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**CONTRIBUTIONS TO GEOMETRIC VISUAL ILLUSIONS**
**(with illustration supplement included)**

**Jacques NINIO**

Included in the web site http://www.lps.ens.fr/~ninio

## TOPICS DISCUSSED HERE



## SEE IN OTHER SECTIONS

Zöllner and other illusions in stereo (in Chapter on stereoscopic vision)
The “depth from illusory disparities” issue (in Chapter on stereoscopic vision)
Shape perception (to be developed in a forthcoming chapter)

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## 1. OVERVIEW

Most of what I have to say on geometrical visual illusions is now in a review article [1] or in books [2,3]. This chapter of the web site is a biographical account of my itinerary in the field. My position in this field is very strange, to say the least. It is a domain to which I devoted intense thinking in 1975-1976, and in which I performed rather systematic psychophysical work, from 1995 to 2004, a great deal of which is still in my drawers. I also designed original stimuli showing counter-intuitive effects (for instance, a diamond looking too small compared to a square – see Fig. 3 in the Illustration supplement and [2]). I kicked off a well-known illusion (the so-called flattening of short arcs), showing that it is definitely not an illusion (Appendix A in [4] or Figures 9a-9c in [1]), I announced a paradoxical result which should have interested neuro-philosophers: two trapeziums, one above the other can never be made perceptually equal (when the wide bases look equal, the small bases don’t, etc.[ 1, 2, 3]). Technically, I made a number of strong statements, based upon psychophysical work, about which types of deformations account best for the illusions: For instance, the Zöllner illusion, considered with all its variants, is best described in terms of an expansion at right angles to the oblique segments [5, 6], while the Poggendorff and related illusions are best described by misangulation biases [7]. The Müller-Lyer illusion, and a large number of illusions which are not usually found in its company are best described with a “convexity rule” (for the layman, take it provisionally as a law of contrast: “small looks smaller, large looks larger”) [1, 4], which is quite different from the often invoked assimilation effect. I also established a number of other points, which are detailed below. All this has been largely unnoticed. Perhaps I failed to make my findings intelligible to my colleagues, and perhaps putting it all together in this web chapter will make my work more obvious.

It is through visual illusions that I started to work thoroughly on visual perception. In 1975, I was a full-time molecular biologist having developed a quantitative understanding of how errors could arise in molecular processes [8, 9]. I contemplated the possibility of extending such analyses to other domains. I bought two books which played an important role: The Intelligent Eye by Richard Gregory [10], that shaped my way of thinking about perception and The Psychology of Visual illusion by J.O. Robinson [11], that presented the geometrical illusions in an exhaustive manner, and provided a critical account of all the scientific literature on the subject. It seemed to me, reading the two books, that so far, geometrical visual illusions had not been treated in a geometrically insightful way.

Some very elementary geometric truths may be counter-intuitive. A good example is the fact that if you take planar sections of a circular cone, you get a circle, an ellipse, a parabola or a hyperbola depending upon the orientation of the plane with respect to the axis of the cone. Therefore, any of these curves can be thought of as the projection of any other of these curves, and this knowledge has practical value, for it allows a geometer to predict some properties of hyperbolas, knowing proven properties of ellipses.

This is not to say that an ellipse or a hyperbola should be perceptually equivalent. However, it may be the case that some strange aspects of 2d shape perception, as revealed by geometric visual illusions are just “legal” but counter-intuitive outcomes of the internal rules used by the brain for performing geometry. I spent several months scrutinizing the geometrical illusions in J.O. Robinson’s book, trying to extract their gist. In Robinson's book, the illusions were grouped into chapters. This amounted to a formal classification of the illusions. What if some illusions had been misplaced? I considered that a good “objective” classification of the illusions was crucial. The analogy I had in mind was Darwin’s theory of evolution, that was greatly facilitated by Linnaeus’ systematic classification work. We learnt from Linnaeus that the most obvious characters are not necessarily the most relevant ones to understand the relatedness between living organisms. Some hidden subtle characters may turn out to be far more relevant. Thus, a dolphin is much closer to land mammals than to fishes, and a bird is closer to a penguin or even a snake than to a flying insect. A very common attribution error in geometric illusions is related to the horizontal-vertical illusion, often illustrated with a letter T, having two segments of equal lengths, the horizontal one looking shorter than the vertical one. Here, most of the illusory effect comes from the fact that the horizontal segment is split in two due to its intersection with the vertical segment (bisection illusion – I have some psychophysical data showing this).

The more I was looking at illusions, the more they reduced to simple inequalities, such as “if there are two segments a and b in a figure with a larger than b, then the a/b ratio is perceptually increased”. Psychologists would call this a law of contrast enhancement. Curiously, I had to invoke a law of this kind mostly in the situations that were previously explained in terms of assimilation, which is nearly the opposite principle. I derived a few principles, that were “pulling in different directions”, yet could work together, and could be embodied into a coherent framework. This theoretical work was ultimately published in 1979 [4]. There were a number of quite good insights in this work, that led me to fruitful developments later, and there were a few attribution errors, now corrected in [1].

One published result attracted my attention. Bela Julesz had shown that most geometrical illusions are maintained when they are presented as random-dot stereograms, the Zöllner illusion being the only notable exception [12]. Seymour Papert had shown earlier that the Müller-Lyer illusion was maintained in camouflaged stereograms [13].Thus, it seemed, stereoscopic vision provided an objective way to classify the illusions into two groups. A first group containing most illusions would occur late in visual processing, after the stage in which the streams from the two eyes are usually combined to provide stereoscopic interpretation. The shape giving rise to the geometric illusion does not exist prior to this stage, since it is, by design, monocularly undetectable. Contradistinctively the Zöllner illusion is destroyed when camouflaged in the form of random-dot stereograms. So, it seemed, it arises rather early in visual processing, before the stage at which the streams from the two eyes are normally combined. (I proved much later that this reasoning was based upon an incorrect hidden assumption - [14]). In any event it seemed to me at that time that stereoscopic vision provided a criterion for an objective classification of geometric illusions, and I thus started to think about the geometric problems of stereoscopic vision (see the web chapter on stereo vision).

After moving to Ecole Normale Supérieure, I started doing some psychophysical measurements on geometrical illusions. I knew Robinson’s book almost by heart, and followed whatever was published in Gregory’s scientific journal “Perception”, but was quite ignorant of what was published elsewhere on the topic. My first work was on Poggendorff illusion. I attempted to measure the effect on several variants at several orientations, not knowing that Weintraub had already done an excellent job [15]. I found that the misalignment effect was essentially that which could be observed in the absence of the parallels. Fortunately, I did not publish this result, it was an artefact: I had used rather long collinear segments, and in this case they are minimally deviated when they intersect the parallels of the Poggendorff figure, so what remains is the “pure misalignment” Zehender illusion, that is observed in the absence of parallels [16].

A few years later, I happened to have frequent discussions with Kevin O’Regan, and we decided to do some psychophysical work on the Zöllner illusion. We constructed several rather unusual variants (people could have described them as “new” illusions) , and studied the strength of the illusion at eight different orientations. We concluded, rather firmly [5] that (i) the illusion occurred at the level of a single stack of oblique segments (ii) the distortion was not an apparent rotation of the stack, but could be a shear, or an expansion at right angles to the obliques (iii) that the way the segments ended was not of primary importance.

At that time, both Kevin and I were also working on memory, but independently. Kevin published his seminal paper on change blindness [17]. I was involved in precise determinations of the capacity of visual memory [18, 19]. We found nevertheless the time to pursue the work on geometrical illusions. This time we performed extensive work on the Poggendorff illusion and several of its variants, including the corner-Poggendorff variant [20]. We did not invent any new pattern, but performed rather systematic measurements, that allowed us to separate the measured illusory effect into two components [7]: a minor one, the pure misalignment illusion, and a major one, that was clearly a misangulation effect (contrary to my theory in [4]). We could also make sense of why the illusion was weak in some patterns (the corner-Poggendorff), and rather strong in others (a Weintraub variant in which a segment is not collinear with another segment, but with a dot).

Kevin then started taking responsibilities as director of the Laboratoire de Psychologie Expérimentale, and I continued the work alone. (Why, within the French context, I could not have students working with me is another story). I designed and carried out to the end several series of psychophysical measurements, along the same lines as the Poggendorff work:  many related stimuli, studied at 8 or 16 different orientations, 10 subjects, each subject going through a whole series ten times. I thus obtained results on the horizontal-vertical illusions, the square-diamond illusions, the trapezium illusions, the Müller-Lyer illusions. I also studied hybrids between Poggendorff and Zöllner.  I did not attempt yet to publish the results (but see the “unpublished results” section at the end of this chapter). However, I made some salient conclusions leak in various places.

In 2002, I attended the 25th ECVP in Glasgow, organized by Pascal Mamassian. I became acquainted there with Baingio Pinna, with whom I immediately sympathized. He showed me a collection of perhaps 50 or 100 of his yet unpublished illusory patterns, and I went through them one by one, trying to form an opinion on what could be reduced to previously known effects, and what  was really puzzling. Among the geometrical effects, I was struck by two drawings. One was a marvellous variant of the Müller-Lyer illusion, that flatly contradicted all published theories of the illusion, except mine: the illusion of the diagonal (Fig. 14-17 in [21] also shown in [1], Figure 6a). Another one represented squares, that, in the neighbourhood of oblique segments, appeared as trapeziums. I also had, among my unpublished patterns, a distorted square illusion. In both cases,  two families of inducing segments or inducing lines at nearly orthogonal orientations were simultaneously present, and seemed to have additive rather than subtractive effects. We put our distorted square patterns in common, and discussed them in the context of the tilt illusions. Looking at the whole set, plus some additional variants, designed on this occasion, we proposed that there was a common theme to most of these illusions, namely “orthogonal expansion” – a tendency for sets of  parallel or nearly parallel lines to expand at a right angle to the lines [6]. So, having eliminated in a first work the “rotation” description of the Zöllner illusion, I eliminated the “shear” description of the illusion in subsequent work, and am now convinced that “orthogonal expansion” is the key to these illusions.

In April or May 2007, I listened to a seminar given by Pierre Pica at the Laboratoire de Psychologie Expérimentale in Paris. Pierre Pica is a linguist who has been spending several months every year with an Indian tribe of Amazonia, the Mundurucus. He made many puzzling observations about the structure of their language, their counting habits [22], their fascination for symmetry, their spatial sense, their understanding of geometry [23]. I designed for him a battery of visual tests, which he embarked in his subsequent expedition in the land of the Mundurucus (dec. 2007 - July 2008). The tests were designed in such a way as to make sense for the Mundurucus, at least to the best of Pierre Pica's knowledge. For instance, there were tests on the geometrical illusions, that I embodied as tests on the appreciation of symmetry with respect to an axis, of the two figures to be compared. It seems that the subjects took pleasure in performing these tests. Their responses to the tests were in most cases (Zöllner, Poggendorff, trapezium, Müller-Lyer, Helmholtz) very similar to those in our contemporary occidental culture. There were however two differences. One difference related to the Delboeuf illusions with circles. The two stimuli were presented on the two sides of a symmetry axis, and the subjects were not sensitive to the traditional effect. This was expected, by analogy with previous observations on Titchener's circles in a Himba population [24]. It seems that the critical cultural issue is whether the subjects focus on the two parts of the stimuli as a single entity, or separate entities. Another result was quite unexpected: The Mundurucus were not subject to the square/diamond illusion. Again, the square and the diamond were presented on the two sides of a symmetry axis, a feature that normally reduces the illusion by one half [25]. This, perhaps can be related to their widespread use of diamond motifs on their body ornaments. I hope that Pierre Pica will write one day about these experiments. Although the raw data are preserved, the notebooks in which Pierre Pica was recording the circumstances of the tests were lost, so it would be hard to publish the results in a regular scientific journal. Later, my co-worker in stereo vision, Svetlana Rychkova e.g., [26], became interested in testing geometrical illusions on people with normal or impaired vision. She used slightly modified variants of the programs I had written for Pierre Pica, and her work is in progress [27].

Another encounter turned out to be quite important. Being an assiduous reader of the journal "Perception" I had noticed two very innovative illusions by Vicario, the sloping step, and the rarefaction illusions: [28, 29], see also da Pos and Zambianchi, 1996 [30] and Vicario [31] for a collection of rarely discussed illusions. A very rewarding correspondence was initiated between Vicario and I. I learnt many effects from him, thanks to his unique, encyclopaedic knowledge of geometric illusions and their history.

So, what is my theory of geometrical illusions, people will ask? Actually, my main work is not centred on why there are illusions, its main focus is an attempt to determine what is the nature of the distortion in the illusion. Typically, what is the correct way to describe the Zöllner illusion: Is it a rotation of the stacks, a relative sliding of the parallel segments (shear distorsion), an increased separation between the segments (orthogonal expansion)? Once the nature of the distortion is correctly described, one can then try to find a rationale for the distortions. The explanation may be purely mechanistic, such as: inhibition between neurons having receptive fields tuned to similar orientations. I do not have the ambition to defend a particular neuronal model. It is enough to know that, in principle, there is at least one neuronal model that may account for the observations. Whatever it is, there is then hope that a more appropriate one will be found later. The explanation may also be a teleological one: there is some purpose for the observed effects. Here, my general idea is that the brain constructs some representation of shapes in which it incorporates geometrical relationships derived from visual processing, and it has to satisfy a number of constraints. The main problem, I propose, which is at the origin of geometrical illusions is the conflict between sizes or orientations, as may be derived when observing a scene with a fixed eye (which then obey the laws of linear perspective) and the sizes and orientations which can be acquired as the eyes move and explore a scene, in which case apparent (angular) sizes dominate (which then follow the laws of curvilinear perspective). How this may lead to Müller-Lyer, and so many other effects is a bit technical, but should become clear at the end. Here also, I am satisfied with the fact that there is at least one plausible teleological theory for at least one large class of illusions.

## 2. THEORETICAL WORK ON GEOMETRICAL ILLUSIONS [1, 4]

### 2.1 The convexity principle

In 1975, I spent months and months scrutinizing the illusions in Robinson's book [11], trying to extract the basic geometrical rules behind these. The book often showed several variants of each illusion, and this protected me from expeditous erroneous generalizations. For instance, people who encounter the Müller-Lyer illusion for the first time are seized with amazement, then they say "of course, this illusion is due to the arrows". It was very clear, from the examples in Robinson's book, that the arrows in the Müller-Lyer pattern may be replaced by squares, circles, or by almost any shape without damage to the illusory effect.

My main working assumption was that the brain was constructing a representation of a scene, in the same spirit with which a geometer constructs a geographical map: Measurements are taken from one landmark to another, with more or less reliable instruments, according to well-defined procedures, then the measurements are combined according to well-defined rules. The resulting representation may appear distorted but the distortions are not necessarily due to carelessness. These are due to fundamental constraints. In the case of geographical maps, there is the constraint of representing a portion of spherical surface on a planar one. I considered that the brain also had to construct a kind of map, combining distance and orientation measurements taken with its neuronal instruments. One of the instruments would provide measurements that were biased in a systematic way, the brain would combine the measurements and apply corrective factors. In the end, a rather satisfactory representation would be constructed, except in a few cases.

I focused first on extent illusions and found several geometrical themes, which turned out to be related. Imagine that the measure of an extent x is a function m(x). Many illusions can be accounted for by a rule of contrast: There are two segments of lengths a and b, with b larger than a, and the figure is perceptually distorted in such a way that the contrast between b and a is increased, which can be spelled out as:

m(b) / m(a) is larger than b / a if b is larger then a (1)

An assimilation effect would be just the opposite, m(b) / m(a) would be closer to one than b / a.

If you add the obvious property that m(0) = 0 (the measure of a null extent is zero), then relation (1) is true for any convex function m(x) going through the origin – a function which instead of being linear, rises with a curvature of constant sign (See Fig. 1 in the illustration supplement, Figure 5a in [1] or Figure 7 in [4]).

This rule of contrast, or "convexity" applies in an obvious way to patterns such as the "contrast illusion" with arcs shown in Fig. 2 of the illustration supplement, which is not a "best-selling" illusions. As an application of the principle, I designed the original pattern with four circles in Fig. 9 of the illustration supplement. Many more illusions, discussed in [4] seem to support the convexity principle.

The convexity principle also applies to an apparently unrelated illusion, the bisection illusion (Fig. 12 in the illustration supplement). Here, you can admire the power of mathematics, because if Eq. (1) is true and $m(0) = 0$, then:

$$m(a+b) > m(a) + m(b) \qquad (2)$$

And in particular, $m(2a) > 2m(a)$ (2a).

Actually, there are two sides to the bisection illusion. The known part of it is that the subdivided segment looks smaller than the undivided one. The hidden part of the illusion is that, paradoxically, an isolated half segment looks smaller than half the subdivided segment (Fig. 12 in the illustration supplement and Figure 5b in [1]). According to my analysis, the real illusion, expressed by Eq. (2a) is the fact that a full undivided segment looks more than twice as long as an isolated half. However, this is not perceived as an illusion. See again Fig. 12.

I have surreptitiously used here a subsidiary principle – a principle of compromise: If measurements are in conflict, then a compromise is made. The Müller-Lyer illusion can be analysed in terms of convexity + compromise principles (See Fig. 6 - 8 in the illustration supplement).

Here is a metaphor for the convexity principle, published in [1]:

*Assume that you are on the sea front, and you wish to represent the layout of a number of floating targets. Your only instrument is a chronometer. You measure the time it takes to swim from one target to the other. When two targets are close, one can swim rapidly from one to the other. When the targets are distant, one swims less rapidly, and the swimming speed diminishes as the targets become more and more distant. Thus, the measured time to connect two targets grows more than proportionately with the distance between targets. This time, provided by the chronometer, overestimates large distances with respect to small ones. In psychological language, it increases the contrast between large and small. In mathematical language, the measure is a convex one. The relationship between an extent x and its measured value m(x) can be represented by a parabola, or any curve starting at the origin, and rising with a curvature of constant positive sign.*

Recently, Baingio Pinna produced the “illusion of the diagonal”, which I think is an excellent illustration of the convexity principle (See Fig. 4 in the illustration supplement, and Figure 6 in [1]). By adding an arrowhead to the diagonal, one obtains a “reversed Müller-Lyer” effect – a figure in which a shaft with an ingoing finn appears larger than expected. The convexity principle accounts in one stroke for the standard Müller-Lyer and the reverse Pinna variant, as it explained in one stroke the standard and the reverse bisection illusion. Kennedy, Orbach and Löffler [32] recently produced an illusion with triangles (See Fig. 11 in the illustration suplement and Figure 5g in [1]) which they christened “isocele/scalene triangle illusion”. In these figures, I explain with a geometrical construction how this effect can be deduced from the convexity principle. The “gravity lens illusion” by Naito and Cole [33] also seems to me to be predictable by the convexity principle (Fig. 7 in the illustration supplement and Figure 5j in [1]).

In order to illustrate the convexity principle, I designed a paradoxical figure with squares and diamonds, showing that the convexity effect is stronger than the square-diamond illusion. It is reproduced here in Fig. 3 of the illustration supplement. A simplified version is shown in Fig. 5 of the supplement. The effect is related to Pinna's diagonal illusion (see [1]).

So, the convexity principle seems to be at work in many apparently unrelated illusory patterns. It allows one to construct patterns that seem to contradict well-known effects, including the bisection illusion, the square-diamond illusion, or the Müller-Lyer illusion.

In my early work [4], I used the convexity principle to explain the Poggendorff illusion, but I am now convinced that the explanation was incorrect (see section on Poggendorff below).

While a large number of illusions were described in terms of a convexity rule, a number of other illusions seemed to go in the opposite direction, and these required two subsidiary principles: A principle about the effect of subdividing a figure, and a principle about the space occupied by the figure. Both acted as corrective terms to the convexity principle. I shall examine the two subsidiary principles in turn.

2.2 The subdivision rule.

If we follow the logic of the convexity principle, and partition a figure in 2, 3, 4 or more parts, the higher the number of subdivisions, the more it should appear contracted. Actually the effect is clearly observed only for n = 2 (bisection illusion). Starting with n = 4 we have clearly the opposite trend, an expansion of subdivided figure, as observed for instance in Helmholtz squares illusions. So, I introduced a "subdivision principle" according to which there is an expansion effect on subdivided figures, the magnitude of the effect depending upon the number of subdivisions. Combining the convexity with the subdivision principle is mathematically possible. One can get both an overall contraction effect for n = 2 and an overall expansion effect for n > 3, as I showed in [4]. I was recently led to re-examine this principle, in the light of the Zöllner and other tilt illusions [6], as will be explained later. See also Fig. 14 in the illustration supplement or Figure 7 in [1].

2.3 The space occupation rule.

A number of geometric illusions(including Titchener's circles and the Ponzo illusion) clearly could not be accounted for by a combination of the convexity and the subdivision principles. I needed to add a "space-occupation" rule according to which a size normalization factor is applied to all figures. The large ones are perceptually reduced, and the small ones are perceptually enlarged. In this way, illusions that are classically explained by a contrast effect are reinterpreted in terms of an almost opposite principle. Fershad Nemati [34] was aware of that, and proposed a principle of expansion when there was empty space around a figure.

Here, my ideas have evolved considerably. For recent, precise formulations, see [1].

**3. UNPUBLISHED WORK**

I have several unpublished results on orientation profiles (Figures 16-23 in the illustration supplement).

In one study, I compared the Zöllner illusion with or without explicit axes along the stacks. The measured effect is larger in the case of explicit axes (Figure 16 of the illustration supplement). 17 subjects took part in the experiment, and each data point represents the average over 170 measurements. In all other studies, reported below, there were 10 subjects, and 10 measurements per subject for each data point.

In another study, I determined orientation profiles in variants of the square-diamond illusion (Figures 17-19 of the illustration supplement). The square-diamond illusion is usually presented with one apex of the diamond pointing towards the square. I found that when the figures were displayed more symmetrically the illusion was significantly reduced. Furthermore, it is surpassed, for all subjects, by an illusion that goes in the opposite direction, in which the diagonal of a small diamond is underestimated with respect to the side of a larger square. The results were presented in a talk at ECVP 2011 (Toulouse), and reported in the corresponding abstract [25].

I also determined orientation profiles for variants of the trapezium illusion (Figures 20-22 of the illustration supplement). The trapezium illusion was maximal when the bases of the trapeziums were horizontal, and minimal when they were vertical. The oblique sides, but not the bases, were essential to the illusion, suggesting the existence of a common component between the trapezium and the Zöllner illusion. The study is made somewhat difficult by the fact that figures with trapeziums often lead to interpretations in perspective that perturb the comparison of trapeziums as flat figures. One philosophically important side-result of the study is that two trapeziums in the standard configuration can never be altered in such a way as to be seen equal! When you try to equalize (by a nulling procedure) the two large bases and the orientations of the two sides, the small bases look unequal, and when you try equalize the two small bases and the orientations of the sides, then the two large bases look unequal! It is thus impossible to draw two trapeziums, one above the other, so that they would look identical! The results were presented in a talk at ECVP 2011 (Toulouse), and reported in the corresponding abstract [25].

The most important result, in my opinion, is that obtained on hybrid Zoellner-Poggendorff patterns (Figure 23 of the illustration supplement). It clearly rules out the “shear” hypothesis for Zöllner, and it is clearly favourable to the “orthogonal expansion” interpretation.

My experiments on orientation profiles with Müller-Lyer patterns were frustrating. I studied stimuli containing Müller-Lyer patterns and visually related stimuli, including the receding arrow illusion (Figure 3h in [1]), and Judd's bisected arrow illusion (Figure 1c in [1]). Unfortunately, my orientation profile experiments failed to show a relationship between the Müller-Lyer, the Judd and the receding arrow illusions. The results with Müller-Lyer patterns were erratic. They were strongly subject-dependent, there was no simplifying symmetry when the patterns were turned upside down, etc. My provisional, not too satisfactory, explanation is that a subject may compare the lengths of the segments between the fins according to various criteria, (for instance, forming a virtual rectangle with a pair of segments, looking at orientations, etc.) and the criterion he/she chooses depends upon the orientation of the stimulus.

## 4. ABSTRACTS OF PUBLISHED WORK.

### An algorithm that generates a large number of geometric visual illusions. Ninio, J. (1979) Journal of Theoretical Biology 167-201.

ABSTRACT

*An algorithm is described which, starting with any geometrical figure, constructs a representation in which the deviations from the model coincide with the known perceptual distortions. First, the algorithm specifies a measurement process: drawing*

*a straight line across the figure and measuring the encountered segments, assigning to every segment of length x its measure m(x). Next, the measures taken along a line D are corrected with a normalizing factor N(D) which is a function of the measures made on this line. Finally, a representation of the analysed figure is constructed, using for every segment its normalised length n(x)=m(x).N(D), instead of its actual length.*

*It is first established that within this general framework, a large number of illusions can be immediately predicted by specifying a property of the measure or of the norm. Only four properties are required to justify most illusions. They are (1) and (2) m(x) and n(x) must be convex functions (3) the norm must increase with the number of segments measured on a line (4) the norm must decrease when the average segment on a line increases in length. It is then shown that the four requirements, conflicting as they may be in some circumstances, can nevertheless be condensed into one single expression of n(x). This simple formula predicts a large number of widely different illusions (Delboeuf, Titchener, Ponzo, trapeze, Müller-Lyer, flattening of short arcs, etc). It permits to predict new illusions and new effects in old illusions, but fails to predict the Zöllner illusion, and the reversal of the Müller-Lyer illusion when the outgoing finns are becoming very large.*

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**The half - Zöllner illusion**

**Ninio, J. and O'Regan, J.K. (1996) Perception 25, 77-94.**

ABSTRACT

*The Zöllner figure contains stacks of short parallel segments oriented obliquely to the direction of the stack. Adjacent parallel stacks of opposite polarity seem to diverge where their top segments form an arrowhead. To probe whether or not the opposite polarities are necessary to the illusion, three 'half-Zöllner' configurations were designed, containing stacks of a single polarity. The 'orientation profile' of these configurations was studied, that is, the way the strength of the perceived illusion varies with the orientation of the stacks. The subjects had to align two stacks or align stacks with target segments situated at a slight distance from them. All three half-Zöllner configurations produced errors that could be assimilated to global-orientation misjudgments. These errors were of opposite sign for the two types of stacks and varied with the orientation of the stacks as in the standard Zöllner illusion.*

*A further study was conducted in which the effects of several configurational parameters was explored for a single observer. The standard Zöllner illusion increases with the separation of the stacks. The illusion is also increased when the orientations of the segments in different stacks are orthogonal, independently of the particular longitudinal orientations of the stacks.*

*When the ends of the short segments are curved so that at their endpoints they become precisely perpendicular to the axes of the stacks, the standard and half-Zöllner illusions are reduced, but not abolished. Therefore, they cannot be entirely accounted for by a mechanism of alignment of illusory contours generated at these endpoints.*

*The results are consistent with the existence of a single common mechanism at work in both the standard and the half-Zöllner illusion. It is suggested that the illusion itself is not a rotation of the stacks but either a shear deformation in which the segments of the stack slide with respect to one another, or an expansion of the stacks orthogonally to the segments.*

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**Characterization of the misalignment and misangulation components in the Poggendorff and corner-Poggendorff illusions.**

**Ninio, J. and O’Regan, J. K. (1999) Perception 28, 949-964.**

ABSTRACT

*In the Poggendorff illusion, two colinear segments abutting obliquely on an intervening configuration (often consisting of two long parallel lines) appear misaligned. We report here the results of a component analysis of the illusion and several of its variants, including in particular the "corner Poggendorff" illusion, and variants with a single arm. Using a nulling method, we determined an "orientation profile" of each configuration, that is, how the illusions varied as the configuration was rotated in the plane of the display. We were able to characterize a pure misalignment component (having peaks and dips around the ±22.5 degree and ±67.5 degree orientations of the arms) and a pure misangulation component of constant sign, having peaks at the ±45 orientations of the arms. Both these components were present in both the classic and the corner Poggendorff configurations. Thus, the misangulation component appears clearly in the classic Poggendorff illusion, once the misalignment component is partitioned out. Similarly, the corner Poggendorff configuration, which essentially estimates a misangulation component, contains a misalignment component which becomes apparent once the misangulation is nulled. While our analysis accounts for much of the variability in the shapes of the profiles, additional assumptions must be made to explain the relatively small misangulation measured in the corner-Poggendorff configuration (1.5 degrees, on average, at peak value), and the relatively large illusion measured in the configurations with a single arm (above 6 degrees, on average, at peak values). We invoke the notion that parallelism and colinearity detectors provide counteracting cues, the first class reducing misangulation in the corner-Poggendorff configuration, and the second class reducing the illusion in the Poggendorff configurations with two arms.*

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**Orthogonal expansion: a neglected factor in tilt illusions.**

**Ninio, J. and Pinna, B. (2006) Psychologia 49, 23-37.**

ABSTRACT

*A broad collection of illusions belonging to the Zöllner and the Poggendorff families, including new variants - in particular, tilted and tilting squares - are examined in the light of two possible formal principles : a principle of regression to right angles (RRA) and a principle of "orthogonal expansion", which is a perceptual expansion of the extent perpendicularly to the inducing lines. The domains of validity of the two principles are compared. We propose that RRA is more pertinent when the target line is explicitely present and makes real intersections with the inducing lines. Orthogonal expansion can produce RRA as a side-effect. It would be more pertinent when there are several parallel or nearly parallel inducing lines, and it does not require the presence of a real target. Both principles may be grounded on neurophysiological mechanisms. Orientation detectors would influence each other in the orientation domain, generating RRA and accounting for the illusions of the Poggendorff family. They would also influence each other in the extent domain, generating orthogonal expansion, and accounting for the illusions of the Zöllner family.*

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**Orientation profiles of the trapezium and the square-diamond geometrical illusions. Ninio, J. (2011) Perception 40 supplement, 45**
ABSTRACT of talk presented at ECVP Toulouse, 2011
*In previous work, "orientation profiles" (describing how the strength of an illusion varies with its orientation in the plane) were determined for several variants of the Zöllner and the Poggendorff illusions (e.g., Ninio and O'Regan, 1999, Perception, 28(8), 949-964). The study is extended here to two other classical illusions. Illusion strengths were determined for 10 subjects at 16 orientations on 4 variants of the trapezium illusion and 8 variants of the square-diamond illusion. The trapezium illusion was maximal when the bases of the trapeziums were horizontal, and minimal when they were vertical. The oblique sides, but not the bases, were essential to the illusion, suggesting the existence of a common component between the trapezium and the Zöllner illusion. The square-diamond illusion is usually presented with one apex of the diamond pointing towards the square. I found that when the figures were displayed more symmetrically, the illusion was reduced by one half. Furthermore, it is surpassed, for all subjects, by an illusion that goes in the opposite direction, in which the diagonal of a small diamond is underestimated with respect to the side of a larger square.*

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**Geometrical illusions are not always where you think they are: a review of some classical and less clasical illusions, and ways to describe them.**
**Ninio, J. (2014) Frontiers in Human Neurosciences volume 8, article 856, 21 pages.**

*Geometrical illusions are known through a small core of classical illusions that were discovered in the second half of the 19th century. Most experimental studies and most theoretical discussions revolve around this core of illusions, as though all other illusions were obvious variants of these. Yet, many illusions, mostly described by German authors at the same time or at the beginning of the 20th century have been forgotten and are awaiting their rehabilitation. Recently, several new illusions were discovered, mainly by Italian authors, and they do not seem to take place into any current classification.*

*Among the principles that are invoked to explain the illusions, there are principles relating to the metric aspects (contrast, assimilation, shrinkage, expansion, attraction of parallels) principles relating to orientations (regression to right angles, orthogonal expansion) or, more recently, to gestalt effects.*

*Here, metric effects are discussed within a measurement framework, in which the geometric illusions are the outcome of a measurement process. There would be a main "convexity" bias in the measures: the measured value m(x) of an extant x would grow more than proportionally with x. This convexity principle, completed by a principle of compromise for conflicting measures can replace, for a large number of patterns, both the assimilation and the contrast effects.*

*We know from evolutionary theory that the most pertinent classification criteria may not be the most salient ones (e.g., a dolphin is not a fish). In order to obtain an objective classification of illusions, I initiated with Kevin O'Regan systematic work on "orientation profiles" (describing how the strength of an illusion varies with its orientation in the plane). We showed first that the Zöllner illusion already exists at the level of single stacks, and that it does not amount to a rotation of the stacks. Later work suggested that it is best described by an 'orthogonal expansion' — an* *expansion of the stacks applied orthogonally to the oblique segments of the stacks, generating an apparent rotation effect. We showed that the Poggendorff illusion was mainly a misangulation effect. We explained the hierarchy of the illusion magnitudes found among variants of the Poggendorff illusion by the existence of control devices that counteract the loss of parallelism or the loss of collinearity produced by the biased measurements. I then studied the trapezium illusion. The oblique sides, but not the bases, were essential to the trapezium illusion, suggesting the existence of a common component between the trapezium and the Zöllner illusion. Unexpectedly, the trapeziums sometimes appeared as twisted surfaces in 3d. It also appeared impossible, using a nulling procedure, to make all corresponding sides of two trapeziums simultaneously equal. The square-diamond illusion is usually presented with one apex of the diamond pointing towards the square. I found that when the figures were displayed more symmetrically, the illusion was significantly reduced. Furthermore, it is surpassed, for all subjects, by an illusion that goes in the opposite direction, in which the diagonal of a small diamond is underestimated with respect to the side of a larger square. In general, the experimental work generated many unexpected results. Each illusory stimulus was compared to a number of control variants, and often, I measured larger distortions in a variant than in the standard stimulus.*

*In the Discussion, I will stress what I think are the main ordering principle in the metric and the orientation domains for illusory patterns. The convexity bias principle and the orthogonal expansion principles help to establish unsuspected links between apparently unrelated stimuli, and reduce their apparently extreme heterogeneity. However, a number of illusions (e.g., those of the twisted cord family, or the Poggendorff illusions) remain unpredicted by the above principles. Finally, I will develop the idea that the brain is constructing several representations, and the one that is commonly used for the purpose of shape perception generates distortions inasmuch as it must satisfy a number of conflicting constraints, such as the constraint of producing a stable shape despite the changing perspectives produced by eye movements.*

**5. CITED REFERENCES**


[1] Ninio, J. (2014) Geometrical illusions are not always where you think they are: a review of some classical and less classical illusions, and ways to describe them. Frontiers in Human Neusosciences volume 8, article 856, 1-21. doi: 10.3389/fnhum.2014.00856

[2] Ninio, J. (1998) La science des illusions. Odile Jacob, Paris. German translation Nacht Schwarz Schlank? Gustav Kiepenhauer Verlag, Leipzig (1999) and Büchergilde Gutenberg, Frankfurt (2000). Greek translation E epistimi ton psevdaithiseon, Katoptro, Athenes. (2001). English translation The science of illusions. Cornell University Press, Ithaca (2002). Japanese translation, Shin'Yosha, Tokyo (2004).

[3] Ninio, J. (2011, last edition) L'empreinte des sens. Odile Jacob, Paris.

[4] Ninio, J. (1979) An algorithm that generates a large number of geometric visual illusions. Journal of Theoretical Biology 167-201.

[5] Ninio, J., and O’Regan, J. K. (1996) The half - Zöllner illusion. Perception 25, 77-94.

[6] Ninio, J. and Pinna, B. (2006) Orthogonal expansion: a neglected factctor in visual illusions. Psychologia 49, 23-37.

[7] Ninio, J. and O'Regan, J.K. (1999) Characterization of the misalignment and misangulation components in the Poggendorff and corner-Poggendorff illusions. Perception 28, 949-964.
[8] Ninio, J. (1974) . A semi-quantitative treatment of missense and nonsense suppression in the strA and ram ribosomal mutants of Escherichia coli . Evaluation of some molecular parameters of translation in vivo. Journal of Molecular Biology 84, 297-313.
[9] Ninio, J. (1975) Kinetic amplification of enzyme discrimination. Biochimie 57, 587-595.
[10] Gregory, R. (1970) The Intelligent Eye. Weidenfeld and Nicholson, London.
[11] Robinson, J.O. (1972) The psychology of visual illusion. Hutchinson University Library, London.
[12] Julesz, B. (1972) Foundations of Cyclopean Perception. Chicago University Press, Chicago, Illinois, USA.
[13] Papert, S. (1961) Centrally produced geometrical illusions. Nature 191, 733.
[14] Ninio, J. and Herbomel, P. (1990) Stereoscopic study of the Zöllner illusion. Perception 19, [ECVP abstract issue], 362.
[15] Weintraub, D. J., Krantz, D. H., and Olson, T. P. (1980) The Poggendorff illusion: consider all the angles. Journal of Experimental Psychology- Human Perception and Performance 6, 718-725
[16] Zehender, W. V. (1899) Über geometrisch-optische Taüschungen. Zeitschrift für die Psychologie 20, 65-117.
[17] Rensink, R.A., O'Regan, J.K. and Clark, J.J. (1997) To see or not to see: The need for attention to perceive changes in scenes. Psychological Science 8, 368-373.
[18] Brunel, N. and Ninio, J. (1997) Time to detect the difference between two images presented side by side. Cognitive Brain Research 5, 273-282.
[19] Ninio, J. (1998) Acquisition of shape information in working memory, as a function of viewing time and number of consecutive images : evidence for a succession of discrete storage classes. Cognitive Brain Research 7, 57-69.
[20] Greene, E. (1988) The corner Poggendorff. Perception 17, 65-70.
[21] Pinna, B. (2003) Riflessione fenomenologiche sulla percezione della qualità emergenti: verso una riconsiderazione critica della Teoria della Pregnanza. Annali della Facoltà di Lingue e Litterature Straniere, 3, 211-256 (Fig. 14-16).
[22] Pica, P., Lemer, C., Izard, V. and Dehaene, S. (2004) Exact and approximate arithmetic in an Amazonian indigene group 306, 499-503.
[23] Dehaene, S., Izard, V., Pica, P. and Spelke, E. (2006) Core knowledge of geometry in an amazonian indigene group. Science 311, 381-384.
[24] de Fockert, J., Davidoff, J., Fagot, J., Parron, C. and Goldstein, J. (2007) Accurate size contrast judgements in the Ebbinghaus illusion by a remote culture. Journal of Experimental Psychology: Human Perception and Performance 33, 738-742.
[25] Ninio, J. (2011) Orientation profiles of the trapezium and the square-diamond geometrical illusions. Perception 40 Supplement, 45.
[26] Rychkova, S., and Ninio, J. (2011) Alternation frequency thresholds for stereopsis as a technique for exploring stereoscopic difficulties. i-Perception 2, 50-68.
[27] Rychkova, S., and Bolshakov, A. (2013) Zollner and Poggendorff illusions in children with ophthalmology. Perception 42 supplement, 70.

[28] Vicario, G.B. (1972) Phenomenal rarefaction and visual acuity under 'illusory' conditions. Perception 1, 475-482.
[29] Vicario, G.B. (1978) Another optical-geometrical illusion. Perception 7, 225-228.
[30] Da Pos, O. and Zambianchi, E, eds. (1996) Visual illusions and effects - A collection. Guerini Studio, Milan.
[31] Vicario, G. B. (2011) Illusioni ottico-geometriche. Una rassegna di problemi. Venezia: Istituto veneto di scienze, lettere ed arti.
[32] Kennedy, G.J., Orbach, H.S. and Löffler, G. (2008) Global shape versus local feature: An angle illusion. Vision Research 48, 1281-1289.
[33] Naito, S. and Cole, J.B. (1994) The gravity lens illusion and its mathematical model. In Contributions to Mathematical Psychology, Psychometrics and Methodology (G.H. Fischer abd D. Laming, Eds). Springer-Verlag, New-York, pp. 39-50.
[34] Nemati, F. (2009) Size and direction of distortion in geometric-optical illusions: conciliation between the Müller-Lyer and Titchener configuratons. Perception 38, 1585-1600.

# ILLUSTRATION SUPPLEMENT

**To the chapter**
**"Contributions to geometric visual illusions"**
**in J. Ninio' web site:**
**http://www.lps.ens.fr/~ninio**

**[document to be assembled in the "portrait"**

**orientation, with stapples on the long sides]**

Convexity rules:

$m(a+b) > m(a) + m(b)$

if $b > a$ then $m(b)/m(a) > b/a$

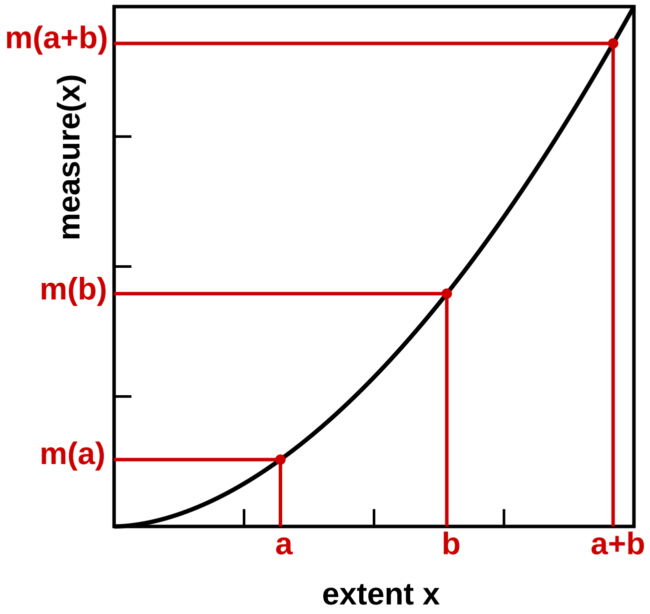



Fig. 1. Convexity effects. The measure of x $y = m(x)$ in ordinate, increases more than linearly with x. Taking two values of x, a and b such that $b > a$, the existence of a convex relationship between m(x) and x implies that $m(b)/m(a) > b/a$.

It also implies that $m(a+b) > m(b) + m(a)$.

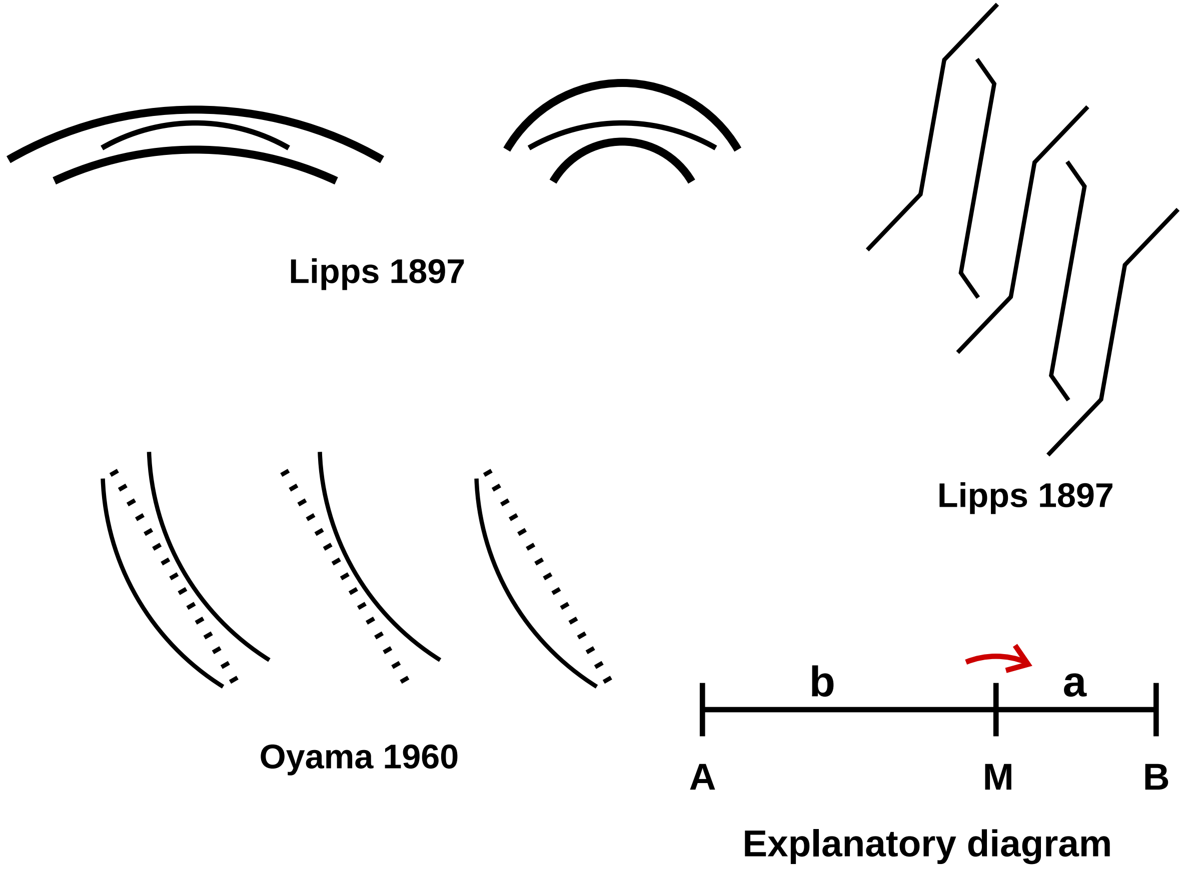
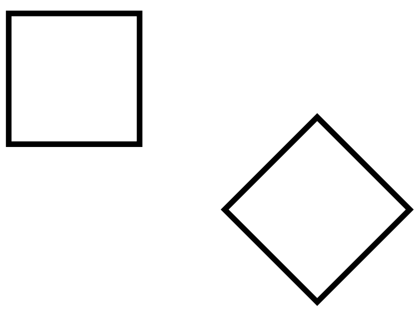
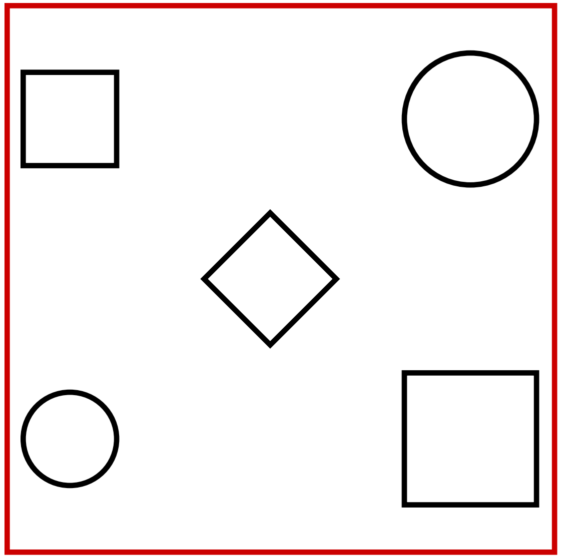



Fig. 2. Illusions explained by $m(b)/m(a) > b/a$ Consider the endpoints of the intermediate arcs, or the endpoints of the small zigzags in the Lipps figures. According to the explanatory diagram an endpoint M is perceptually displaced towards the nearest neighbouring line.



The square-diamond illusion



Ninio "The science of illusions"
1998-2004

Fig. 3. The side of the big square, bottom right is equal to the diagonal of the central square yet it appears larger, in agreement with the convexity rule m(b)/m(a) > b/a when b>a. The opposite would have been predicted, on the basis of the square-diamond illusion (top).

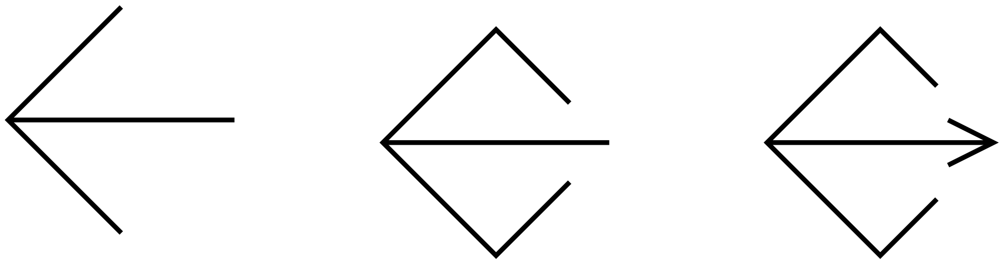
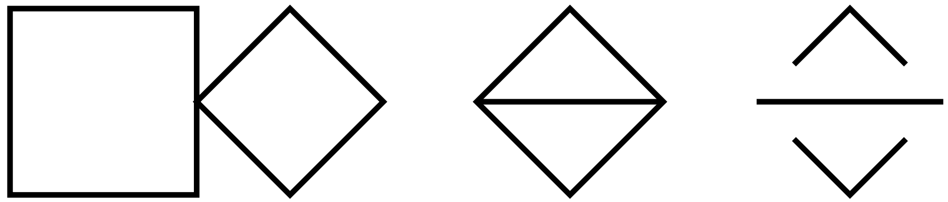
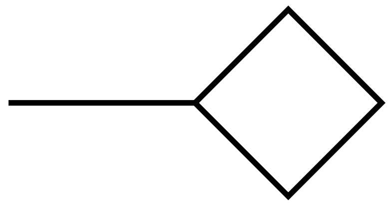


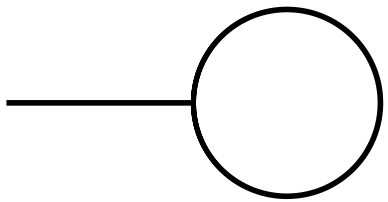
Fig. 4. Pinna's diagonal illusion (2003) Again, an illustration of the convexity rule. When the diagonals are free to expand, they are perceptually enlarged with respect to the sides.



Ninio, Aalen talk 2009



Schumann (1900)



Titchener (1901)

Fig. 5. Further variations on the theme of sides and diagonals.

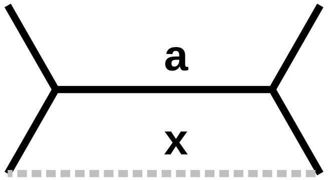



Fig. 6. Muller-Lyer illusion. According to the convexity principle, the "real" illusion is in the perceptual enlargement of x with respect to y: m(x)/m(y) > x/y. By construction, a = b but a looks larger than b as a side-effect of consistency, which works in the direction of "assimilation".

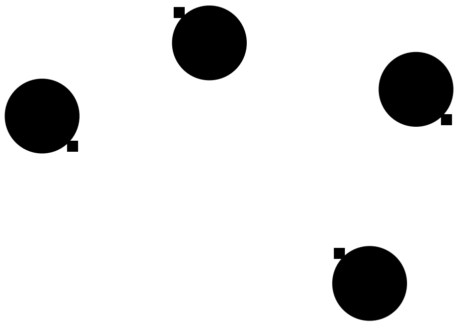
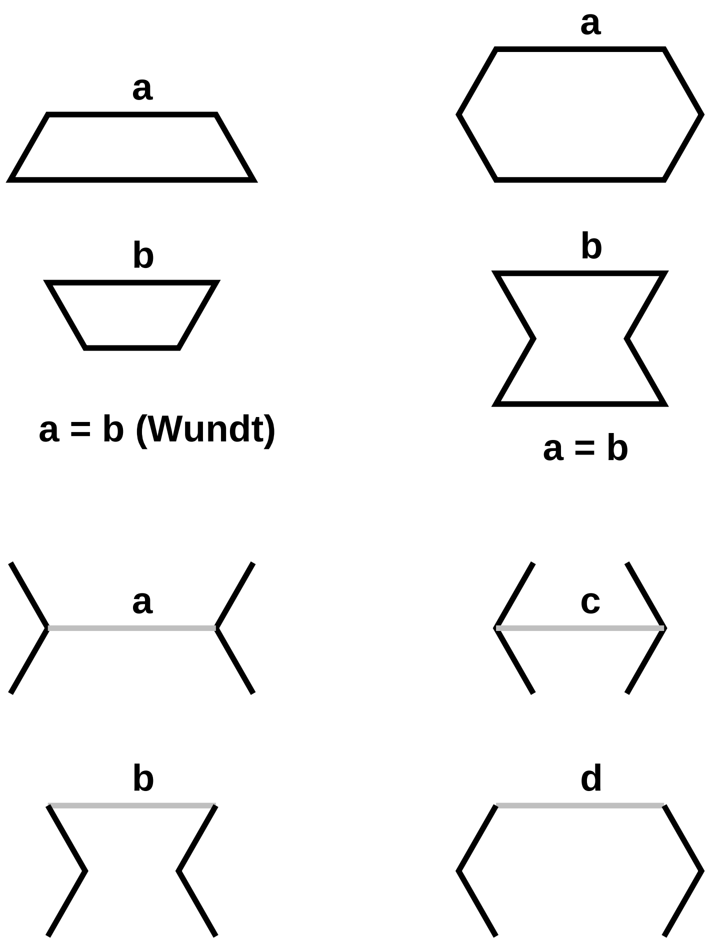
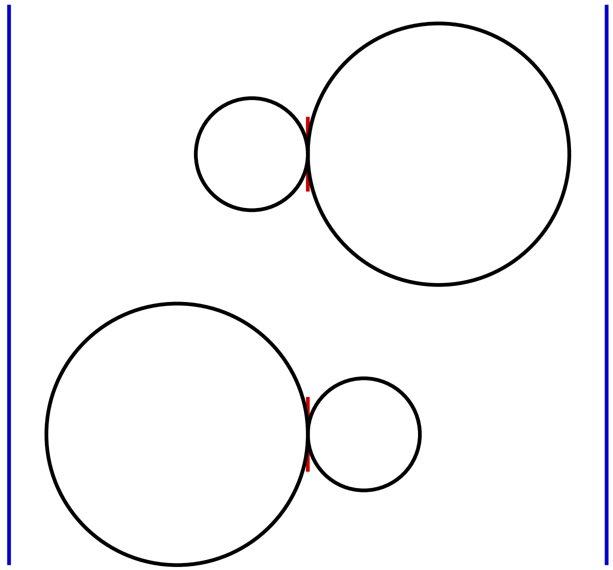


Fig. 7. The gravity lens illusion, by Naito and Cole, 1994. The 4 small squares form the the apexes of a parallelogram. It seems to be a variation on the theme of the Muller-Lyer illusion: equal segments shink or expand according to the proximity of smaller or larger neighbouring segments.

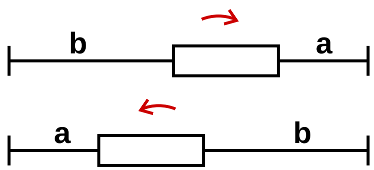



a = b = c = d

Fig. 8. More variations on the Muller-Lyer theme



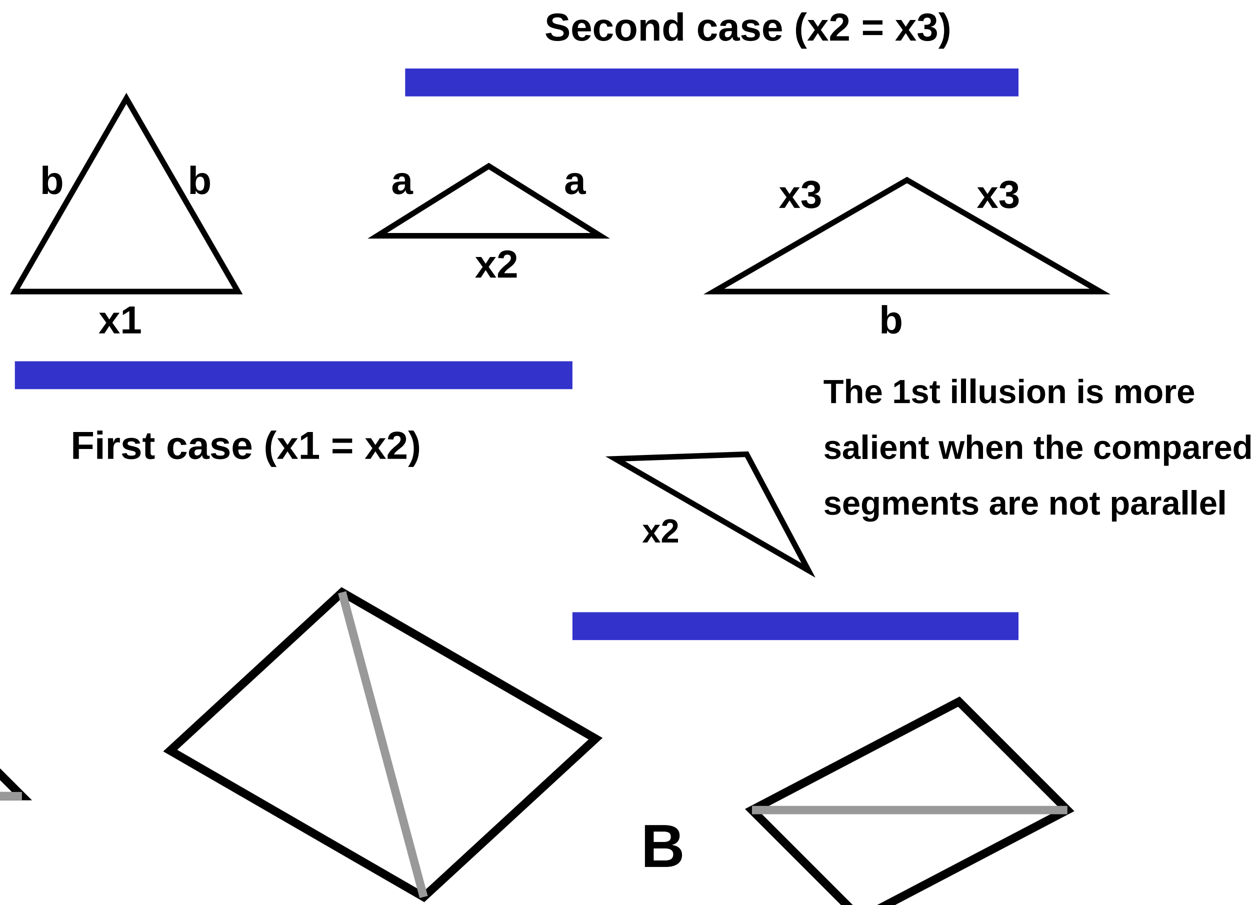
Ninio, 1979

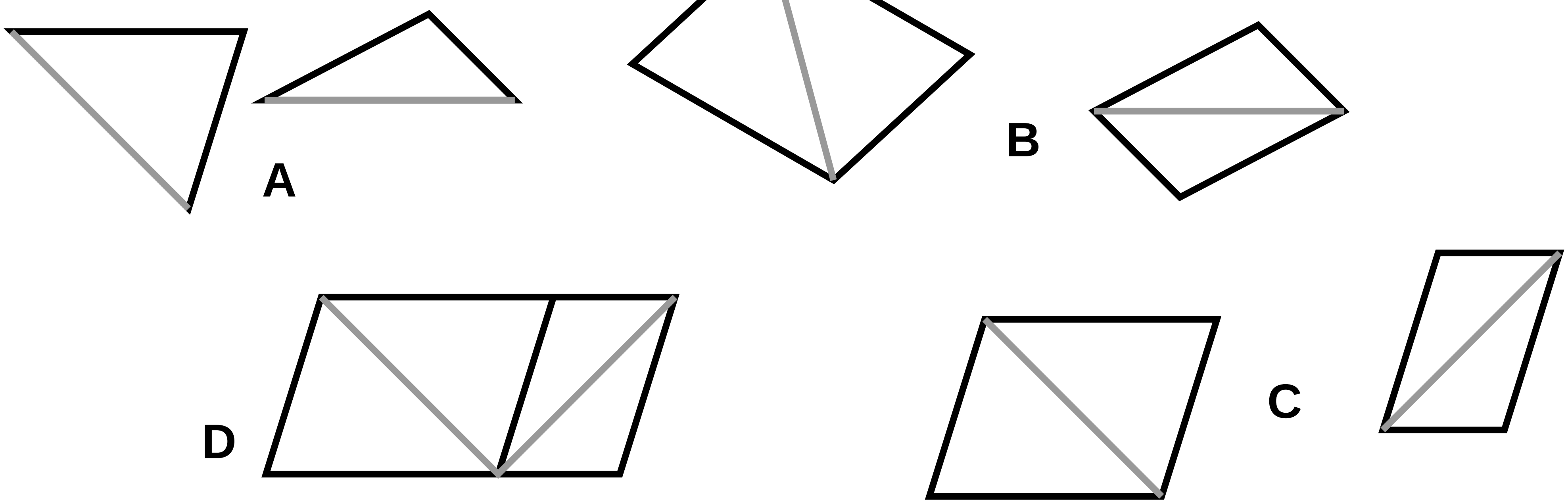


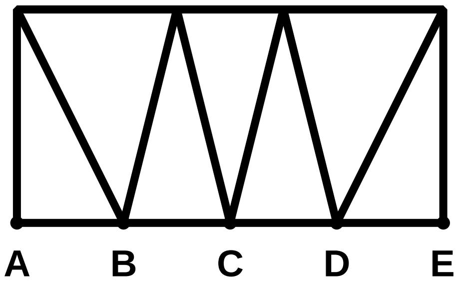

Explanatory diagram

**Fig. 9. Attraction to the borders effect. The rectangles in the explanatory diagram represent two adjoining circles. The perceptual displacements to the left or to the right are predicted by a convexity rule, m(b)/m(a) > b/a, if measurements are made with respect to virtual borders.**

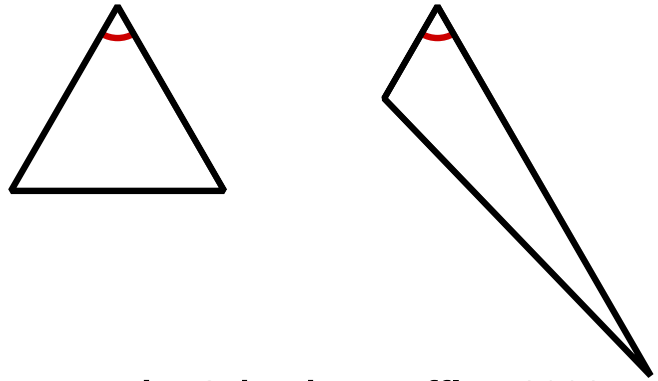
Fig. 10. Illusions with triangles.
There are two prototypical cases,
shown on the right.
in both examples b > a
therefore m(b)/m(a) > b/a
then x2 is perceived as < x1
in the first case
and x2 is perceived as < x3
in the second case.
Sander's parallelogram (below)
can be explained by a triangle
illusion of the first type.
(Follow the sequence A, B, C, D.)
All grey segments are equal.



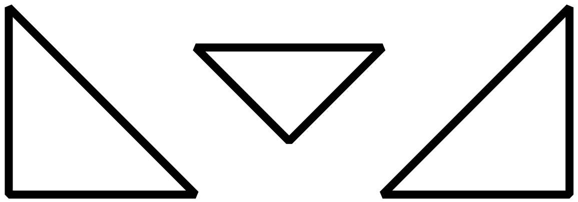





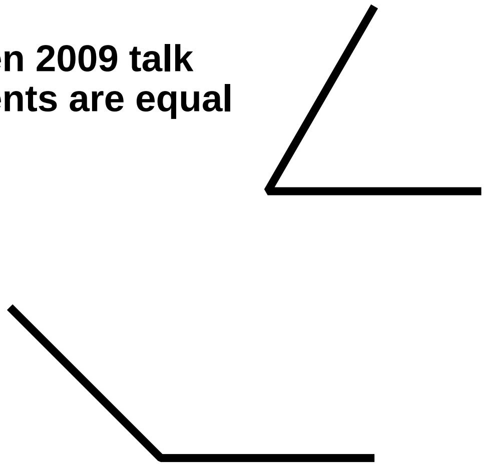

AB = BC = CD = DE
Dr Fee, 1888, in "The science of illusions"



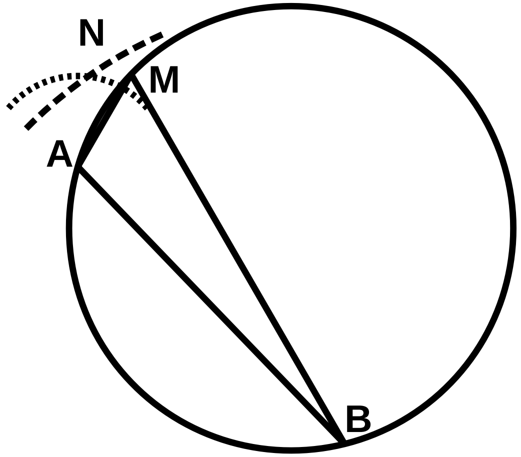
Kennedy, Orbach & Loffler, 2008
The left angle appears larger
than the right one.



Simplified pattern, Aalen 2009 talk
The 3 horizontal segments are equal



Laska 1890
The sides of the obtuse
angle appear larger
than those of the acute angle.

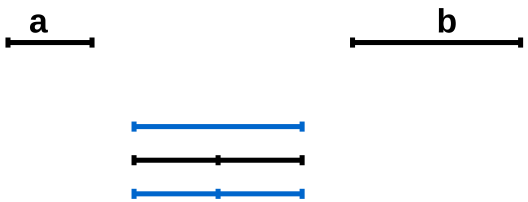



Explanation of the K-O-L illusion.
If the short side is made shorter
and the long side made longer
M should be replaced by N, and
the angle ANB is smaller than AMB
(classical geometry).

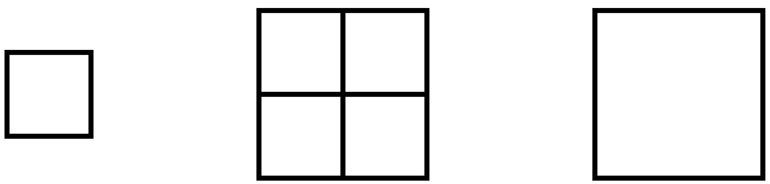
**Fig. 11. Further illusions with triangles.**


Standard bisection illusion.
b = 2c but the divided segment
appears smaller than the undivided one.

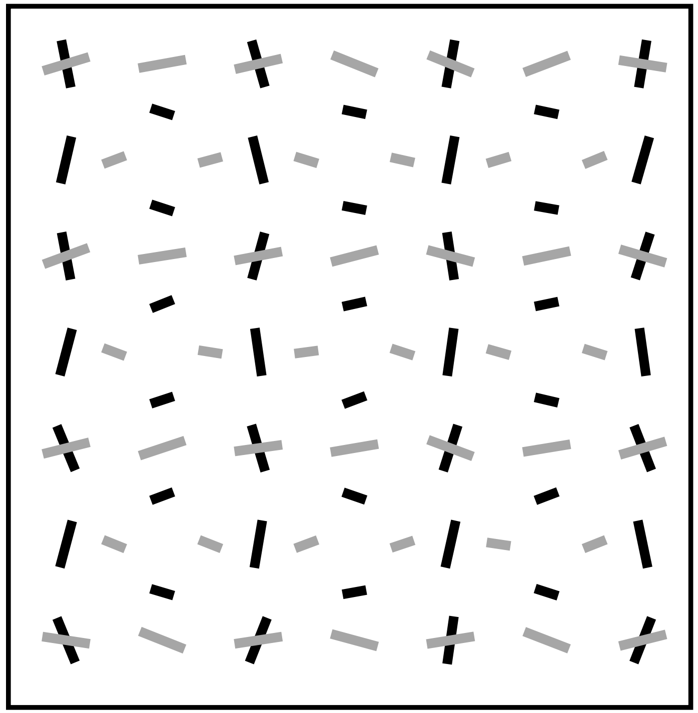

"inverse" bisection illusion
a = c but a appears smaller than c




"Real" bisection illusion
b = 2a, but m(b)/m(a) > 2



The central subdivided square appears larger than the square on the right, and each of its 4 components squares appear smaller than the square on the left (Aalen 2009 talk).



Ninio, "The science of illusions"
(modified)

**Fig. 12. Bisection effects. According to the convexity principle there is a single "real" bisection illusion. It is produced by the perceptual enlargement of the undivided segment b with respect to a: m(b)/m(a) > b/a. The standard and the "inverted" variants would have the status of pedagogical displays.**

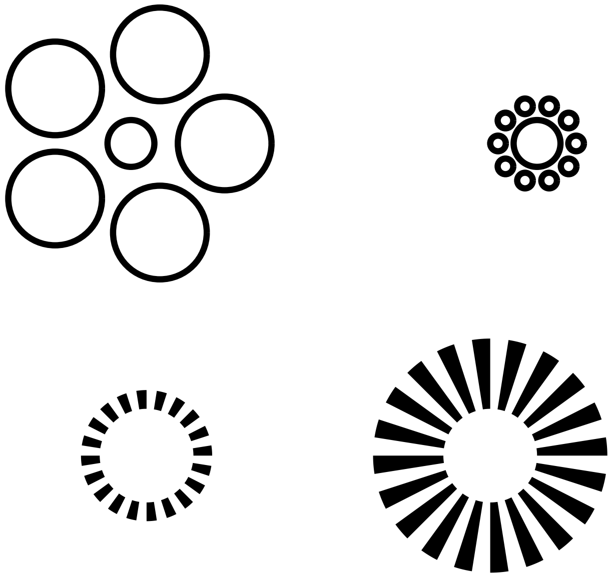
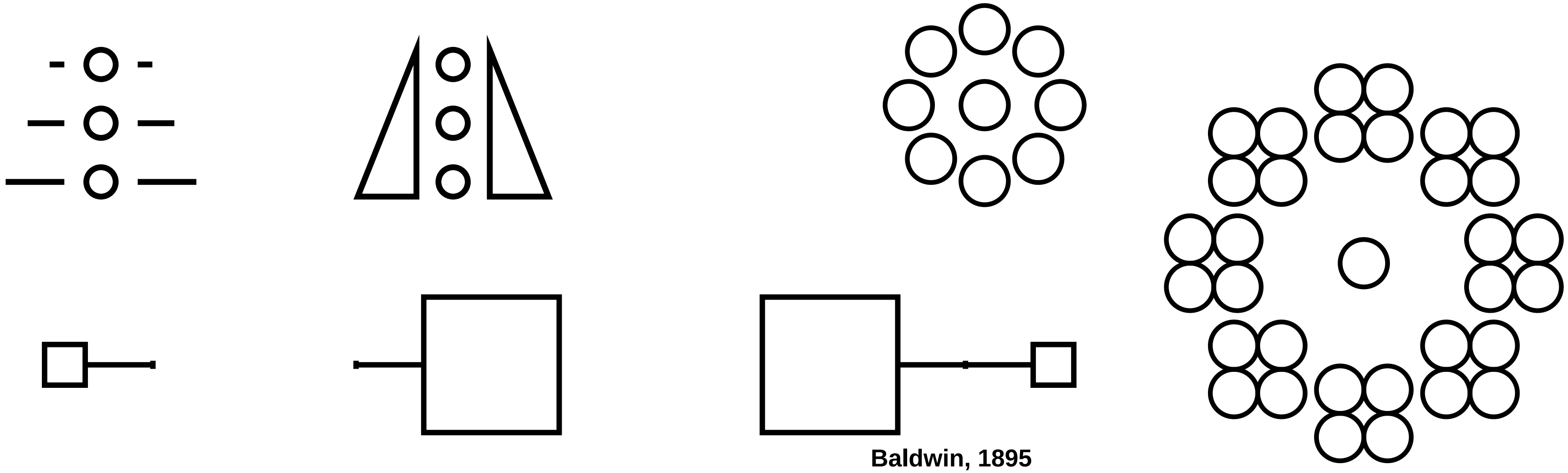


**Fig. 13. Space occupation rule.**
**According to this rule, a normalizing factor is applied to whole figures. The large ones are perceptually reduced, and the small ones are perceptually enlarged.**
**Thus, in the left part of the Ebbinghaus pattern (top left) ALL the circles would be reduced, and and in the right part, ALL the circles would be enlarged. The effect is expected to work in all directions. It seems that the illusion works in the bottom right pattern with circles, in which all the circles have the same size. Furthermore, it goes against the classical explanation by a contrast effect.**



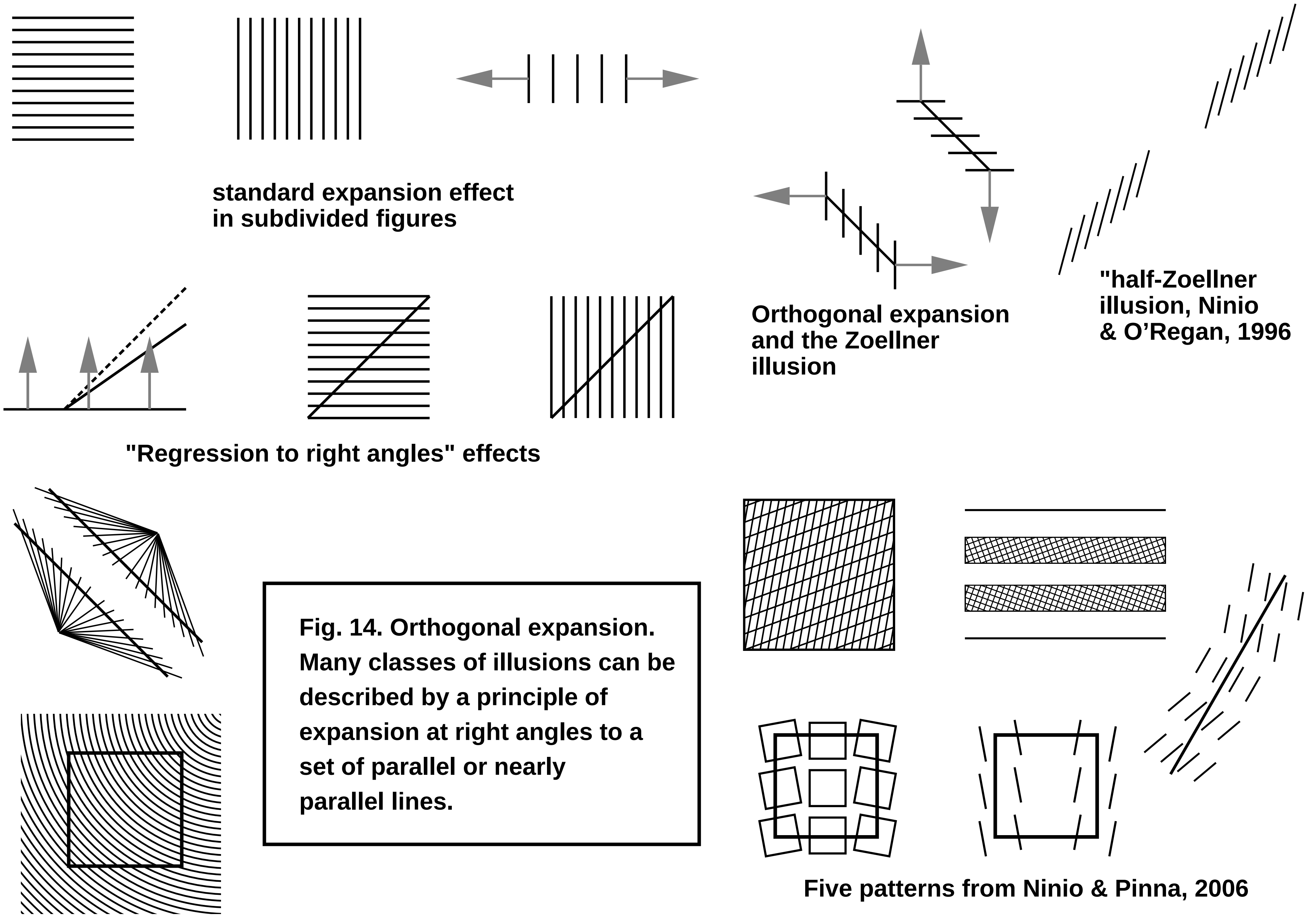
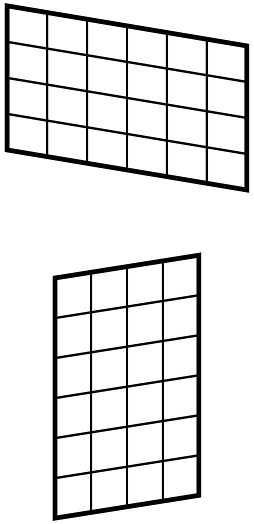
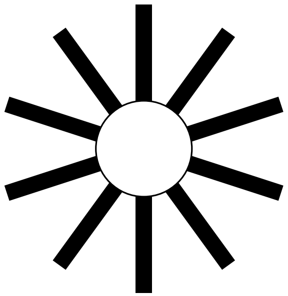



**Fig. 14. Orthogonal expansion.** Many classes of illusions can be described by a principle of expansion at right angles to a set of parallel or nearly parallel lines.

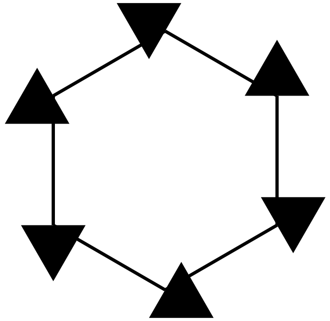
## Figure 15 : Some illusions that I do not understand



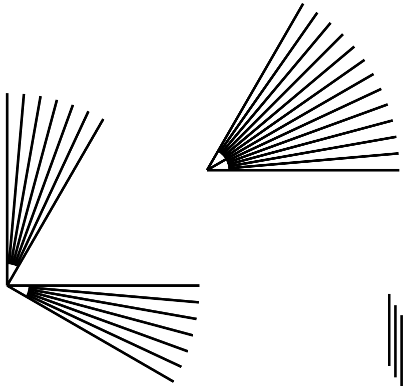
Shepard's tables



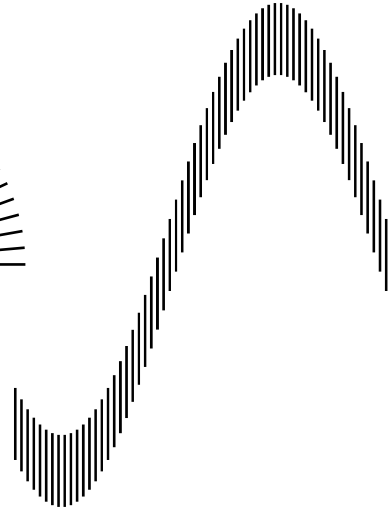
Angularity illusion
Pinna, 1991



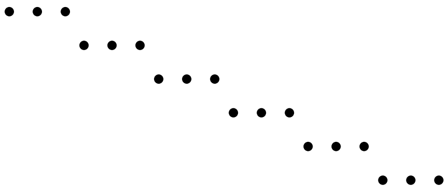
Gerbino



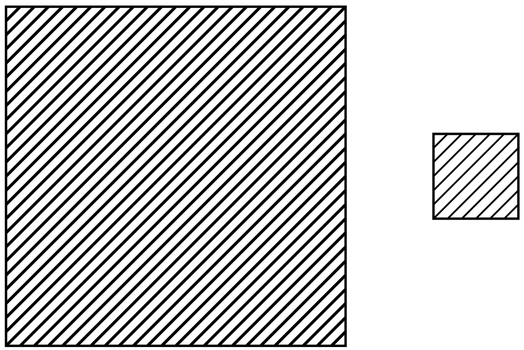
Tolanski



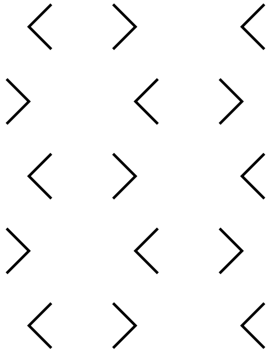
Botti illusion, 1909
Day and Stecher's pattern, 1991



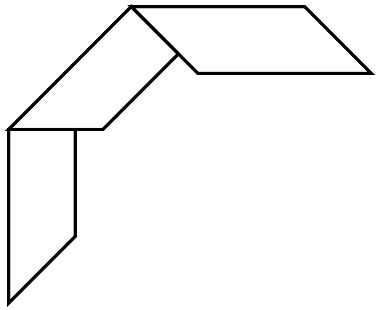
Sloping steps illusion
Vicario, 1978



Rarefaction illusion
Vicario

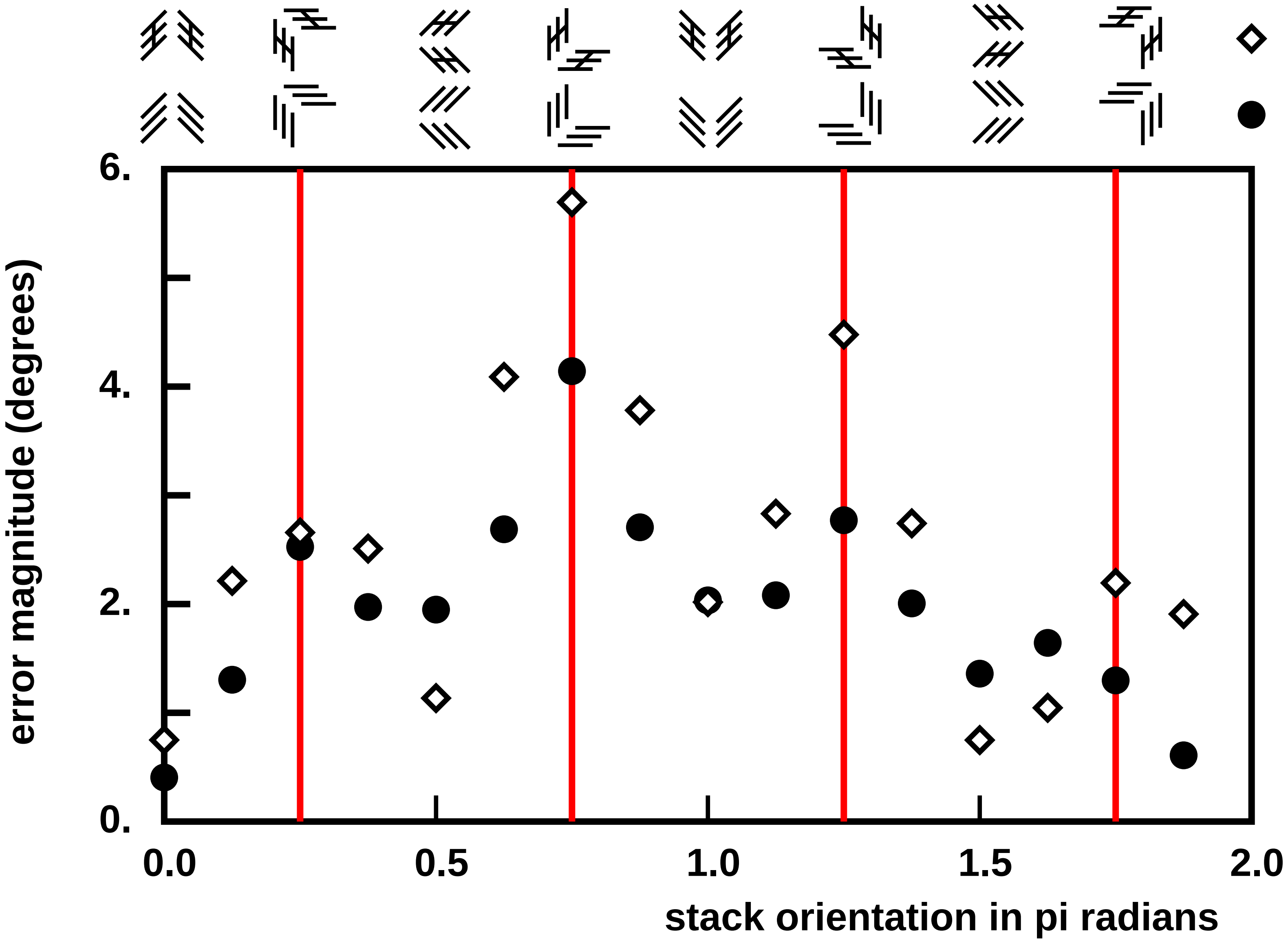


Displacement illusion
Morinaga, 1954



Bressanelli and
Massironi, 2006

**Figure 16 : Orientation profiles in the Zoellner illusion with or without axes (with 5 bars per stack)**




**17 subjects / weighted average / 170 measures per data point**

**Figure 17 : Orientation profiles in the square-diamond illusions , with or without symmetry**

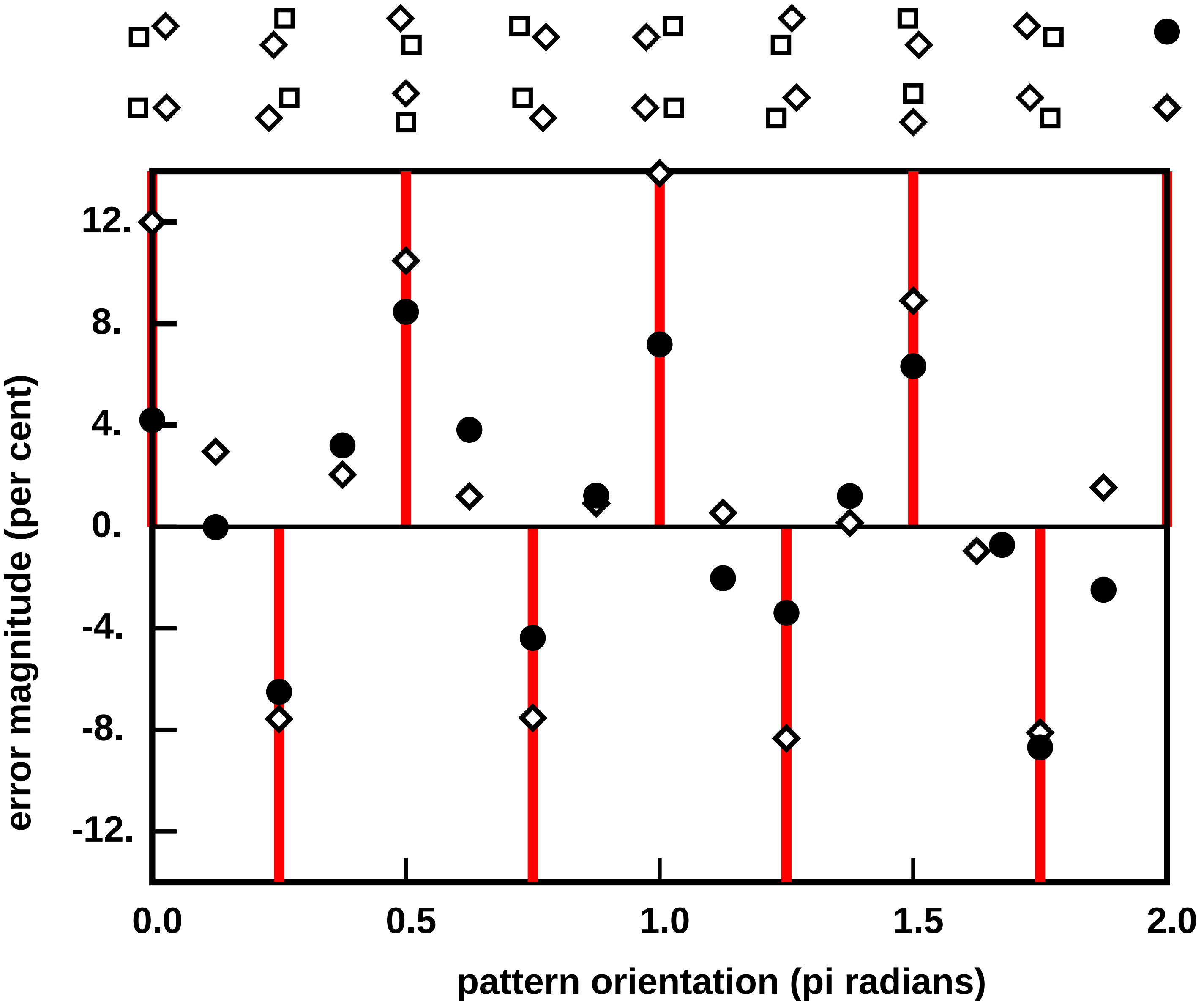


**Figure 18 :**
**Orientation profiles in the square-diamond illusions**
**square versus diamond within a larger square**

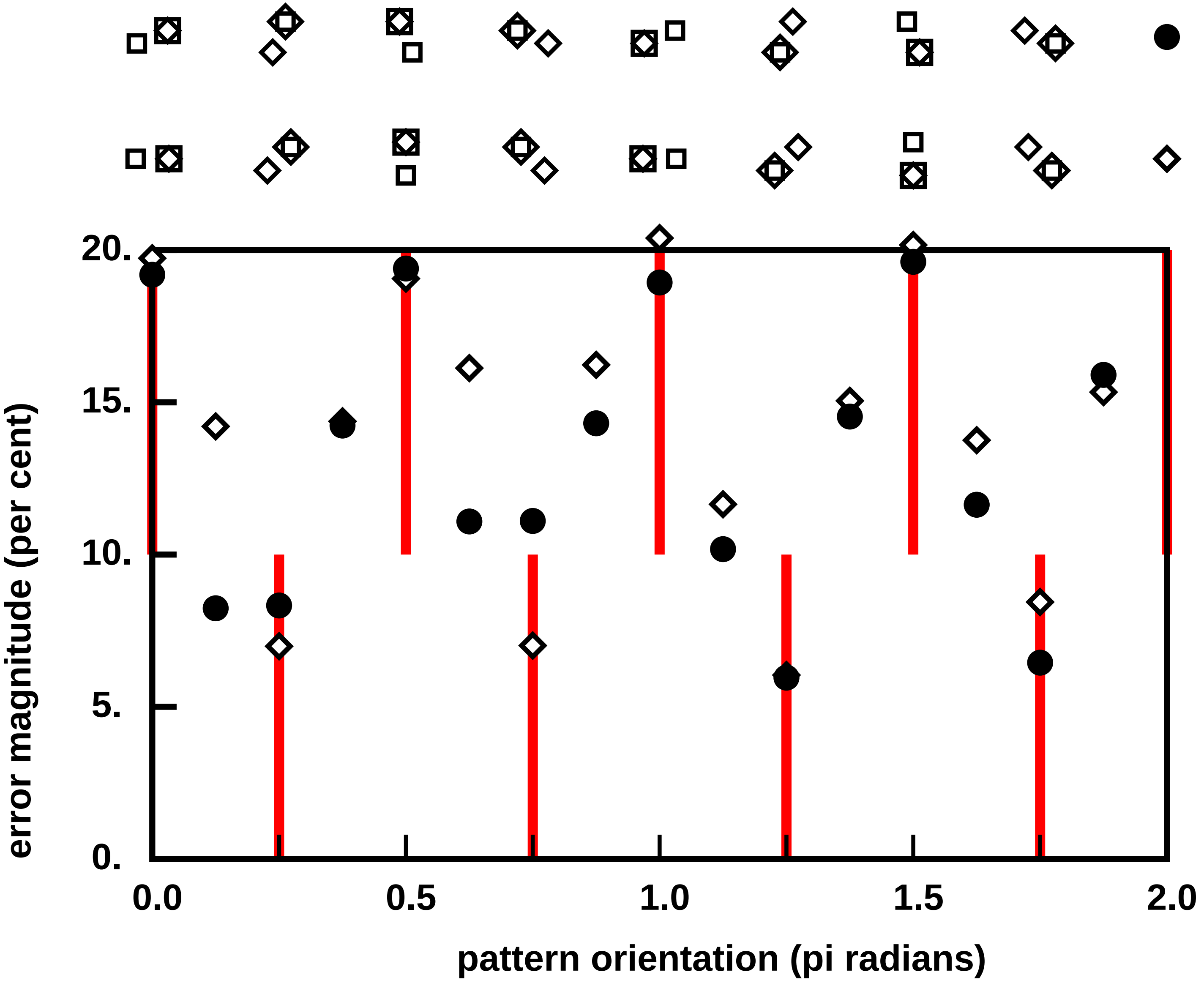
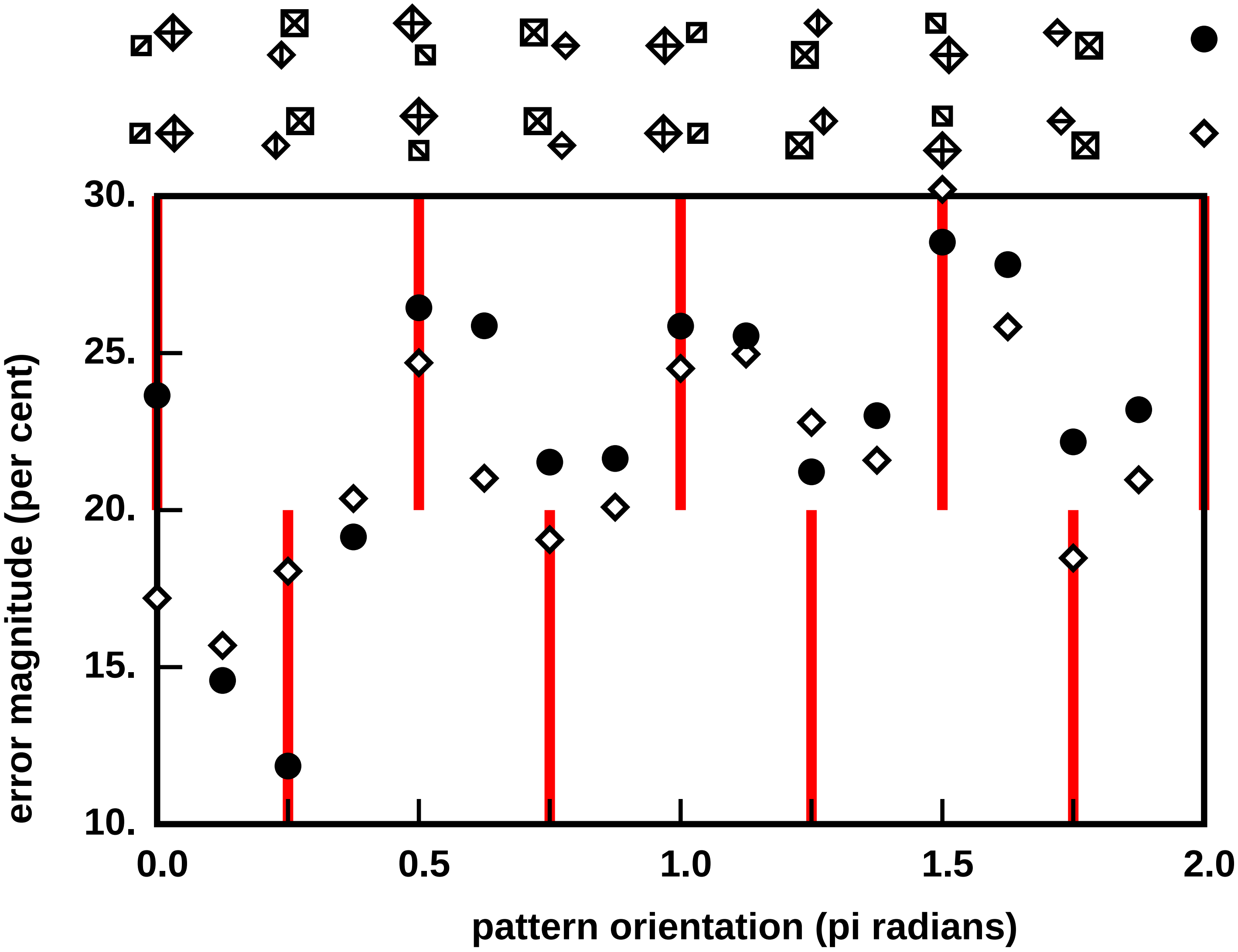


**Figure 19 :**
**Orientation profiles in the square-diamond illusions**
**diagonal of small square versus side of larger square**



# Figure 20 :
## Orientation profiles in the trapezium illusions
## 1-apparent inequality of the large bases

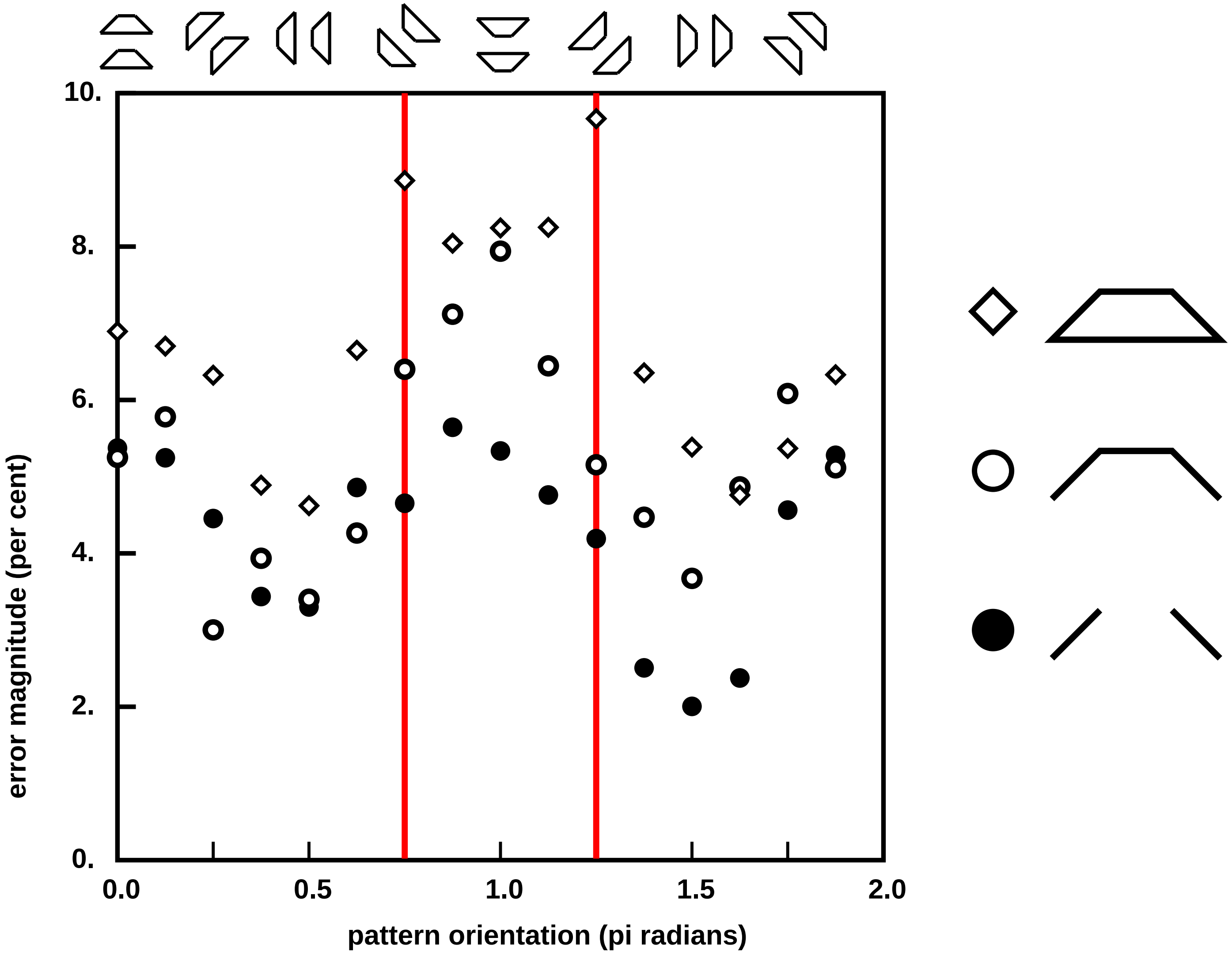


**Figure 21 :**
**Orientation profiles in the trapezium illusions**
**2--apparent inequality of sides or heights**

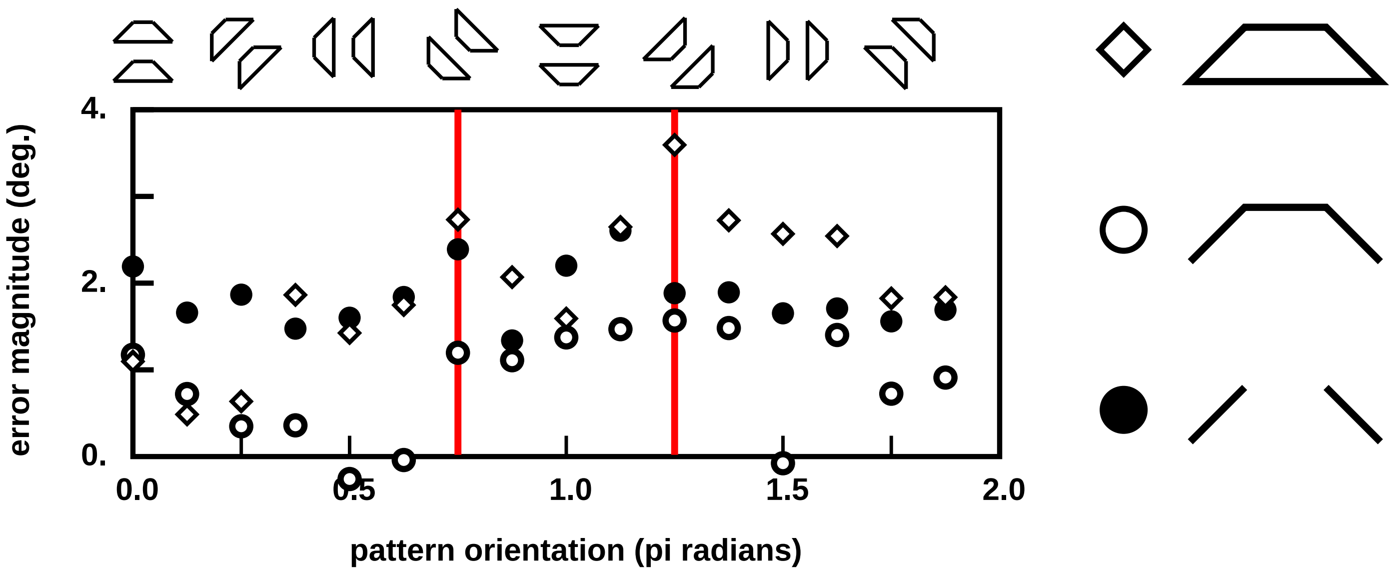


**Figure 22 :**
**Orientation profiles in the trapezium illusions**
**3-a study of the configuration without sides**

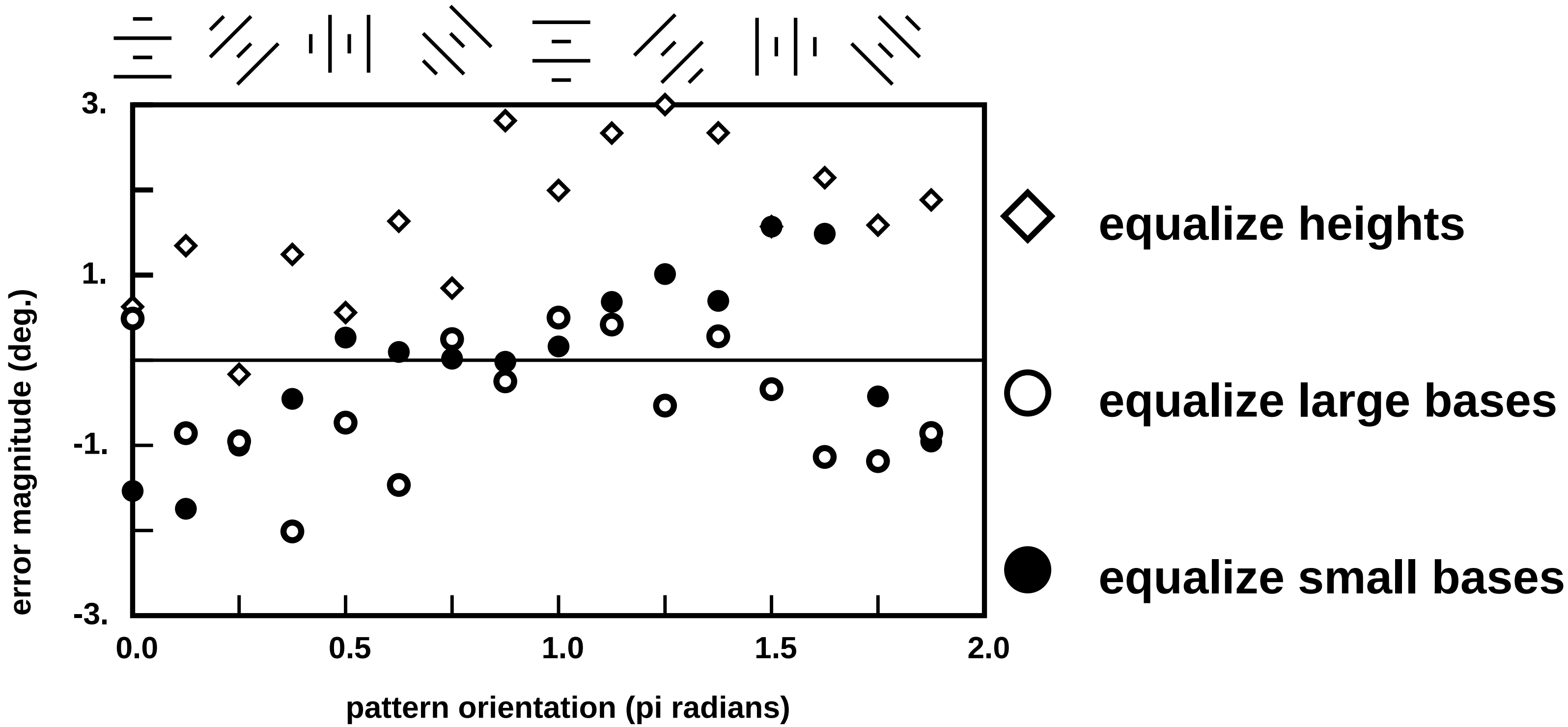


**Figure 23 Hybrid Zoellner-Poggendorff patterns :**
**After subtraction of the Zehender component**