Tutorial 7, Advanced MCMC
2020/21 ICFP Master (second year)

Werner Krauth
ENS Paris
(Dated: 09 March 2022)

1. **Simulated tempering for the V-shaped stationary distribution**

   In lecture 7 (part 7.2), we discussed the simulated tempering algorithm. Here, we study this algorithm for the V-shaped stationary distribution. In all the below versions, one move corresponds to one displacement followed (with probability ϵ) a replica move.

   (a) Set up the simulated tempering algorithm for the case $k = 2$ (flat distribution and V-shaped distribution) on the path graph P_n as a function of n and of the switching probability ϵ.

   (b) Compute the conductance of this algorithm for the case $k = 2$ (flat distribution and V-shaped distribution) on the path graph P_n as a function of n and of the switching probability ϵ. Locate the bottleneck as a function of n and ϵ.

   (c) Set up the direction-lifted simulated tempering algorithm for this same case.

   (d) Set up the transition matrices and compute the dominant and sub-dominant (left) eigenvalues both for the simulated tempering and for the direction-lifted simulated tempering.

   (e) Compute the time-dependent TDV, and from there the mixing time of the reversible Metropolis algorithm on the path graph with the V-shaped stationary distribution as a function of n and of ϵ. Generalize to the direction-lifted Metropolis algorithm, if possible.