Choice under Social Influence: the Curse of Coordination

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Orléans 15 mars 2007

Collaborators

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« Complex Systems in Human and Social Sciences » (CNRS-MENR)

collective behavior

- From a « microscopic » level (description of the agents and their interactions) to a « macroscopic » level (description of collective behavior).
- Examples from economics, theoretical neuroscience, statistical physics:

Elementary units	Interactions	Collective level
agents *preferences*	social influences (externalities)	market : equilibrium price
neurons *activation rule*	synaptic weights	psychophysics: associative memory
spins (magnetic moments)	interactions	thermodynamics: ferromagnetism

The dying seminar

- T. C. Schelling
  « Micromotives and Macrobehavior », Norton & Cy, 1978)
  (« La tyrannie des petites décisions », PUF, 1980)

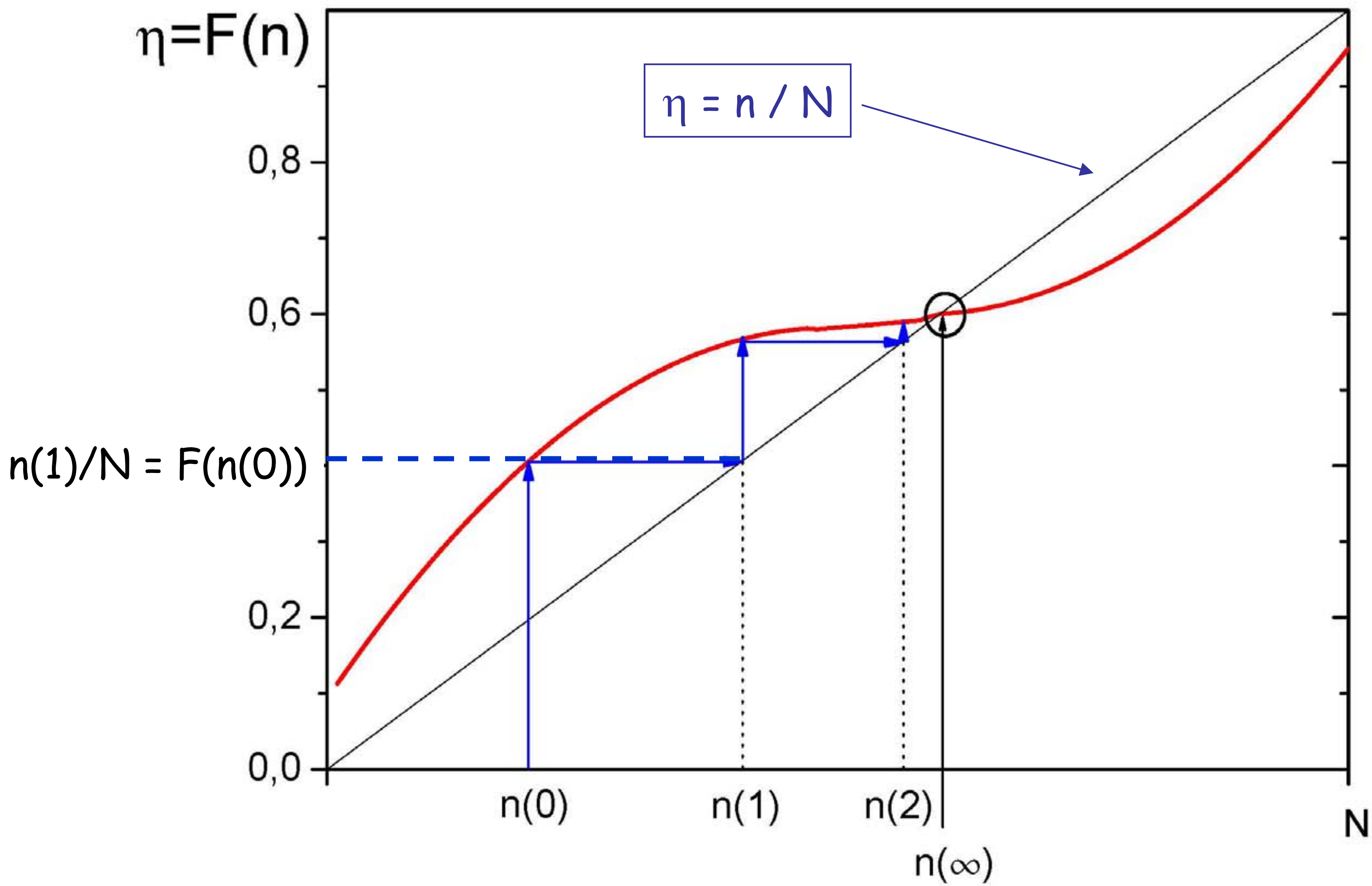
« critical mass model » :

• N scientists are asked to participate to a seminar/working group, meeting every Saturday

•each week every one knows what was the attendance of the last Saturday

• every one has his own willingness to participate: scientist i wants to participate if the number who attend is larger than n_i

$F(n)$ = fraction of agents who want to attend if: $n_i \leq n$



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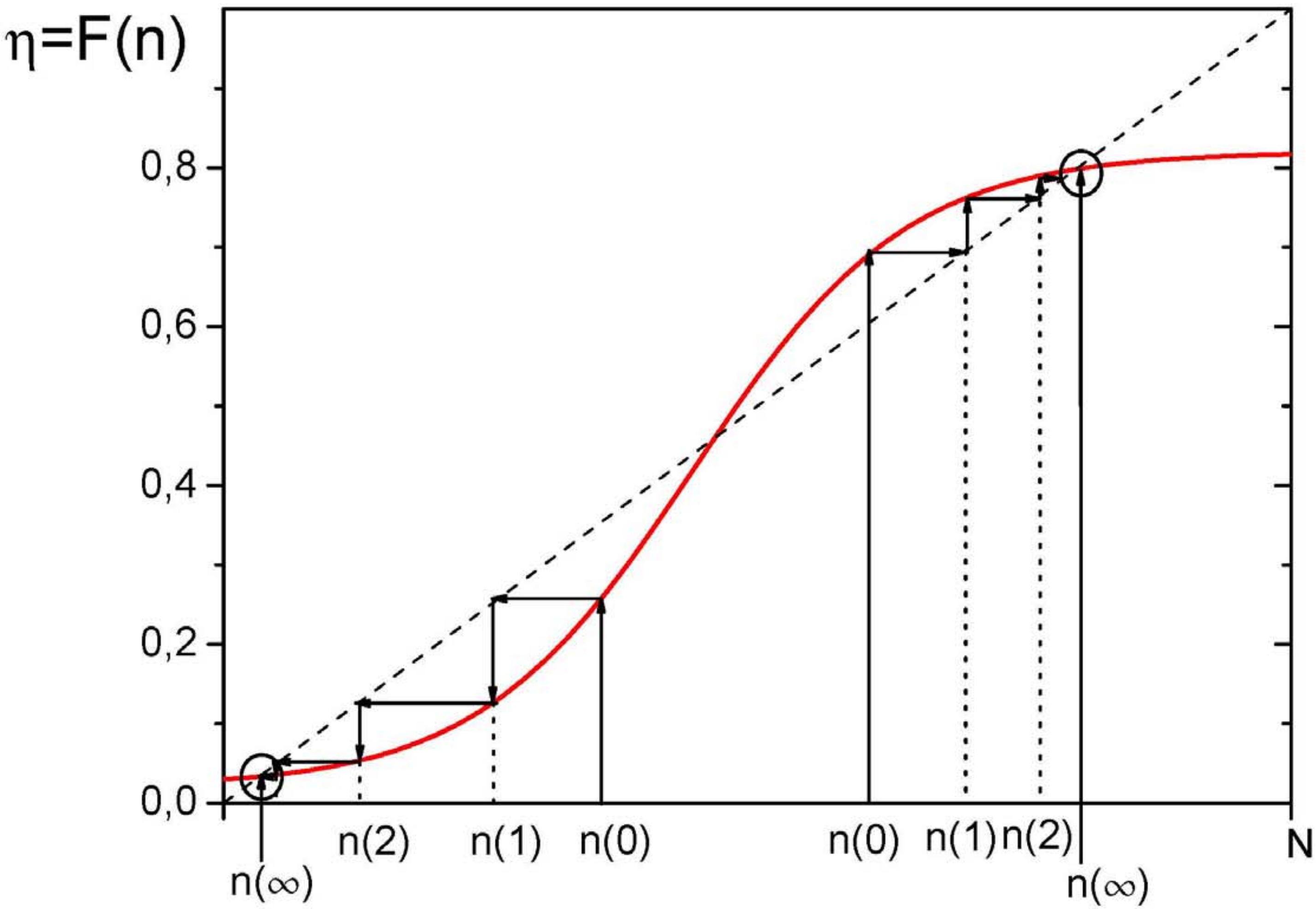

The dying seminar

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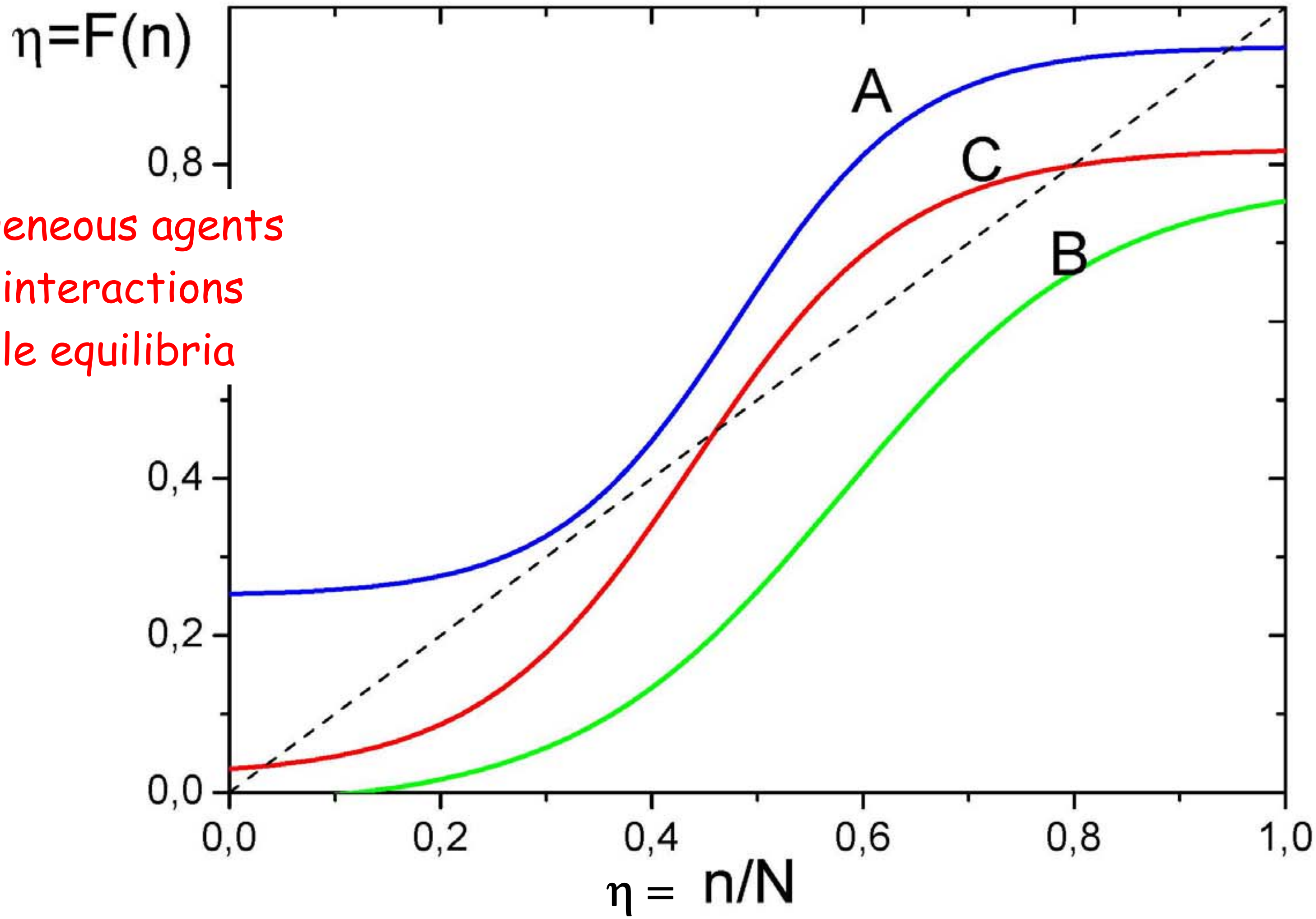

heterogeneous agents
+ social interactions
= multiple equilibria

Discrete choices

Economics Discrete Choice Theory		Statistical Physics Ising and Potts models
Random Utility Models (RUM)		Random Field Ising Models (RFIM)
agents (customers)	N	Ising spins
binary choice: buy/not buy one unit of a good	$\omega_i = 0, 1$	binary state $S_i = \pm 1$
posted price	P	global external field
individual willingness-to-pay (IWP) (reservation price)	h_i	local external field
social influence - positive externality $J_{ij} = J/$(nber of neighbors)	J_{ij} $J > 0$	interactions ferromagnetic coupling
rational agents: i is happy to buy if $V_i \equiv h_i - P + \sum_j J_{ij}\omega_j > 0$	$T = 0$	ground state

Two different points of view considered as equivalent in economics

(standard case: no interaction)

'economic approach' (McFadden, Manski)	'psychological approach' (Thurstone)
heterogeneous, time independent reservation prices $\{h_i, i = 1, ..., N\}$	(possibly) homogeneous reservation price h
deterministic choice agent i buys if $V_i > 0$ $V_i = h_i - P$	stochastic choice agent i buys with probability $1/[1 + \exp(-\beta V)]$ $V = h - P$
an observer has not access to the individual h_i only to the fraction of buyers	an observer can only see the fraction of buyers
RFIM, $T = 0$ quenched disorder	standard Ising model, $T > 0$ annealed disorder

Use of the Ising framework in socio-economic modelling by physicists... and economists

- Galam S., Gefen Y., Shapir Y. (1982) "Sociophysics: A Mean Behavior Model for the Process of Strike", Mathematical Journal of Sociology
- Orléan A., (1995) "Bayesian Interactions and Collective Dynamics of Opinion: Herd Behaviour and Mimetic Contagion", Journal of Economic Behavior and Organization
- Durlauf S.N., (1997) "Statistical Mechanics Approaches to Socioeconomic Behavior"
- Weisbuch G., Stauffer D. (2003) "Adjustment and social choice", Physica A
- ...

plan

Market (and non market) model with a single good and externalities
(~ T. C. Schelling, « the dying seminar » and RFIM at T=0)

- equilibrium properties and collective states
  - customer's phase diagram
  - monopolist's phase diagram
    - **JPN, D. Phan, M.B. Gordon, J. Vannimenus** (2005)
      Multiple equilibria in a monopoly market with heterogeneous agents and externalities
      Quantitative Finance Vol.5, No. 6, 557-568 - preprint 2003 arXiv cond-mat/0311096
    - **M. B. Gordon, JPN, D. Phan and J. Vannimenus**
      Seller's dilemma due to social interactions between customers , Physica A, 2005
    - **M. B. Gordon, JPN, D. Phan and V. Semeshenko**
      Discrete Choices under Social Influence: Generic Properties, preprint 2007
      http://halshs.archives-ouvertes.fr/halshs-00135405
    - **M. B. Gordon, JPN, D. Phan and V. Semeshenko**
      The perplex monopolist: optimal pricing with customers under social influence, in prepapration
- repeated game framework:
  - hysteresis
  - behavorial learning by the customers
    - **V. Semeshenko, M B Gordon, JPN & D Phan**, proceedings of ECCE1, Springer 2006
    - **JPN, V. Semeshenko, M B Gordon**, in preparation

Market model

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simplifying hypothesis

- strategic complementarity : $J_{ik} > 0$

  making the same choice as the others is advantageous

- homogeneous social influence ($J_{ik}=J$):

$$J \frac{1}{\|\vartheta_i\|} \sum_{k \in \vartheta_i} \omega_k = J\,\eta_i$$

weight of neighbours' choices

fraction of i's neighbours that adopt

- global neighbourhood and large N :

$$\eta_i = \frac{1}{N-1} \sum_{\substack{k=1 \\ (k \neq i)}}^{N} \omega_k \cong \frac{1}{N} \sum_{k=1}^{N} \omega_k \equiv \eta$$

= fraction of buyers

η insensitive to fluctuations:

single agents cannot influence individually the collective term $J\eta$

Nash equilibria

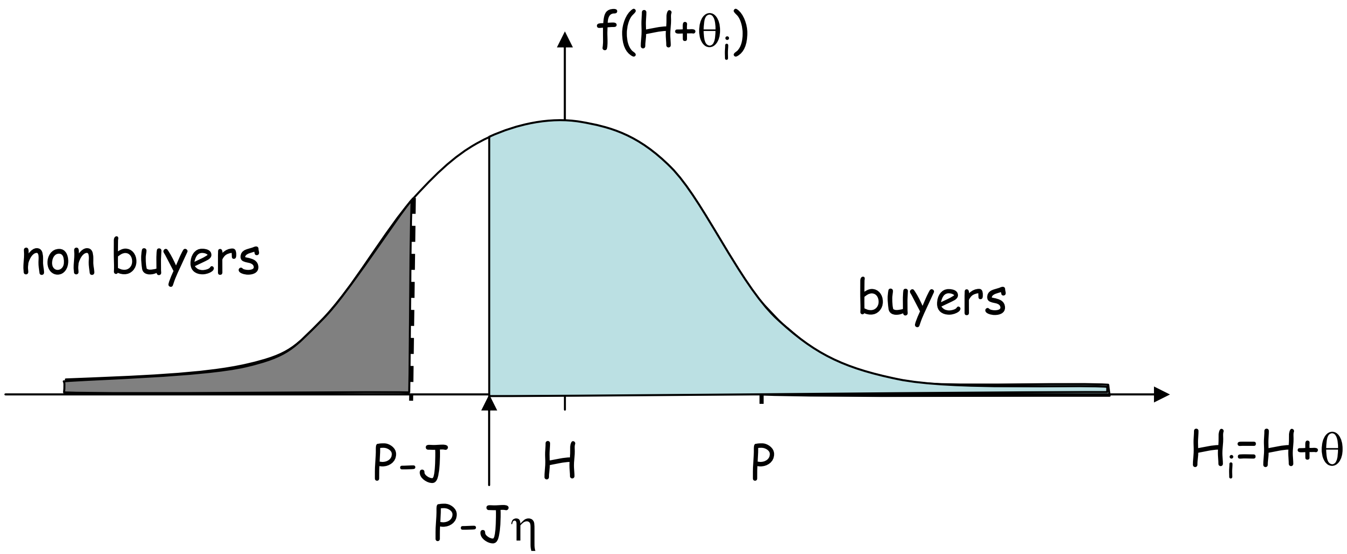
- individual i's choice : buy if $V_i - P = H + \theta_i + J\eta - P > 0$

  ↑ fraction of buyers (η)

- fraction of buyers :

  $\text{if } \eta = 0 \Rightarrow \text{for } H + \theta_i > P \; : \omega_i = 1$

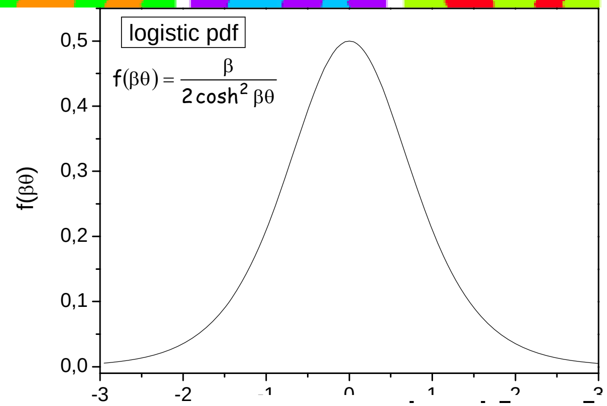
  $\text{if } \eta = 1 \Rightarrow \text{for } H + \theta_i < P - J \; : \omega_i = 0$



- Nash equilibrium : $\eta = \int_{P-J\eta}^{\infty} f(H+\theta)\, d\theta$

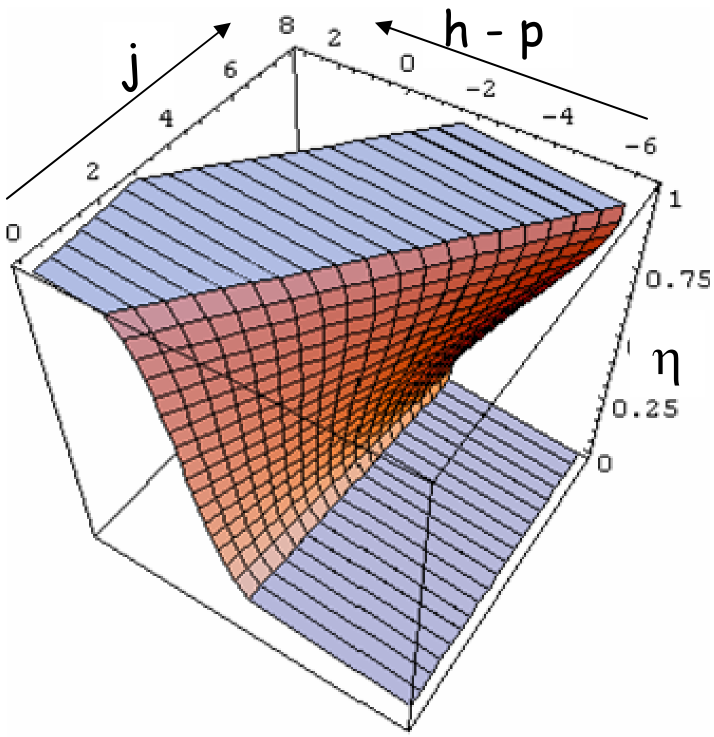
IWP: logistic distribution

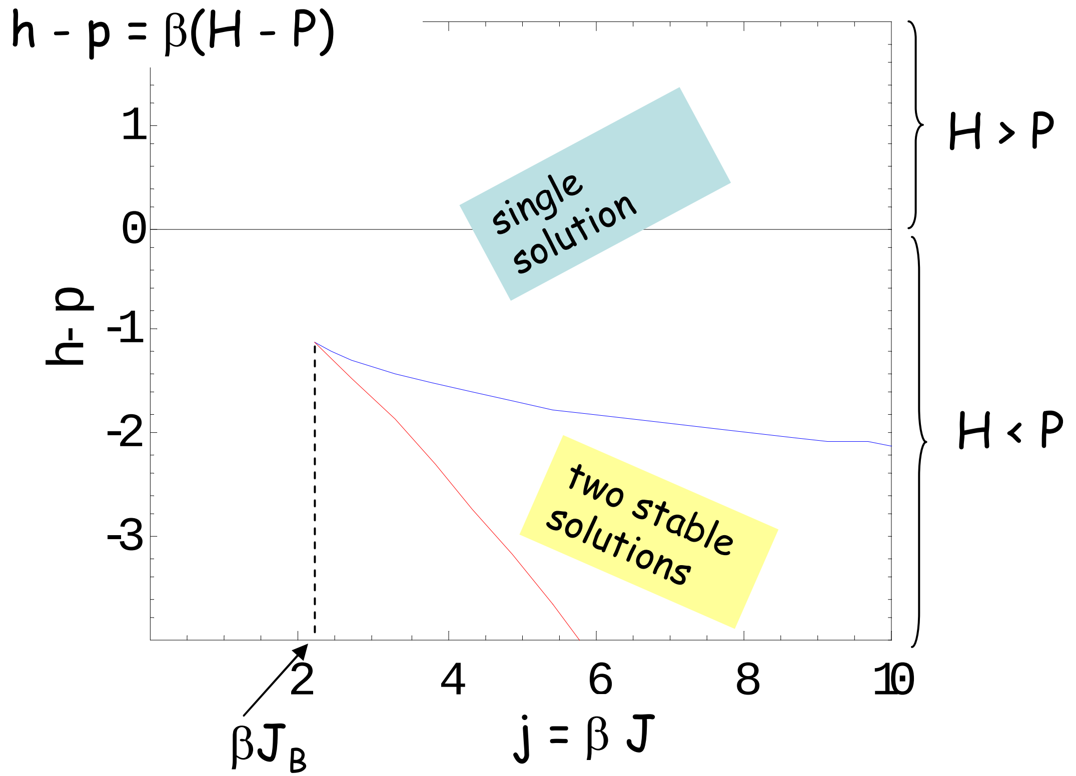
$$f(\theta) = \frac{\beta}{2\cosh^2 \beta\theta}$$



η = fraction of buyers

customers phase diagram





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smooth mono-modal distribution of the IWP

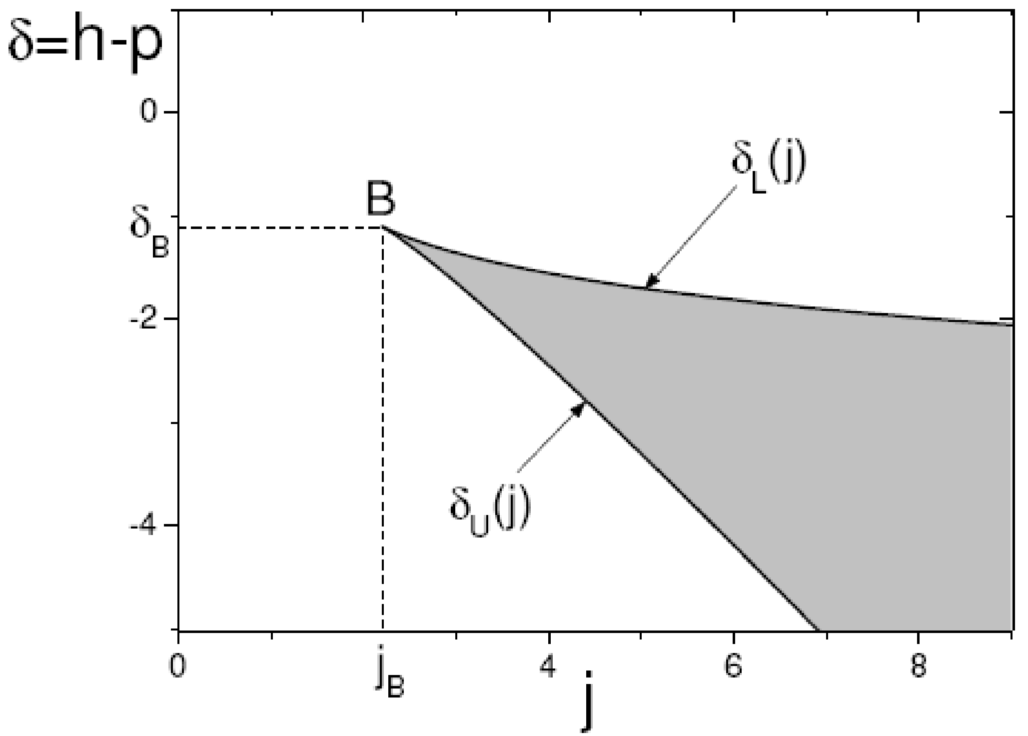


Figure 4: *Demand phase diagram on the plane* ($j = J/\sigma$, $\delta = (H - P)/\sigma$), *for a smooth IWP distribution (here the logistic). In the shaded region the demand presents multiple Nash equilibria. Outside this region, the demand is a single valued function of* j *and* δ.

customer's phase diagram

- alternative representation

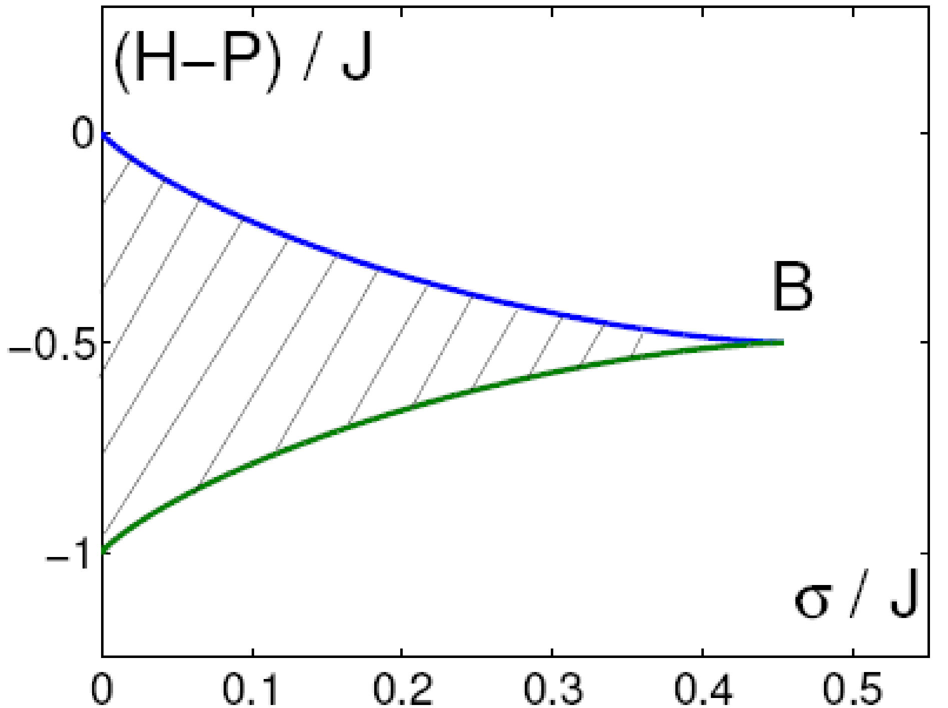
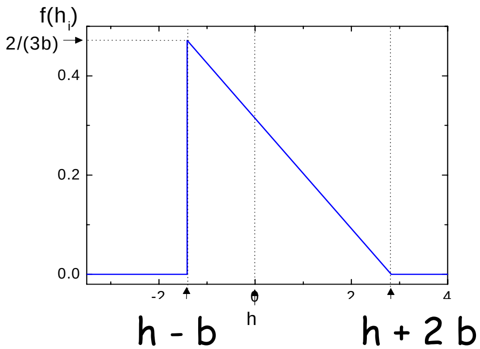


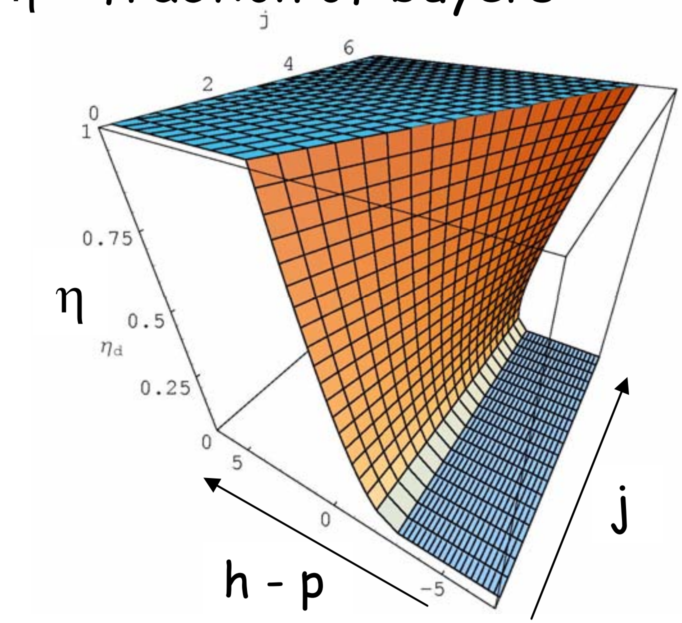
Figure 5: *Demand phase diagram in the plane* $(\tilde{\sigma} = \sigma/J,\ \tilde{\delta} = (H-P)/J)$, *for a smooth IWP distribution (here the logistic). Inside the dashed region the demand presents multiple Nash equilibria. Outside this region, the demand is a single valued function of* $\tilde{\sigma}$ *and* $\tilde{\delta}$.

IWP: triangular distribution

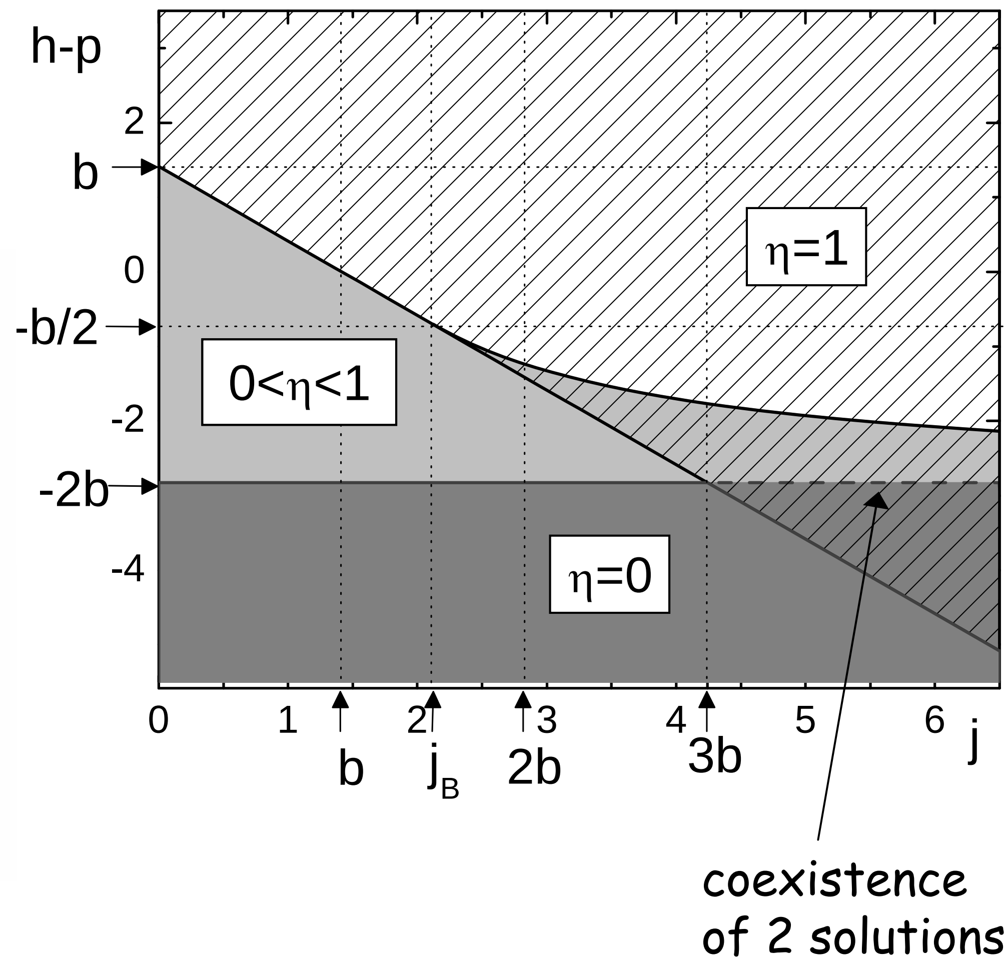
distribution :



η = fraction of buyers



customers phase diagram :



Customer's phase diagram

- triangular distribution of the IWP

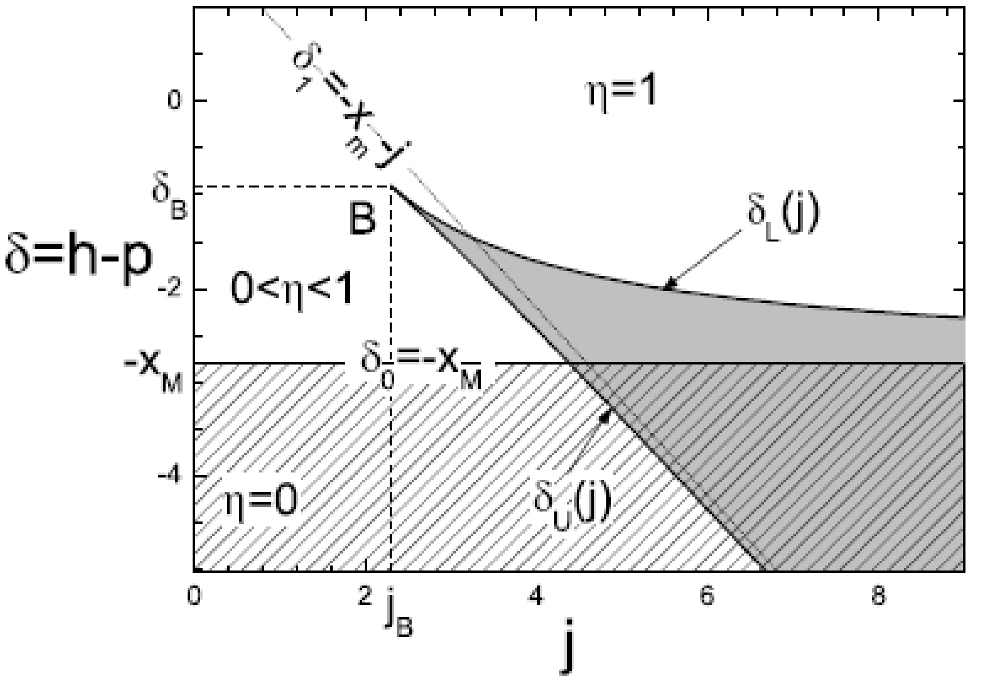


Figure 8: *Triangular pdf of unitary variance and a maximum at $x_B = -1$: customers phase diagram.*

Customer's phase diagram

- multimodal distribution: exple. bi-modal case

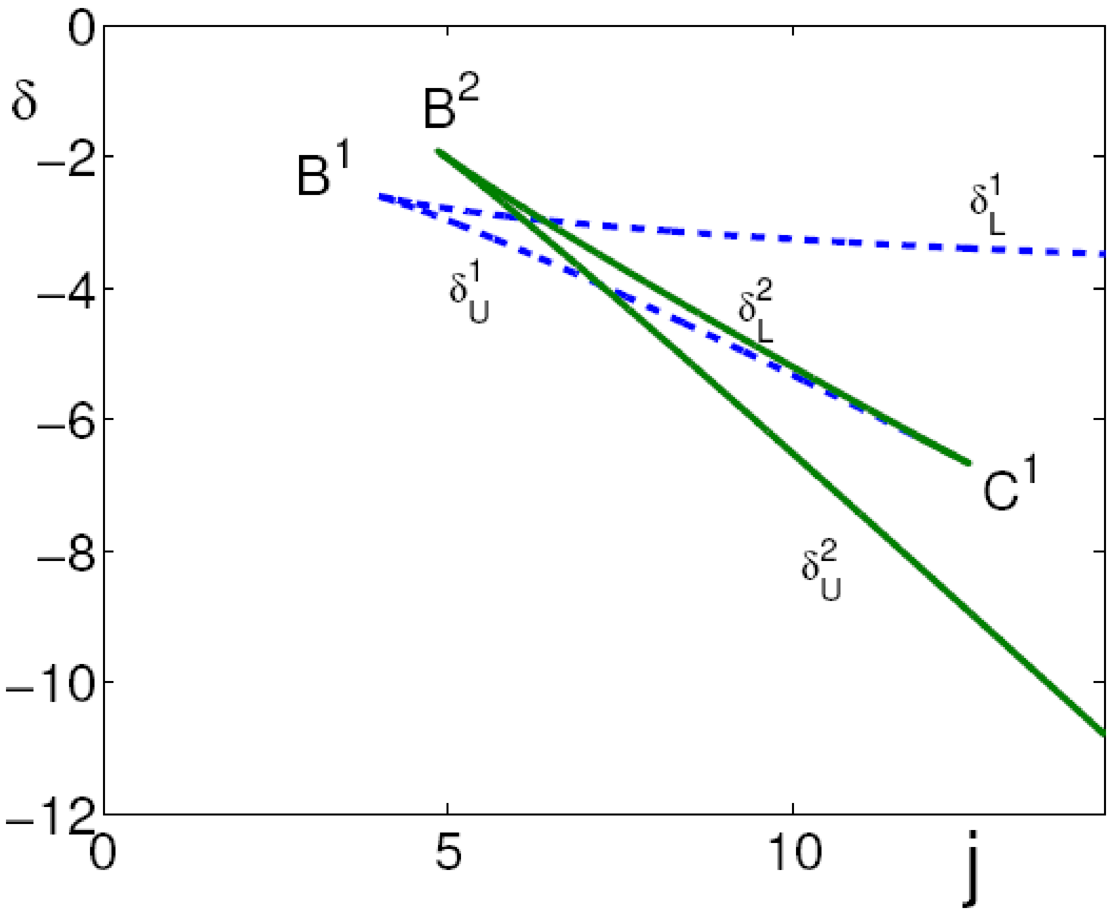
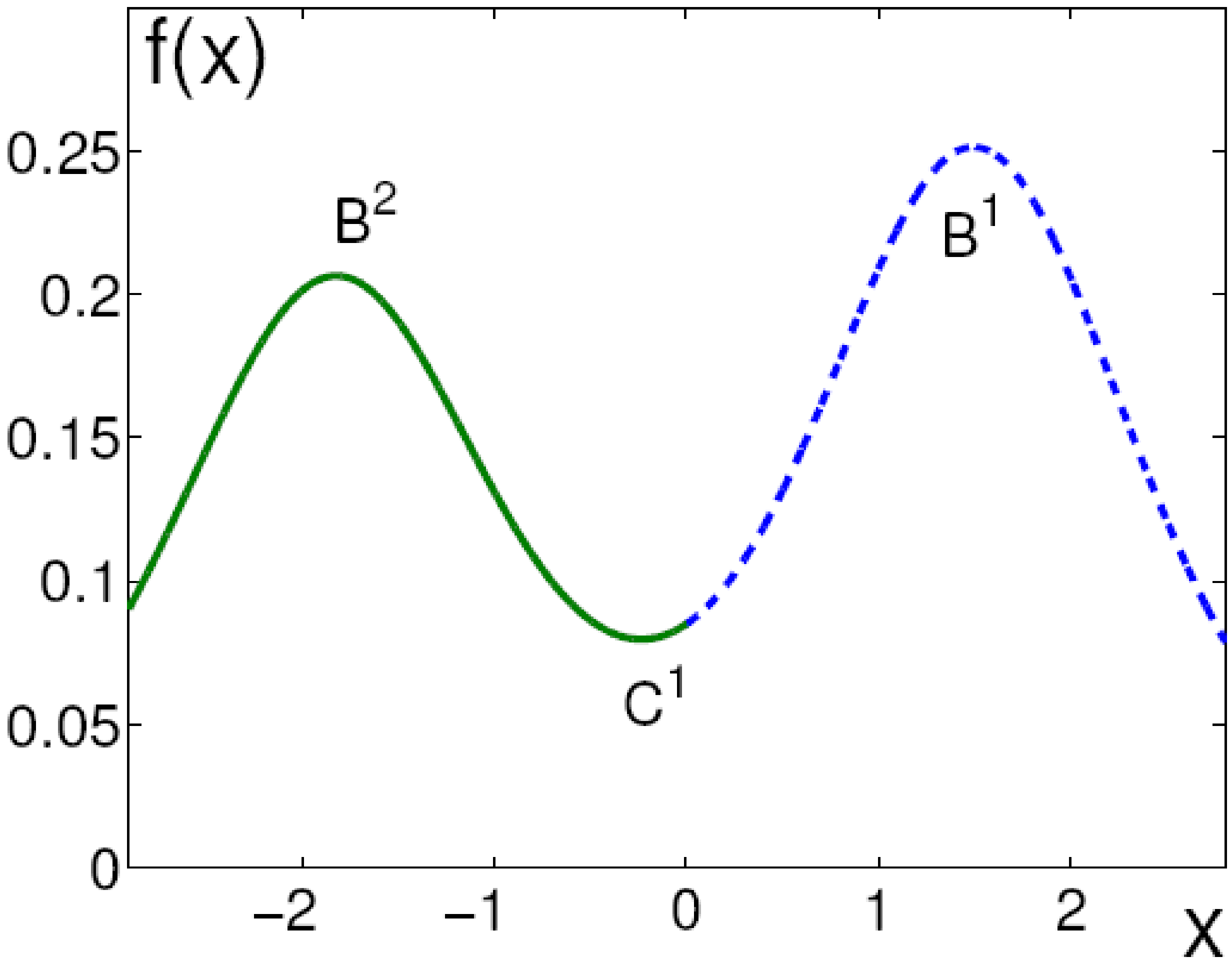


Figure 12: *Phase diagram (aggregate demand) for the case of the smooth bimodal pdf shown on figure 11.*

Customer's phase diagram

- bi-modal distribution



plan

- market model with a single good and externalities
  J.-P. Nadal, D. Phan, M.B. Gordon, J. Vannimenus (2003) Multiple equilibria in a monopoly market with heterogeneous agents and externalities

- equilibrium properties and collective states
  - multiple solutions
  - customer's phase diagram
  - **Monopolist's phase diagram**

- repeated game framework :
  - hysteresis
  - learning by the customers

  experience Weighted Attraction (EWA) learning scheme (Camerer, 2003)
  results with different EWA learning models

- empirical data
- perspectives

Monopolist's phase diagram

- Monopolist:

Dilemma: sell at high price to a small number of customers or at low price to a large number of customers?

Maximisation of the profit

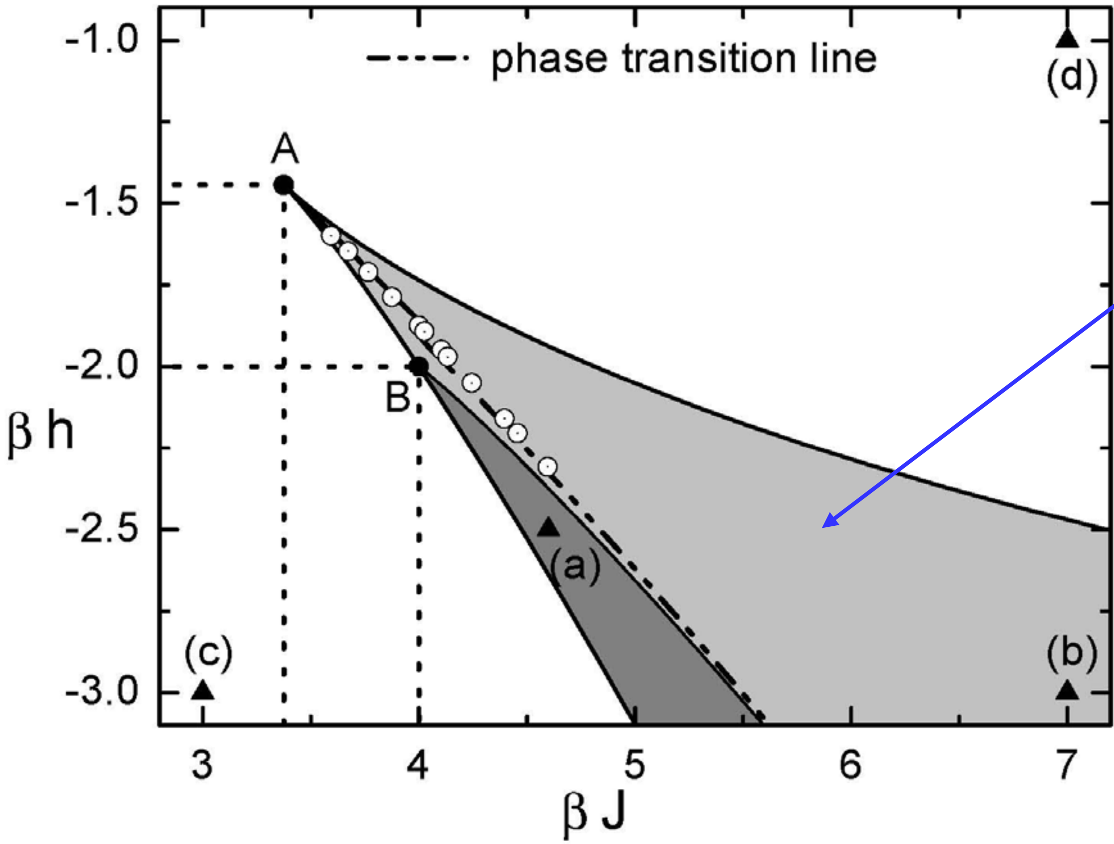
P = price of one unit of the good

C = cost per unit

Choose the price P which maximises

$$\Pi = (P - C)\, \eta(P)\, N$$

Monopolist's dilemma

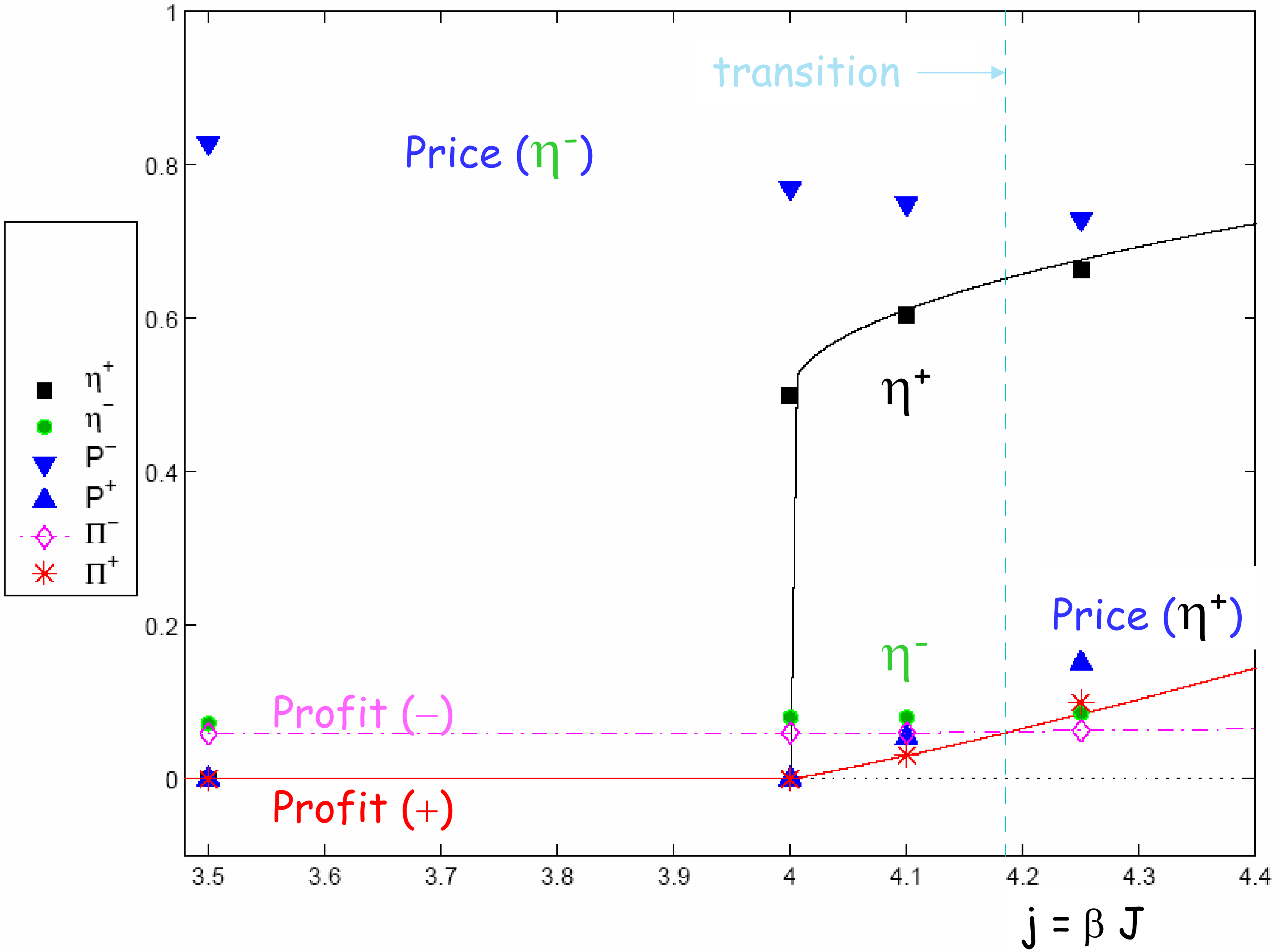


coexistence of 2 solutions

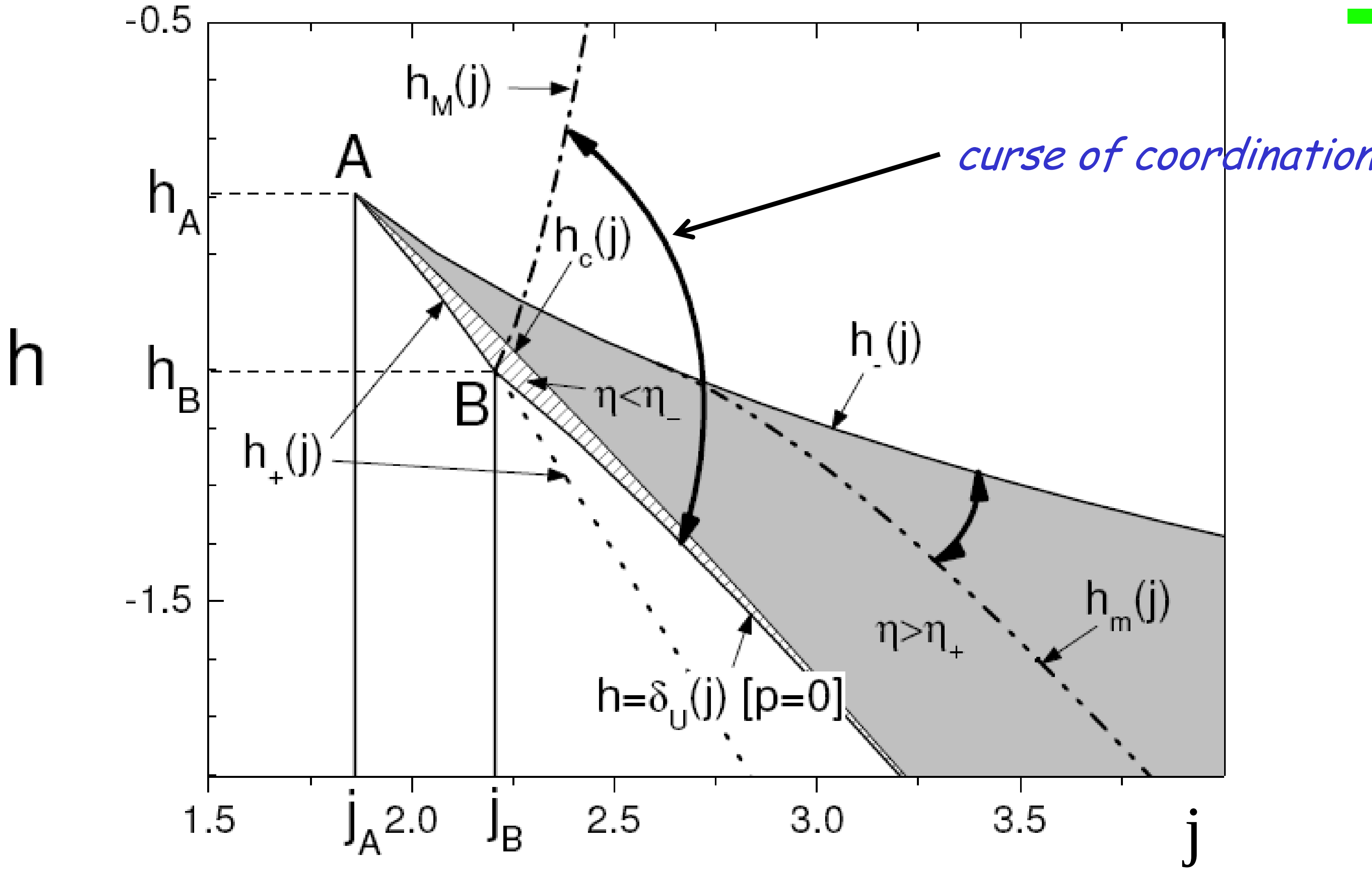
This figure: logistic distribution for the IWP

But = generic phase diagram for any smooth monomodal distribution

Monopolist's phase diagram



monopolist's phase diagram



curse of coordination: the optimal seller's strategy corresponds to a price for which the demand is not unique; the optimal profit will not be obtained if the population do not coordinate

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Hysteresis

- Myopic Best-Response Dynamics

- Evolution of the demand as the price increases or decreases
- (or as the mean willingness to pay/adopt, h, decreases or increases)

- Known from statistical physics:

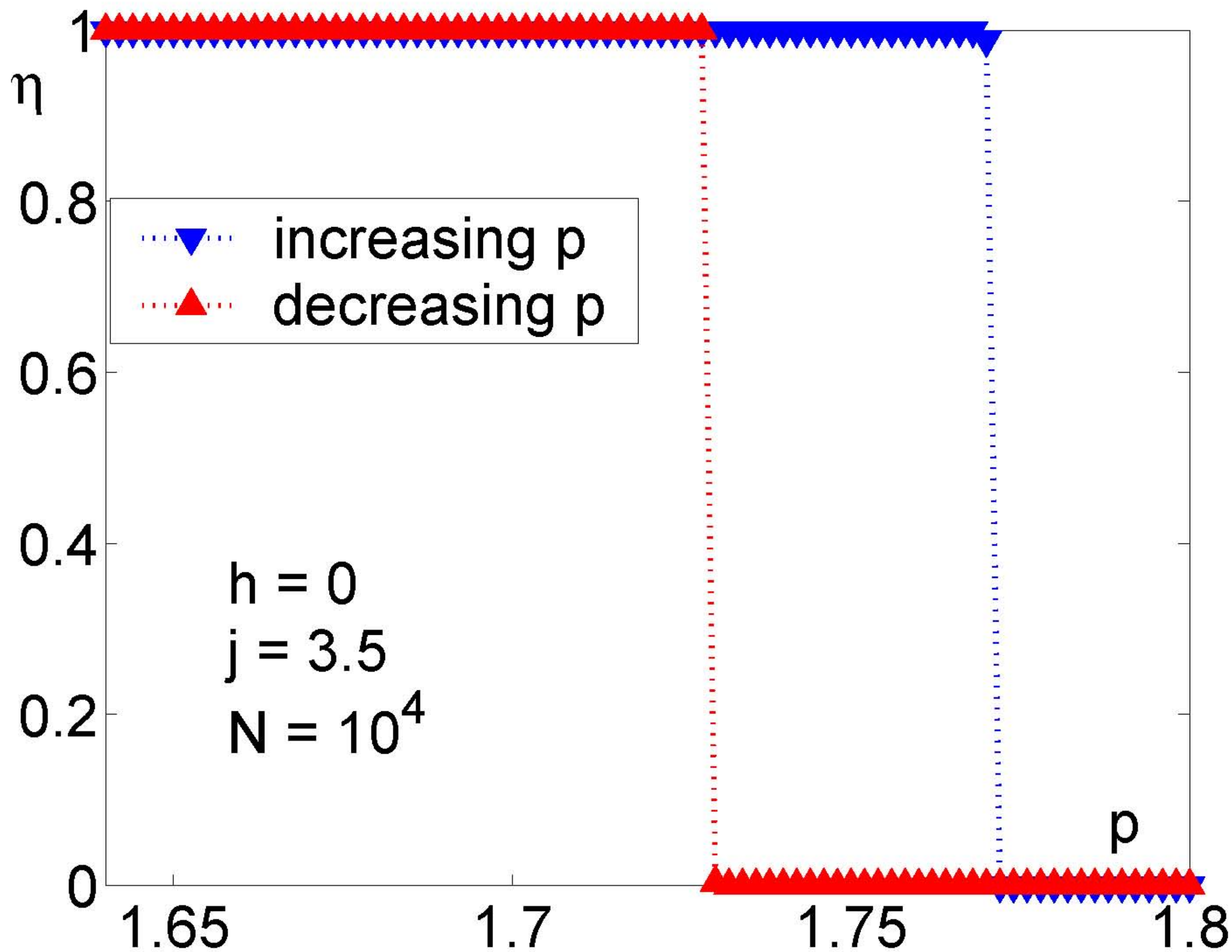
  hysteresis and avalanches
  (ref. : Sethna *et al*, 1993)

Thomas C. Schelling

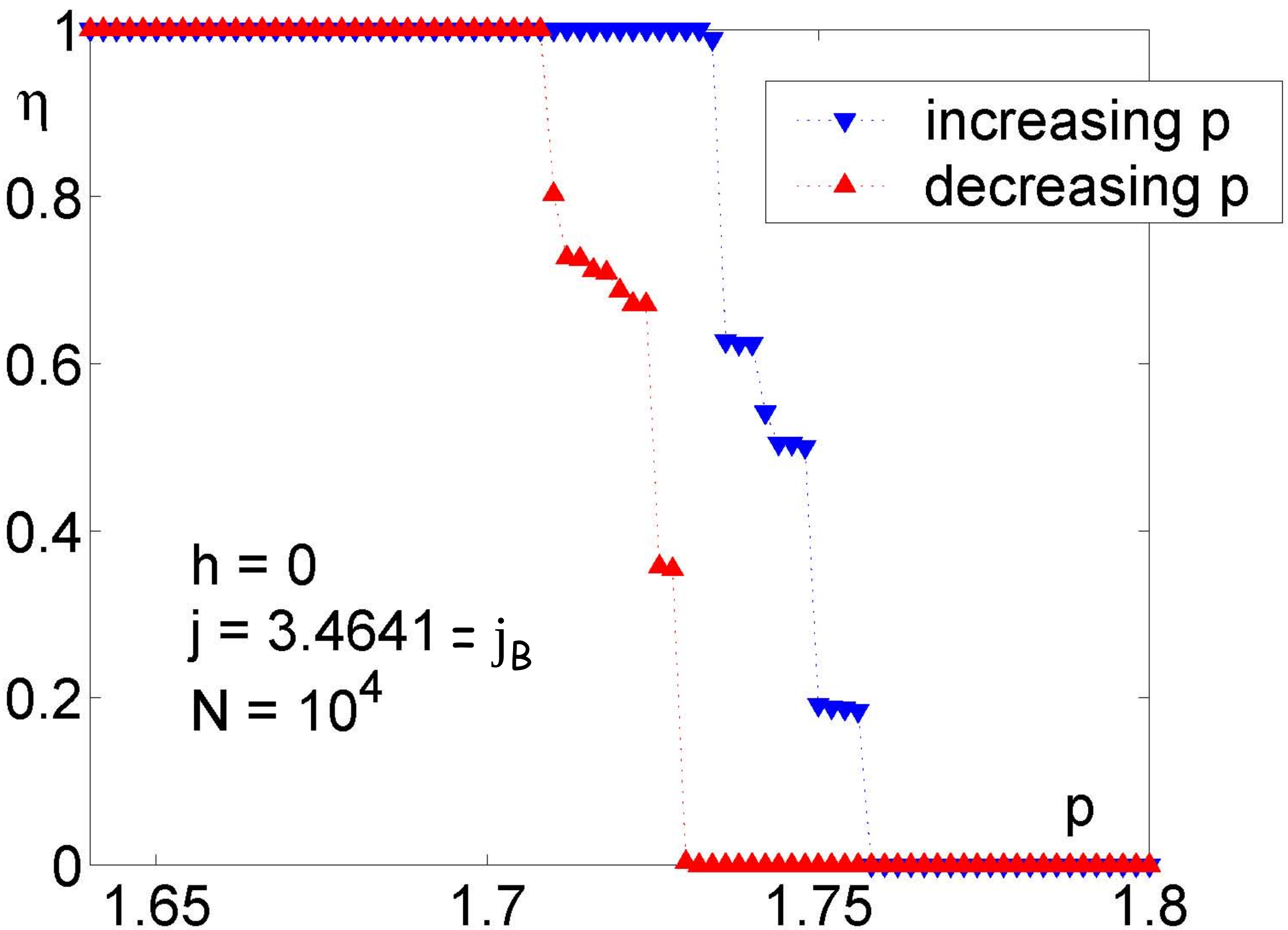
« Some vivid dynamics can be generated by any reader with a half-hour to spare, a roll of pennies and a roll of dimes, a tabletop, a large sheet of paper, a spirit of scientific inquiry, or, lacking that spirit, a fondness for games. »

in *From micromotives to macrobehavior* (1978)

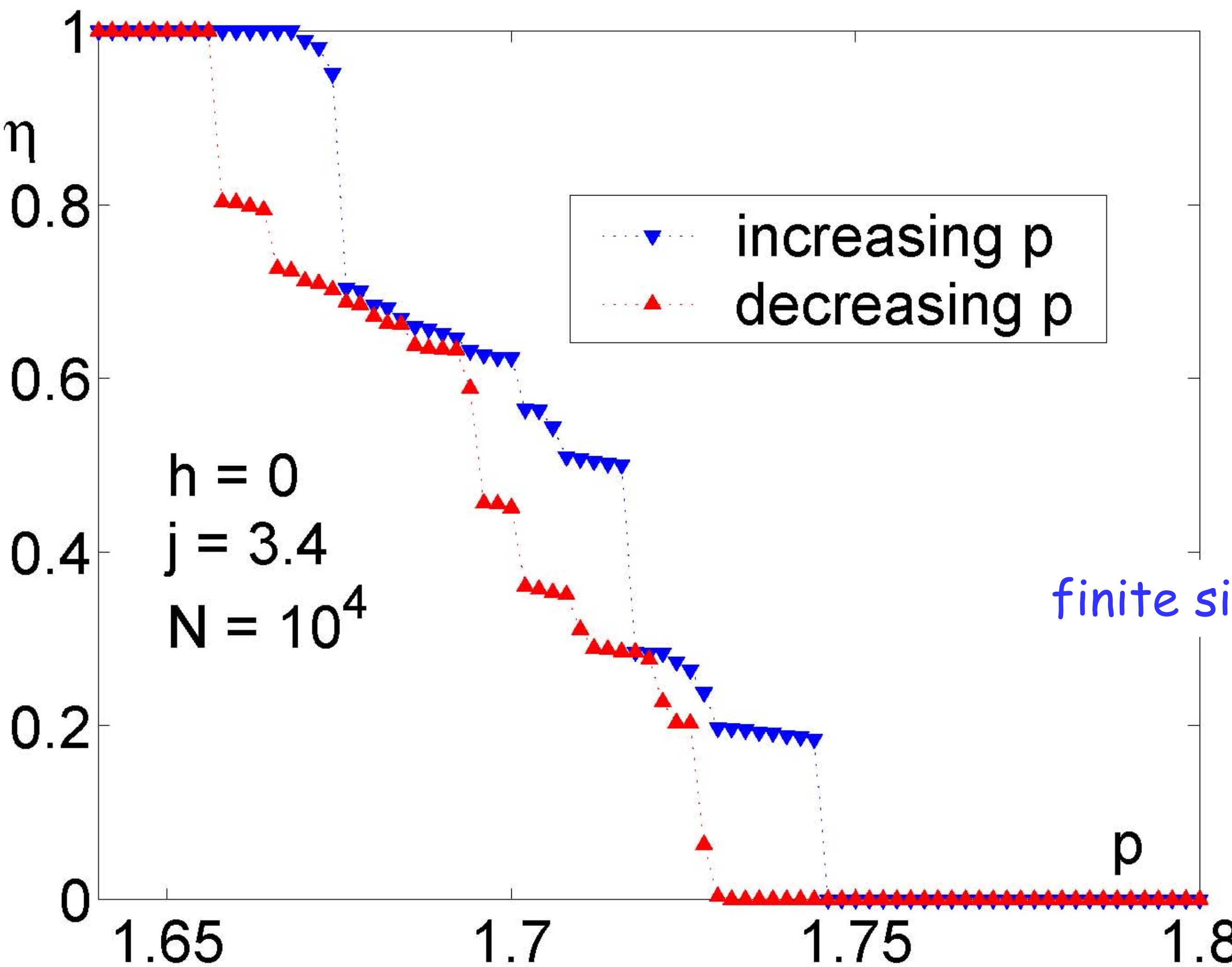
Myopic Best-Response Dynamics



Myopic Best-Response Dynamics

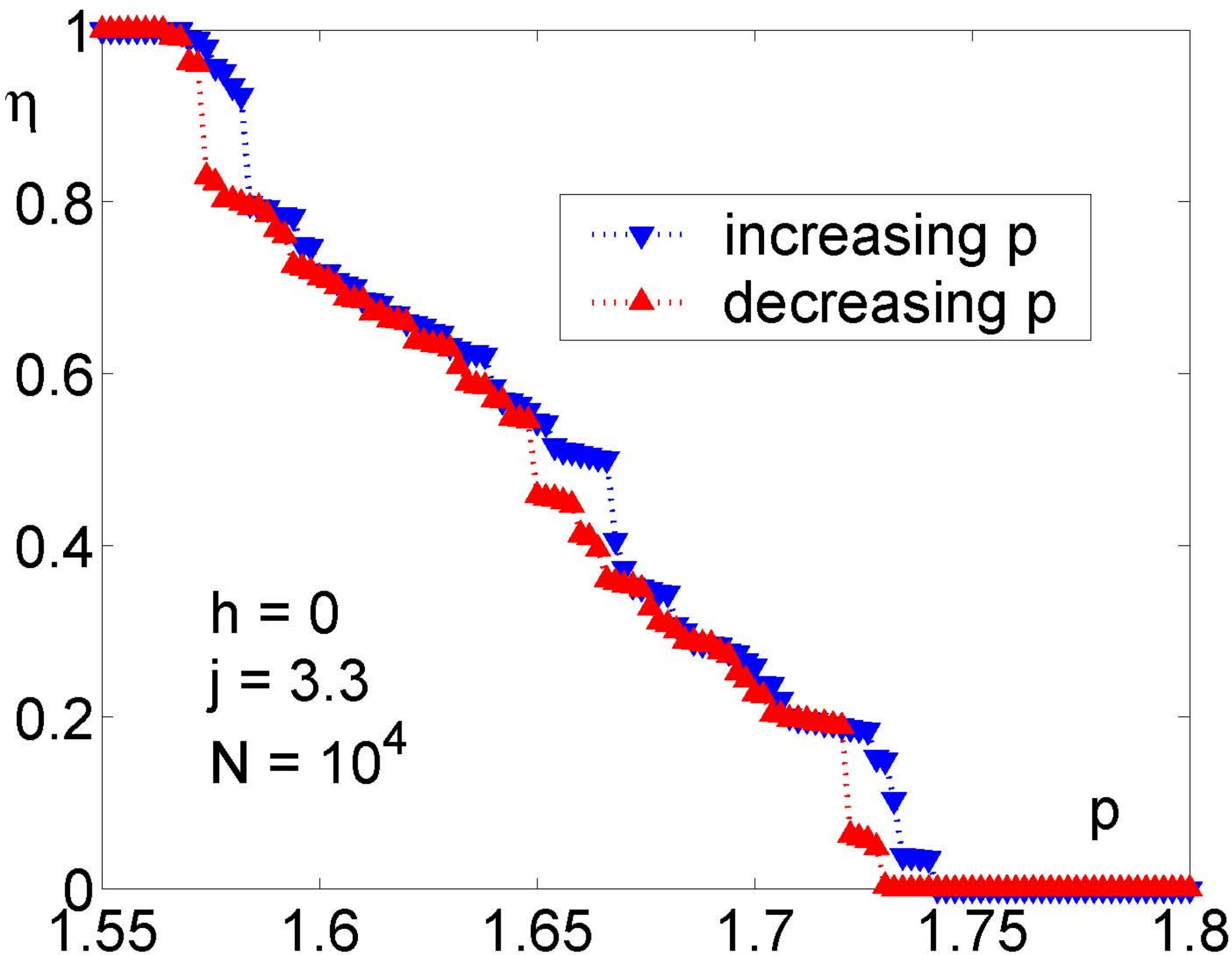


Myopic Best-Response Dynamics



finite size effect

Myopic Best-Response Dynamics



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  - multiple solutions
  - customer's phase diagram
  - monopolist's phase diagram

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  results with different learning models

  V. Semeshenko, M B Gordon, JPN & D Phan, 2006, proceedings of ECCE1, Springer

  JPN, V. Semeshenko, M B Gordon, in preparation

- empirical data
- perspectives

hypothesis

- the demand adapts to the price faster than the time scale of price revision:

  price is assumed to be fixed during the customers' learning.

- each agent must decide whether to buy or not
  - under imperfect information (he doesn't know the decisions of the others)
  - and incomplete information (he doesn't know his actual payoff)

- based on the "attraction" of buying or not buying

- attraction values are updated (learned) based on the actual fractions of buyers

learning dynamics

- At each time step each agent i makes a binary decision :

$$\text{Proba}[\omega_i(t)=1] = f(\Delta_i(t)), \quad \text{with} \quad \Delta_i(t) \equiv A_i^1(t) - A_i^0(t)$$

← relative attraction for buying

  - best response:

$$\omega_i(t) = \begin{cases} 1 & \text{if} \quad \Delta_i(t) - P > 0 \\ 0 & \text{if} \quad \Delta_i(t) - P < 0 \end{cases}$$

  - trembling hand:

$$P[\omega_i(t)=1] = 1 - \varepsilon(\Delta_i(t) - P)$$

$$\text{If logistic: } P[\omega_i(t)=1] = \frac{1}{1+\exp-\beta[\Delta_i(t)-P]}$$

- Learns his relative attraction [Camerer: Experience Weighted Attractions]

  from the observation of the actual fraction of buyers $\eta(t)$:

$$\Delta_i(t) = H_i + J\hat{\eta}_i(t) \qquad \hat{\eta}_i(t) = \text{ agent i's estimate of } \eta$$

$$\hat{\eta}_i(t+1) = \hat{\eta}_i(t) + \mu(t+1)\{[\delta + (1-\delta)\omega_i(t)]\,\eta(t) - \hat{\eta}_i(t)\}$$

$$\mu(t+1) = (1-\kappa)\frac{\mu(t)}{\mu(t)+\phi} + \kappa(1-\phi)$$

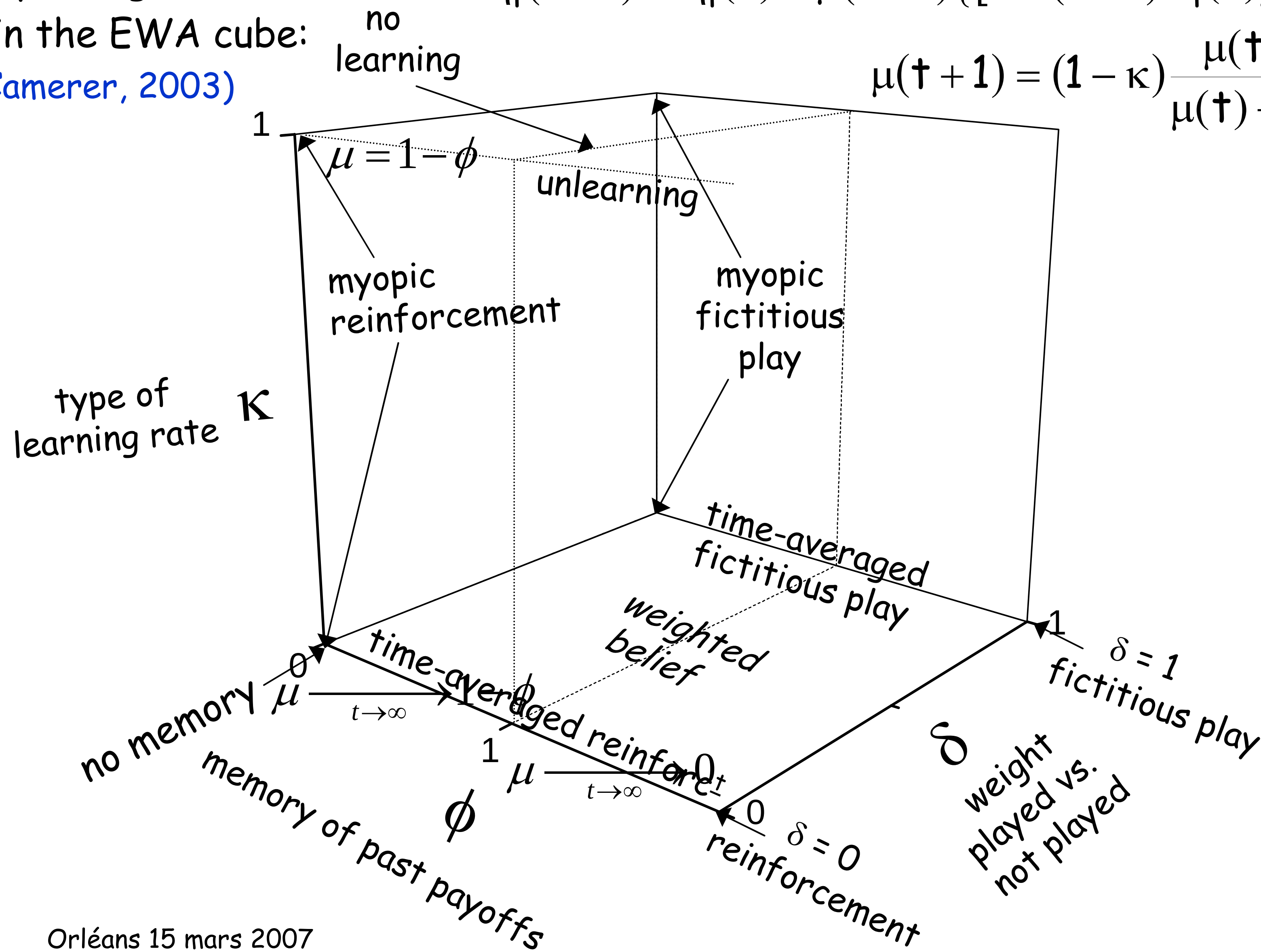
Experience Weighted Attraction

- updating attractions in the EWA cube:

(Camerer, 2003)

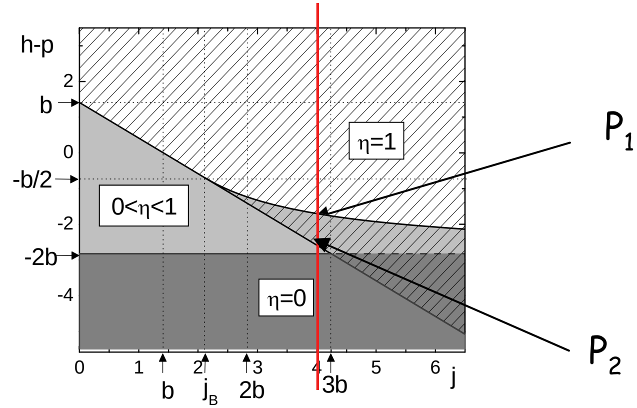
$$\hat{\eta}_i(t+1) = \hat{\eta}_i(t) + \mu(t+1)\left\{\left[\delta + (1-\delta)\omega_i(t)\right]\eta(t) - \hat{\eta}_i(t)\right\}$$

$$\mu(t+1) = (1-\kappa)\frac{\mu(t)}{\mu(t)+\phi} + \kappa(1-\phi)$$

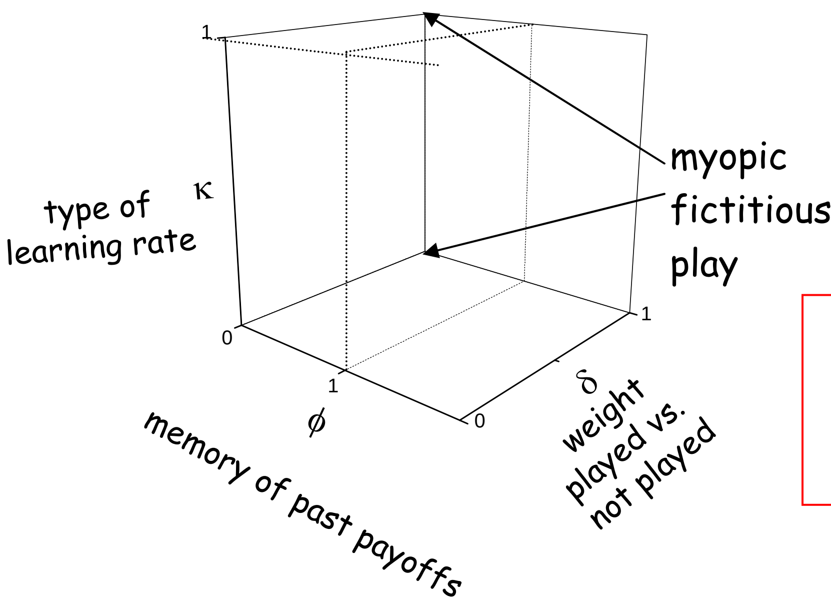


simulations

phase diagram
for the
triangular distribution :



J=4



- Simplest case :

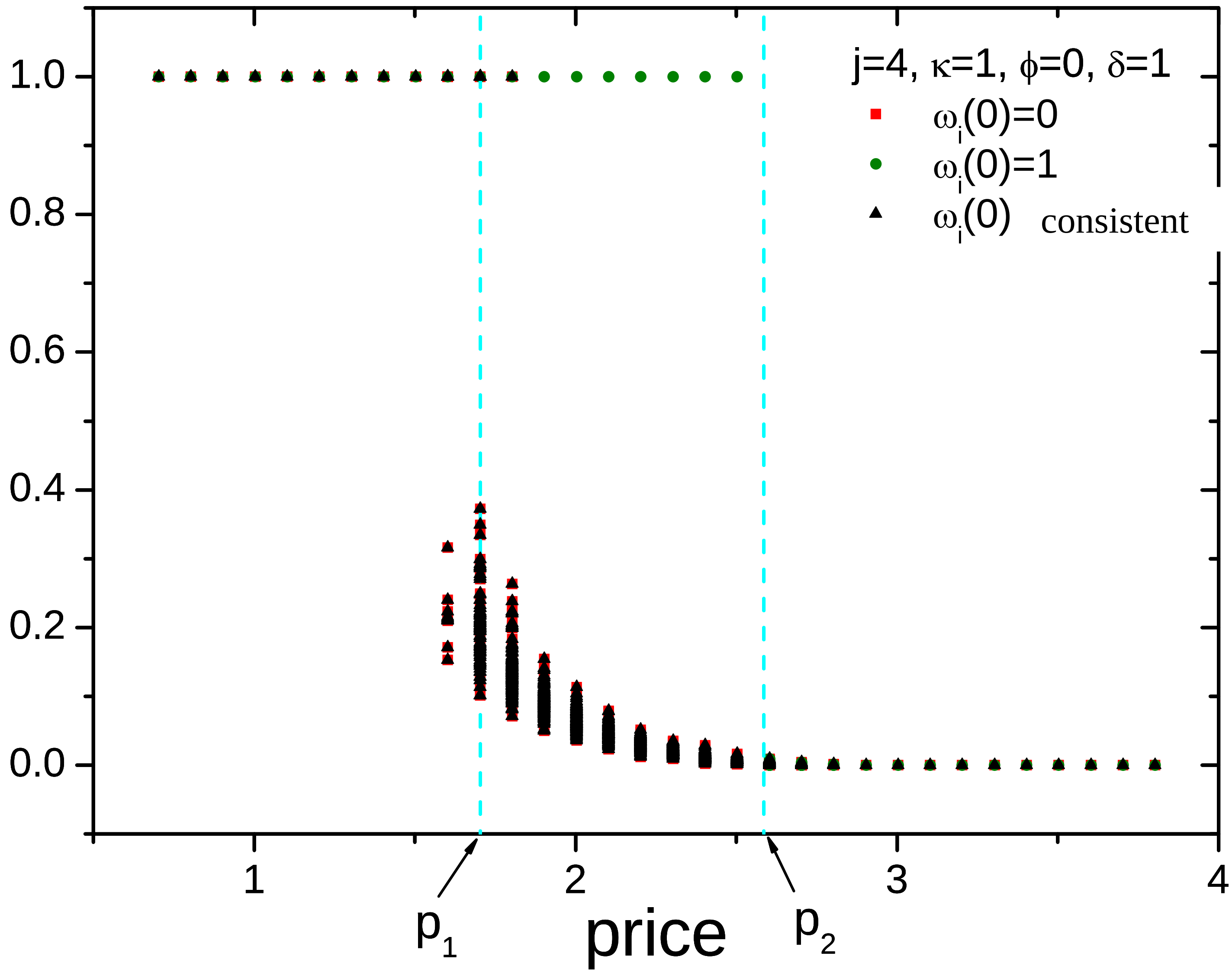
$$\mu(t+1)=1$$

$$\hat{\eta}_i(t+1)=\hat{\eta}_i(t)+\mu(t+1)\left[\eta(t)-\hat{\eta}_i(t)\right]$$

$$\hat{\eta}_i(t+1)=\eta(t)$$

myopic fictitious play

η = fraction of buyers



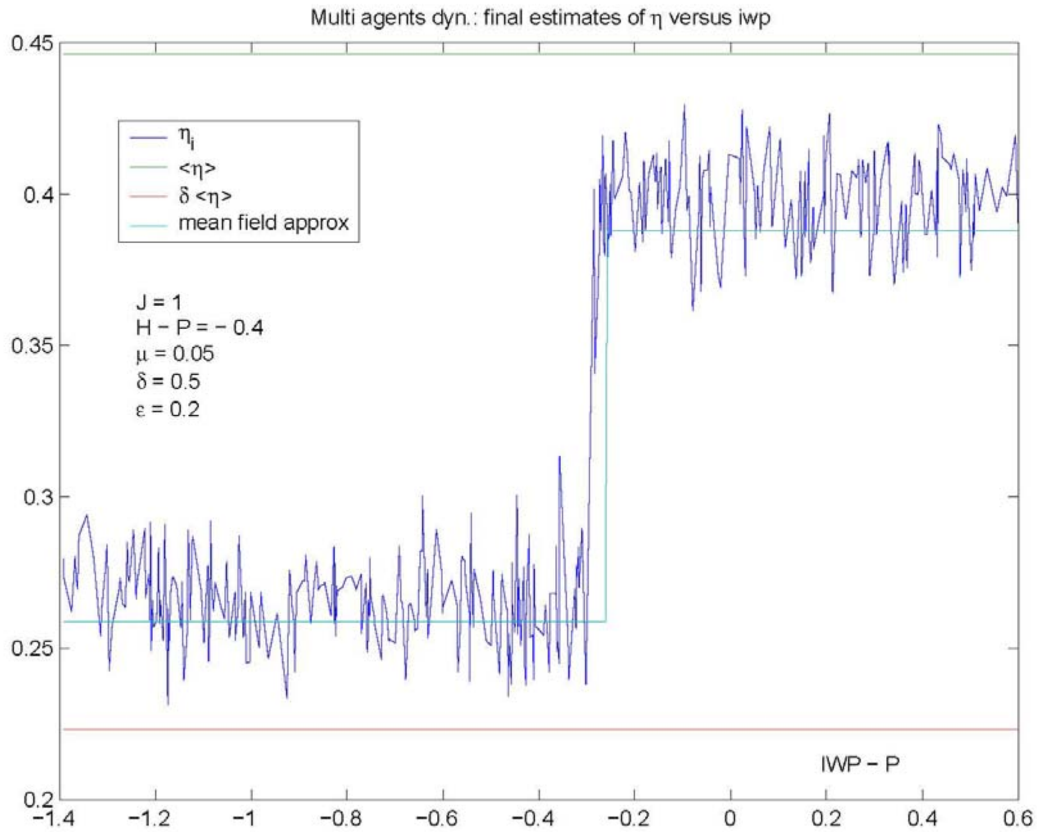
discussion

- asymptotics for $\phi < 1$
  - if $\delta > 0$
    - buyers : $\hat{\eta}_i \rightarrow \eta$
    - for non buyers: $\hat{\eta}_i \rightarrow \delta\eta < \eta$: attractions are underestimated
    - discontinuity at Δ = P

- asymptotics for $\phi > 1$
  - the time decay of $\mu(t)$ may hinder the learning process

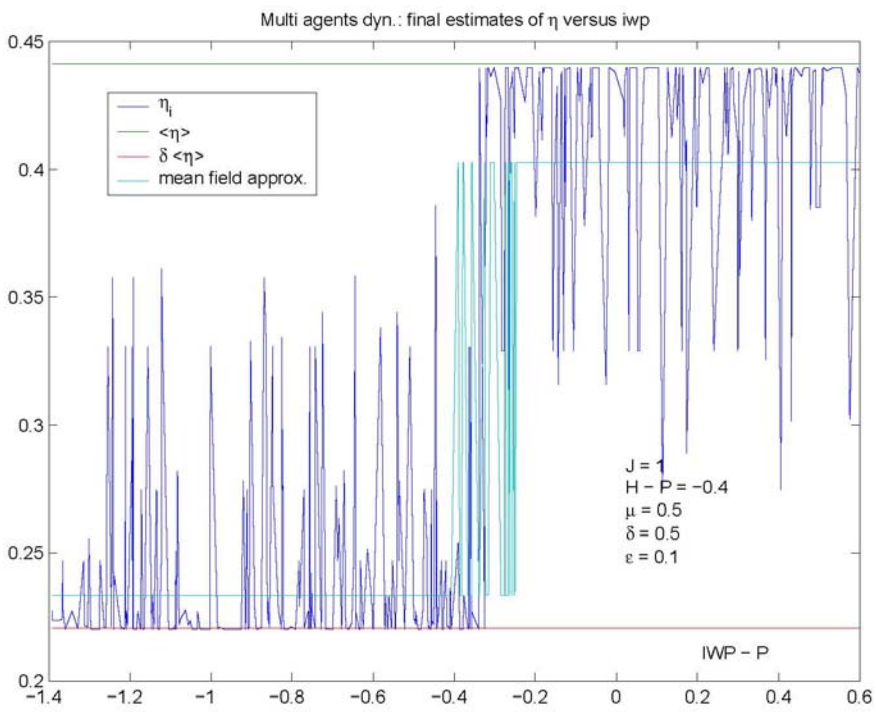
- more results:
  - decision through a trembling hand: interference between ε and μ
  - analytic results
    - the learning process as a special random walk
  - learning directly the attractions (estimations of $H_i+J\eta-P$) leads to very different results

Trembling hand

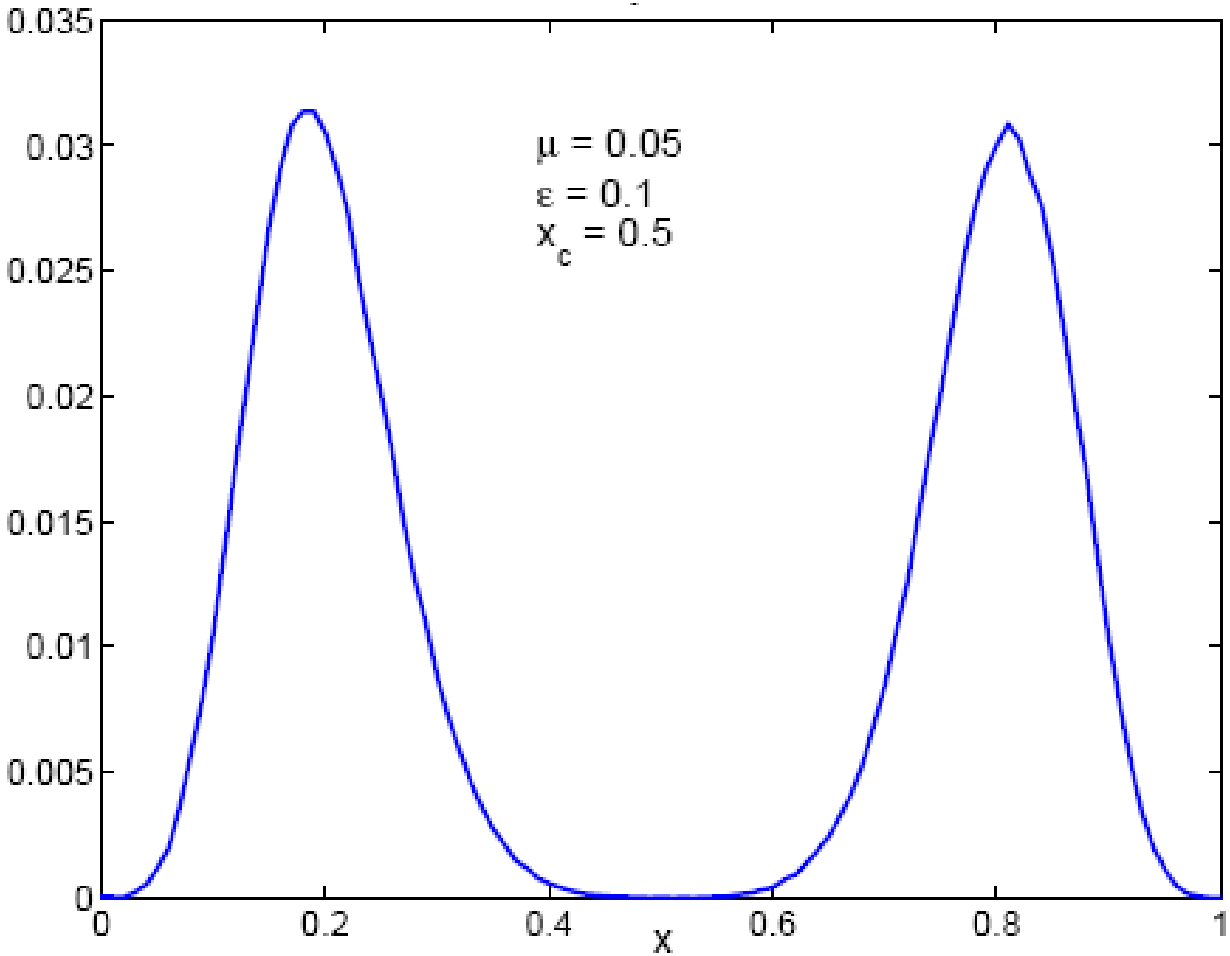


IWP - P

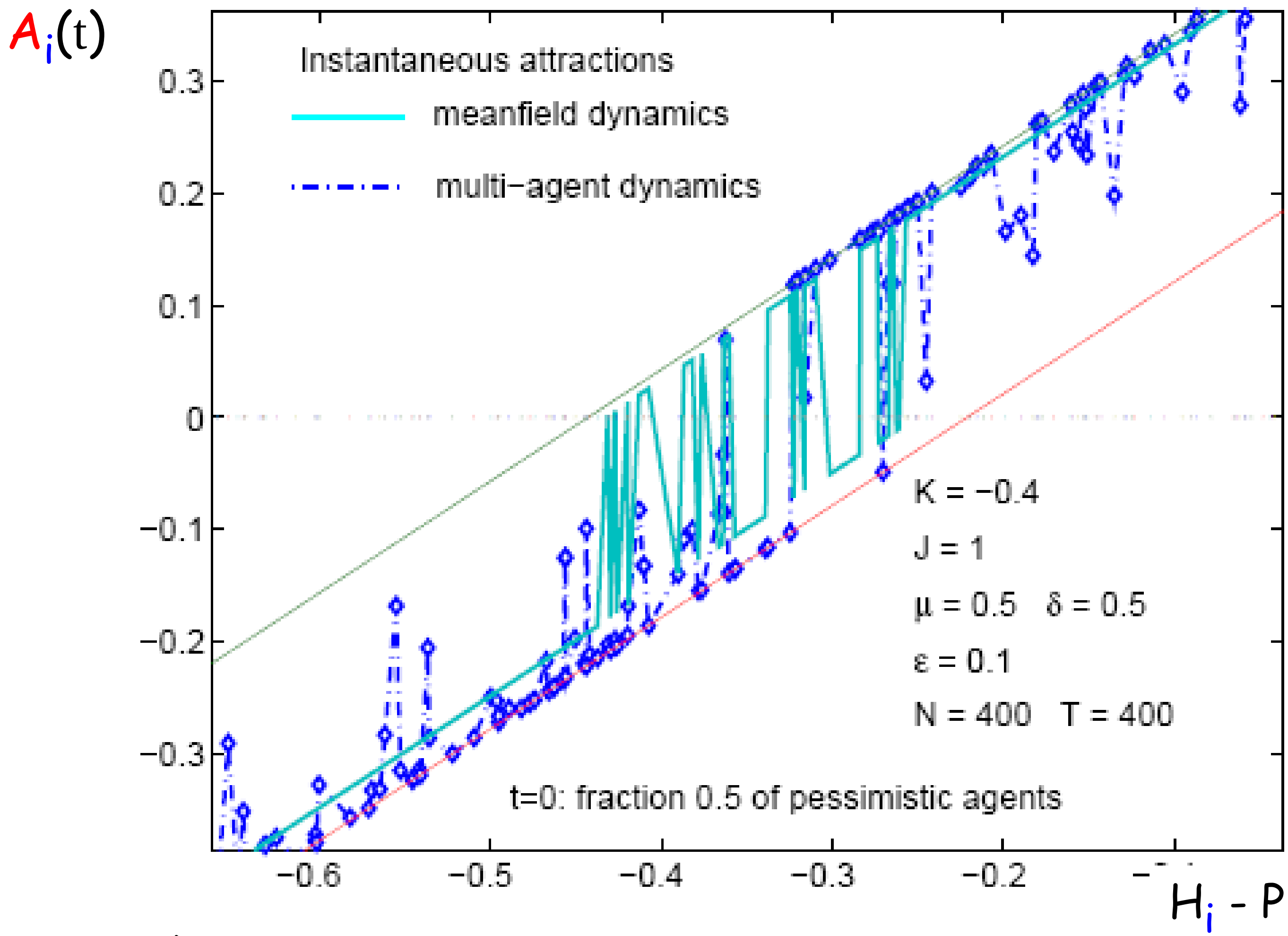
Trembling hand



Stationary distribution of the attraction of a single agent

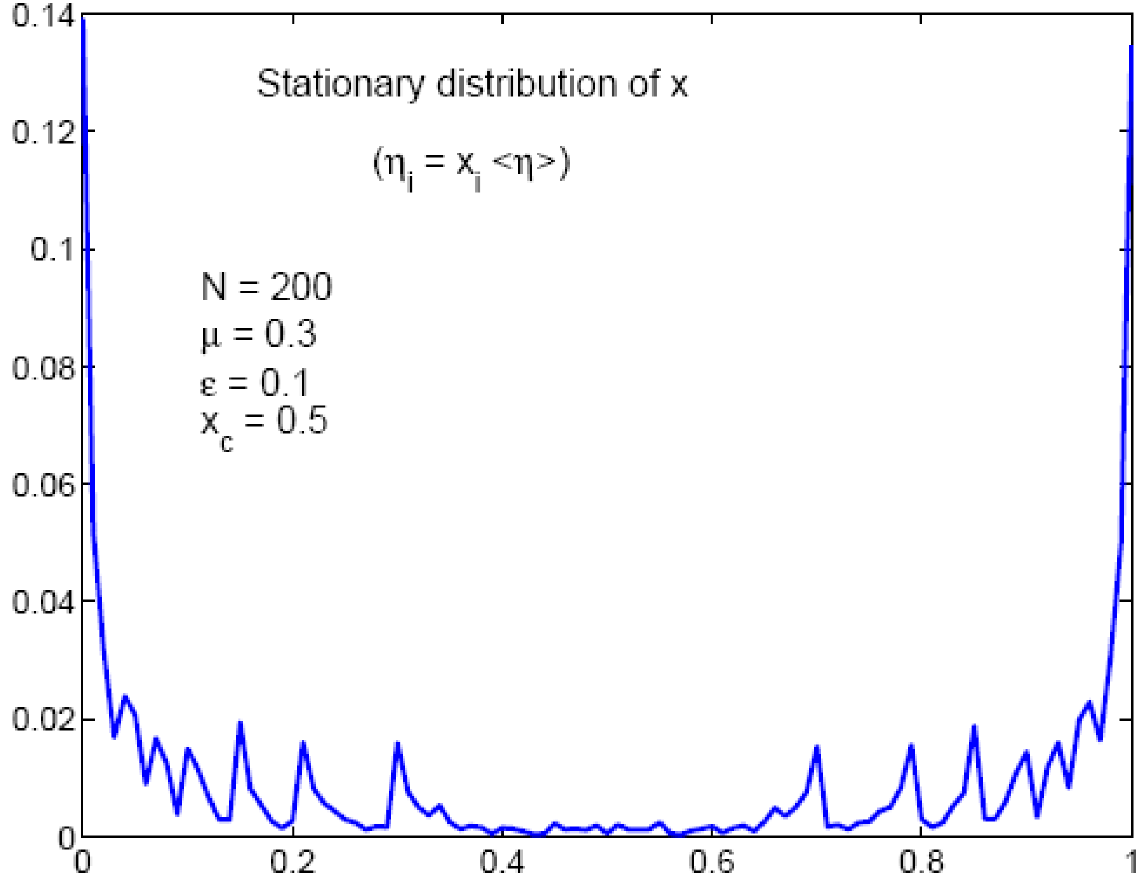


0 <(normalized attraction) < 1



(singular regime)

Stationary distribution of the attraction of a single agent



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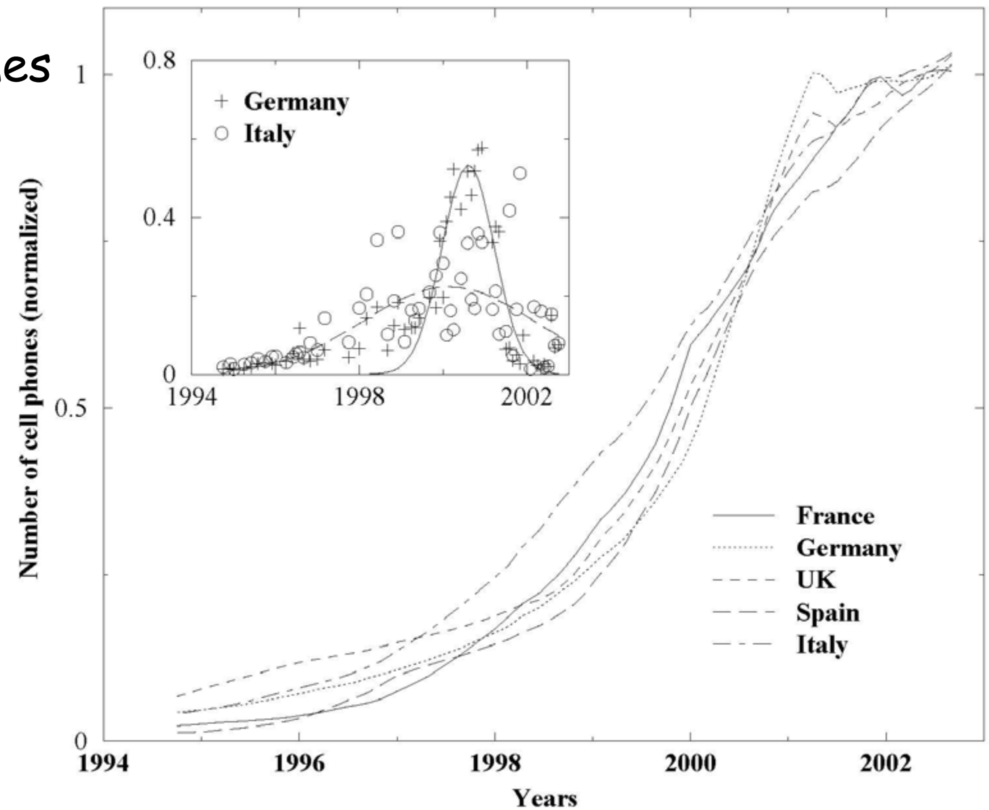
Empirical Data

Number of cell phones

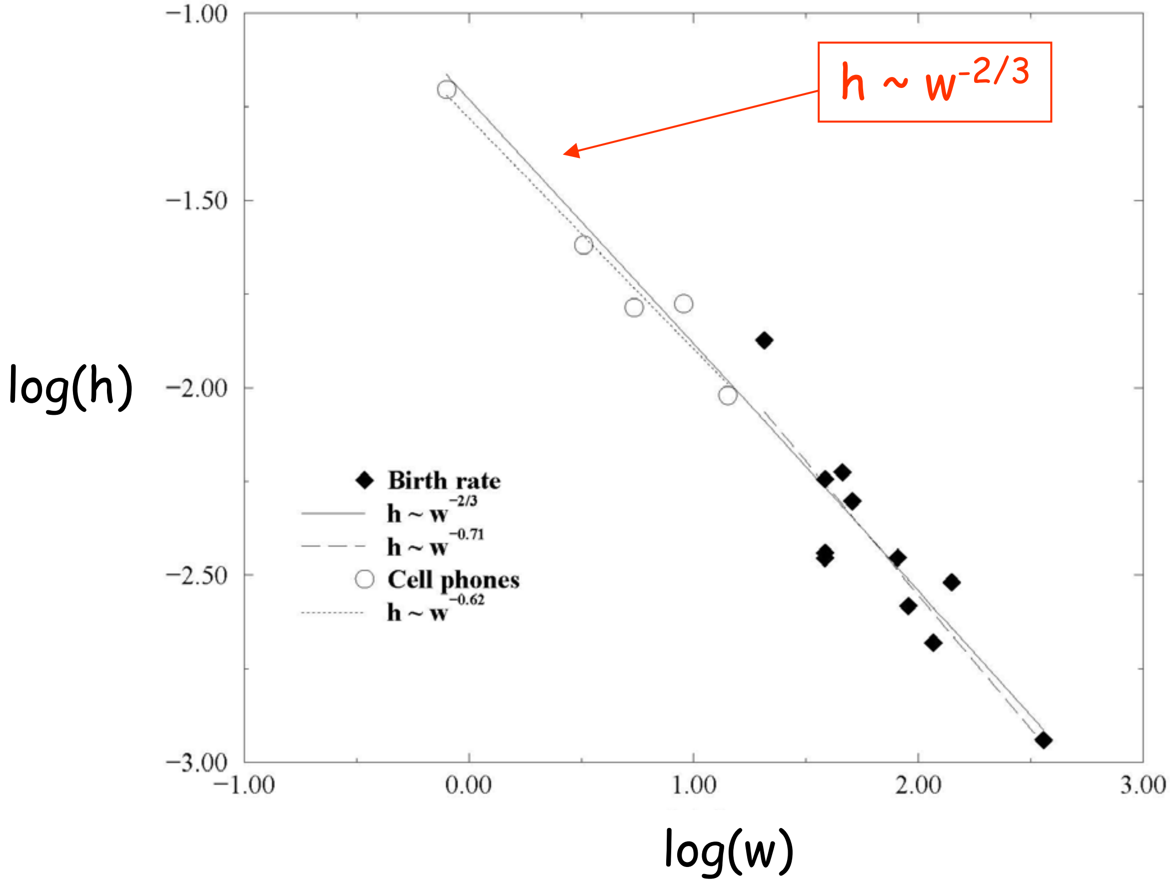


Q. Michard & J.-P. Bouchaud, preprint 2005

« Theory of collective opinion shifts: from smooth trends to abrupt swings »

http://arxiv.org/abs/cond-mat/0504079

Empirical Data: scaling



Q. Michard & J.-P. Bouchaud, preprint 2005

« Theory of collective opinion shifts: from smooth trends to abrupt swings »

http://arxiv.org/abs/cond-mat/0504079

« Of Songs and Men: a Model for Multiple Choice with Herding »

Christian Borghesi and Jean-Philippe Bouchaud
arXiv:physics/0606224 v1 (June 2006)

modelling/analysis of data:

web-based music market experiment,

M. J. Salganik, P. S. Dodds, D. J. Watts, *Experimental Study of Inequality and Unpredictability in an Artificial Cultural Market*, Science 311, 854-856 (2006)

Current works and Perspectives

- Heterogeneous agents + social interactions + learning

- more on the monopolist model and its variants (joint adaptive dynamics: seller and customers) ; more on adaptive dynamics (market and non market contexts)

  [+ M. B. Gordon, V. Semeshenko (TIMC, Grenoble), J. Vannimenus (LPS ENS); D. Phan (GEMAS), R. Waldeck (ENST Bretagne)]

- diffusion of criminality – Project supported by the City of Paris

  [+ L. Kebir, H. Berestycki (CAMS), V. Semeshenko, M. B. Gordon (TIMC, Grenoble), S. Franz (ICTP, Trieste)]

- social interactions in medical care

  [+ E. Tanimura (CAMS), J. Scheinkman (Princeton)]

- adaptive dynamics and evolution of phoneme categories

  [+ J. Pierrehumbert (Northwestern Univ.), Laurent Bonnasse-Gahot (CAMS)]

physics - economics - mathematics - linguistics