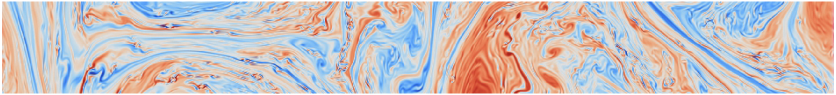
Turbulence
Kolmogorov Phenomenology



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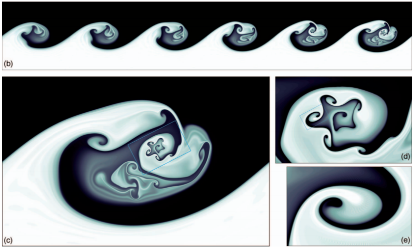
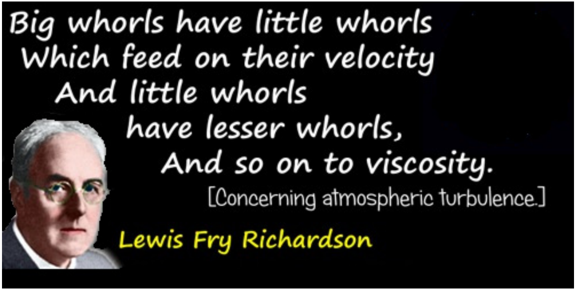
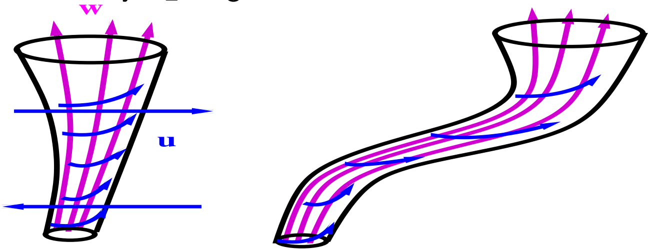
Richardson’s Poem





Vortex stretching

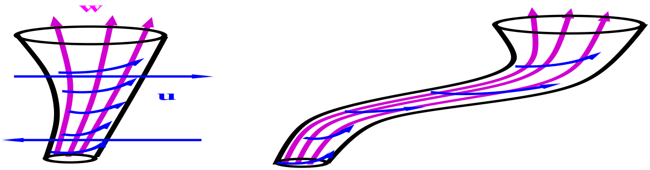
Consider an eddy of velocity u_ℓ and lengthscale ℓ being sheared by an eddy of velocity U_L lengthscale L



The rate energy moves to smaller scales is:

$$\frac{dE_\ell}{dt} \propto \frac{U_L}{L} u_\ell^2$$

Vortex stretching



The rate energy moves to smaller scales is:

$$\frac{dE_\ell}{dt} \propto \frac{U_L}{L} u_\ell^2$$

Assuming

1. the flux of energy across scales is constant and equal to ϵ
2. the most effective interactions are among similar size eddies

$$\epsilon \propto \frac{u_\ell^3}{\ell} \quad \text{or} \quad u_\ell \propto \epsilon^{1/3} \ell^{1/3}$$

Fourier Space (Finite box)

$$\mathbf{u}(\mathbf{x},t) = \sum \tilde{\mathbf{u}}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad \tilde{\mathbf{u}}_{\mathbf{k}}(t) = \left\langle \mathbf{u} e^{-i\mathbf{k}\cdot\mathbf{x}} \right\rangle$$

Energy Spectrum:

$$E(k) = \frac{1}{2\delta k} \sum_{k \le |\mathbf{k}| < k+\delta k} |\tilde{\mathbf{u}}_{\mathbf{k}}|^2 \sim \frac{\text{Energy}}{\text{per unit wavenumber}}$$

$$k \propto 1/\ell, \qquad E(k)k \propto u_\ell^2$$

$$\boxed{E(k) \propto \epsilon^{2/3} k^{-5/3}}$$

Kolmogorov's Spectrum!

Kolmogorov scale

$$\boxed{E(k) \propto \epsilon^{2/3} k^{-5/3}}$$

Viscosity will become important when

$$\epsilon \propto \nu \frac{u_\nu^2}{\ell_\nu^2} \propto \nu \frac{(\epsilon^{1/3}\ell_\nu^{1/3})^2}{\ell_\nu^2} \propto \nu \frac{\epsilon^{2/3}}{\ell_\nu^{4/3}}$$

$$\boxed{\ell_\nu = \frac{\nu^{3/4}}{\epsilon^{1/4}}} \quad \text{or} \quad \boxed{k_\nu = \frac{\epsilon^{1/4}}{\nu^{3/4}}}$$