# Hits and flops dynamics

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**Abstract**

We discuss the influence of various mechanisms of information contagion on the dynamics of choices in social networks of decision makers. We show that both polling and contagion processes often end up in herding behaviour, with most agents taking uniformly either one view or the other, even in disordered systems. In the parameter transition regime, the outcome of the dynamical process depends on the precise instanciation of disorder. For instance, when agents face a ternary choice between similar products A and B with the possibility of taking none, the initial distribution of early buyers drastically influences the outcome of the selection process.

## 1 Introduction

The theory of Social Choices applies to conditions when agents are facing choices and lack complete information about the possible consequences of their choices. This approach differs from standard economics which concentrates on the hypothesis of unbounded rationality implying that agents have a full understanding about the consequences of their choices.

It is often assumed that agents lacking full information try to take into account other agents' choices to orientate their own choice. The choice process then becomes a social process, hence the name of Social Choice.

The theory has been applied for instance to:

- certain markets such as toys and gadgets (Farrell 1998), and of course to stock markets;
- the movie industries;
- the adoption of technological changes,
- political and economical measures,
- the political arena where voters choose parties and political options.

The early literature seems to be inspired from epidemiology: choices are binary, for instance to buy or not to buy, initial conditions correspond to very few initial buyers and any time a non-buyer "meets" a buyer she becomes "infected" and buys, thus being able to infect other non-buyers. The aggregate dynamics (sum of all previous sales) yields an S shaped curve for market share as a function of time. When one waits long enough, all potential buyers buy the product. Such processes are often referred to by economists as herding behaviour or fads and they are often held responsible for the observation of bubbles in financial markets. The discussion of this approach and of its application to marketing can be found in Goldenberg et al (2000). Let us recall and discuss its basic assumptions:

- Binary infecting encounters can occur between any pair of agents. Neither social nor geographical connection structure constrains the encounters.
- Agents are all of the same kind with respect to their "susceptibility" for "infection".
- The choice of initial buyers or adopters is always exogenous.
- Choices are binary, and there is no competition for instance between different offers with the possibility of taking none.

Latané and Nowak 1997 in social sciences and Föllmer 1974 in economics were among the first to discuss the importance of social connection structures on aggregate choice dynamics.

## 2 Polling processes

The "microscopic" process (Levy et al 2000) introduced by these authors to describe individual choices resembles a "polling process". Agents interact with their "social neighbours", the connection structure often being a square lattice, and take a decision which follows a weighted sum of their neighbours' opinions. The weighting factors represent the influences of their neighbours. The rationale for such a process can be either

- a polling mechanism based on a priori uncertainty and search from others' advice ;
- or the fact that choices are not independent of each other (existence of externalities). This is the case for instance when the choice concerns the adoption of a standard: having the same standard as the majority allows you more exchanges.

A physicist can readily guess that the homogeneous case, all agents being identical, can be mapped directly onto zero temperature ferromagnetism and crystal growth in terms of dynamics (Galam 1997). Taking into account inhomogeneities of agents characteristics brings us to compare the polling dynamics with the dynamics of disordered systems such as spin glasses. In fact, since in most cases only positive interactions are taken into account, there is no frustration and once more we are considering the equivalent of ferromagnets.

The problem is often an issue in dynamics and metastable states such as crystal growth and domains in ferromagnets, rather than the search for equilibrium states. One can certainly discuss the opportunity of taking into account time fluctuations and assimilate these to non-zero temperature, which would bring the system to thermal equilibrium. But we are considering social systems for which it would be hard to make a distinction between short term fluctuations bringing the system to equilibrium and long term trends which might change the equilibrium states of the systems, while such a distinction can be made in material science. In view of constant evolution of the social environment, the search for metastability makes more sense than the quest of a thermodynamic equilibrium.

Let us illustrate some of the concepts on the issue of choices concerning new technologies from a previous work by Weisbuch and Boudjema 1999 concerning the adoption of environmental measures by a population of farmers.

## 2.1 From utilities to threshold automata

At each time step farmers have the choice to accept (1) or to refuse a contract (0) according to some estimated utility functions, taking into account two terms:

- an absolute (or private) utility, or total incomes of the farm system, in the two cases when contracts are accepted ($u_1$) or refused ($u_0$);
- a social term taking into account the choice of his neighbours $Jf_0$ or $Jf_1$, $f$ corresponding to the fraction of neighbours having made the corresponding choice $i$.

The utilities expressions are then:

$$U_0 = u_0 + Jf_0 \tag{1}$$

$$U_1 = u_1 + Jf_1 \tag{2}$$

and the agent decides upon the sign of their difference

$$\Delta U = \Delta u + J(2f_1 - 1) \tag{3}$$

taking the choice of highest utility.

This condition on continuous utilities can be transformed for fixed private utility difference $\Delta u_i$ into a discrete condition on the number of neighbours $v_1$ having made the adoption choice. Let us take the example of a square lattice with 8 neighbours $v$ per site. The condition for adoption becomes:

$$\Delta u + \frac{v_1}{4} > 1 \tag{4}$$

The following diagram then represents the discrete thresholds in number of adopting neighbors corresponding to the absolute differences utilities:

```
Adoption thresholds:
5       4       3       2       1     0
____|_______|_______|_______|_______|__
    0      0.25    0.5     0.75    1.0
Private utility differences.
```

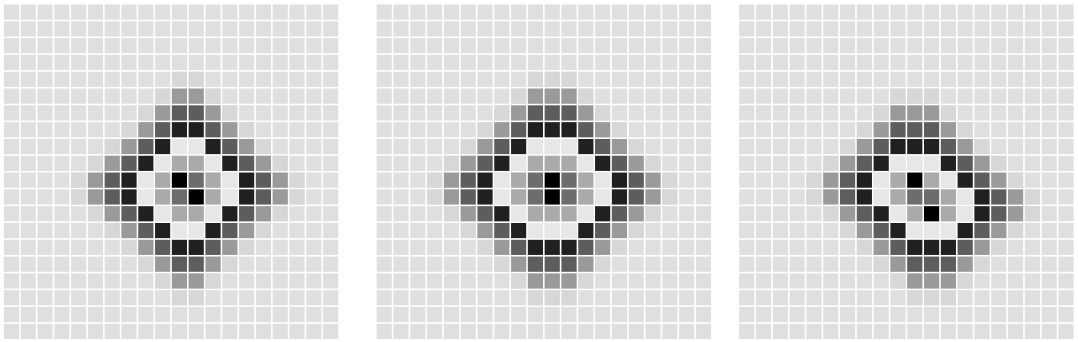

Figure 1: The 3 figures show the first 6 time steps of adoption dynamics starting from 3 seed configurations made of two adopters (the central pair of black cells). Gray levels code time steps. The threshold for growth is 2 adopting neighbours.

For instance, when all private utility differences obey $0.5 < \Delta u \leq 0.75$, the network is made of cellular automata of counter type with threshold two (Weisbuch 1990). As seen on figure 1 any seed of two initial adopters distant by less than 2 grows to fill the lattice. Growth is fast, occurs regularly and growing domains have diamond shapes.

For threshold two, growth of adoption in a social network of farmers then does not only depend upon the private utilities but also upon the presence of seed configurations of pairs of initial adopters. If none is present there is no growth. If one starts from an initial density of adopters in a lattice, the probability of observing growth is an S shaped curve as a function of the initial density of adopters. We have shown (1998) that the crossover between absence of growth and growth is obtained for

$$n_c \simeq \sqrt{\frac{N}{12}} \tag{5}$$

on a square lattice with 8 neighbours, and

$$n_c = \frac{1}{(k-1)(k-2)} \tag{6}$$

for a random network of connectivity $k$. Similar expressions are obtained for threshold 3, but with different scaling laws. But the general conclusion seems to be generic:

For social networks of any type, regular or random, the polling dynamics with threshold 2 and 3 has homogeneous attractors of 0 or 1, depending upon the density of initial adopters. In the crossover region, the same density results into either unlimited growth or absence of growth, depending upon the presence or absence of initial seed configurations.

## 2.2 INCA: Inhomogeneous cellular automata

However, we have no reason to assume that all agents share the same characteristics. After all, we have in mind farmers whose income, expenses and criteria of choice vary largely. The variety of agents characteristics introduces some randomness in the distribution of utilities and connections of the agents. We only deal here with *frozen disorder, i.e.* we suppose that some distribution of characteristics is chosen at the beginning of a simulation and that all chosen parameters are kept constant for the time of the simulation (Vichniac G., 1986).

Let us first study the influence of a distribution of different $\Delta u$ among agents. The equivalent physical system is then ferromagnetism in the presence of an external random field. We have chosen to add to the average $\Delta u_a$, a random term drawn from a Gaussian distribution with a given width $\sigma u$. The random term is constant in time.

A direct on-line examination of computer simulations show that in the region of intermediate average $\Delta u_a$ growth is favored by the random terms. Initial clusters that would otherwise disappear or be limited to small regular size have "seeds" on their periphery, due to agents with $\Delta u$ larger than average. Interfacial growth can then proceed from these seeds. The process is illustrated on figure 3.

Depending upon the density of initial adopters and the spread of the distribution of $\Delta u$, growth can fill most of the lattice or remain limited to only a fraction of it. Statistics averaged on disorder show the increase of the final fraction of adopters $d_f$ with the magnitude of the random terms in conditions when average $\Delta u$ and initial densities $d$ would not permit growth without these random terms.

But the averaged fraction only gives partial information: an average of $d_f = 0.5$ could correspond to half of the densities being 0 and the other half being 1 (perfect bimodality), or to the opposite case of a distribution uniform on [0,1]. We investigated the issue by plotting the histograms of $d_f$ and found that intermediate values of the average final densities can correspond to very

different distributions as observed on histograms represented in figure 2.

The bins for the histograms were the ten decimal intervals plus the two extremal discrete bins corresponding to $d_f = 0$ and $d_f = 1$. Once more, the most striking feature to observe is the dispersion of results for equivalent randomness distributions. The distribution of utilities is insufficient to predict the outcome of the imitation process; for this purpose one would have to know the precise spatial distribution.

The online observations explains the phenomenon. $\Delta u_a = 0.47$ is close to a threshold for growth of two neighbours (0.5). All the cells on the periphery of a cluster are then "candidate" interfacial seeds for surface growth since they already have two adopting neighbours, but they are "actual" seeds when their $\Delta u$ is larger than 0.5. which occurs with probability $\pi$.

For certain seed configuration such as the left of figure 3, if the center position between the 2 early adopters is filled, which occurs with probability $\pi$, total invasion proceeds with probability one following the earlier stages represented on figure 3.

For other configurations, such as the right one on figure 3, growth probability of the second layer while not equal to one, is much higher than the probability of growth of the first layer. The probability of growth on any site at the interface is written as:

$$\pi = 1 - q \tag{7}$$

where $q$ is the probability that the site stays empty because its $\Delta u$ is smaller than 0.5. The probability $P1$ that at least one site among $s1$ opportunities at the first step is an actual inter-facial seed is then:

$$P1 = 1 - q^{s1}. \tag{8}$$

The probability $P2$ that any site among $s2$ opportunities at the second step is actual inter-facial seed is then:

$$P2 = 1 - q^{s2}. \tag{9}$$

When $\frac{s1}{s2}$ is small, $P2$ can get pretty close to one, even for intermediate values of $q$: for example, when $q = 0.5$, $P1 = 0.5$ and $P2 = 1$ for the left configuration of figure 3 and $P1 = 0.75$ and $P2 = 0.994$ for the right configuration. If $\pi$ is large enough to allow growth to start at the first layer outside the seed, invasion will be observed with very high probability.

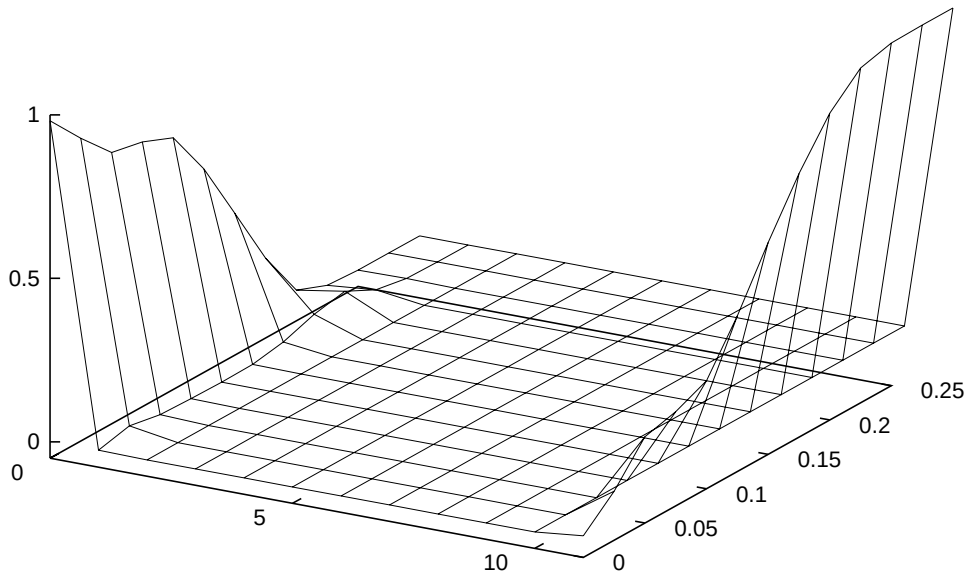


Figure 2: Histograms of fractions of adopters $d_f$ at "infinite" time as a function of width of distribution $\sigma u$ (y axis). Averages are taken from 500 samples. Initial densities are $d = 0.01$. The bins along the x axis are the ten decimal intervals plus the two extremal discrete bins corresponding to $d_f = 0$ and $d_f = 1$. $\Delta u_a = 0.47$ is just below threshold 2.

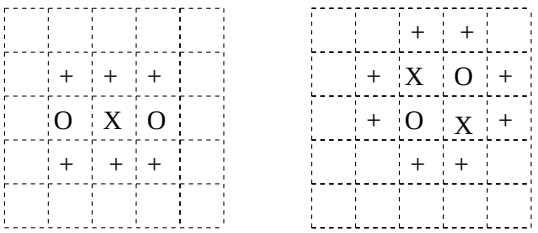


Figure 3: Initial steps of growth for two adopters seeds. The 'O's are initial adopters, the 'X' are growth opportunities at the first level, and '+' are opportunities at the second level. For the right configuration s1=2, s2=8.

All the results discussed in this section concerning the observation of extreme fraction of choices, close to either 0 or 1, are based on the notion of initial seed configurations: they generalise to any case of disorder in utilities and coupling intensities and in connections (to random networks for instance), (Weisbuch and Boudjema 1999).

# 3 Social percolation

## 3.1 Simple models

Another possible "microscopic" mechanism for individual decision is a contact process. A decision is taken by the agent when at least one of her neighbours has taken a positive decision; until that time, she remains undecided. The rationale for this choice is information contagion: agents take their decision when they get some private information about the consequences of their choice from other connected agents who learnt about consequences after having made the corresponding choice. The contact process is equivalent to a polling process with threshold 1. Heterogeneity of agents is introduced by taking into account the fact that information is only one necessary condition to take a decision; if it happens that the information from their neighbours is "the quality of the product (or the party, or the movie) is less than what you demand", the agents take a negative decision. A simple way to implement such a decision process is to suppose that information about the choice is represented by a scalar quantity $q$ (for quality) and that agents $i$ each have expectations $p_i$ (for preference).

An agent will buy a product (or adopt a new technology, or vote for a party) when:

- she obtains information about the quality $q$ from one of her neighbours who made a positive choice;
- if

$$q > p_i$$

As presently posed the problem becomes equivalent to site percolation: we can predict a priori that none of the agents with preferences larger than the quality of the product will participate to the above described contagion process across the social network; they can then be removed as totally inactive.

Let us consider a network, lattice or with random connections, with a random distribution of $p_i$'s. If one starts from initial conditions with a large majority of undecided agents (by lack of information) and a small fraction of "early birds" who have already taken a positive decision, contagion proceeds across the net according to the fraction $f$ of agents with

$$p_i < q.$$

The early birds could be individuals who got the product for free or who happened to get direct relevant information in the case of markets, promoters of a new technology in case of the adoption of new technologies or party members in politics. The comparison of $f$ to the percolation threshold $p_c$ on the social network allows to predict the success or failure of the product with a very high probability:

- When $f > p_c$ most sites with preferences smaller than $q$ are invaded, which results in a large success for the seller (or the promoters of the new technology, or the political party presenting the platform).
- By contrast when $f < p_c$, contagion soon stops and the product is a commercial failure with very small market share.

Let us recall some well known figures (Stauffer and Aharony 1994) in the case of uniform distributions of $p_i$'s: for a square lattice with four neighbours per site $p_c = 0.593$, with eight neighbours $p_c = 0.407$, and for a random network with large connectivity $k$, $p_c = 1/(k-1)$ (the connectivity of a network is the number of neighbours per site). The above contrasted dynamic behaviours resulting from the necessity of contagion for adoption of a new product are very different from the prediction of "perfectly rational economics" which would yield a market share exactly equal to $f$. Furthermore, this contrasted behaviour between the two dynamical regimes is a generic property independent of the details of the network connectivity.

We also observed it by numerical simulations with models of vectorial preferences. In management science, a standard approach to decision making is multi-criteria analysis. Several criteria can influence agent decisions: they might take into account not only immediate or delayed profits but also time spent, risks, or even non-economic considerations such as moral, aesthetic or social values. Rather than considering a scalar quality, we are now having a vector quality

$$\mathbf{q} = \{q_0, ...q_j, ...q_n\} \tag{10}$$

The above described results (a sharp phase transition separating a percolation regime from a regime with very limited propagation) apply to any "social" version of multicriteria analysis as long as the contagion process immediately make agents fully aware of all qualities of the product with respect to the criteria.

This is the case for instance when different criteria $q_j$ are weighted by the decision making agent $i$ and added up according to an expression:

$$q_t(i) = \sum_j w_{ij} q_j, \tag{11}$$

where the $w_{ij}$ are the weighting factors reflecting the subjective importance of qualities $q_j$ to agent $i$. $q_t$ is then compared to the agents preference and the standard decision procedure applies. The same elimination process of those agents such that $p_i > q_t$ occurs and information contagion only proceeds through the rest of the agents: the same percolation transition is observed.

We went further along the multi-criteria idea by supposing that different neighbours $k$ might transfer an information biased by their own preference vector because they might have different interests for different qualities. Let us consider for instance the case of movie goers. The qualities of a movie might include aesthetics, action, actors performance, musics etc. and not all agents weight equally these qualities when taking a decision or when transmitting their views. We then suppose that rather than transmitting the true qualities of the movie $q_0, ..., q_j, ....$, a neighbour $k$ transmits a biased opinion $w_{k0}q_0, ...w_{kj}q_j, ...$ about the movie. The set of biasing weights $w_{k0}, ...w_{kj}, ...$ reflecting his own interests are taken in the $[0, 1]$ interval.

When facing a decision to be taken, an agent then often gets several different opinions about a movie from different neighbours $k$, due to their different biases. Let us suppose that he appreciates each quality $q_j$ of the movie by selecting the highest transmitted signal $\max_k\{w_{kj}q_j\}$ concerning that quality $q_j$ given by any of his informed neighbours. The rationale for this procedure is that the agent is aware that his neighbours have different weighting factors and that the highest signal $\max_k\{w_{kj}q_j\}$ is also the closest to the intrinsic quality $q_j$. Another rationale is that to get an authorised opinion about some quality, you better ask to someone who cares about it. The decision process of agent $i$ then proceeds according to the following 3 steps:

Balance the opinion of his informed neighbours according to:

$$\mathbf{q}' = \{\max_k\{w_{k0}q_0\}, ... \max_k\{w_{kj}q_j\}, ....\} \tag{12}$$

Weigh the above evaluation $\mathbf{q}'$ according to his own criteria:

$$q'_t = \sum_j w_{ij}q'_j; \tag{13}$$

Compare $q'_t$ to his preference $p_i$. If the weighted quality is larger than the preference, the agent takes the positive decision and goes to see the movie (resp. buys the product or adopts the technology).

Since agent $i$ now becomes fully informed, he re-adjusts his evaluation of the quality vector $\mathbf{q}'$ to the intrinsic quality vector $\mathbf{q}$; but he will again transmit a biased quality vector according to his own views to his neighbouring "virgin" neighbours.

Simulations done with the above hypotheses applied to a 2-d quality vector, and a square lattice with a neighbourhood of 8 as the social network, also yield the abrupt transition between a percolation and a failure regime. The observed changes only concern the necessity of having larger $q_j$ and to wait for longer times before the percolation is achieved with respect to the case of un-biased information contagion.

When then conclude that the existence of two regimes separated by an abrupt transition is a generic property of social systems with information propagation along social links and decision taken according to available information.

## 3.2 Adjustment meta-dynamics

Among the situations that we try to model some are occurring rarely such as major technological changes, while some others are recurrent as in the movie industry: people often visit movie theaters (or restaurants) in their home town, and producers are producing new movies for the market at a frequency high enough for both parties to adjust supply and demand according to their previous experience. Let us try to translate the "tatonnement" (or adjustment) process familiar to economists to the case of movie goers.

- after opportunities during which they went to the movie, the agents will be more demanding and typically increase their expectations (here

the preferences $p_i$); on the opposite, those who did not go, lower their expectations and preferences.

- after hits (resp. flops) the movie producers will decrease (resp. increase) the quality $q$ of the produced movie(s), in their effort to remain above the threshold while minimizing expenses.

We have made computer simulations based on a faster contagion process leading eventually to percolation, embedded in a slower adjustment process as described above: we iterated a series of steps each one composed of a contagion process which was left evolving until percolation or its absence was checked, and of the resulting adjustment process. All details concerning the algorithms that were used, including the Leath algorithm, are described in Solomon et al (2000).

After a transient adjustment period, we observed an alternance of hits and flops. Such a dynamics is often described as self organised criticality: the adjustment process brings and maintains the system parameters $p_i$'s and $q$ in the neighborhood of the percolation threshold. The situation is very different from a Gaussian distribution of fluctuations in the neighbourhood of equilibrium (a situation which would satisfy most economists interested in the General Equilibrium theory!). On the contrary, the large-amplitude variations of the fraction of movie goers is reminiscent of the fat tails observed in the distribution of return in financial markets. The above analysis is consistent with the interpretation of fat tails as due to strong cooperative effects, (here the constraints imposed by information contagion on the buy/sell decisions of the agents), in the neighbourhood of a transition[1].

## 3.3 Competition among several options

Quite often, in markets or in politics, agents are facing several options, between for instance product A and B. When the connectivity of the social network is large enough, several percolating subnetworks can coexist (Aizenman 1997; Stauffer 1999 with earlier literature), and in the case of the two nearly equivalent products with quality larger than the percolation threshold, both can in principle reach a sizable fraction of the market. The higher

[1]We do not imply here that the General Equilibrium theory is wrong, but simply that some of its assumptions about convexity don't apply to situations where the decision process gives such large variations as observed around the percolation transition

the connectivity of a network, the larger is the probability to have more than one spanning cluster at the percolation threshold.

In the simple case of successive exclusive presentations of options of equivalent quality, the order of presentation makes the difference and the first presented option wins as demonstrated by Ahmed and Abdusalam (2000).

When the options are presented at the same time, whether one option, two options, or none, will percolate is not a priori clear as we show by computer simulations. These simulations were run on a square lattice of connectivity eight. Two options, say two movies A and B, are presented at the same time at one boundary of the lattice (namely, one boundary line is populated with "early birds" which went to see either A or B movie). Movies A and B are "nearly" equivalent in quality, in the sense that their quality $q$ is the same but whenever a competition appears when two neighbours of a decision making agent transmit different signals, one about A and the other about B, A is systematically chosen by the agent (if the quality is larger than his preference). On the other hand, when only one signal is received, whether A or B, the agent chooses accordingly A or B (again if $q > p_i$). Biased reversibility is also achieved: an agent that has previously chosen B changes to A when he later receives an A signal, but the opposite is not true.

Table 1 summarizes the results obtained with 63937 trials on a 4001x4001 square lattices with uniform random distributions of $p_i$ between 0 and 1, and for $q_A = q_B = p_c = 0.407$. The initial conditions correspond to filling the boundary line with a random distribution of early adopters with a 50 percent probability of choosing A or B.

| percolation of | fraction of sites | | | | | occurences |
|---|---|---|---|---|---|---|
| | bad | good | refused | virgin | flippers | |
| good | 0.001 | 0.207 | 0.303 | 0.487 | 0.00008 | 39817 |
| bad | 0.141 | 0.065 | 0.299 | 0.494 | 0.00008 | 653 |
| both | 0.090 | 0.136 | 0.330 | 0.442 | 0.00009 | 147 |
| none | 0.002 | 0.099 | 0.147 | 0.751 | 0.00008 | 23320 |

Table 1: Percolation when two nearly equivalent products "good" and "bad" are in competition on a square lattice with connectivity eight, for random distribution of A and B among the early birds at the boundary. The first column described which clusters percolate, either the good product, the bad product, both or none. The five next columns indicate the respective

fractions of bad, good, refusing, virgin and flipper sites (a flipper changed from B to A during the simulation). The last column is the number of occurrences of the observed situation.

The most striking result is that the slight advantage of A is enough to make A clusters percolate nearly on 62 percent of the time (from the results of last column), while B clusters do it 1 percent of the time. Obviously in most cases, the large success of A prevents B to make it alone or even in conjunction with A: double percolation is seldomly observed, 0.23 percent of the time. The local mechanism for domination of A is simple; B's only survive when they are separated from A's by lines of refusers. The average fractions reported in the middle column show that the competition is always in the defavour of the competitors: for fully rational agents presented with one choice the fraction of movie goers would be 0.407, and for social agents just above the percolation threshold, the percolation cluster should contain most of the sites with $p_i < q$. We can see here that the result of simulation is a strong proportion of virgin sites who never "hear" about the movies' quality. The virgin sites can have any preference $p_i$ and are of course of two kinds, those who would never had gone because their expectation is larger than the quality of the movie and those who would have gone if they had known.

On the other hand, we tried another set of initial conditions with clusters of "early birds". The boundary line is composed of two equal homogeneous segments of A's and B's movie goers:

AAAAAAAAAAAAAAAAAABBBBBBBBBBBBBBBBBB

The rationale for this "birds of a feather flock together" hypothesis is that in many occasions we can imagine the initiators to be connected with each other: think for instance of people who get some decisive advantage or some private information such as privileged customers in the market, critics in the media industry, party members in politics etc.

The results which are observed with this initial configuration are noticeably different: there are less conflicts between the A and B options, since conflicts occur at the interface of clusters. The fact that B's have the opportunity to start as a homogeneous cluster strongly increases their chance to percolate. In fact we observe that the cases of percolation of either A or B clusters are each roughly half of the chances of A clusters for random initial conditions. One also observes that the occurrence of double percolation is

increased by a factor 22, but only to reach a 5 percent level.

We also checked the importance of the reversibility by monitoring the fractions of flipper sites which first chose B and then switched to A after receiving new information. The fraction varies from less than 0.5 percent for 101x101 lattices to less than 0.01 percent for 4001x4001 lattices for both types of initial conditions; this implies that reversibility does not play a large role in the selection of which dynamics actually occurs, apart for the early birds. We might then expect similar results for irreversible choices in the same parameter's region.

| percolation of | fraction of sites | | | | | occurences |
|---|---|---|---|---|---|---|
| | bad | good | refused | virgin | flippers | |
| good | 0.039 | 0.168 | 0.301 | 0.492 | 0.000003 | 18762 |
| bad | 0.168 | 0.039 | 0.301 | 0.491 | 0.000003 | 18575 |
| both | 0.114 | 0.113 | 0.330 | 0.443 | 0.000005 | 3200 |
| none | 0.051 | 0.051 | 0.148 | 0.750 | 0.000002 | 23400 |

Table 2: Percolation results for networks similar to those of table 1, except that the initial conditions are regular: the line of initial movie goers is divided into two equal homogeneous segments of agents having respectively chosen the "good", and the "bad" movie.

We then see that the results of the competitive percolation process depends crucially on the initialization of the boundary.

Huang (2000) looked at many consecutive information waves spreading over the lattice and used them to create an ensemble of different attitudes in the stock market model of Cont and Bouchaud. He also, for a single movie, clarified the reasons for stability and instability in the dynamics of section 3.2. A. Novak, private communication, has criticized our approach of assuming random distribution of $p_i$ without any correlations between neighbours. Thus Kar Gupta et al (2000) have looked at the (in-)stability of Ising-correlated percolation, when only the up spins in a critical Ising model are regarded as possible customers transmitting information.

# 4 Conclusions

In conclusion, social interactions in inhomogeneous networks of agents can often drive social systems far from any a priori predictable equilibrium.

When the average properties of the system bring it close to a transition, two types of unpredictiveness can occur:

- Small changes of parameters can move the system across the transition from one dynamical regime to another one;
- At the transition, the knowledge of the parameters defining the distribution of local properties is insufficient for accurate prediction, as shown for the polling dynamics and for the contagion dynamics; the dynamics depends on some local knowledge about initial configurations.

These phenomena are generic; they are observed:

- For regular lattices and random networks.
- For contagion dynamics (threshold 1) and poll dynamics (larger thresholds).
- For different kinds of disorder.

These observations about dynamics can be used not only to describe empirical phenomena, but also to build "selling" strategies for sellers or policy implementers.

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