# Mixing beliefs among interacting agents

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ABSTRACT. *We present a model of opinion dynamics in which agents adjust continuous opinions as a result of random binary encounters whenever their difference in opinion is below a given threshold. High thresholds yield convergence of opinions towards an average opinion, whereas low thresholds result in several opinion clusters: members of the same cluster share the same opinion but are no longer influenced by members of other clusters.*

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## 1. Introduction

Many models about opinion dynamics [FOL 74], [ART 94], [ORL 95], [LAT 97], [GAL 97], [WEI 99], are based on binary opinions which social actors update as a result of social influence. One issue of interest concerns the importance of the binary assumption: what would happen if opinion were a continuous variable such as the worthiness of a choice (a utility in economics), or some belief about the adjustment of a control parameter? In some European countries, the Right/Left political choices were often considered as continuous and were represented for instance by the geometrical position of the seat of a deputy in the Chamber.

Binary opinion dynamics under imitation processes have been well studied, and we expect that in most cases the attractor of the dynamics will display uniformity of opinions, either 0 or 1, when interactions occur across the whole population. This is the "herd" behaviour often described by economists [FOL 74], [ART 94], [ORL 95]. Clusters of opposite opinions appear when the dynamics occur on a social network with exchanges restricted to connected agents. Clustering is reinforced when agent diversity, such as a disparity in influence, is introduced, [LAT 97], [GAL 97], [WEI 99].

The a priori guess for continuous opinions is also homogenisation, but tending towards average initial opinion [LAS 89]. The purpose of this paper is to present results concerning continuous opinion dynamics subject to the constraint that convergent opinion adjustments only proceed when opinion difference is below a given threshold. We will give results concerning homogeneous mixing across the whole population and mixing across a social network. Preliminary results about binary vectors of opinions will also be presented.

## 2. Complete Mixing

### 2.1. *The basic model*

Let us consider a population of $N$ agents $i$ with continuous opinions $x_i$. At each time step any two randomly chosen agents meet. They re-adjust their opinion when their difference of opinion is smaller in magnitude than a threshold $d$. Suppose that the two agents have opinion $x$ and $x'$ and that $|x - x'| < d$; opinions are then adjusted according to:

$$x = x + \mu \cdot (x' - x)$$
$$x' = x' + \mu \cdot (x - x')$$

where $\mu$ is the convergence parameter whose values during the simulations may range from 0 to 0.5.

The rationale for the threshold condition is that agents only interact when their opinion are already close enough; otherwise they do not even bother to discuss. The reason for such behaviour might be for instance lack of understanding, conflicts of interest, or social pressure. Although there is no reason to suppose that openness to discussion, here represented by threshold $d$, is constant across population or even symmetric on the occasion of a binary encounter, we will nontheless take it as a constant simulation parameter in the present paper. (We conjecture that the results we get would remain similar provided that the distribution of $d$ accross the whole population is sharp rather than uniform).

The evolution of opinions may be mathematically predicted in the limiting case of small values of $d$ [NEA 00]. Density variations $\delta\rho(x)$ of opinions $x$ obey the following dynamics:

$$\delta\rho(x) = \frac{d^3}{2} \cdot \mu \cdot (\mu - 1) \cdot \frac{\partial^2(\rho^2)}{\partial x^2}$$

This implies that starting from an initial distribution of opinions in the population, any local higher opinion density is amplified. Peaks of opinions increase and valleys are depleted until very narrow peaks remains among a desert of intermediate opinions.

### 2.2. *Results*

Figures 1 and 2, obtained by computer simulations, display the time evolution of opinions among a population of $N = 1000$ agents for two values of the threshold $d$. Initial opinions were randomly generated across a uniform distribution on [0,1]. At each time step a random pair is chosen and agents re-adjust their opinion according to equations 1 and 2 if their opinions are closer than $d$. Convergence of opinions is observed, but consensus is only achieved for the larger value of $d$.

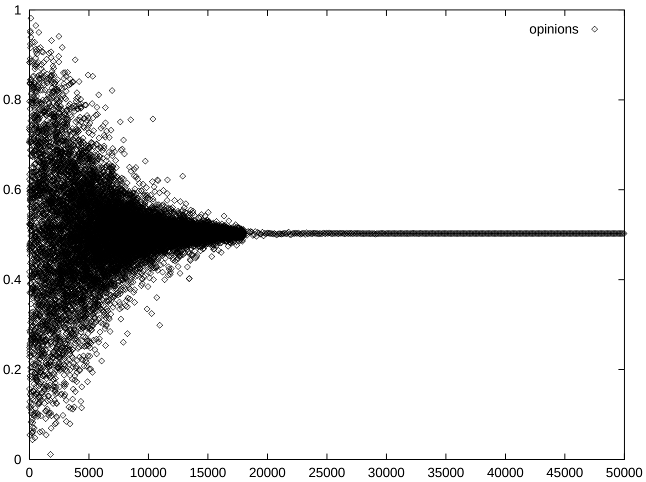


**Figure 1.** *Time chart of opinions ($d = 0.5 \quad \mu = 0.5 \quad N = 2000$). One time unit corresponds to sampling 1000 pairs of agents.*

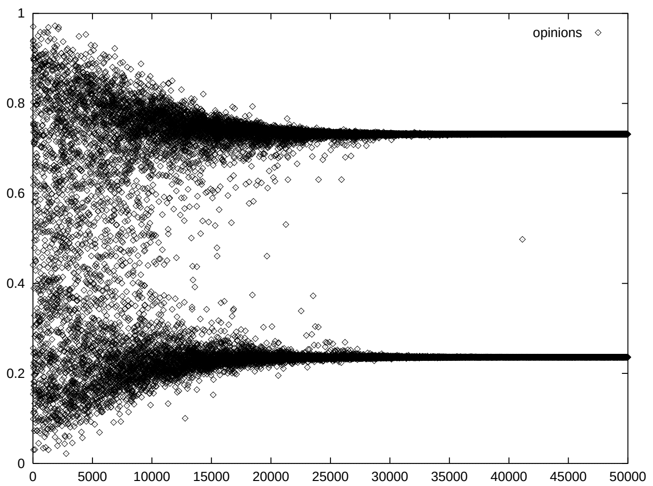


**Figure 2.** *Time chart of opinions ($d = 0.2 \quad \mu = 0.5 \quad N = 1000$). One time unit corresponds to sampling 1000 pairs of agents.*

Another way to monitor agents' opinion dynamics is to plot final opinions as a function of initial opinions. Figure 3 shows how final opinions "reflect" initial opinions for $\mu = 0.5$. One may observe that some agents with initial opinions roughly equidistant from final peaks of opinions may end up in either peak: basins of attractions in the space of opinions overlap close to the clusters boundaries. The overlap observed when $\mu = 0.5$ is strongly reduced when $\mu = 0.05$ (not represented here): agents then have more time to make up their mind as opinions are changing 10 times more slowly and their final opinions are those of the nearest peak.

A large number of simulations was carried out and we found that the qualitative dynamics depend most strongly on the threshold $d$. $\mu$ and $N$ only influence the convergence time and the width of the distribution of final opinions (when a large number of different random samples are made). $d$ controls the number of peaks of the final distribution of opinions as shown in figure 4. The maximum number of peaks, $p_{max}$, decreases as a function of $d$. A rough evaluation of $p_{max}$, based on a minimal distance of $2d$ between peaks (all other intermediate opinions being attracted by one of the peaks), plus a minimal distance of $d$ of extreme peaks from 0 and 1 edges, gives $p_{max} = \frac{1}{2d}$, in accordance with the observations of figure 4.

The finite size of the population allows for slight variations of the number of peaks for intermediate values of $d$, as a result of random samplings. These size effects were confirmed when studying larger and smaller population sizes. In the intermediate regions one also observes small populations of "wings" (corresponding to a few percent of the population) in the vicinity of the extreme opinions 0 and 1 (the term wings refers to asymmetric peaks with a vertical bound of either 0 or 1).

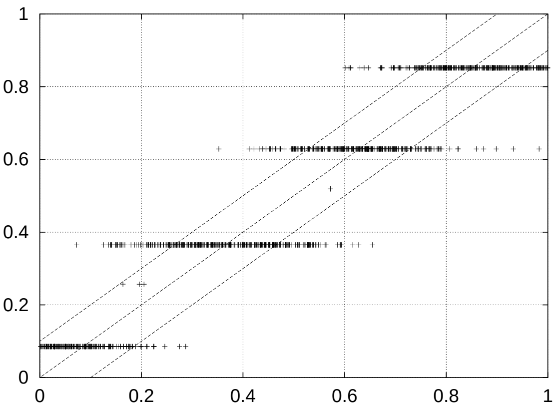


**Figure 3.** *Diagram of final opinions vs initial opinions (*$\mu = 0.5, \quad d = 0.1$*)*

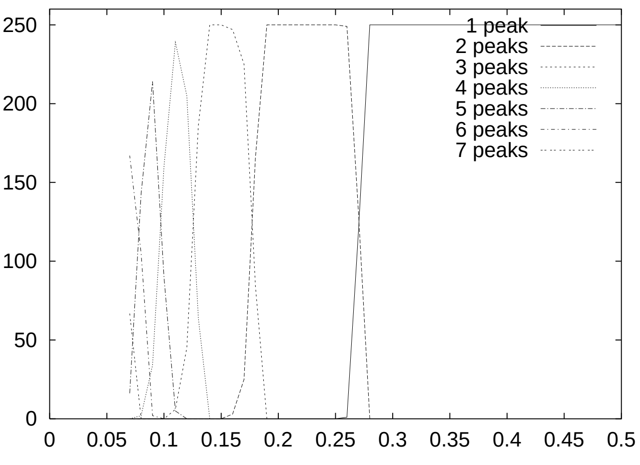


**Figure 4.** *Statistics of the number of peaks of opinions as a function of* $d$ *on the x axis for 250 samples (*$\mu = 0.5, \quad N = 1000$*). Wings are excluded from the statistics.*

## 3. Social Networks

The literature on social influence and social choice also considers the case when interactions occur along social connections between agents [FOL 74] rather than randomly across the whole population. Apart from the similarity condition, we now add to our model a condition on proximity, i.e. agents only interact if they are directly connected through a pre-existing social relation. Although one might certainly consider the possibility that opinions on certain unsignificant subjects could be influenced by complete strangers, we expect important decisions to be influenced by advice taken either from professionals (doctors, for instance) or from socially connected persons (e.g. through family, business, or clubs). Facing the difficulty of inventing a credible instance of a social network as in the literature on social binary choice [WEI 99], we here adopted the standard simplification and carried out our simulations on square lattices: square lattices are simple, allow easy visualisation of opinion configurations and contain many short loops, a property that they share with real social networks.

We then started from a 2 dimensional network of connected agents on a square grid. Any agent can only interact with his four connected neighbours ( N, S, E and W). We used the same initial random sampling of opinions from 0 to 1 and the same basic interaction process between agents as in the previous sections. At each time step a pair is randomly selected among *connected agents* and opinions are updated according to equations 1 and 2 provided of course that their distance is less than $d$.

The results are not very different from those observed with non-local opinion mixing as described in the previous section, at least for the larger values of $d$ ($d > 0.3$, all results displayed in this section are equilibrium results at large times). As seen in figure 5, the lattice is filled with a large majority of agents who have reached consensus around $x = 0.5$ while a few isolated agents have "extremists" opinions closer to 0 or 1. The importance of extremists is the most noticeable difference with the full mixing case described in the previous section.

Interesting differences are noticeable for the smaller values of $d < 0.3$ as observed in figure 6. When several values are possible for clusters of converging opinions, consensus can only be reached on connected clusters of agents.

For connectivity 4 on a square lattice, only one cluster can percolate [STA 94] across the lattice. All agents of the percolating cluster share the same opinion. Otherwise, non percolating clusters have homogeneous opinions inside the cluster and these opinions correspond to groups of non-connected clusters with similar but not exactly equal opinions as observed on the histogram (figure 7) and on the pattern of opinions on the lattice (figure 6). The differences of opinions between groups of clusters relates to $d$, but the actual values inside a group of clusters fluctuate from cluster to cluster because homogenisation occurred independently among the different clusters: the resulting opinion depends on fluctuations of initial opinions and histories from one cluster to the other. The same increase in fluctuations compared to the full mixing case is observed from sample to sample with the same parameter values.

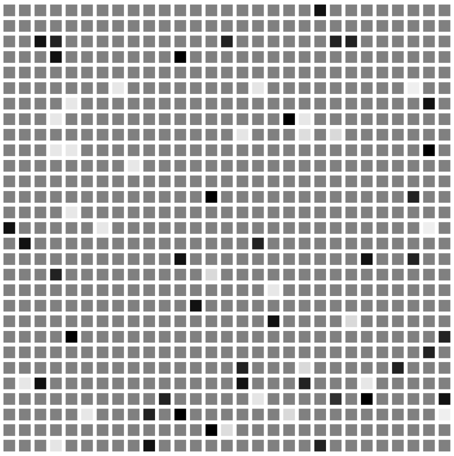

**Figure 5.** *Display of final opinions of agents connected on a square lattice of size 29x29 (*$d = 0.3 \quad \mu = 0.3$ *after 100 000 iterations) . Opinions between 0 and 1 are coded by gray level. Note the percolation of the large cluster of homogeneous opinion and the presence of isolated "extremists".*

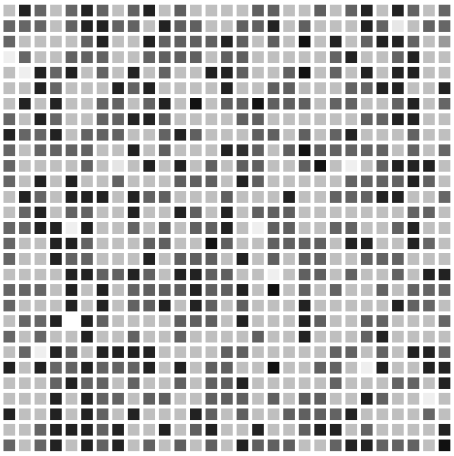

**Figure 6.** *Display of final opinions of agents connected on a square lattice of size 29x29 (*$d = 0.15 \quad \mu = 0.3$ *after 100 000 iterations) . One still observes a large percolating cluster of homogeneous opinion as well as the presence of smaller non-percolating clusters with similar but not identical opinions.*

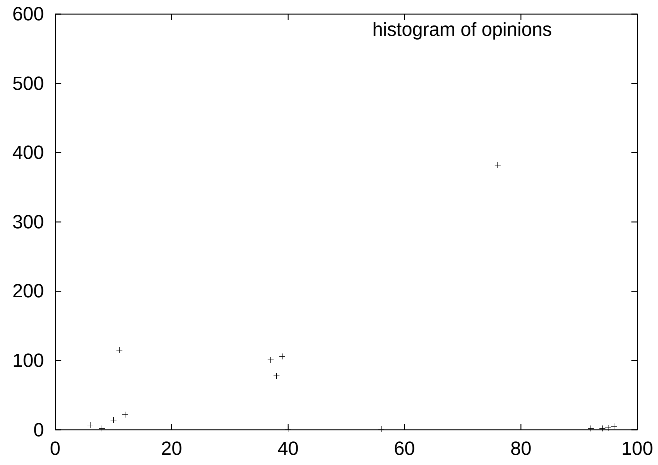


**Figure 7.** *Histogram of final opinions corresponding to the pattern observed in figure 6 . The 101 bins numbered from 0 to 100 correspond to a hundred times the final opinions which vary between 0 and 1.0. Note the high narrow peak at 76 which corresponds to the percolating cluster and the smaller and wider peaks which correspond to non-percolating clusters.*

The qualitative results obtained with 2D lattices should be observed with any connectivity, either periodic, random, or small world since they are related with the percolation phenomenon [STA 94].

## 4. Vector opinions

### 4.1. *The model*

Another subject for investigation is vectors of opinions. Usually people have opinions on different subjects, which can be represented by vectors of opinions. In accordance with our previous hypotheses, we suppose that one agent interacts concerning different subjects with another agent according to some distance with the other agent's vector of opinions. In order to simplify the model, we revert to binary opinions. An agent is characterised by a vector of $m$ binary opinions about the complete set of $m$ subjects, shared by all agents. We use the notion of Hamming distance between binary opinion vectors (the Hamming distance between two binary opinion vectors is the number of different bits between the two vectors). Here, we only treat the case of complete mixing; any pair of agents might interact and adjust opinions according to how many opinions they share. The adjustment process occurs when agents agree on at least $m - d$ subjects (i. e. they disagree on $d - 1$ or fewer subjects). The rules for adjustment are as follows: all equal opinions are conserved; when opinions on a subject differ, one agent (randomly selected from the pair) is convinced by the other agent with probability $\mu$. Obviously this model has connections with population genetics in the presence of sexual recombination when reproduction only occurs if genome distance is smaller than a given threshold. Such a dynamics results in the emergence of species (see [HIG 91]).

We are again interested in the clustering of opinion vectors. In fact clusters of opinions here play the same role as biological species in evolution. A first guess is that vector opinions dynamics might be intermediate between the binary opinion case and continuous opinions.

### 4.2. *Results*

We observed once again that $\mu$ and $N$ only modify convergence times towards equilibrium; the most influencial factors are threshold $d$ and $m$ the number of subjects under discussion. Most simulations were done for $m = 13$. For $N = 1000$, convergence times are of the order of 10 million pair iterations. For $m = 13$:

– When $d > 7$, the radius of the hypercube, convergence towards a single opinion occurs (the radius of the hypercube is half its diameter which is equal to 13, the maximum distance in the hypercube).

– Between $d = 7$ and $d = 4$ a similar convergence is observed for more than 99.5 per cent of the agents with the exception of a few clustered or isolated opinions distant from the main peak by roughly 7.

– For $d = 3$, one observes from 2 to 7 significant peaks (with a population larger than 1 per cent) plus some isolated opinions.

– For $d = 2$ a large number (around 500) of small clusters is observed (The number of opinions is still smaller than the maximum number of opinions within a distance of 2).

The same kind of results are obtained with larger values of $m$: two regimes, uniformity of opinions for larger $d$ values and extreme diversity for smaller $d$ values, are separated by one $d_c$ value for which a small number of clusters is observed (e.g for $m = 21, d_c = 5$. $d_c$ seems to scale in proportion with $m$ ).

Since there is no a priori reference opinion as in the previous cases of continuous opinions, information about the distribution of opinions is obtained from the histogram of distances among couples of opinions represented in figure 8. The results of the two next figures were obtained by averaging over 200 samples.

After all agents have been involved in 1000 possible exchanges of opinions on average, most pairs of opinions are different. One important result is that polarisation of opinions (opposite vectors of opinions) is not observed. The clustering process rather results in orthogonalisation of opinions with an average distance around 6: opinion vectors have no correlation, positive or negative, whatsoever. Similar results were observed concerning distances of binary strategies in the minority game [MAR 97].

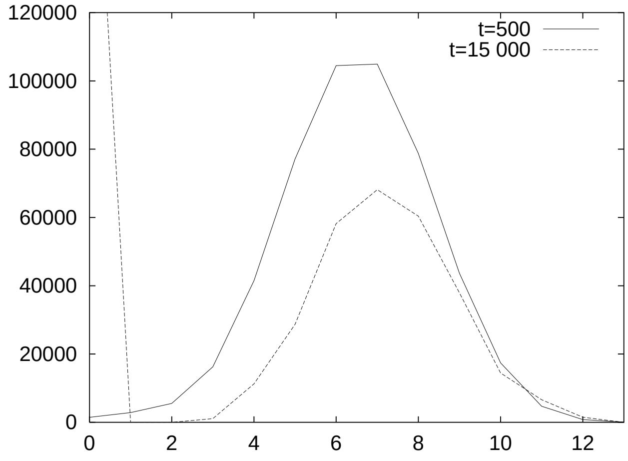


**Figure 8.** *Histogram of distances in vector opinions for* $N = 1000$ *agents and thus 500 000 pairs of agents. (*$d = 3, \quad \mu = 1$*). Times are given in 1000 iterations units.*

We were also interested in the populations of the different clusters. Figure 9 represents these populations at equilibrium (iteration time was 12 000 000) in a log-log plot according to their rank-order of size. No scaling law is obvious from these plots, but we observe the strong qualitative difference in decay rates for various thresholds $d$.

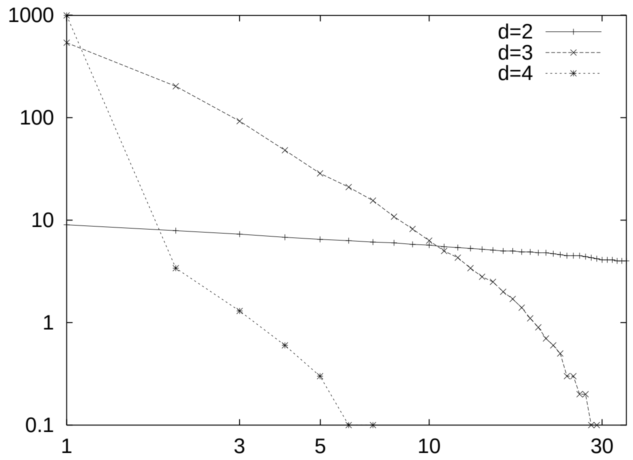


**Figure 9.** *Log-log plot of average populations of clusters of opinions arranged by decreasing order. for* $N = 1000$ *agents (*$\mu = 1$*).*

## 5. Conclusions

We thus observe than when opinion exchange is limited by similarity of opinions among agents, the dynamics yield isolated clusters among initially randomly distributed opinions. In the end, exchange only occurs inside clusters as a the result of the exchange dynamics; initially all agents were communicating either directly or indirectly through several directly connected agents. The concertation process as described here is sufficient to ensure clustering even in the absence of difference in private interests or in experience about the external world.

We have studied three very basic models and observed the same clustering behaviour, at least for some parameter regimes, which make us believe that the observed clustering is robust and should be observed in more complicated models, not to mention political life! Many variations and extensions can be proposed, including further opinion selection according to experience with some external world (“social reinforcement learning”). An interesting extension would be a kind of historical perspective in which subjects (or problems) would appear one after the other: position and discussions concerning an entirely new problem would then be conditioned by the clustering resulting from previous problems.

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**References**

Arthur, B. W. (1994) “Increasing Returns and Path Dependence in the Economy”, (University of Michigan Press).

Föllmer H. (1974) ”Random Economies with Many Interacting Agents”, Journal of Mathematical Economics, 1/1, 51-62.

Galam S. (1997) ”Rational group decision making: A random field Ising model at T=0”, Physica A, 238, 66-80.

Higgs P.G. and Derrida, B. (1991) “Stochastic models for species formation in evolving populations”, J. Phys. A: Math. Gen., 24, 985-991.

Laslier, J.F. (1989) “Diffusion d’information et évaluations séquentielles” Economie appliquée.

Latané, B. and Nowak, A. (1997) ”Self- Organizing Social Systems: Necessary and Sufficient Conditions for the Emergence of Clustering, Consolidation and Continuing Diversity”, in Barnett, G. A. and Boster, F. J. (eds.) Progress in Communication Sciences.

Neau, D (2000), “Révisions des croyances dans un système d'agents en interaction”, rapport d'option de l'école polytechnique, available at http://www.lps.fr/ weisbuch/rapneau.ps.

Orléan A. (1995), ”Bayesian Interactions and Collective Dynamics of Opinions: Herd Behavior and Mimetic Contagion”, *Journal of Economic Behavior and Organization*, 28, 257-274.

Stauffer D. and Aharony A., (1994) “Introduction to Percolation Theory”, Taylor and Francis, London.

Weisbuch G. and Boudjema G. (1999), “Dynamical aspects in the Adoption of Agri-Environmental Measures”, Adv. Complex Systems, 2, 11-36.