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Self-sustained slip pulses of finite size between dissimilar materials

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Abstract

The problem of a steady-state slip pulse of finite size between dissimilar materials is studied. It is shown that for a Coulomb friction law, there is a continuous set of possible solutions for any slip propagation velocity and any slip length. These solutions are, however, nonphysical because they show a singular behaviour of the slip velocity at one extremity of the pulse, which implies a crack-like behaviour. In order to regularize these solutions, we introduce a modified friction law due to Prakash and Clifton (Experimental Techniques in the Dynamics of Deformable Solids, Vol. AMD-165, pp. 33–48; J. Tribol. 120 (1998) 97), which consists in introducing in the Coulomb friction law a relaxation time for the response of the shear stress to a sudden variation of the normal stress. Then, we show that even for a slip velocity-dependent characteristic time, the degeneracy of the solutions is not suppressed and a physical pulse is not selected. This result shows the absence of steady state self-healing pulses within the modified friction law and is consistent with recent finite-difference calculations (J. Geophys. Res. 107 (2002) 10).

Keywords: A. Dynamics; B. Friction; Elastic material; C. Well-posedness

1. Introduction

Ruptures in fault zones separating dissimilar materials may provide a naturally unified explanation to some fundamental observations on earthquake and fault behaviour (Andrews and Ben-Zion, 1997; Ben-Zion and Andrews, 1998; Ben-Zion and Huang, 2002; Cochard and Rice, 2000). Effectively, there is a number of problems that are

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not completely explained in terms of ruptures in a homogeneous solid and that might be related to ruptures along bi-material interfaces. The interest of bi-material studies in seismology is reinforced by the fact that many earthquakes seem to have rise times much shorter than would be expected from classical crack models (Heaton, 1990). Furthermore, processes induced by heterogeneous fault zones allow earthquake ruptures to propagate at shear stresses which are low compared to friction threshold. This provides a possible explanation to the apparent lack of observed heat flow from some major faults (Lachenbruch and Sass, 1992). See the introduction of Andrews and Ben-Zion (Ben-Zion and Andrews, 1998) and of Cochard and Rice (2000) for a thorough discussion of other involved issues.

Weertman (1980) has shown that a coupling between slip and normal stress exists in a frictional interface between dissimilar materials. He concluded that a self-healing pulse can propagate along the frictional interface between dissimilar elastic solids, even when the remote shear stress is less than the frictional stress of the interface. A family of steady-state pulses at a bi-material interface under Coulomb friction law has been computed by Adams (1995, 1998, 2001). However, he has also shown that these solutions are often linearly unstable (Adams, 1995). Ranjith and Rice (2001) have shown a connection between the existence of the generalized Rayleigh wave speed and the ill-posed nature of the problem. When the material pair is such that the generalized Rayleigh wave speed is defined, the problem is ill-posed for any value of the friction coefficient, whereas when it is not defined the problem remains ill-posed for values of the friction coefficient larger than a critical value.

In a numerical study, Andrews and Ben-Zion (Andrews and Ben-Zion, 1997; Ben-Zion and Andrews, 1998) examined wrinkle like propagation using Coulomb friction law, and encountered numerical problems. Cochard and Rice (2000) found that the Adams instability was responsible for those numerical problems: the cases studied by Andrews and Ben-Zion fall precisely in the range in which the generalized Rayleigh wave speed is defined, and are thus certainly ill-posed. In order to regularize the problem, the Coulomb friction law has been replaced by an experimentally based friction law due to Prakash and Clifton (Prakash and Clifton, 1993; Prakash, 1998). This law smooths into a continuous transition with time or slip the otherwise instantaneous variation of shear strength that would follow from an instantaneous variation in normal stress if the Coulomb law was used. Ranjith and Rice (2001) have shown that this law can provide a regularization for the linear stability analysis. However, when solving the full time-dependent problem, the different numerical results (Andrews and Ben-Zion, 1997; Ben-Zion and Andrews, 1998; Ben-Zion and Huang, 2002; Cochard and Rice, 2000) do not all lead to a rupture generated by the propagation of steady state self-sustained slip pulses of finite size.

Once regularized, the physical problem is no longer exactly the same as it was originally when the Coulomb law was used. The main purpose of this paper is to provide a complete analytical study of this problem. In the next section, we present the formulation of the steady state slip pulse problem. In Section 3, we show that for a Coulomb friction law, there is a continuous set of solutions for any slip propagation velocity and any slip length. As expected, these solutions turn out to be nonphysical. In Section 6, we use the so-called Prakash–Clifton friction law in order to regularize these solutions. Then, we show that this law *does not suppress* the degeneracy of the solutions and *does not select* a physical pulse.

2. The steady-state slip pulse problem

We consider the dynamic problem of 2D in-plane slip (plane strain deformation, Mode II rupture) along a frictional interface on the plane $Y = 0$ separating two linear isotropic elastic half-spaces (Fig. 1).

The loading, particle motion, and rupture propagation are in the X-direction and all variables are functions of X, Y and t only. Shear and dilatational wave velocities are $c_{sn} = \sqrt{2\mu_n/\rho_n}$ and $c_{dn} = \sqrt{(\lambda_n + 2\mu_n)/\rho_n}$, where ρ_n is mass density, λ_n and μ_n are Lamé coefficients, and subscripts $n = 1, 2$ denote the top ($Y > 0$) and bottom ($Y < 0$) materials, respectively. Shear and normal stresses on the fault are $\tau(X,t) = \sigma_{xy}(X, Y = 0, t)$ and $\sigma(X,t) = \sigma_{yy}(X, Y = 0, t)$. Applied shear stress and normal stress at the remote boundaries are τ^∞ and $-\sigma^\infty$, such that $\tau^\infty < f\sigma^\infty$, where f is the Coulomb dynamic coefficient of friction. Slip and slip velocity across the fault are $\delta(X,t) = u_x(X, Y = 0^+, t) - u_x(X, Y = 0^-, t)$ and $V(X,t) = \partial\delta/\partial t$.

Let us consider an in-plane rupture that propagates with a constant subsonic velocity c along a material interface. Rupture propagation occurs in this problem only in one direction, that of slip in the more compliant material. Since the problem is steady state, the solution depends only on $x = X - ct$ and Y. The shear and normal stress on the fault in the solution of Weertman are (Weertman, 1980)

$$\sigma(x) = -\sigma^\infty + \frac{\mu^*}{c} V(x), \tag{1}$$

$$\tau(x) = \tau^\infty - \frac{\bar{\mu}}{c} \int_{-\infty}^{\infty} \frac{V(x')}{x' - x} \frac{\mathrm{d}x'}{\pi}, \tag{2}$$

where the integral is taken in the sense of Cauchy principal value, and

$$\mu^* = \frac{1}{\Delta}[(1 + b_1^2 - 2a_1 b_1)\,\mu_2 D_2 - (1 + b_2^2 - 2a_2 b_2)\,\mu_1 D_1], \tag{3}$$

$$\bar{\mu} = \frac{1}{\Delta}\,[(1 - b_1^2)\,a_1 \mu_2 D_2 + (1 - b_2^2)\,a_2 \mu_1 D_1] \tag{4}$$

with

$$a_n = \sqrt{1 - c^2/c_{dn}^2}, \tag{5}$$

$$b_n = \sqrt{1 - c^2/c_{sn}^2}, \tag{6}$$

$$D_n = 4a_n b_n - (1 + b_n^2)^2 \tag{7}$$

and Δ is a known function of the propagation velocity and of the materials parameters (Weertman, 1980), whose explicit behaviour does not influence the subsequent analysis.

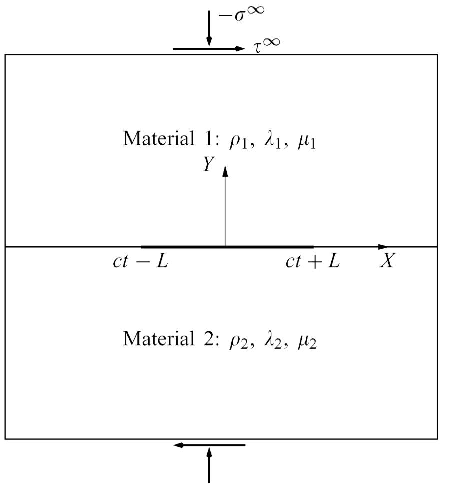

Fig. 1. The model problem for 2D in-plane steady-state rupture along the interface between different elastic solids.

The coefficient $\bar{\mu}$ decreases with increasing c and its zero defines a generalized Rayleigh speed, $c_{\rm GR}$, for the materials pair. When the two materials are the same, $c_{\rm GR}$ reduces to the regular Rayleigh velocity. As the velocity contrast increases, the zero intersects of $\bar{\mu}$ increases and for large enough contrast $\bar{\mu}$ is positive for all subsonic values of c and $c_{\rm GR}$ is not defined. A steady-state Weertman pulse propagating at $c = c_{\rm GR}$, when such a speed exists, produces no changes of shear stress on the fault. When the two materials are identical, $\mu^* = 0$, and there is no coupling between slip and changes of normal stress on the fault. However, when the two materials differ, $\mu^* > 0$ and nonuniform slip produces a dynamic reduction of normal stress that is proportional to the local slip velocity.

Eqs. (1) and (2) result from applying the conditions of continuity of normal displacements and shear and normal stresses along the interface $Y = 0$. These boundary conditions allow to write the stresses at the interface as functions of the slip velocity only. In order to solve the slip pulse problem, one has to prescribe the slip conditions and/or the friction law along the interface. We impose that the pulse has a finite size $2L$. Thus outside this region, the slip velocity $V(x)$ identically vanishes,

$$V(x) = 0, \qquad |x| > L. \tag{8}$$

Along the pulse, a friction law which relates the shear stress to the normal loading at the interface should be prescribed. For this, we use a simplified version of the Prakash–Clifton friction law (Prakash and Clifton, 1993; Prakash, 1998), which consists on introducing in the Coulomb friction law a slip velocity dependent relaxation time $t_0(V)$ for the response of the shear stress to a sudden variation of the normal stress

$$-\,t_0(V)\frac{\mathrm{d}}{\mathrm{d}t}\,\tau(x) \equiv L_0(V)\frac{\mathrm{d}}{\mathrm{d}x}\,\tau(x) = \tau(x) + f\sigma(x), \qquad |x| < L, \tag{9}$$

where $L_0 \equiv ct_0$ is a characteristic length scale. A thorough discussion of the Prakash–Clifton law can be found in (Cochard and Rice, 2000; Ranjith and Rice, 2001). Note that if renewal of the asperity contact population is the underlying mechanism leading to loss of frictional "memory" of prior strength, then one expects that $t_0(V)$ should vary inversely with V at high slip rates, basically as $L^*/|V|$, where L^* is a characteristic sliding distance to renew the contact population. In order to match the behaviour at high slip rates without introducing singular behaviour at low slip rates, the Prakash–Clifton law used in the previous studies (Ben-Zion and Huang, 2002; Cochard and Rice, 2000; Ranjith and Rice, 2001) replaces the characteristic time scale $t_0(V)$ by a slip velocity dependent function $L^*/(V^* + |V(x)|)$, where L^* and V^* are characteristic length and velocity scales.

The slip velocity can now be determined from Eqs. (1), (2), (8) and (9). Let us take L as the length scale, and define a nondimensioned slip velocity $S(x)$ by

$$V(x) = \frac{(f\sigma^\infty - \tau^\infty)c}{\mu^* f} S(x). \tag{10}$$

Note that for similar materials the scaling (10) is not adequate, since μ^* vanishes in this case. Using Eqs. (1) and (2), conditions (8) and (9) become

$$S(x) = 0, \quad |x| > 1, \tag{11}$$

$$S(x) = 1 + K\left(1 - \eta(S)\frac{\mathrm{d}}{\mathrm{d}x}\right)\int_{-1}^{1} \frac{S(x')}{x' - x}\frac{\mathrm{d}x'}{\pi}, \quad |x| < 1, \tag{12}$$

where $\eta(S)$ and K are the pertinent parameters of the present problem. They are defined by

$$\eta(S) = \frac{L_0}{L}, \tag{13}$$

$$K(c) = \frac{\overline{\mu(c)}}{f\mu^*(c)}. \tag{14}$$

The function $\eta(S)$ can be seen as the inverse of the pulse size in units of the characteristic length scale L_0. The influence of the elastic parameters of the two materials are embedded in the parameter $K(c)$, whose behaviour is shown in Fig. 2. When the generalized Rayleigh speed is not defined, $K(c)$ is always positive. On the other hand, when the generalized Rayleigh speed exists, $K(c)$ is positive for a wide range of propagation velocities $0 < c < c_{\mathrm{GR}}$, and it takes finite negative values for propagation velocities $c_{\mathrm{GR}} < c \leqslant c_{\mathrm{s1}}$.

The main equations of this section are well known. They have been used for stability studies of homogeneous slips along the interface between different materials (Adams, 1995, 1998, 2001; Ranjith and Rice, 2001). In contrast, our goal consists in finding the properties of inhomogeneous slips along the interface, by introducing a slip length which is different from the fault size. Of course, the final goal would be to relate the properties of the self-sustained slip pulses to the self-healing pulses deduced by Heaton from geological observations (Heaton, 1990). Indeed, recent numerical studies (Andrews and Ben-Zion, 1997; Ben-Zion and Andrews, 1998; Ben-Zion and Huang,

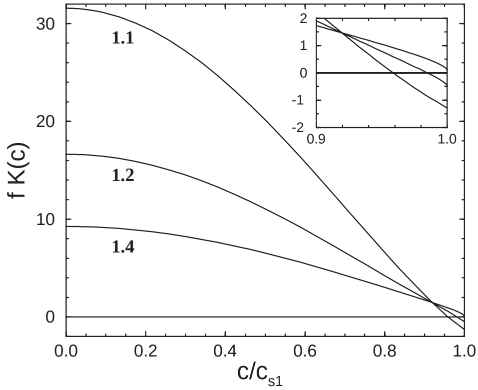

Fig. 2. The behaviour of $K(c)$, defined by Eq. (14), as a function of the subsonic propagation speed c, for ratios of c_{s2}/c_{s1} equal to 1.1, 1.2, and 1.4. The other materials coefficients are taken such that $\mu_1=\mu_2,\ c_{d1}=\sqrt{3}c_{s1},\ c_{d2}=\sqrt{3}c_{s2}$. The inset is a close-up of the curves in the region $0.9c_{s1}\leqslant c\leqslant c_{s1}$.

2002; Cochard and Rice, 2000) have been controversial about the conditions of existence of steady-state self-sustained slip pulses of finite size. It was found in (Ben-Zion and Huang, 2002) that the self-sharpening and divergent behaviour found earlier by Cochard and Rice (2000) with Coulomb friction law exists also with regularized friction for large enough propagation distance, or equivalently for long times. The parameters of the regularized friction law had to be fine tuned to produce apparent stability for a given propagation distance. However, eventually, the pulse always dies or diverges.

When the material pair is such that the generalized Rayleigh wave speed is defined, Adams (1998) has shown that in the framework of the classical Coulomb friction law, there exists a continuous family of steady-state pulses at a bi-material interface propagating at $c=c_{GR}$. However, Ranjith and Rice (2001) have shown that these solutions are linearly unstable for any value of the friction coefficient. In the following, we look for possible steady-state solutions by focussing on the cases when $K(c)\neq 0$. We study the singular integro-differential Eqs. (11) and (12), using a pure Coulomb friction law ($\eta=0$), and the Prakash–Clifton law ($\eta\neq 0$) for two model cases: $\eta(S)=\eta_0$, $\eta(S)=\eta_0+\eta_1 S$, where η_0 and η_1 are constants. We also point out that the use of a more general form of $\eta(S)$ give similar results to the latter cases. Note, however, that the possible solutions of Eqs. (11) and (12) always coexist with the trivial solution $S(x)=0$ for all x, since we prescribed $\tau^\infty < f\sigma^\infty$. Then, the absence of solutions for these equations implies an absence of slipping along the whole interface.

3. The Coulomb friction law (η = 0)

For this case, Eq. (12) is reduced to

$$S(x)=1+K\int_{-1}^{1}\frac{S(x')}{x'-x}\,\frac{\mathrm{d}x'}{\pi},\qquad |x|<1. \tag{15}$$

Using usual techniques of singular integral equations (Muskhelishvili, 1953), the solution of Eq. (15) is straightforward. Define a complex function $F(z)$; $z = x + iy$; such that

$$F(z) = \int_{-1}^{1} \frac{S(x)}{x - z} \frac{dx}{2i\pi}, \tag{16}$$

whose behaviour for $|z| \to \infty$ is readily given by

$$F(z) \sim -\frac{1}{2i\pi z} \int_{-1}^{1} S(x)\,dx. \tag{17}$$

The function $F(z)$ is holomorphic everywhere except on the interval $[-1, 1]$ of the real axis, where it satisfies

$$F(x + i0) - F(x - i0) = S(x), \tag{18}$$

$$F(x + i0) + F(x - i0) = \int_{0}^{1} \frac{S(x')}{x' - x} \frac{dx'}{i\pi}. \tag{19}$$

Then combining these two conditions with Eq. (15) yields

$$e^{-i\alpha} F(x + i0) - e^{i\alpha} F(x - i0) = \cos\alpha, \tag{20}$$

where the parameter α is related to K by

$$\tan\alpha = K, \quad -\pi/2 < \alpha < \pi/2. \tag{21}$$

The holomorphic function that satisfies the jump condition (20) and the asymptotic behaviour (17) is readily given by (Muskhelishvili, 1953)

$$F(z) = \frac{i}{2\tan\alpha} \left[1 - \left(\frac{z - 1}{z + 1} \right)^{\alpha/\pi} \right] \tag{22}$$

and the solution $S(x)$ follows directly from Eq. (18)

$$S(x) = \cos\alpha \left(\frac{1 - x}{1 + x} \right)^{\alpha/\pi}, \quad |x| < 1. \tag{23}$$

Therefore, a Coulomb friction law leads to a continuous set of solutions, where neither the length of the pulse nor its propagation velocity are selected. For each value of the parameter α, corresponding to a given value of K, there exists a "mathematical" solution $S(x)$ satisfying the Coulomb friction law. However, these solutions are clearly nonphysical ones, since $S(x)$ diverges near $x = -1$ (resp. $x = 1$) for $\alpha > 0$ (resp. $\alpha < 0$). Moreover, as seen in Fig. 3, due to the singularity of the slip velocity, the normal stress changes its sign, which induces an opening loading, and thus a crack-like behaviour, in a certain region of the pulse. This clearly violates the boundary condition of continuity of the normal displacement embedded in the solution of Weertman (1980). Therefore, a pure Coulomb friction law is inconsistent with a condition of slip without opening.

Moreover, when the material pair is such that the generalized Rayleigh wave speed is defined, one has $\alpha = 0$. For this special case, Eq. (23) shows that $S(x) = 1$ for

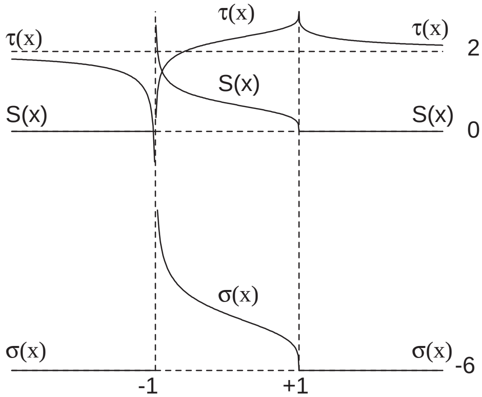

Fig. 3. The behaviour of the slip velocity $S(x)$, the normal and shear stress $\sigma(x)$ and $\tau(x)$ at the interface. The values are taken such that $\alpha = \pi/4$, $\tau^{\infty} = \sigma^{\infty}/3$ and $f = 0.5$.

$|x| < 1$ and it is discontinuous at $x = \pm 1$. The corresponding solutions do not contain infinities of the normal stress or slip rate. Such discontinuities are acceptable since they propagate at the relevant wave speed. Therefore, there exists a continuous family of steady-state pulses at a bi-material interface propagating at $c = c_{\mathrm{GR}}$ (Adams, 1998; Rice, 1997). However, Ranjith and Rice (2001) have shown that these solutions are linearly unstable for any value of the friction coefficient.

In the following section, we will use the Prakash–Clifton friction law as a possible regularizing procedure of the simple Coulomb friction law. However, we will see that even when using a slip velocity-dependent characteristic time, the degeneracy of the solutions, found for $\eta = 0$, is not suppressed and a physical pulse cannot be selected.

4. The Prakash–Clifton friction law ($\eta \neq 0$)

The presence in Eq. (12) of the differential operator, $\partial/\partial x$, introduces an additional degree of freedom in the problem, since one has to fix a constant of integration; this operator does not appear in the case of the Coulomb friction law ($\eta = 0$). These two ingredients together are a signature of a possible eigenvalue problem, where in the case of the existence of solutions, the parameter K should be determined as a function of η.

For $\eta \neq 0$, we assume that the propagation speed does not coincide with c_{GR}, and thus $K(c) \neq 0$. Moreover, we do not explore solutions which allow loss of contact along the interface, since we are interested in studying the Weertman pulse (Weertman, 1980) for which the normal displacement is continuous. Since Eq. (11) imposes that the slip velocity vanishes outside the rupture region, one expects continuous matching of $S(x)$ at the rupture edges. Indeed, the slip rate $S(x)$, which by definition is always positive, should not diverge at $x = \pm 1$, since it leads to an opening loading $\sigma(x)$, which violates the boundary condition of continuity of the normal displacement. Moreover, if $S(\pm 1) \neq 0$, the integral part of Eq. (11) induces a singular logarithmic behaviour in the vicinity of the rupture edges. Therefore, necessary conditions for physical solutions are given by

$$S(\pm 1) = 0. \tag{24}$$

These conditions will be used to fix the integration constant of Eq. (30), and the parameter K as functions of η. Therefore, if physical solutions exist, the pulse size would be selected as a function of its propagation speed and of the physical parameters. This is in contrast with the case $\eta = 0$, where K was undetermined and the continuity conditions of the slip velocity could not be satisfied simultaneously at the two rupture edges. This problem is similar to the so-called Saffman–Taylor problem (McLean and Saffman, 1981), where a fluid penetrates into a thin cell that contains a more viscous liquid. The introduction of a nonzero η in the present problem is similar to the introduction of capillary effects in the Saffman–Taylor problem, which suppresses the degeneracy of the solutions found at vanishing capillary number (McLean and Saffman, 1981).

Let us first fix the asymptotic behaviour of $S(x)$ in the vicinity of the endpoints $x = \pm 1$. Conditions (24) impose to $S(x)$ to behave as

$$S(x) \sim (1+x)^{\beta_0}, \quad x \to -1, \tag{25}$$

$$S(x) \sim (1-x)^{\beta_1}, \quad x \to 1, \tag{26}$$

where β_0 and β_1 are real positive constants. Without loss of generality, we can prescribe that $0 < \beta_0 < 1$ and $0 < \beta_1 < 1$. Let us also note the following results:

$$\int_{-1}^{1} \frac{(1-x')^{\beta_1}}{x'-x}\,\mathrm{d}x' = -\frac{2^{\beta_1}}{\beta_1} + \pi(1-x)^{\beta_1}\cot\pi\beta_1 - \frac{2^{(\beta_1-1)}(1-x)}{\beta_1-1} + O((1-x)^2), \tag{27}$$

for $x \to 1$, and

$$\int_{-1}^{1} \frac{(1+x')^{\beta_0}}{x'-x}\,\mathrm{d}x' = \frac{2^{\beta_0}}{\beta_0} - \pi(1+x)^{\beta_0}\cot\pi\beta_0 + \frac{2^{(\beta_0-1)}(1+x)}{\beta_0-1} + O((1+x)^2), \tag{28}$$

for $x \to -1$. Using Eq. (30), one can determine the values of the constants β_0 and β_1. For this, one has to fix the form of $\eta(S)$. In the following, we will study some particular cases.

4.1. Case $\eta(S) = \eta_0$

For this simple case, Eq. (11) is transformed into

$$S(x) = 1 + K(c)\left(1 - \eta_0\frac{\mathrm{d}}{\mathrm{d}x}\right)\int_{-1}^{1}\frac{S(x')}{x'-x}\frac{\mathrm{d}x'}{\pi}, \quad |x| < 1. \tag{29}$$

It is easy to verify that whatever are the values of β_0 and β_1, Eq. (29) cannot be fullfiled in the vicinity of the endpoints $x = \pm 1$. This is due to the presence of the differential operator which gives the highest-order singular contribution that is not balanced by any other term in Eq. (29). Therefore, one concludes that a constant $\eta(S)$ in the Prakash–Clifton law does not allow physical solutions with the appropriate asymptotic behaviour imposed by conditions (24). In the following, we modify slightly the Prakash–Clifton law by introducing a weak nonlinearity in $\eta(S)$.

4.2. Case $\eta(S) = \eta_0 + \eta_1 S$

Then, Eq. (11) is transformed into

$$S(x) = 1 + K(c)\left(1 - (\eta_0 + \eta_1 S(x))\frac{\mathrm{d}}{\mathrm{d}x}\right)\int_{-1}^{1}\frac{S(x')}{x'-x}\frac{\mathrm{d}x'}{\pi}, \quad |x| < 1, \tag{30}$$

Using identities (27) and (28), one finds that Eq. (30) may admit solutions that satisfy conditions (24) if and only if

$$\beta_0 = \beta_1 = \tfrac{1}{2}, \tag{31}$$

which leads to a square root behaviour of the slip velocity at the rupture edges. Since the function $S(x)$ is defined in the interval $[-1, 1]$, one can decompose the slip velocity in terms of Chebyshev polynomials by writing

$$S(x) = \sqrt{1-x^2}\sum_{n=0}^{\infty} a_n U_n(x), \tag{32}$$

where $U_n(x)$ are the Chebyshev polynomials of the second kind. Let us recall the following Hilbert transform property of this class of polynomials

$$\int_{-1}^{1}\frac{\sqrt{1-x'^2}U_n(x')}{x'-x}\frac{\mathrm{d}x'}{\pi} = -T_{n+1}(x), \tag{33}$$

where T_n are Chebyshev polynomials of the first kind. Using decomposition (32) and identity (33), Eq. (30) becomes

$$\sqrt{1-x^2}\sum_{m=0}^{\infty} a_m U_m(x)\left[1 - K\eta_1\sum_{n=0}^{\infty} a_n T'_{n+1}(x)\right]$$
$$= 1 - K\sum_{n=0}^{\infty} a_n T_{n+1}(x) + K\eta_0\sum_{n=0}^{\infty} a_n T'_{n+1}(x). \tag{34}$$

Isolating the square-root behaviour from the integer power behaviour in Eq. (34) leads to the following identities:

$$K\eta_1\sum_{n=0}^{\infty} a_n T'_{n+1}(x) \equiv K\eta_1\sum_{n=0}^{\infty}(n+1)a_n U_n(x) = 1, \tag{35}$$

$$K\sum_{n=0}^{\infty} a_n T_{n+1}(x) - K\eta_0\sum_{n=0}^{\infty}(n+1)a_n U_n(x) = 1. \tag{36}$$

Eq. (35) admits the unique solution $K\eta_1 a_0 = 1$, and $a_n = 0$ for all $n > 0$. However, this solution is not satisfied by Eq. (36). Therefore, a weak nonlinearity in the friction law as given above is not sufficient for regularizing the slip pulse solution at the endpoints $x \pm 1$. Since the form $\eta(S) = \eta_0 + \eta_1 S$ can be seen as an expansion of any nonlinear behaviour one can wonder if the absence of physical solutions persists when one takes into account a more general friction law.

4.3. Case of a general $\eta(S)$

It is rather unlikely that the nonlinear integrodifferential equation (11) has a solution with enough regular endpoints. As an example, let us write

$$\eta(S) = \eta_0 + \eta_1 S(x) F(S(x)), \tag{37}$$

where η_0 and η_1 are arbitrary constants, and F is any function of $S(x)$ that satisfies $F(S(\pm 1)) \equiv F(0) = 1$. The asymptotic analysis in the vicinity of $x = \pm 1$ is similar to the case where $F(S(x)) = 1$ and Eqs. (35) and (36) are transformed into

$$K\eta_1 F(S(x)) \sum_{n=0}^{\infty} a_n T'_{n+1}(x) = 1, \tag{38}$$

$$K \sum_{n=0}^{\infty} a_n T_{n+1}(x) - K\eta_0 \sum_{n=0}^{\infty} a_n T'_{n+1}(x) = 1. \tag{39}$$

Introducing the function defined for $|x| < 1$:

$$Q(x) = \sum_{n=0}^{\infty} a_n T_{n+1}(x). \tag{40}$$

Eq. (39) is then a linear differential equation of first order for $Q(x)$ which has an explicit solution

$$Q(x) = \frac{1}{K}[1 - \exp(x/\eta_0)]. \tag{41}$$

Therefore, the coefficients in the series expansion following Eq. (40) can be determined, and so the function $S(x)$. On the other hand, Eq. (38) gives

$$F(S(x)) = -\frac{\eta_0}{\eta_1}\exp(-x/\eta_0). \tag{42}$$

At $x = \pm 1$, Eq. (42) gives $F(S(\pm 1)) = -(\eta_0/\eta_1)\exp(\mp 1/\eta_0)$, which is in contradiction with the condition $F(S(\pm 1)) \equiv F(0) = 1$. Therefore, we conclude that even within a general nonlinear friction law, solutions of finite size steady-state slip pulses are not allowed.

Finally, whatever the nonlinearities included in the friction law (we also checked a law of the form $\eta(S) = \eta_0/S$), excluding some peculiar and probably unphysical cases, it seems hopeless to find a regular solution to the steady-state slip pulse.

5. Discussion

We have studied the problem of the existence of solutions for steady-state slip pulse of finite size between dissimilar materials. We have shown that for a Coulomb friction law, there is a continuous set of solutions that are however nonphysical because they show a singular behaviour of the slip velocity. We have shown that even within the Prakash–Clifton friction law, the degeneracy of the solutions is not suppressed and a physical pulse is not selected. This analytical result is consistent with recent finite-difference calculations (Ben-Zion and Huang, 2002). Of course, when the material pair is such that the generalized Rayleigh wave speed is defined, there exists a family of steady-state pulses at a bi-material interface propagating at $c = c_{GR}$ (Adams, 1998; Rice, 1997). However, these solutions are nonphysical within a Coulomb friction law because they are linearly unstable (Ranjith and Rice, 2001).

When the two materials on each side of a planar fault are identical, unstable slip is impossible if the interface is governed by the classical Coulomb friction law; it requires more elaborate friction laws for which, under constant normal stress, the friction stress at some point decreases as the slip displacement or slip velocity increases (Perrin et al., 1995). A simple argument of the crack-like behaviour can be found in the steady-state slip pulse solution between similar materials. Using a pure Coulomb friction law, one can easily show that these solutions are given by

$$V(x) \propto \frac{1}{\sqrt{1-x^2}} \tag{43}$$

and the use of the Prakash–Clifton law will not regularize the problem. The presence of the square root singularity reflects such a crack-like behaviour, which means that once the slip pulse exceeds a critical length, it will propagate through the whole fault plane. Thus, such models cannot produce complexity since they introduce one characteristic length scale only; the nucleation size.

Our main conclusion is that the dissimilarity between the materials on each side of the planar fault is not sufficient to produce steady state self-sustained slip pulses of finite size, because it does not introduce an additional length scale against which the pulse size can be scaled. Two recent approaches have been proposed in order to explain the existence of self-healing slip pulses, by adding a new length scale in their models. The first approach assumes that rupture occurs within an interface between a compliant fault zone layer and a stiffer surrounding solid (Ben-Zion and Huang, 2002). The additional length scale in this approach being the thickness of the layer. The second approach does not impose a priori the continuity of the normal displacement in the rupture region (Gerde and Marder, 2001). This allows the rupture to occur by opening in certain regions and slipping in others.

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