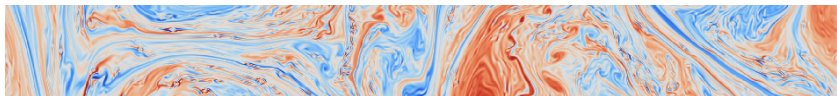


Turbulence

Kolmogorov Phenomenology

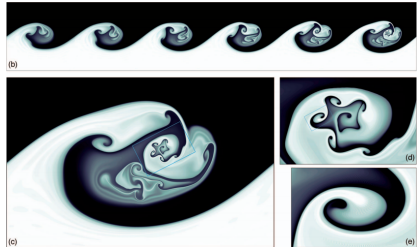


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Richardson's Poem



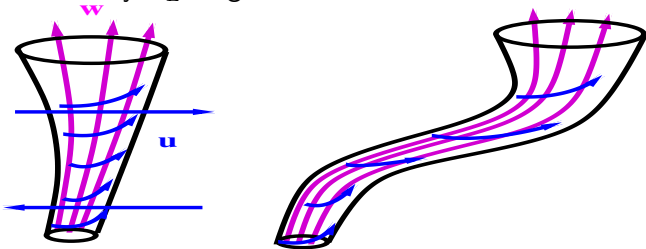
Big whorls have little whorls
Which feed on their velocity
And little whorls
have lesser whorls,
And so on to viscosity.
[Concerning atmospheric turbulence.]



Lewis Fry Richardson

Vortex stretching

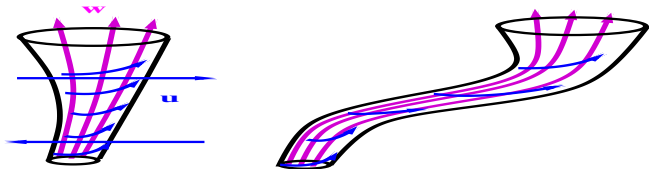
Consider an eddy of velocity u_ℓ and lengthscale ℓ being sheared by an eddy of velocity U_L lengthscale L



The rate energy moves to smaller scales is:

$$\frac{dE_\ell}{dt} \propto \frac{U_L}{L} u_\ell^2$$

Vortex stretching



The rate energy moves to smaller scales is:

$$\frac{dE_\ell}{dt} \propto \frac{U_L}{L} u_\ell^2$$

Assuming

- 1 the flux of energy across scales is constant and equal to ϵ
- 2 the most effective interactions are among similar size eddies

$$\epsilon \propto \frac{u_\ell^3}{\ell} \quad \text{or} \quad u_\ell \propto \epsilon^{1/3} \ell^{1/3}$$

Fourier Space (Finite box)

$$\mathbf{u}(\mathbf{x}, t) = \sum \tilde{\mathbf{u}}_{\mathbf{k}}(t) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad \tilde{\mathbf{u}}_{\mathbf{k}}(t) = \langle \mathbf{u} e^{-i\mathbf{k} \cdot \mathbf{x}} \rangle$$

Energy Spectrum:

$$E(k) = \frac{1}{2\delta k} \sum_{k \leq |\mathbf{k}| < k + \delta k} |\tilde{\mathbf{u}}_{\mathbf{k}}|^2 \sim \frac{\text{Energy}}{\text{per unit wavenumber}}$$

$$k \propto 1/\ell, \quad E(k)k \propto u_\ell^2$$

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Kolmogorov's Spectrum!

Kolmogorov scale

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Viscosity will become important when

$$\epsilon \propto \nu \frac{u_\nu^2}{\ell_\nu^2} \propto \nu \frac{(\epsilon^{1/3} \ell_\nu^{1/3})^2}{\ell_\nu^2} \propto \nu \frac{\epsilon^{2/3}}{\ell_\nu^{4/3}}$$

$$\ell_\nu = \frac{\nu^{3/4}}{\epsilon^{1/4}} \quad \text{or} \quad k_\nu = \frac{\epsilon^{1/4}}{\nu^{3/4}}$$

