Exercises 04

Consider again the equation

$$\partial_t \mathbf{b} + \mathbf{v} \times \mathbf{b} = -\nabla P' - \alpha \mathbf{b} + \nu_m \nabla^{2m} \mathbf{b} \tag{1}$$

in a periodic box where $\nabla \cdot \mathbf{b} = 0$.

$\mathbf{v}$ is related to $\mathbf{b}$ as $\mathbf{v} = (\nabla\times)^n \mathbf{b}$ for some $n \in \mathbb{N}$ so that
$(\nabla\times)^1 \mathbf{b} = \nabla \times \mathbf{b}$,
$(\nabla\times)^2 \mathbf{b} = \nabla \times \nabla \times \mathbf{b}$,
$(\nabla\times)^3 \mathbf{b} = \nabla \times \nabla \times \nabla \times \mathbf{b}$ and so on
For $n = 1$ the system reduces to the Navier Stokes with $\mathbf{b} = \mathbf{u}$.
For any n Energy $\mathcal{E} = \langle \frac{1}{2} |\mathbf{b}|^2 \rangle$ & Helicity $\mathcal{H} = \langle \mathbf{b} \cdot \mathbf{v} \rangle$ are conserved for $\nu_m = 0$ and $\alpha = 0$ (see last homework)?

1. For a given n what is the sign of $\mathcal{H}$?
2. Predict the direction of cascade of $\mathcal{E}$ and $\mathcal{H}$ when $\mathcal{H} > 0$.
3. Assuming self-similarity predict the energy spectra for every n.