BRANCHING INSTABILITY OF BRITTLE FRACTURE

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Summary A new method for determining the elastodynamic stress fields associated with the propagation of antiplane branched cracks is presented. The exact dependence of the stress intensity factor just after branching is given as a function of the stress intensity factor just before branching, the branching angle and the instantaneous velocity of the crack tip. The jump in the dynamic energy release rate due to the branching process is also computed. Applying a growth criterion for a branched crack, it is shown that the minimum speed of the initial single crack which allows branching is equal to $0.39c$, where c is the shear wave speed. At the branching threshold, the corresponding bifurcated cracks start their propagation at a vanishing speed with a branching angle of approximately 40 degrees. Using these exact results, the branching of a single propagating crack under mode I loading is also considered and the critical velocity for branching is computed.

INTRODUCTION

An issue of great interest in crack dynamics is the question of understanding the instabilities associated with the appearance and evolution of kinked or bifurcated cracks. We are particularly interested in this phenomenon in brittle materials, in which several experiments [1, 2, 3] have shown evidence of these instabilities. A main experimental distinction can be made between micro–branches and macro–branches. The fact is that over a crack velocity threshold, the so called micro–branching instability sets in : the broken surface left by the crack shows a dense array of micro–kinks that were incipient cracks that aborted quickly their inroad into the material. The angle that these micro–cracks make with the main crack direction depends on the scale at which these structures are observed. Also, there is the other experimental subset corresponding to macro–branches, that could occur at velocities of the crack of the order of $0.5 - 0.6c_R$ (the threshold for this instability is not precise) and that usually initiates the total rupture of the sample.

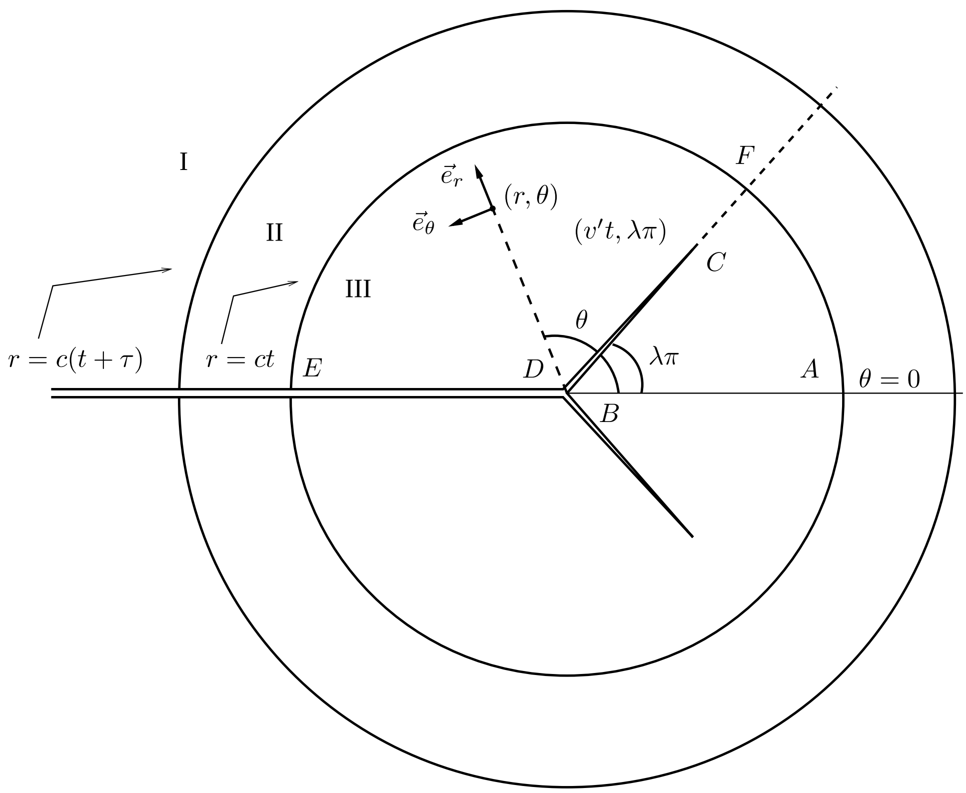

Figure 1. A half–plane crack that propagates at a speed v for $t < -\tau$ suddenly stops at $t = -\tau$. For $t > 0$, the new branches propagate straightly from the position $r = 0$ at a velocity v', following the directions $\pm\lambda\pi$. The orthonormal basis $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$ corresponding to the cylindrical coordinate system (r, θ, z) is shown. The cylindrincal waves originated at the crack's arrest ($r = c(t + \tau)$) and at the start of branches propagation ($r = ct$) divide the material into three regions. In the region labelled I, the stress field comes only from the dynamical straight crack propagation, since the effects of the crack's arrest and the crack's branching are not experienced there. The region II is influenced by the crack arrest only, thus the stress field there is a static one [4, 6]. It is only the region III that is influenced by the crack's branching. Thus, one has to solve the branched crack problem only in this latter region.

Given this experimental phenomenology, a theoretical approach that has been suggested is to describe kinked or bifurcated cracks in the framework of Linear Elastic Fracture Mechanics [4, 5], which has proven its validity for single straight crack dynamics in brittle materials [6, 7, 8]. The main goal of this approach is to obtain expressions for the stress intensity factors at the crack tips of the emergent bifurcated cracks. The idea is to compare their associated energy release rates with the unperturbed configuration, i.e. a crack from which no branches emanated. The dependence of these energy release rates in the crack kink angles, velocities, etc.. may elucidate the origin of the instability. This line of work involves solving a dynamic linear elasticity problem where the crack boundaries develop a kinked shape that makes the theoretical analysis quite involved. Our final goal is to solve the problem of determining the stress intensity factors of a kinked crack that starts at a given angle and velocity from a main crack whose velocity is also given. The present work is an intermediate step towards that final goal.

The aim of this presentation is to present a new method to calculate the dynamic stress intensity factors associated with the propagation of anti–plane kinked or branched cracks [9, 10]. This approach allows the exact calculation of the corresponding dynamic stress intensity factors. The latter are very important quantities in dynamic brittle fracture mechanics, since they determine the crack path and eventual branching instabilities. As an illustration, we consider the configuration depicted in Fig. 1 of an initially propagating antiplane crack that branches into two cracks that merge symmetrically. The explicit dependence of the stress intensity factor just after branching is given as a function of the stress intensity factor just before branching, the branching angle and the instantaneous velocity of the crack tip. The jump in the dynamic energy release rate due to the branching process is also computed. Similarly to the single crack case, a growth criterion for a branched crack is applied. It is based on the equality between the energy flux into each propagating tip and the surface energy which is added as a result of this propagation. It is shown that the minimum speed of the initial single crack which allows branching is equal to $0.39c$, where c is the shear wave speed. At the branching threshold, the corresponding bifurcated cracks start their propagation at a vanishing speed with a branching angle of approximately 40 degrees.

Using these exact results, the branching of a single propagating crack under mode I loading is also considered. It is shown that the formulation of the corresponding model problem is identical to the anti–plane case. The difficulty for solving completely the branching problem under plane loading configurations lies in the existence of two characteristic wave speeds and in the vectorial nature of the displacement field. However, this analogy allows the reasonable hypothesis that under plane loading configurations, the jump in the energy release rate due to branching is also maximized when the branches start to propagate quasi–statically. Using the Eshelby's approach, the branching of a single propagating crack under mode I loading is found to be energetically possible when its speed exceeds a threshold value. The critical branching parameters depend of the criterion applied for the selection of the paths followed by the bifurcated cracks. For instance, the maximum energy release rate criterion [11] or the principal of local symmetry [12]. It is found that the critical velocity for branching of the initial single crack is weakly sensitive to the applied criterion. However, the principal of local symmetry gives a larger branching angle, which is more consistent with experimental observations. Finally, it is shown that an increasing fracture energy with the velocity results in a decrease in the critical velocity at which branching is energetically possible.

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