Lectures on Statistical Mechanics

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Lecture 1

Probability theory

In statistical mechanics, probabilities occupy, quite naturally, a central place. They are also present in daily life. In the present chapter, we present key concepts of the theory of probabilities. This provides us with a language and an understanding of the nature of probabilities.

in order to avoid misconceptions that arise from carrying over daily-life concepts of probabilities to where they do not apply. In Lecture 2, we will do the same for statistics, before plunging into statistical mechanics proper.

1.1   Probabilities

Probability theory and statistics are both concerned with the relation between data-generating processes and observed data (see Fig. 1.1, adapted from [1]):

- Probability theory describes properties of observations given a data-generating process. Such a data-generating process may describe the repeated throwing of two dice or the Boltzmann probability distribution of statistical physics, itself related to the time evolution of mechanical systems. It may also correspond to a process that modifies beliefs about next week's weather in Paris, or about the mass of protons given 17 experimental findings and theoretical constraints. Probabilities appear in everyday-life thinking and decision-making. Humans' "... judgment and decision-making under uncertainty" [2] is a powerful factor in social psychology and economics. This insight led to the Nobel prize in Economics in 2002.[1]

- Statistics analyzes the observed data from an unknown data-generating process, and possibly from additional information. In statistical inference, one then tries to analyze the data-generating process. Statistics makes "hard", mathematically proven statements. Nevertheless, each application of statistics to concrete data is outside of mathematics, as concrete data are not mathematical objects.

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[1]see   https://www.nobelprize.org/prizes/economic-sciences/2002/kahneman/biographical