The Astrophysical Journal, 667: L93–L96, 2007 September 20

NONLOCAL PHENOMENOLOGY FOR ANISOTROPIC MAGNETOHYDRODYNAMIC TURBULENCE

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Received 2007 June 6; accepted 2007 August 2; published 2007 August 31

ABSTRACT

A nonlocal cascade model for anisotropic magnetohydrodynamic (MHD) turbulence in the presence of a uniform magnetic field $\boldsymbol{B}$ is proposed. The model takes into account that (1) energy cascades in an anisotropic manner, and as a result a different estimate for the cascade rate in the direction parallel and perpendicular to the $\boldsymbol{B}$ field is made, and (2) the interactions that result in the cascade are between different scales. Eddies with wavenumbers $k_\parallel$ and $k_\perp$ interact with eddies with wavenumbers $q_\parallel$, $q_\perp$ such that a resonance condition between the wavenumbers $q_\parallel$, $q_\perp$ and $k_\parallel$, $k_\perp$ holds. As a consequence, energy from the eddy with wavenumbers $k_\parallel$ and $k_\perp$ cascades due to interactions with eddies located in the resonant manifold whose wavenumbers are determined by $q_\parallel \simeq \epsilon^{1/3}k_\perp^{2/3}/B$ and $q_\perp \simeq k_\perp$, and energy will cascade along the lines $k_\parallel \simeq k_0 + k_\perp^{2/3}\epsilon^{1/3}/B$. For a uniform energy injection rate in the parallel direction, the resulting energy spectrum is $E(k_\parallel,k_\perp) \simeq \epsilon^{2/3}k_\parallel^{-1}k_\perp^{-5/3}$. For a general forcing, however, the model suggests a nonuniversal behavior. The connections with previous models, numerical simulations and weak turbulence theory are discussed.

Subject headings: magnetic fields — MHD — solar wind — turbulence

1. INTRODUCTION

Magnetic fields are met very often in astrophysics: the interstellar medium, accretion disks, and the interiors of stars and planets. In most of these cases, the magnetic fields are strong enough to play a dynamical role in the evolution of the involved astronomical objects (Zeldovich et al. 1990). The fluid and magnetic Reynolds numbers are large enough so that a large number of scales are exited and coupled together, making it very difficult to calculate the evolution of these systems, even with the power of present-day computers. As a result, a turbulence theory that models the behavior of the small unresolved scales is needed. The simplest set of equations that describes the evolution of the flow and the magnetic field when the two are coupled together are the magnetohydrodynamic (MHD) equations that are written in the Elsässer formulation as

$$\partial_t \boldsymbol{z}^\pm = \pm\boldsymbol{B}\cdot\boldsymbol{\nabla}\boldsymbol{z}^\pm - \boldsymbol{z}^\mp\cdot\boldsymbol{\nabla}\boldsymbol{z}^\pm - \boldsymbol{\nabla}P + \nu\boldsymbol{\nabla}^2\boldsymbol{z}^\pm, \qquad (1)$$

where $\boldsymbol{z}^\pm = \boldsymbol{u} \pm \boldsymbol{b}$ with $\boldsymbol{u}$ as the velocity, $\boldsymbol{b}$ as the magnetic field, ν as the molecular viscosity, assumed here equal to the magnetic diffusivity $\eta = \nu$, $\boldsymbol{B}$ is a uniform magnetic field, and incompressibility $\boldsymbol{\nabla}\cdot\boldsymbol{z}^\pm = 0$ has been assumed.

For zero viscosity the above equations conserve three quadratic invariants: the magnetic helicity (which we are not going to be concerned with in the present work) and the two energies $E^\pm = \int (z^\pm)^2\, dx^3$. The question then arises in the limit of infinite Reynolds number; is there a physical process under which the two energies cascade to sufficiently small scales so that they can be dissipated?

For hydrodynamic turbulence a description of such a process exists and was given by Kolmogorov (1941, hereafter K41). In his phenomenological description the energy z_l^2 at a scale l interacts with similarly sized eddies and cascades in a timescale l/z_l. As a result in a statistically steady state, the energy cascades in a scale-independent way at a rate $\epsilon \simeq z_l^3/l$ that leads to the prediction $z_l \sim l^{1/3}$ or, in terms of the one-dimensional energy spectrum, $E(k) \sim k^{-5/3}$. Since the phenomenological description of the energy cascade in hydrodynamic turbulence, there have been attempts to derive similar results for MHD flows. However, nontrivial difficulties arise when a uniform magnetic field is present.

1. The MHD equations are no longer isotropic, resulting in an anisotropic energy flux and energy spectrum. Simple dimensional arguments cannot be used to estimate the degree of anisotropy that is a dimensionless quantity.
2. The MHD equations are no longer scale invariant; as a result simple power-law behavior of the energy spectrum is expected only in the small or large B limit that scale similarity is recovered.
3. It is not clear that interactions of similarly sized eddies (local interactions) dominate the cascade. Differently sized eddies could play an important role in cascading the energy.

The first model for MHD turbulence was proposed by Iroshnikov (1964, hereafter I64) and by Kraichnan (1965, hereafter K65). The K65 model assumes isotropy and that the timescale of the interactions of two wavepackets of size l is given by the Alfvén timescale $\tau_A \sim l/B$. The energy cascade due to a single collision is given by $\Delta z^2 \sim (z^3/l)\tau_A$. The number of random collisions that would be required then to cascade the energy is going to be $N \sim (z^2/\Delta z^2)^2$. As a result, the energy will cascade in a rate $\epsilon \sim E/N\tau_A \sim z^4/(Bl)$, and therefore, $z_l \sim (\epsilon B)^{1/4}l^{1/4}$. The resulting one-dimensional energy spectrum is then given by $E(k) \sim (\epsilon B)^{1/2}k^{-3/2}$. The assumption of isotropy however has been criticized in the literature and anisotropic models have been proposed for the energy spectrum. Goldreich & Sridhar (1995, hereafter GS95) proposed that in strong turbulence the cascade happens for eddies such that the Alfvén timescale $\tau_A^{-1} \sim Bk_\parallel$ is of the same order as the nonlinear timescale $\tau_{NL}^{-1} \sim zk_\perp$ (the so-called critical balance relation), where $k_\parallel$ and $k_\perp$ are the parallel and perpendicular to the mean magnetic field wavenumbers, respectively. Repeating the Kolmogorov arguments, one then ends up with the energy spectrum $E(k_\parallel,k_\perp) \sim k_\perp^{-5/3}$ with the parallel and perpendicular wavenumbers following the relation $k_\parallel \sim k_\perp^{2/3}$. A generalization of this result was proposed by Galtier et al. (2005), where the ratio of the two timescales τ_A/τ_{NL} was kept fixed, but not necessarily of order 1, in an attempt to model MHD turbulence in both the weak and the strong limits. Bhattacharjee & Ng (2001, hereafter BN01) repeated the K65 model arguments replacing the nonlinear timescale by $\tau_{\rm NL} \sim l_\perp / z_l$ and the Alfvén timescale by $\tau_{\rm A} \sim l_\parallel / B$. Further assuming that the cascade is only in the $k_\perp$-direction, they obtained the energy spectrum $E(k) \sim (\epsilon B)^{1/2} k_\perp^{-2} k_\parallel^{-1/2}$. Finally, Zhou et al. (2004) suggested using as a timescale that given by the inverse average of the Alfvén and nonlinear timescales $\tau^{-1} = (\tau_{\rm A})^{-1} + \tau_{\rm NL}^{-1}$ to obtain a smooth transition from the K41 to the I64 and K65 models, and the anisotropic BN01 result depends on the amplitude of $\boldsymbol{B}$.

Although this large variety of models exists, the agreement with observations (Goldstein et al. 1995) and with the results of numerical simulations (Mason et al. 2007; Müller & Grappin 2005; Ng et al. 2003; Dmitruk et al. 2003; Maron & Goldreich 2001; Cho & Vishniac 2000; Biskamp & Müller 2000; Ng & Bhattacharjee 1996) is only partially satisfactory and seems to be case dependent. Furthermore, all these models assume locality of interactions (i.e., only similarly sized eddies interact and only one length scale is needed in each direction in the phenomenological description). Locality of interactions, however, has been shown to be in question by both theoretical arguments and analysis of data in numerical simulations, even in the isotropic case (Alexakis et al. 2005; Debliquy et al. 2005; Verma et al. 2005; Yousef et al. 2007), and there have been attempts to capture these nonlocal effects in recent shell and closure models (Plunian & Stepanov 2007; Gogoberidze 2007). Furthermore, in weak turbulence theory (Galtier et al. 2000, 2002), to first-order approximation for the interaction of three eddies (say z_k^+, z_p^+, z_q^-), only the modes that satisfy the resonance condition $\boldsymbol{k} + \boldsymbol{p} + \boldsymbol{q} = 0$ and $Bk_\parallel + Bp_\parallel - Bq_\parallel = 0$ are effective in cascading the energy. This restricts the values of $q_\parallel = 0$. For sufficiently large B, one then expects $k_\parallel \gg q_\parallel \simeq 0$. In other words, short eddies ($k_\parallel^{-1}$) interact with long eddies ($q_\parallel^{-1}$) in the parallel direction. In that sense the cascade is nonlocal. It seems reasonable, therefore, that nonlocality is an essential ingredient of MHD turbulence that needs to be taken into account in a model. In addition we expect that the energy will not cascade isotropically, so not only the amplitude of the energy cascade rate is of importance but also the direction. With these two points in mind (anisotropy and nonlocality), we try to construct a nonlocal model for the energy cascade.

2. NONLOCAL CASCADE MODEL

To begin the new model, let us consider a MHD flow in a statistically steady turbulent state forced at large scales in the presence of uniform field $\boldsymbol{B}$. We denote the two, two-dimensional energy spectra as $E^+(k_\perp, k_\parallel)$ and $E^-(k_\perp, k_\parallel)$, where the total energy is given by $E_T^\pm \equiv \int E^\pm \, dk_\perp \, dk_\parallel$. For simplicity, assume $E^-(k_\perp, k_\parallel) \sim E^+(k_\perp, k_\parallel)$ (i.e., negligible cross helicity) and drop the $\pm$ indexes, leaving the case $E^- \ll E^+$ to be investigated in the future. To shorten the notation, we write $E_k = E(k_\perp, k_\parallel)$. The index k denotes that E_k depends on the wavenumbers $k_\parallel$ and $k_\perp$.

Let us now consider two eddies of different scales z_k^+ and z_q^- interacting. Let us assume that the z_k^+ eddy has wavenumbers $\sim k_\perp, k_\parallel$ and the z_q^- eddy has wavenumbers $\sim q_\perp, q_\parallel$. Here we focus on the cascade of the energy of the z_k^+ eddy; the cascade of z_q^- can be obtained by changing the indexes k, q, and $\pm$. From the form of the nonlinear term we expect by a dimensional analysis argument that the rate of energy cascade of the z_k^+ eddy will be $\mathcal{E}(k) \sim (z_k^+)^2 (z_q^-) q \sim (k_\perp k_\parallel E_k)(q_\perp q_\parallel E_q)^{1/2} q$ and likewise for the energy cascade rate of the z_q^- eddy. However, in such an interaction the energy will not cascade isotropically, but it will depend on the value of $\boldsymbol{q}$. In a interaction of the two eddies, the energy of the z_k^+ eddy will move from the wavenumber $\boldsymbol{k}$ to the wavenumber $\boldsymbol{k} + \boldsymbol{q}$. If $q_\parallel \ll q_\perp$, then most of the cascade will be in the $q_\perp$-direction. As a result, we need to separately define the rate energy cascades to larger $k_\perp : \mathcal{E}_\perp(k)$ and the rate energy cascades to larger $k_\parallel : \mathcal{E}_\parallel(k)$ as

$$\mathcal{E}_\perp \sim (k_\perp k_\parallel E_k)(q_\perp q_\parallel E_q)^{1/2} q_\perp, \tag{2}$$

$$\mathcal{E}_\parallel \sim (k_\perp k_\parallel E_k)(q_\perp q_\parallel E_q)^{1/2} q_\parallel. \tag{3}$$

Note that in writing the equations above we have not taken into account possible scale-dependent correlations between the two fields that could reduce the energy cascade. Such an effect has been taken into account by Boldyrev (2005) based on the GS95 model and could be incorporated in the present model. However, we do not make such an attempt here, since we want to present the model in its simplest form. Equations (2) and (3) express the rate energy cascades in the absence of a uniform magnetic field $\boldsymbol{B}$ and are valid only when $|\boldsymbol{q}| < |\boldsymbol{k}|$, because small eddies, although they have a stronger shear rate $z_q q$, decorrelate, making them less effective in cascading the energy. However, in the presence of $\boldsymbol{B}$, not all wavenumbers $\boldsymbol{q}$ are as effective in cascading the energy E_k. Because the two eddies z_k^+, z_q^- travel in opposite directions, the time available for the eddy z_q^- to cascade the energy of the eddy z_k^+ is the Alfvén time $\tau_{\rm A} \sim [q_\parallel B]^{-1}$, where we have assumed here that $q_\parallel \leq k_\parallel$. On the other hand the time needed to cascade the energy is $\tau_{\rm NL} \sim (z^- \, |q|)^{-1} \sim [|q| \, (q_\perp q_\parallel E_q)^{1/2}]^{-1}$. Therefore, from all the available wavenumbers, only the wavenumbers with $\tau_{\rm A} \gtrsim \tau_{\rm NL}$ will be effective in cascading the energy. This restriction leads to

$$q_\parallel B \lesssim q_\perp \sqrt{q_\perp q_\parallel E_q}, \tag{4}$$

where we used $|\boldsymbol{q}| \simeq q_\perp$ as a first-order approximation of $|q|$ for large B. This relation looks very similar to the critical balance relation of the GS95 model. However, in this case the relation (4) gives the wavenumbers that the eddy z_k^+ will interact with and does not restrict the location of z_k^+ in spectral space. In this sense this model is nonlocal, because it allows eddies of different sizes in the parallel direction to interact. We are going to refer to the set of wavenumbers that satisfy the relation above as the resonant manifold and use this relation as an equality. Finally, since the mean magnetic field does not directly effect the perpendicular direction, we assume that similarly sized $k_\perp$ and $q_\perp$ are the most effective in cascading the energy $q_\perp \sim k_\perp$ (in that sense we assume locality in the $k_\perp$-direction). Equation (4) is written as

$$q_\parallel \simeq k_\perp^3 E_q / B^2. \tag{5}$$

We are now ready to impose the constant energy flux condition that would lead to a stationery spectrum. Because the cascade is anisotropic, constant energy flux now reads

$$\partial_{k_\perp} \mathcal{E}_\perp + \partial_{k_\parallel} \mathcal{E}_\parallel = 0. \tag{6}$$

Equations (2), (3), (5), and (6) form the basis of our model. It is worth noting that in this model the cascade of energy decreases with the introduction of $\boldsymbol{B}$, not because the individual interactions weaken, but because the number of modes that are able to cascade the energy decreases due to the resonance condition (eq. [5]). A sketch of the mechanisms involved in the model is shown in Figure 1.

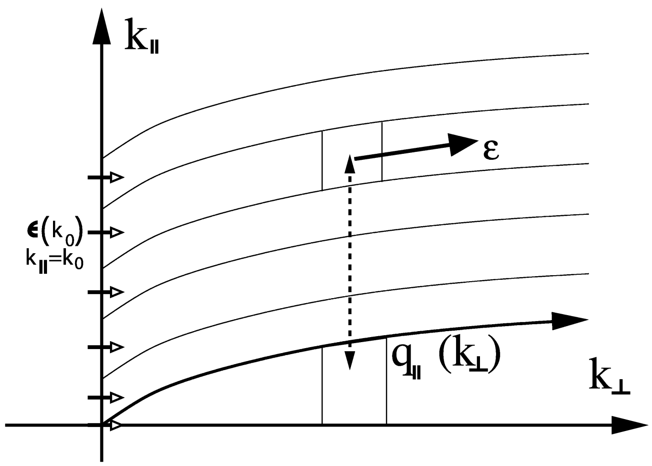

FIG. 1.—Illustration of the nonlocal model. An eddy at $(k_\parallel,k_\perp)$ interacts (nonlocally) with an eddy at the resonant manifold $(q_\parallel(k_\perp), k_\perp)$ cascading the energy in the direction of the arrow. The energy injected at the $k_\perp = 0$ axis at a rate $\epsilon(k_\parallel = k_0)$ cascades along the drawn lines.

First, let us consider the weak turbulence limit that is obtained in the limit $B \to \infty$. For large B based on equation (5), the resonant manifold becomes very thin, $q_\parallel/k_\perp \ll 1$. Furthermore, because $\mathcal{E}_\parallel/\mathcal{E}_\perp \sim q_\parallel/k_\perp \ll 1$, we can neglect the cascade in the parallel direction. If E_k is also nonsingular at $k_\parallel = 0$, we have that $E_q \simeq E(k_\perp,0)$. Substituting $q_\parallel$ from equation (5) in equation (2) and imposing the constant flux condition (eq. [6]), we obtain

$$\mathcal{E}_\perp \sim k_\perp^4 k_\parallel E_k E(k_\perp,0)/B = \epsilon(k_\parallel), \tag{7}$$

where $\epsilon(k_\parallel)$ is the energy injection rate at each plane $k_\parallel =$ constant. Note that the spectrum $E(k_\perp,k_\parallel)$ depends on the energy of the resonant manifold $E(k_\perp,0)$ just like the weak turbulence result and unlike what the BN01 local theory for weak turbulence predicts. If the energy spectrum for $E(k_\perp,k_\parallel)$ and for the resonant manifold $E(k_\perp,0)$ scale like $k_\perp^n$ and $k_\perp^m$, respectively, then we end up with the weak turbulence prediction (Galtier et al. 2000)

$$m + n = -4. \tag{8}$$

Assuming that the two spectra are smooth around $k_\parallel = 0$, we obtain

$$E^\pm \sim k_\perp^{-2}\sqrt{B\epsilon(k_\parallel)/k_\parallel}. \tag{9}$$

For large but not infinite B, we need to take into account that the energy cascade in the parallel direction is nonzero. In this case, energy does not cascade in the perpendicular direction, but cascades along the lines that are tangent to the direction of $\mathcal{E}$ and satisfy $dk_\parallel/dk_\perp = \mathcal{E}_\parallel/\mathcal{E}_\perp = q_\parallel/k_\perp$, or

$$k_\parallel = \int_0^{k_\perp} \frac{q_\parallel(k'_\perp)}{k'_\perp}\, dk'_\perp + k_0, \tag{10}$$

where k_0 expresses the intersection of these lines with the $k_\perp = 0$ axis (see Fig. 1). Different k_0 correspond to different lines. It is important to note that since these lines do not cross the $k_\perp = 0$ axis at the origin, but at some point ($k_\perp = 0$, $k_\parallel = k_0$), we need to assume that energy is injected at that point. Let λ be the length along such a curve; then we can move to a new coordinate system given by (λ,k_0). In this new coordinate system, the constant flux relation to first order in $q_\parallel$ reads

$$\frac{d|\mathcal{E}|}{d\lambda} = \frac{d}{d\lambda}\left([k_\perp k_\parallel E_k][q_\perp q_\parallel E_q]^{1/2}|q|\right) = 0, \tag{11}$$

$$\text{or } [k_\perp k_\parallel E_k][k_\perp q_\parallel E_q]^{1/2}k_\perp = \epsilon(k_0), \tag{12}$$

where only terms up to order $q_\parallel$ are kept and $\epsilon(k_0)$ expresses the rate that energy is injected into the system at small $k_\perp \simeq 0$ and $k_\parallel \simeq k_0$. Letting $k_\parallel \to q_\parallel$, we obtain the equation for the resonant manifold,

$$(k_\perp q_\parallel E_q)\,[k_\perp q_\parallel E_q]^{1/2}\, k_\perp = \epsilon\,(0) \equiv \epsilon_0. \tag{13}$$

Substituting $q_\parallel$ from (eq. [5]) and solving for E_q, we obtain

$$E_q = E(q_\parallel(k_\perp),k_\perp) = \epsilon_0^{1/3} B k_\perp^{-7/3}. \tag{14}$$

The equations (5) and (10) then give us

$$q_\parallel = k_\perp^{2/3}\epsilon_0^{1/3}/B \text{ and } k_\parallel = \tfrac{3}{2}k_\perp^{2/3}\epsilon_0^{1/3}/B + k_0. \tag{15}$$

Returning to the equation for the energy (eq. [12]), we get

$$E_k = \epsilon(k_0)\epsilon_0^{-1/3}k_\perp^{-5/3}k_\parallel^{-1}, \tag{16}$$

where k_0 is given by equation (10) and the predicted spectrum (eq. [16]) is valid in the range $q_\parallel < k_\parallel < \infty$. For smaller values of $k_\parallel$, the condition $k_\parallel < q_\parallel$ that we initially assumed is not satisfied. The energy of the modes inside the resonant manifold is given by equation (14), and no singularity at $k_\parallel = 0$ exists.

In the special case in which $\epsilon(k_0)$ is a constant, $\epsilon(k_0) = \epsilon_0$, that corresponds to a uniform injection rate per unit of wavenumber ($k_\parallel$) at the large scales $k_\perp \to 0$, the spectrum reduces to

$$E_k = \epsilon_0^{2/3}k_\perp^{-5/3}k_\parallel^{-1}, \tag{17}$$

but in general the spectrum will depend on the way that energy is injected in the system. The nonuniversality that the model suggests is due to the fact that we assumed that the energy cascades in a deterministic way only along the lines in the $(k_\parallel,k_\perp)$-plane given by (eq. [10]). In reality, energy will not cascade strictly along the lines (eq. [10]), but there is going to be some exchange of energy between lines that could bring the energy spectrum in the form of equation (17). However, if and how fast a universal spectrum can be obtained in MHD is not an easy question to answer. This question is related to the return to isotropy of an anisotropic forced flow in hydrodynamic turbulence, which is still an open question. If in MHD turbulence in the presence of a uniform magnetic field there is a universal spectrum, this is expected to happen at smaller scales than in hydrodynamic turbulence, because nonlinear interactions are weaker.

So far we concerned ourselves with only large values of B. In principle, we could extend our results to smaller values of B without making some of the approximations used to arrive at the results (eqs. [16] and [17]). However, such procedure leads to more complex equations that prevent us from deriving the energy spectrum in a compact form, and we do not make such an attempt at present.

It is worth emphasizing the similarities the current model has with the GS95 model. Both models emphasize the role of the manifold $k_\parallel \simeq k_\perp^{2/3}\epsilon_0^{1/3}/B$, obtained by the resonance condition (eq. [5]) in out model or the critical balance condition in the GS95 model. However, in this model the cascade is not restricted in this manifold, but instead all modes in the $(k_\parallel,k_\perp)$-plane cascade due to nonlocal interactions with the modes in this manifold.

Another point we need to emphasize is that taking $B \to \infty$ does not reduce the predicted spectra in equation (16) to the weak turbulence limit (eq. [9]). This is due to the two different limiting procedures followed. In the first case (eq. [9]), first the limit $B \to \infty$ was taken, and then the one-dimensional flux was determined; while in the second case (eq. [16]), we first obtained the two-dimensional energy flux, and then the limit $B \to \infty$ was taken. Note, however, that the condition (8) is satisfied in both cases and the spectrum is also smooth at $k_\parallel = q_\parallel$, because although the resonant manifold scales like $k_\perp^{-7/3}$ while the rest of the spectrum scales like $k_\perp^{-5/3}k_\parallel^{-1}$, the resonant manifold widens as $k_\perp$ increases. It is possible, as we discuss below, that in different (numerical) setups either of the two limiting procedures can be valid and different spectra could be obtained.

In numerical simulations a finite discrete number of modes is kept. Based on this model, the cascade rate is reduced in the presence of a mean magnetic field, not because the individual interactions themselves are weakened, but because the number of modes that interact effectively is reduced. If the modes in a numerical simulation with the smallest nonzero wavenumber $k_1 = 2\pi/L$ (where L is the box height) is larger than the resonant manifold ($k_1 > q_\parallel$), then if B is further increased, the scaling of the energy dissipation rate with B will be lost, and the spectrum exponents could change, since the number of modes in the resonant manifold already have taken their minimum value (i.e., the number of modes that have $k_\parallel = 0$). A difference in the energy spectrum exponents can also be expected in numerical simulations if the modes inside the resonant manifold are not forced. The sensitivity of the model to the way the system is forced could in part explain the disagreement in the measured spectrum exponents.

The author thanks H. Politano for useful discussions. Support from Observatoire de la Côte d'Azur and the Rotary Club's district 1730 are gratefully acknowledged.

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