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Elastic effects in torsional oscillators containing solid helium

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A number of recent experiments have used torsional oscillators to study the behavior of solid helium. The oscillator frequencies increased at temperatures below 200 mK, an effect attributed to decoupling of a fraction of the helium mass—the signature of a "supersolid" phase. However, helium's shear modulus also increases below 200 mK and the frequency of a torsional oscillator depends on its elastic properties, as well as on its inertia. In many experiments helium is introduced via a hole in the torsion rod, where its shear modulus contributes to the stiffness of the rod. In oscillators with relatively large torsion rod holes, changes in the helium's shear modulus could produce the entire low temperature frequency shifts that have been interpreted as mass decoupling. For these oscillators we also find that the known elastic properties of helium in the torsion rod can explain the observed TO amplitude dependence (which has been interpreted as a critical velocity) and the TO dissipation peak. However, in other oscillators these elastic effects are small and the observed frequency changes must have a different origin.

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In its simplest form, a torsional oscillator (TO) consists of a rigid "head" (with moment of inertia I) attached to a stationary base by a torsion rod (with torsional stiffness K). Its resonant frequency is given by $f = (1/2\pi)\sqrt{K/I}$ and can be measured very precisely for a high Q oscillator. If the torsion rod's stiffness is constant, this provides a direct and sensitive technique to measure the moment of inertia. Such oscillators have been widely used to study superfluidity in liquid ^{4}He and ^{3}He, by confining the helium in narrow channels or small pores in the TO head.[1–3] In appropriate geometries, the zero viscosity superfluid fraction decouples from the walls of the cavity, reducing the effective moment of inertia. The increase in the TO frequency is then a direct measurement of the superfluid density ρ_s.

The TO technique has recently been used to study the behavior of solid ^{4}He.[4–18] At temperatures below 200 mK, the TO frequency increases and, in analogy to measurements with liquid helium, this has usually been interpreted in terms of mass decoupling—the "nonclassical rotational inertia" (NCRI) which would characterize a supersolid. However, the behavior of a torsional oscillator can be sensitive to a number of effects in addition to the solid helium's inertia, for example, the pressure dependence of the TO background, possible slip at the walls, dissipation in the helium and, most importantly, the solid helium's shear rigidity.[19–21] Any increase in the shear modulus of the helium will stiffen the oscillator and raise its frequency, an effect which could be misinterpreted as mass decoupling. Recent low frequency measurements showed that the shear modulus of solid ^{4}He, $\mu_{\rm He}$, increases significantly below 200 mK, with the same dependence on temperature, ^{3}He concentration, and frequency as the TO anomaly.[22–25] This behavior has been attributed to dislocations, which are mobile and soften the crystal at high temperatures but are pinned by ^{3}He impurities below 200 mK. The shear modulus also has an amplitude dependence[26] which closely resembles[7] that seen in TO measurements. These similarities raise the possibility that the TO behavior is an artifact of elastic changes which mimic mass decoupling in a supersolid. These elastic effects are not expected to be large, since the shear modulus of solid helium is much smaller than that of typical TO materials such as beryllium copper ($\mu_{\rm He}/\mu_{\rm BeCu} \approx 2.8 \times 10^{-4}$). However, the frequency changes attributed to NCRI can also be quite small so it is important to compare the two.

The magnitude and frequency dependence of the elastic effects of helium in a TO depend on the design of the oscillator. If the oscillator head is not completely rigid, e.g., if it is a cylinder with relatively thin walls, then solid helium can increase the head's torsional stiffness and raise the TO frequency. Elastic effects can be even larger in heads with thin annular sample spaces where the solid helium acts as a "glue" between the inner and outer walls of the annulus.[19,20] Even if the oscillator head is rigid, solid helium is very soft and some of it will "elastically decouple," i.e., will oscillate with larger amplitude than the walls of its container. Any stiffening of the helium reduces this overshoot, raising the TO frequency and mimicking mass decoupling. This effect is reduced when the helium is confined in a narrow annulus and has a characteristic f^2 frequency dependence[21,27] that distinguishes it from a true change in inertia. Simple estimates, as well as detailed numerical modeling of particular TO geometries, suggest that this effect is too small to explain the apparent NCRI in most experiments.[27]

The most direct way in which solid helium can raise the TO frequency is through its contribution to the stiffness of the torsion rod.[28–31] Most oscillators introduce helium into the sample space via a hole through this rod. When this helium freezes, its shear modulus will stiffen the torsion rod, as will any subsequent increase in the solid helium's shear modulus. This effect is independent of frequency and so is difficult to distinguish from mass decoupling. If the torsion rod's outer radius is r_o and its center hole has radius r_i, the shear modulus of solid helium in the rod raises the TO frequency by an amount[32]

$$\frac{\Delta f_{\rm elastic}}{f_0} = \frac{1}{2}\frac{\mu_{\rm He}}{\mu_{\rm rod}}\frac{1}{\left(\frac{r_o}{r_i}\right)^4 - 1}. \quad (1)$$

TABLE I. Torsional oscillator dimensions and frequency shifts from shear modulus changes in helium in their torsion rods.

Experiment	Torsion rod	r_o (mm)	r_i (mm)	f_0 (Hz)	$\frac{\Delta f_{\text{elastic}}}{f_0}$	$\frac{\Delta f_{\text{measured}}}{f_0}$	$\frac{\Delta f_{\text{measured}}}{\Delta f_{\text{elastic}}}$
Hunt, Pratt (Refs. 6 and 7)	BeCu	0.5	0.4	575	9.8×10^{-5}	4.7×10^{-5}	48%
Aoki f_- (Refs. 8–10)	BeCu	0.95	0.4	496	4.6×10^{-6}	1.4×10^{-6}	29%
Aoki f_+ (Refs. 8–10)	BeCu	0.95	0.4	1173	4.6×10^{-6}	1.8×10^{-6}	40%
Penzev (Ref. 11)	BeCu	1.1	0.4	1002	2.5×10^{-6}	5.8×10^{-7}	23%
Kondo (Ref. 12)	BeCu	1.0	0.4	1500	3.7×10^{-6}	4.7×10^{-6}	1.26
Zmeev (Ref. 13)	BeCu	1.0	0.3	854	1.2×10^{-6}	1.5×10^{-5}	13
Kim (Ref. 5)	BeCu	1.1	0.19	911	1.3×10^{-6}	2.0×10^{-5}	150
Rittner (Ref. 15)	Al	2.55	0.25	484	2.2×10^{-8}	5.8×10^{-6}	260
Choi (Refs. 16 and 17)	BeCu	1.45	0.05	911	7.1×10^{-11}	5.0×10^{-5}	700 000
Fefferman (Ref. 18)	Ag	1.62	0.05	910	4.1×10^{-11}	2.2×10^{-6}	9700
Paalanen (Ref. 28)	BeCu	0.5	0.2	331	3.7×10^{-6}		20%–40%

The size of this elastic effect depends strongly (as the fourth power) on the ratio between the outer and inner radii of the torsion rod, a parameter which varies between 1.25 (Refs. 6 and 7) and 10 (Ref. 15) in different groups' oscillators. However, the difficulties associated with machining and drilling small torsion rods introduce significant uncertainties into these dimensions. Thinner sections in the torsion rod or off-center filling holes would introduce weak sections in the rod which would make the elastic contributions of the solid helium in the hole even more significant. Some TO designs[16–18,33–36] use a separate fill capillary rather than a hole through the torsion rod. In this case, Eq. (1) is modified to

$$\frac{\Delta f_{\text{elastic}}}{f_0} = \frac{1}{2}\frac{\mu_{\text{He}}}{\mu_{\text{rod}}}\frac{L}{L_{\text{cap}}}\frac{r_i^4}{r_0^4}, \tag{2}$$

where L (L_{cap}) is the length of the torsion rod (capillary). Given typical capillary dimensions (e.g., $r_i = 0.05$ mm, $L = 42$ mm in Refs. 16 and 17), the stiffening effect should be very small in these oscillators.

Table I gives dimensions of the torsion rods and other parameters for TOs used in experiments by a number of different groups. Column 6 is the fractional frequency change $\Delta f_{\text{elastic}}/f_0$ calculated from Eq. (1), using the shear moduli of the torsion rods (53 GPa for BeCu, 31 GPa for Al) and of solid helium [we chose a value, 15 MPa, corresponding to polycrystals at pressures around 30 bars, but this increases with pressure, e.g., by about 25% at 50 bars (Ref. 37)]. This gives an upper limit to this possible elastic contribution to the TO frequency. It should be compared to the measured low temperature TO frequency changes, $\Delta f_{\text{measured}}/f_0$, which are given in column 7. Column 8 is the ratio of the measured to the maximum elastic frequency change or, equivalently, the fractional change in the helium's shear modulus $\Delta\mu_{\text{He}}/\Delta\mu_0$ needed to produce the observed frequency shift.

Figure 1 summarizes this analysis. The horizontal axis is the ratio r_o/r_i (which determines the relative size of torsion rod elastic effects) and the vertical axis is the TO frequency shift. The points are measured low temperature frequency shifts (usually interpreted as mass decoupling) for the experiments included in Table I. The solid line is the maximum possible elastic frequency shift due to helium in the torsion rod (corresponding to a 100% change in shear modulus), given by Eq. (1). For oscillators close to this line, elastic effects in the torsion rod are very important and, in the three oscillators[6–11] to the left of it, could produce the entire frequency change. The fractional frequency shifts measured for the two modes of the compound oscillator of Aoki *et al.*[8] are slightly different, corresponding to shear modulus changes of 29% and 40%. This could reflect additional contributions to the oscillator's response, for example, NCRI or elastic changes in the torsion head. It could also arise from variations of the shear modulus along the torsion rod created during blocked capillary growth, since the two modes are sensitive to the stiffness of different sections of the torsion rod.

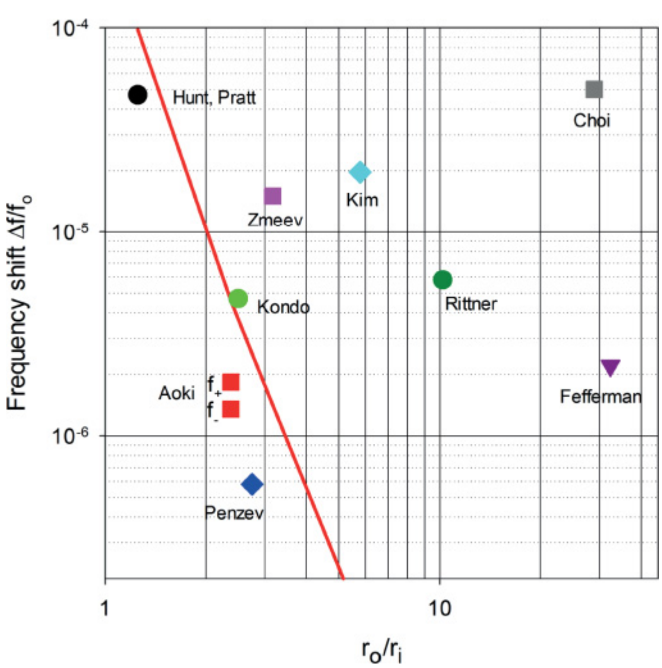

FIG. 1. (Color online) Torsional oscillator frequency changes due to the shear modulus of helium inside their torsion rods. The symbols are measured low temperature frequency shifts, usually interpreted as mass decoupling (NCRI). The different groups/experiments are identified in the legend and beside the points: Hunt, Pratt (Refs. 6 and 7), Aoki (Refs. 8–10), Penzev (Ref. 11), Kondo (Ref. 12), Zmeev (Ref. 13), Kim (Refs. 4 and 5), Rittner (Ref. 15), Choi (Refs. 16 and 17), and Fefferman (Ref. 18). The solid (red) line is the maximum possible elastic effect from the solid helium in the torsion rod, calculated using Eq. (1).

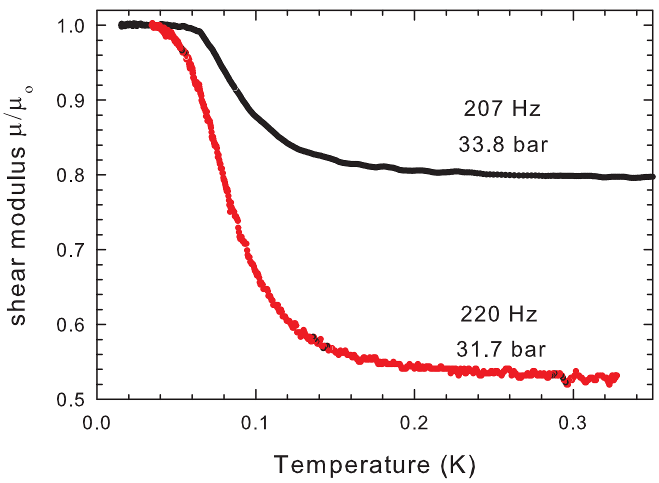

FIG. 2. (Color online) Temperature dependence of the shear modulus (normalized by low temperature values) for helium crystals grown by the blocked capillary technique. The upper (black) curve was measured at 207 Hz in a 33.8-bar polycrystal grown in an optical cell. The lower (red) curve was measured at 220 Hz in a 31.7-bar sample in a different cell (Ref. 39).

The fractional change in shear modulus that would produce the observed frequency shift in these three oscillators varies from 23% to 48%. This is somewhat larger than the shear modulus changes, in the range 7%–15%, which have been reported[22,23,38] in helium crystals grown by the blocked capillary method. However, even larger modulus changes are possible, as shown in Fig. 2. The upper (black) curve is the shear modulus (normalized by its low temperature value) recently measured in a blocked capillary crystal at a pressure of 33.8 bars. This crystal was grown in an optical cell, which allowed us to confirm that it was polycrystalline, and the measurements were made using the technique described in Ref. 22. The modulus decrease was larger than 20%. The lower curve is the shear modulus in a crystal grown in a different cell[39] and shows that modulus changes of nearly 50% are possible in blocked capillary crystals. For single crystals, the elastic contribution of the helium depends on crystal orientation but can be even larger—the elastic constant c_{44} can soften by as much as 86%.[24] Given the size of these changes, it is clear that the contribution of solid helium to the torsion rod's stiffness is important in many TO experiments. However, it is also clear from Fig. 1 that, in other torsional oscillators, the maximum possible change from Eq. (1) is much too small to explain the apparent NCRI.

If the shear modulus of helium in the torsion rod is an important contribution to the TO frequency changes, can this also explain other aspects of the TO behavior? Both the TO and the shear modulus show strong amplitude dependence, although this has been interpreted differently (as a critical velocity in the TO experiments and as a critical strain in the elastic measurements).[26] Helium's shear modulus will affect the amplitude dependence of a TO since the oscillator's motion produces a twist in the torsion rod and therefore creates a shear strain in the helium inside it. The maximum shear strain in the helium $\epsilon_{\rm He}$ (at the outer radius r_i of the hole) is related to the "rim velocity" v (measured at the radius R of the helium annulus or cylindrical volume in the torsion head) and to the length L of the torsion rod or capillary:

$$\epsilon_{\rm He} = \frac{r_i}{R}\frac{v}{2\pi f L}. \qquad (3)$$

Table II gives the relevant parameters for the different oscillators. Column 6 gives the value of the helium shear strain $\epsilon_{\rm He}$ corresponding to the critical velocity v_c measured in the different TO experiments. For the two modes of the double TO of Refs. 8–10, we use the strain in the section of the torsion rod where it is largest (in the upper rod for the low frequency mode, f_-, the lower rod for the high frequency mode, f_+), calculated using the relative angular displacements for the two torsion rods. Where critical velocities were not explicitly stated, we estimated them from the onset of amplitude dependence shown in the papers. Since the critical velocity does not vary widely ($v_c \approx$ 10–25μm/s in these measurements), the corresponding values of the strain are similar in different experiments, e.g., $\epsilon_{\rm He} \approx 5$–$6 \times 10^{-8}$ for the first two experiments in Table II. These values are close to the critical strain ($\epsilon_c \approx 4 \times 10^{-8}$) observed in shear modulus[22,23,26] and acoustic[24] measurements, supporting the idea that the behavior in these TO experiments arises from changes in the shear modulus of the helium in the torsion rod.

Torsional oscillator measurements also show a dissipation peak $Q_{\rm TO}^{-1}$ in the region where their frequency increases. This is not expected for a simple superfluid and has been taken as evidence of a more complicated mechanical response, for example, that of a glassy or viscoelastic solid.[6,40,41] Shear

TABLE II. Torsional oscillator parameters, critical strains and dissipation from helium in their torsion rods.

Experiment	L ($L_{\rm cap}$) (mm)	R (mm)	f_0 (Hz)	v_c (μm/s)	$\epsilon_{\rm He}$ at v_c	$Q_{\rm TO}^{-1}$	$Q_{\rm He}^{-1}$
Hunt, Pratt (Refs. 6 and 7)	10	4.5	575	25	6.2×10^{-8}	7×10^{-6}	0.036
Aoki f_- (Refs. 8–10)	6.35	5.1	496	22	5.4×10^{-8}	3.5×10^{-7}	0.038
Aoki f_+ (Refs. 8–10)	6.35	5.1	1173	15	5.5×10^{-8}	2.1×10^{-7}	0.023
Penzev (Ref. 11)	15	5.0	1002	15	1.3×10^{-8}	1.9×10^{-7}	0.038
Kondo (Ref. 12)	10	4.0	1500	14	1.5×10^{-8}		
Zmeev (Ref. 13)	8.5	6.8	854				
Kim (Ref. 5)	7.5	5.0	911	10	8.8×10^{-9}	4×10^{-6}	16
Rittner (Ref. 15)	19.1	7.9	484	20	1.1×10^{-8}		
Choi (Refs. 16 and 17)	15 (42)	8.05	911	10	2.0×10^{-11}	2.4×10^{-6}	6000
Fefferman (Ref. 18)	9.8 (55)	5.5	910			1×10^{-6}	2200
Paalanen (Ref. 28)	10	N/A	331				

modulus measurements[22,23,38] have a similar dissipation peak $Q_{\rm He}^{-1}$ which has been interpreted in terms of thermally activated relaxation associated with unpinning of dislocations. We can estimate the effect of this elastic dissipation on the quality factor Q of a TO since it scales with the ratio of the elastic energies of the helium and the torsion rod. Neglecting the TO's small background dissipation (typically $\approx 10^{-6}$), this gives

$$Q_{\rm TO}^{-1} = \frac{\mu_{\rm He}}{\mu_{\rm rod}} \frac{1}{\left(\frac{r_o}{r_i}\right)^4 - 1} Q_{\rm He}^{-1}. \quad (4)$$

For the three oscillators in which torsion rod elastic effects are most important, the measured TO dissipation peaks (column 7 of Table II) vary by a factor of more than 35. The corresponding helium dissipation needed to produce these peaks (column 8) varies by less than a factor of 2. It is comparable to the value seen in shear modulus experiments (e.g., $Q_{\rm He}^{-1} \approx 0.05$ at 200 Hz in a blocked capillary crystal with a 15% softening).[38]

It is clear that changes in the shear modulus of solid helium in the torsion rod are important in many torsional oscillators. The experiment by Paalalen *et al.*[28] directly studied this type of elastic effects using an oscillator with a solid head (i.e., containing no helium). When the hollow torsion rod contained solid helium, the TO frequency showed changes which corresponded to shear modulus changes between 20% and 40%. These are comparable to directly measured modulus changes and to those needed to explain the low temperature frequency changes in the three oscillators discussed above. These measurements[28] were also interpreted in terms of dislocation motion and pinning by ^{3}He. The temperature at which the shear modulus changed in these experiments was significantly higher than that of the frequency shifts in more recent torsional oscillator experiments. However, as the authors point out in a footnote, lower transition temperatures were observed in all their other crystals and the higher temperatures in this sample was probably because its ^{3}He concentration was substantially larger than the nominal 0.3 ppm. Also, these measurements were made at strains greater than 10^{-7}, i.e., well above the critical strain, so the dissipation cannot be directly compared to the low amplitude values from more recent experiments.

Torsion rod elastic effects may explain the frequency changes in several TO experiments, but it is clear from Fig. 1 that they are much too small to explain the apparent NCRI in other oscillators. However, elastic changes in the helium can enter in other ways. For example, in experiments by Reppy and co-workers using annular TOs with relatively "floppy" heads, stiffening of the helium in the annular sample space dominates the TO response.[19,20] In other experiments,[13–17,33–36] there is no obvious way in which elastic effects can produce the observed frequency changes. In the earliest TO experiments by the Penn State group,[4,5] the torsion rod dimensions ($r_o = 1.1$ mm, $r_i = 0.2$ mm) were chosen to minimize the elastic contribution of the helium in the torsion rod. They also built a "dummy" TO (with a solid head) and directly confirmed that any elastic contributions from helium in the torsion rod were small compared to the NCRI measured in their oscillators. In this group's work,[4,5,33,34] and in experiments by the KAIST group,[13,16,17,35,36] it appears that another explanation is required.

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