultrapure crystals in the presence of some liquid regions which had accumulated ^{3}He impurities during growth, Rojas found that the stiffness of these crystals increased, instead of decreasing as usually observed. Rojas et al. [13] proposed that this stiffening was due to ^{3}He impurities entering the solid from the liquid regions. Once inside the solid the ^{3}He atoms could hardly escape except if large mechanical vibrations or large amplitude sound waves were applied. As a result, their crystals recovered the usually observed behavior, that is a softening from a stiff state at low T to a sample dependant soft state at T larger than typically 100 mK. The stiff state at low T was found consistent with old measurements of the elastic coefficients by Crepeau et al. [49] and by Greywall [48]. The measurements by Crepeau and by Greywall were made at high T (1.2 K) were the damping of the dislocation motion

by thermal fluctuations made it impossible for the dislocations to move at the high ultrasound frequency used (10 MHz). This comparison confirmed that the stiff state at low T corresponded to immobile dislocations.

Although Rojas' results are well consistent with the results by Beamish et al. [3–5] and the ^{3}He pinning model, it is not impossible that another type of pinning is at play in such experiments. Indeed, Aleinikava et al. [50] have predicted that the lattice potential also named "Peierls potential" should be able to pin the dislocation lines in valleys separated by energy barriers at sufficiently low temperature and stress amplitude. These "Peierls barriers" should be able to inhibit the creation of kinks and their motion, consequently the motion of dislocation lines. This is a very important prediction which should be studied with samples absolutely free of impurities as Rojas started doing but probably also at lower T and lower amplitude. This promises to be difficult but interesting.

4.1 Kinks and Jogs

Dislocation lines may glide in planes containing the line itself and the Burgers vector. As explained by Aleinikava et al. [50], this gliding does not involve any net mass transport, so that, at low T, one usually assumes that it is the only one type of motion allowed. It is mediated by the displacement of "kinks" in the gliding plane. Note that there exist kinks and anti-kinks that correspond to a displacement of the dislocation line in one direction and in the opposite direction (see Fig. 3). But dislocations may have another type of point defects named "jogs" (and anti-jogs) which are in the extra lattice plane whose end is the dislocation line. The motion of jogs allows the line to go up (one says "climb") or down in this extra plane. Now the motion of jogs needs mass transport so that it is usually not possible at low T in the absence of vacancies. But the dislocation core may be superfluid [10], in which case mass transport becomes possible without dissipation. The jog motion may thus allow an anomalous compressibility that is perhaps what is observed in the experiments by Ray and Hallock [30]. But this motion is probably not possible on large distances at sound frequencies, so that jogs should usually be pinning sites for dislocations.

Fig. 3 Jogs and kinks on an edge dislocation line (here perpendicular to the image plane). The climbing plane is the half lattice plane ending with the dislocation line. The gliding plane contains the dislocation line and its Burgers vector. The motion of kinks allows climbing and that of kinks their gliding (Color figure online)
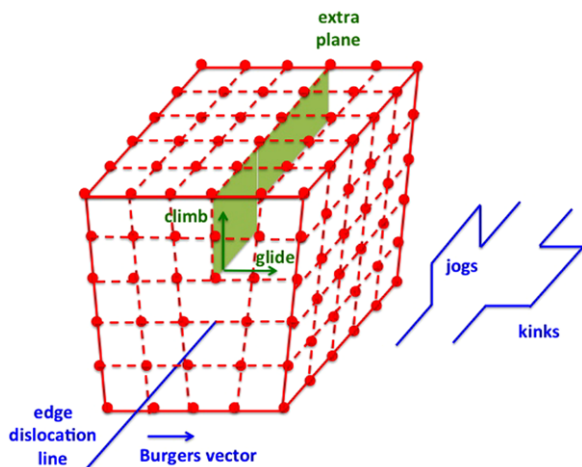
When a large stress is applied to a crystal, it may create jogs and consequently reduce the softening observed. On the contrary, if the crystal is annealed at high enough temperature, the jogs may be removed. Apparently, this is what has been observed by Day and Beamish who noticed that warming up to 0.5 K was sufficient to recover a large softening after the application of a large stress had reduced it. Rojas et al. [13] also observed that the softening of their crystals was largest after annealing near 1 K. The energy of jogs has still to be measured somehow but it might be of the order 10 K, the vacancy energy, since one vacancy arriving on a dislocation should create a jog-antijog pair. How this could be related to the onset of dc-mass flow in Hallock's experiment is another interesting question whose answer is not known to us.

4.2 The ^{3}He Pinning Mechanism

Now the mechanism by which ^{3}He impurities pin dislocation lines is not so obvious even though the measured value of the binding energy (0.73 ± 0.45 K) is now accurately known [4] and found in agreement with calculations (0.8 ± 0.1 K) [51]. In the bulk of the ^{4}He crystal, and in the low T and low concentration X_3 limits, ^{3}He apparently move by coherent tunneling. The tunneling frequency ν depends strongly on the density but near the melting curve it was found to be about 4 MHz by Allen, Richards and Schratter [52, 53]. This means that ^{3}He atoms move ballistically in the bulk solid at a velocity $v \approx 1$ mm/s. At low T and in the absence of ^{3}He, the dislocations also move quite freely as we have seen above from Rojas' results. Given this, how can it be that once bound together, a dislocation line decorated with ^{3}He atoms cannot keep moving freely? This question is mentionned by Anderson [54] among various authors.

We propose the following answer. The tunneling probability of a particle through an energy barrier depends exponentially on its mass. This is true both for ^{3}He atoms and for kinks on dislocations [55, 56]. Corboz et al. [51] assume that ^{3}He atoms should first bind to crossings between dislocations. This is probably because the crystal is more strained at crossings than in the middle of the free length of a dislocation. But crossings are usually assumed to be fixed points where dislocations do not move with respect to each other, so that adding ^{3}He there should not really modify the crystal stiffness. Now, it is also likely that ^{3}He atoms bind more strongly to kinks than away from kinks. As a result, for the dislocation to move when decorated with ^{3}He, what is needed is quantum tunneling of ^{3}He-kink pairs. But this pair should be a heavy composite particle whose tunneling probability along the dislocation line may be much lower than that of individual kinks or ^{3}He atoms. Once decorated with ^{3}He atoms, kinks and consequently dislocations would become immobile. It would be interesting to test this idea either numerically or experimentally.

Another possible explanation of the effect of ^{3}He could be that the mobility of dislocations relies on the possibility that kinks and anti-kinks cross each other without anihilating. Could it be that, once decorated with ^{3}He, kinks and antikinks reflect each other and never cross? Corboz et al. [51] have calculated the binding energy of ^{3}He atoms to dislocations and asked a different question: how is the binding energy dissipated when the ^{3}He atoms falls in its potential well? This is another interesting question to solve. It seems to us that several questions are still open in the physics of dislocations in the presence of ^{3}He impurities.

5 Difficulties with the Supersolid Scenario

It has been noticed that it is rather surprising to find the same kind of rotational anomaly in a bulk solid sample and in the pores of Vycor whose size is very small (typically 70 Å in diameter). Indeed, in the usually proposed scenario, the superflow takes place inside the cores of dislocations which need to be connected together. Rojas et al. [13] suggested that dislocations are grouped together as parallel lines in low angle grain boundaries. This is consistent with X-ray photographs by Iwasa et al. [57] and with the structures observed in Bragg peaks by Burns et al. [58]. Given this structure, how are dislocations connected together in three dimensions?

Furthermore, supersolidity takes place in a dislocation network that is pinned by ^{3}He impurities so that the crystal is stiff. But dislocations in Vycor should already be strongly pinned to the walls and not as sensitive to ^{3}He as in the bulk. It is actually possible that, due to the very strong influence of the walls, the dislocations are not well defined nor the crystal structure. So, why is it that the transition in Vycor takes place at the same temperature as in the bulk? It would be very interesting to measure the stiffness of solid ^{4}He inside Vycor. This is not easy but probably feasible.

Another difficulty with the usual supersolid scenario is that in order to build a supersolid fraction even as small as 0.1 %, one would need a very large dislocation density Λ. This objection has been raised many times. The numerical calculation by Boninsegni [10] shows that for one dislocation, the supersolid mass corresponds to not more than one row of atoms. As a consequence, the total supersolid fraction should be of order $\Lambda/(n_A)^{2/3}$ where $n_A = \rho N/3 = 4 \times 10^{22}$ cm^{-3} is the 3D number density of atoms in the solid. For a total supersolid fraction of 0.1 %, one finds $\Lambda = 1.2 \times 10^{12}$ cm^{-2}, a very large value since it has been measured to be in the range from 10^2 to 10^6, many orders of magnitude smaller. It would mean one dislocation every 90 Å or every 30 atoms. Now, if one estimates that the coherence length ξ along a dislocation line is of order aT^*/T where a is the atomic spacing (3 Å) and $T^* = 2$ K, and since it should be larger than $\Lambda^{-1/2} = 90$ Å, one finds a possible supersolid transition temperature of order 70 mK. This looks correct but, once more, it is hard to believe that the dislocation density is as large as 10^{12} cm^{-2}.

There are more problems with this scenario. Consider a polycrystal with super-fluid dislocation cores. How would the superflow cross grain boundaries? The dislocations cannot reasonably be connected to each other through these grain boundaries. One would then need to imagine that the grain boundaries themselves are supersolid. Since this has been also predicted in some numerical calculations [9]—although not for all grain boundaries—why not? But if this is true, then there is no reason for the supersolid transition in grain boundaries to occur at the same temperature as in dislocations, except perhaps in low angle grain boundaries made of individual dislocations aligned parallel to each other at a large enough distance. One should thus observe a supersolid transition temperature in polycrystals that is not the same as in single crystals... As we see, the supersolid scenario in terms of a supersolid network of dislocations is far from free of difficulties.

When Fefferman et al. [14] compared the rotation anomaly of single crystals to polycrystals, they used a transparent sapphire "minibottle" in order to monitor the growth. Instead of finding as all other authors that the rotation anomaly has a larger

magnitude with polycrystals than with single crystals, they found the opposite. They also found that the anomaly takes place at a higher temperature than usual. This is quite surprising and does not look consistent with the usual supersolid scenario. It points more in the direction of an elastic anomaly being responsible for the frequency shift of this particular TO because the elastic anomaly of single crystals has been found larger in single crystals than in polycrystals. But there must be some extra parameter that needs to be considered, otherwise Fefferman should have found the same anomaly as all other authors. What can it be? Is it the average dislocation length L_n between nodes in their network, which is much larger than usual? Could it be the nature of the helium to solid wall (polished sapphire does not pin dislocations strongly?)? A different kind of disorder in the samples because the growth does not proceed the same way, due to a different thermal conductivity of sapphire? The results by Fefferman et al. [14] are important in the sense that rotation anomalies appear to be not as reproducible as one thought before, but they are puzzling and not really understood.

6 Conclusion

Does supersolidity exist in solid ^{4}He? In several experimental situations like the non-rigid torsional oscillators, it appears that the stiffness change can mimic supersolidity by changing the rigidity of the whole TO and consequently reduce its period as if some of the mass becoming superfluid it had decoupled from the TO walls. The question to consider in each experiment is now whether the observed phenomena are really due to the appearance of supersolidity or to the disappearance of quantum plasticity, two different phenomena which may have a common origin, namely the binding/unbinding of ^{3}He impurities to dislocations. At least two types of experiments show very strong evidence that supersolidity exists. They are first the experiment by West et al. [59] with a very rigid cell where a period change was seen with solid hcp ^{4}He and not with solid hcp ^{3}He while a stiffness change was seen in both ^{3}He and ^{4}He. The second one is the experiment by Choi et al. [60] as a function of a superimposed dc-rotation. In this experiment that is reminiscent of superconductors under dc-magnetic field, the TO period was seen to depend on the velocity of the dc-rotation while no change could be seen in the stiffness of the same sample.

Supposing then that supersolidity was at work in the two above experiments, it remains rather difficult to understand how it depends on ^{3}He impurities, why it occurs at the same temperature in single crystals, in polycrystals and in porous materials like Vycor, and also why it looks different in a polished sapphire TO with no corners as shown most recently by Fefferman et al. [14]. Without a clear model, the existence of supersolidity might stay controversial.

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