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ON HEAVY ELEMENT ENRICHMENT IN CLASSICAL NOVAE

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ABSTRACT

Many classical nova ejecta are enriched in CNO and Ne. Rosner et al. recently suggested that the enrichment might originate in the resonant interaction between large-scale shear flows in the accreted H/He envelope and gravity waves at the interface between the envelope and the underlying C/O white dwarf. The shear flow amplifies the waves, which eventually form cusps and break. This wave breaking injects a spray of C/O into the superincumbent H/He. Using two-dimensional simulations, we formulate a quantitative expression for the amount of C/O that can be entrained into the H/He at saturation. The fraction of the envelope that is enriched depends on the shear velocity and density contrast between the C/O white dwarf and the H/He layer but is roughly independent of the shape of the shear profile. Using this parameterization for the mixed mass, we then perform several one-dimensional Lagrangian calculations of an accreting white dwarf envelope and consider two scenarios: that the wave breaking and mixing is driven by the convective flows; and that the mixing occurs prior to the onset of convection. In the absence of enrichment prior to ignition, the base of the convective zone, as calculated from mixing-length theory, does not reach the C/O interface. As a result, there is no additional mixing, and the nova is slow. In contrast, the formation of a mixed layer during the accretion of H/He, prior to ignition, causes a more violent runaway. For large shear velocities (Mach numbers $\gtrsim 0.3$) the envelope can be enriched to $\gtrsim 20\%$ of C/O by mass, consistent with that observed in some ejecta.

Subject headings: hydrodynamics — nuclear reactions, nucleosynthesis, abundances — novae, cataclysmic variables — methods: numerical — waves

1. INTRODUCTION

Classical novae are a manifestation of thermonuclear runaways in accreted hydrogen shells on the surfaces of white dwarf stars in close binary systems (see, e.g., the review by Gehrz et al. 1998). Compelling observational data indicate that the material ejected by some classical novae can be significantly enriched in C, N, O, and Ne, by $\gtrsim 30\%$ by mass (Gehrz et al. 1998; Livio & Truran 1994). Since the abundance of CNO catalysts constrains the rate of energy release, such high levels of CNO enrichment are required in the fastest novae (Truran 1982), for which the hydrogen burning reactions (via the CNO cycle) increase the entropy of the envelope faster than it can adjust. It was recognized early that if some of the underlying white dwarf matter[10] could be mixed into the accreted envelope prior to the final stages of the thermonuclear runaway, then the explosion would be more energetic and the ejecta more enriched in CNONe elements (Starrfield et al. 1972). One-dimensional models that best reproduce observations typically accrete material "seeded" with a supersolar composition (e.g., Hernanz et al. 1996; Starrfield et al. 1998).

The question of how the accreted envelope is enriched has challenged theory for several decades (see, e.g. the review by Livio & Truran 1994). For very slow accretion ($\lesssim 10^{-10}\,M_\odot\,\mathrm{yr}^{-1}$), the downward diffusion of H into the underlying C/O or O/Ne layers (Kovetz & Prialnik 1985; Prialnik & Kovetz 1984) could trigger ignition in the H diffusive tail and drive convective mixing of heavy elements into the envelope during the early stages of runaway. It is unclear, however, whether there is time at higher accretion rates for this process to be relevant. The possibility of mixing via convective overshoot was first considered by Woosley (1986). Numerical simulations of convective penetration and mixing at the core-envelope interface have been carried out in both two dimensions (Glasner, Livne, & Truran 1997; Kercek, Hillebrandt, & Truran 1998) and three (Kercek, Hillebrandt, & Truran 1999). From comparisons of two- and three-dimensional simulations and a careful resolution study, Kercek et al. (1998, 1999) concluded that convective penetration would not significantly enrich the accreted envelope.

The general problem of shear mixing was considered by many (Fujimoto 1988; Kippenhahn & Thomas 1978; MacDonald 1983), but to date no self-consistent calculations of these effects have been performed. Kippenhahn & Thomas (1978) considered shearing from differential rotation in the envelope and estimated the amount of mass mixed by assuming that the envelope was marginally stable according to the Richardson criterion. MacDonald (1983) considered non-axisymmetric perturbations and the resulting redistribution of matter and angular momentum. Fujimoto (1988) emphasized the role of barotropic and baroclinic instabilities in transporting angular momentum through the envelope. A variation on this was envisaged by Sparks & Kutter (1987) and Kutter & Sparks (1989), who suggested that convection just prior to the runaway would transport angular momentum inward and lead

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[10] By "white dwarf," we mean the predominantly C/O substrate; we shall refer to the accumulated H/He layer as the "atmosphere" or "envelope."

to a large horizontal shear above the C/O white dwarf.

While these studies examined the generation of shear in the envelope, the details of how C/O is mixed into the envelope and the quantity of mass mixed were not resolved. In this paper, we examine a promising mechanism for effecting the entrainment: a resonant interaction between large-scale shear flows in the accreted envelope and interfacial gravity waves (Rosner et al. 2001). The greater compositional buoyancy in the C/O white dwarf means that the interface sustains gravity waves. Miles (1957) showed that in the presence of a shear flow (i.e., a "wind"), gravity waves with a group velocity matching a velocity in the shear flow are resonantly amplified. These waves eventually form cusps and break. When the waves break, they inject, analogously to ocean waves, a spray of C/O into the H/He atmosphere. The source of the shear could arise from a number of mechanisms, such as those described above: what this paper provides is a calculation of the rate of mixing and the maximum mixed mass as a function of the wind velocity.

In § 2 we present the two-dimensional models and demonstrate how the mixed mass depends on the velocity of the flow. We then, in § 3, discuss two scenarios: that mixing only occurs if the convective zone reaches the C/O interface; and that the mixing occurs prior to the onset of convection. We implement these results as a sub-grid model in one-dimensional novae simulations performed with a stellar evolution code and discuss the implications, in § 4, for the ejected mass of C/O.

2. BREAKING GRAVITY WAVES

To calculate the amount of mass mixed by wave breaking as a function of time, we performed a suite of numerical simulations of wind-driven gravity waves. The initial configuration consists of two stably stratified layers of fluids in hydrostatic equilibrium in a uniform gravitational field g. The upper fluid is composed of H/He at a density ρ_1; the lower fluid is composed of C/O at a density ρ_2. We impose a wind in the upper fluid with velocity

$$U(z) = U_{\max}(1-e^{-z/\delta}), \qquad (1)$$

where z is the vertical direction and $z = 0$ corresponds to the location of the initial interface; $U_{\max}$ is the asymptotic maximum velocity at $z \gg \delta$, and δ is the length scale of the shear boundary layer. The form of the wind profile (eq. [1]) was motivated by studies of the wind-wave resonant instability in oceanography (Miles 1957). This instability is the principal mechanism that drives gravity waves and air-water mixing in oceans. In the limit $\delta \to 0$, one recovers the Kelvin-Helmholtz wind profile. Without a study of angular momentum transport in the accreting envelope, we do not know the form of the wind profile and in this work investigate the mixing for a range of δ.

Alexakis, Young, & Rosner (2002) performed a linear stability analysis of the wave amplification for the novae problem, and in a forthcoming paper (A. Alexakis et al., in preparation) we will present extensive numerical studies of wave-wind interactions. Here we apply those results to the conditions at the base of the white dwarf envelope. The simulations are performed using the FLASH code (Calder et al. 2000, 2002; Fryxell et al. 2000), a parallel, multi-dimensional, adaptive-mesh code for simulating compressible, inviscid flow. The computational grid was a uniform mesh of resolution 1024×1024 with periodic boundary conditions along the sides of the box and hydrostatic boundaries (Zingale et al. 2002) at the top and bottom. The equation of state is an ideal gamma-law of internal energy $\rho\epsilon = P/(\gamma-1)$. We choose $\gamma = 5/3$ and for the initial condition we construct an isentropic envelope in hydrostatic equilibrium. This choice makes the Brunt-Väisälä frequency zero, i.e., there are no internal gravity waves, which isolates the interface modes for study. Furthermore, we only consider vertical lengthscales $\delta/H \ll 1$, where $H = \gamma^{-1}c_s^2/g$ is the pressure scale height and c_s is the adiabatic sound speed. The interface is perturbed with specified modes and a velocity perturbation added to the surface of the WD representing the motion of the gravity waves. The wind profile is specified as an initial condition and in particular is not forced. For this study, we set the Mach number $\mathrm{Ma} = U_{\max}/c_s = 0.5$ and consider four values of δ/H: 0.005, 0.01, 0.02, and 0.04. These values vary the reciprocal Froude number $(\delta g/U_{\max}^2)^{1/2} = (\delta/H)/(\gamma \mathrm{Ma}^2)$, which measures the available kinetic energy for mixing.

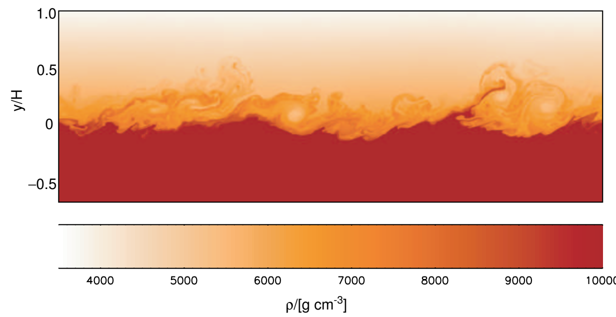

FIG. 1.— Breaking CO waves, as determined by simulations in two dimensions. Gravity points towards the bottom of the figure, with the vertical distance y in units of the pressure scale height H, as evaluated just above the interface. The color scale indicates the mass density in units of $\mathrm{g\,cm^{-3}}$.

The unstable modes have wavenumbers (Alexakis et al. 2002) $k \gtrsim g/U_{\max}^2(1-\rho_1/\rho_2)$. Figure 1 shows fully developed waves breaking, generation of the "spray", and mixing of the white dwarf substrate up into the atmosphere. Mixing is not only effected by the overturning of the wave but also by the wind breaking off the tip of the wave. Figure 2 shows contours (*black lines*) of the carbon mass fraction at 0.49, 0.20, and 0.02, respectively, at a later time in the simulation shown in Figure 1. The contour at 0.49 corresponds to the carbon mass fraction of the underlying white dwarf, while the smaller contour values indicate how far outward into the accreted material the white dwarf substrate is mixed. The contour at 0.2 delimits the region that contains most of the initial enrichment.

Figure 3 shows the surface mass density (the mass of C/O in the mixed layer per unit area), dM_{CO}/dS, averaged over the horizontal direction. The mixed layer is defined here as the region in which the carbon mass fraction is between 0.49 and 0.01. The C/O is mixed rapidly until it saturates; further mixing occurs on diffusive timescales. Our two-dimensional simulations show, for the range of parameters examined, that the total mass of white dwarf material that becomes mixed is independent of the lengthscale δ. The rate of mixing, i.e., the initial slopes of the curves in Figure 3, does show, however, some dependence on δ. Using dimensional analysis and the numerical results, we find that for a fixed density ratio $\rho_1/\rho_2 = 0.6$ (appropriate for the degenerate plasma prior to ignition) the total mass per unit area, dM_{CO}/dS, mixed into the accreted

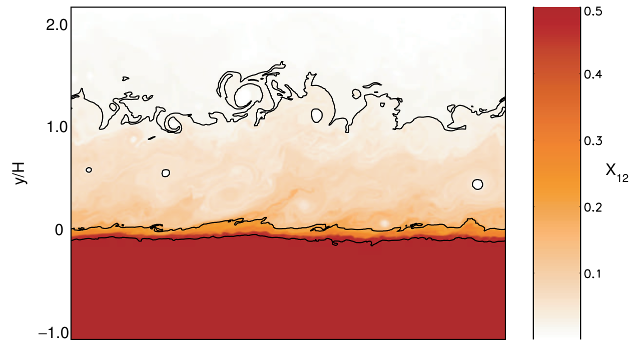

FIG. 2.— Mass fraction of ^{12}C for $\delta/H = 0.04$ after $t = 3500\delta/U$. The vertical dimension is scaled to the pressure scale height H as evaluated just above the interface. The contours for ^{12}C mass fractions of, from the top, 0.02, 0.20, and 0.49.

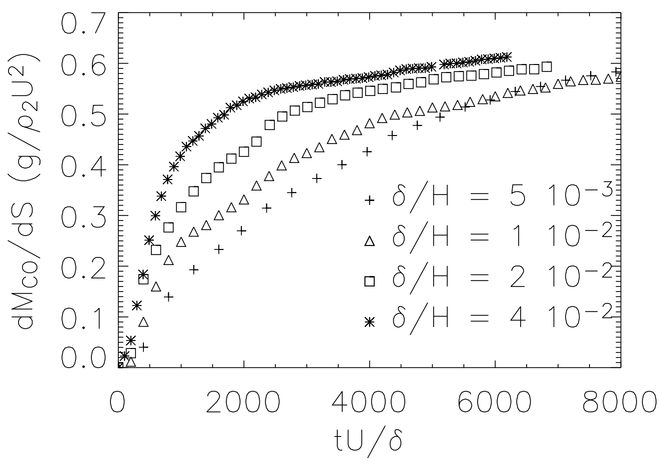

FIG. 3.— The mixed mass of C/O, per unit area, as a function of time. This was computed by averaging over the horizontal direction in the simulations. Time is scaled to $\delta/U_{\rm max}$ and $M_{\rm CO}$ is scaled to $\rho_2 U_{\rm max}^2/g$ (see eq. [2]). Four different values of $g\delta/U_{\rm max}^2$ were used, 0.01, 0.02, 0.04, and 0.09.

H/He saturates at

$$\frac{dM_{\rm CO}}{dS} = \alpha \frac{U_{\rm max}^2}{g} \rho_{\rm WD}, \qquad (2)$$

where α is a non-dimensional constant[11] that we determine from the simulations to be $\alpha \simeq 0.6$ (see Fig. 3). The timescale to reach saturation is far shorter than the timescale of either the accretion phase ($> 10^4$ yr) or the pre-peak convective phase (~ 100 yr) of a typical classical nova. For example, if $\delta/H = 0.01$ and Ma = 0.5, then the saturation timescale is of order seconds for our nova setup.

3. ONE-DIMENSIONAL NOVA MODELS

We now incorporate our simulations of the wave breaking and mixing into simulations of the thermonuclear burning of a nova. In order to explore the global properties of this local mixing mechanism, we compute several one-dimensional models of novae with a modified version of the KEPLER stellar evolution code (Weaver, Zimmerman, & Woosley 1978) that includes a large network of ~ 100 light isotopes (Rauscher et al. 2002) implicitly coupled into the stellar structure solver. This allows us to follow throughout the entire convective envelope the radioactive decays of nuclei that are formed at the base of the burning zone. The underlying white dwarf has a mass of $1.0\,M_\odot$, a radius of 5000 km, a luminosity of 10^{31} ergs s^{-1}, and is composed of a 50%/50% ^{12}C/^{16}O mixture. We retain the outer $0.005\,M_\odot$ on the computational grid to follow the thermal inertia of the outer WD layers and allow the model to relax until it has a constant luminosity before we start the accretion. The accretion rate is $10^{-9}\,M_\odot\,{\rm yr}^{-1}$, and in all but one model (see item 3 below), the accreted material consists of 70 % ^{1}H, 28.7 % ^{4}He, 0.3 % ^{12}C, 0.1 % ^{14}N, and 0.9 % ^{16}O, by mass. Convection is modeled using the Ledoux criterion for stability and mixing length theory. We assume no convective overshooting and that semiconvection is too slow to cause mixing.

We investigate three scenarios, as summarized in Table 1, for generating the gravity wave induced mixed layer.

TABLE 1
PROPERTIES OF THE 1D NOVA MODELS

Case	$M_{\rm enrich}$ ($M_\odot$)	$M_{\rm envel}$ ($M_\odot$)	$\dot{E}_{\rm nuc,max}$ (ergs s^{-1})
no pre-enrichment	0	6.93×10^{-5}	6.68×10^{42}
wave pre-enrichment	1.26×10^{-9}	7.08×10^{-5}	7.24×10^{42}
wave pre-enrichment	1.26×10^{-8}	7.05×10^{-5}	7.65×10^{42}
wave pre-enrichment	1.26×10^{-7}	6.78×10^{-5}	7.71×10^{42}
wave pre-enrichment	1.31×10^{-6}	5.42×10^{-5}	1.27×10^{43}
wave pre-enrichment	3.77×10^{-6}	4.67×10^{-5}	2.68×10^{43}
wave pre-enrichment	1.16×10^{-5}	4.85×10^{-5}	1.26×10^{44}
enriched accretion	(50 %)	2.08×10^{-6}	5.28×10^{40}

1. In the first scenario, the only shearing considered is that from the convective cells driving a wind at the interface between the H-rich atmosphere and the C/O substrate. This is the scenario envisaged by Rosner et al. (2001), in which the C/O, after being entrained, is then distributed throughout the convective zone. Figure 4 summarizes the evolution of the accreted layer. After accretion for about 6.9×10^4 yr a convective zone forms about $2.5 \times 10^{-5}\,M_\odot$ above the WD interface. The total envelope mass accreted at ignition is $\approx 7 \times 10^{-5}\,M_\odot$, in good agreement with the estimates of Fujimoto (1982a,b). Within ~ 100 yr from the onset of convection, the convective zone extends upward to the surface and downward to about $5 \times 10^{-6}\,M_\odot$ above the interface, at which time the runaway reaches its peak rate of energy generation. This peak phase evolves on a timescale of ~ 1 h, but the convection does not reach the WD interface. Only days later, when the nova envelope is already significantly expanding, does the burning layer reach the WD. This downward movement, in mass, of the burning layer is mediated by heat conduction into these deeper H-rich layers. The convective zone also moves deeper, but never quite reaches the WD interface[12]. As a result, there is no injection of C/O into

[11] In general, α depends on ρ_2/ρ_1 and possibly Ma. A parameter study, outside the scope of this paper, is necessary to determine this dependence.

[12] At the very bottom of the H shell the nuclear luminosity from H burning can never become large enough to drive convection, as it goes to zero at the interface; the only possibility for mixing to occur is if a steep temperature gradient arises from the thermal inertia of the heated WD core below an expanding H envelope.

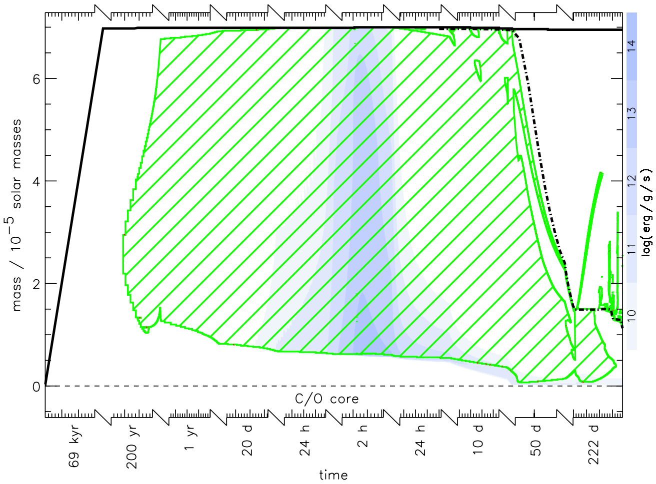

FIG. 4.— Kippenhahn diagram of a nova without enrichment. The *x*-axis indicates time intervals for the different evolution stages, and the *y*-axis gives the mass above the C/O WD substrate. *Green hatching* (framed by a *green line*) indicates convection, *blue shading* indicates nuclear energy generation for which each level of darker blue denotes an increase by one order of magnitude, starting at 10^{10} ergs g^{-1} s^{-1}. The *thick black line* shows the total mass of star (including ejecta), increasing because of accretion; the *dash-dotted line* indicates the mass outside of 10^{12} cm; and the *dashed line* marks the interface between the white dwarf C/O substrate and the accreted layers.

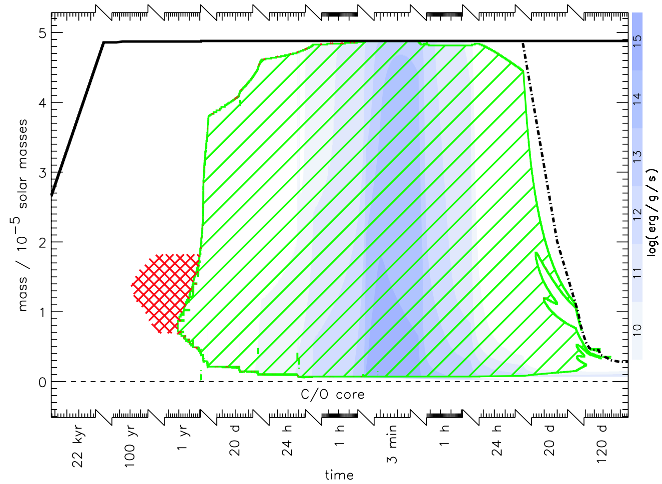

FIG. 5.— Same as Fig. 4 but for a model in which the inner $M_{\rm mix} = 4.6 \times 10^{28}$ g are enriched in C/O with a linear composition gradient (with respect to the Lagrangian mass coordinate) between the WD composition (C/O) at the base and the accretion composition (solar) at the upper edge. Note that the convective zone does not reach the interface with the WD substrate, and that a significant semi-convective region, indicated by *red hatching*, develops prior to the onset of convection.

the H-rich envelope, by construction.

The envelope expands to large radii ($> 10^{12}$ cm, large enough to engulf the secondary) by the end of the simulation, but the result will only be a slow nova, in rough agreement with the semi-analytical calculations of Fujimoto (1982a,b). We do not find any contact of the convective region with the WD substrate; as a result, we do not expect that any convection-driven wave mixing would occur, unless there is a large redistribution of angular momentum in the accreted envelope (Kutter & Sparks 1989; Sparks & Kutter 1987).

2. In the second scenario, the shearing originates from an accretion-driven wind blowing across the surface of the white dwarf. We assume that the wind persists throughout the H/He layer with velocity sufficient to drive mixing on a timescale much less than that to accrete a critical mass of fuel. In this case the mixed layer is generated prior to ignition. We assume a linear (in Lagrangian mass coordinate) gradient in the mass fraction of C/O between the WD and the accreted envelope. Because the shear profile, and hence the amount of mass mixed, is unknown, we consider a range of mixed masses (see Table 1).

   Figure 5 shows the case with the largest pre-enrichment: the mixed material comprises 4.6×10^{28} g (i.e., a total of 2.3×10^{28} g of WD material is being mixed with H-rich material). This is about 25% of the envelope mass, which is less than that generated by the calculation described in section 2. From equation (2), a 25% enrichment corresponds to Ma = 0.4. The mixed layers are added to the surface using the same accretion rate as the rest of the envelope ($10^{-9} M_\odot$ yr^{-1}); to prevent a runaway while these layers are being added, we suspend nuclear energy generation during the accretion of the first 5×10^{28} g. About 50 yr prior to peak energy generation a semiconvective region forms in the outer 60 % (by mass) of the enriched layer; about 8 months before the peak a convection zone starts at the base of this semiconvective layer (at $7 \times 10^{-6} M_\odot$ above the WD interface) that extends upward to the surface and downward to about $5 \times 10^{-7} M_\odot$ above the interface. In contrast to the first case, the pre-mixed envelope with the largest enriched mass ignites at a smaller accumulated mass, $\approx 5 \times 10^{-5} M_\odot$.

   The energy generation in the enriched layer is dominated by ^{12}C(p,γ)^{13}N(β^+)^{13}C. The peak energy generation timescale is about 3 min—20 times faster than in the first case—with a specific energy generation rate about 20 times higher. Figure 6 shows the specific nuclear energy generation in the accreted envelope, with the time centered about the peak energy generation rate. The larger rate of energy generation roughly corresponds to the ~ 20 times greater metal enrichment. In this scenario the entire 25 % of pre-enriched C/O material is spread throughout the hydrogen envelope during the runaway. This leads to a a corresponding enrichment in the ejecta. The ejecta are unbound, i.e., they have a positive velocity at very large radii.

   As in the first case, the convection never reaches the WD interface (although the base of the convective zone is much deeper), so there is no additional convection-induced wave mixing of the C/O substrate with the hydrogen-rich envelope. We did experiment with pre-runaway, pre-enriched layers as small as 1/10,000 the mass used in the case described above (Table 1). As before, we disregarded nuclear burning for the first 10^{28} g of accreted material, with the exception of the case with $3.77 \times 10^{-6} M_\odot$ of enrichment, where we took twice that mass to accommodate the entire enriched layer. The difference in masses for which nuclear burning is suspended is the reason for the case with the largest pre-enriched mass, $1.15 \times 10^{-5} M_\odot$ having a slightly larger envelope mass at runaway than that of the case with $3.77 \times 10^{-6} M_\odot$ of enriched mass.

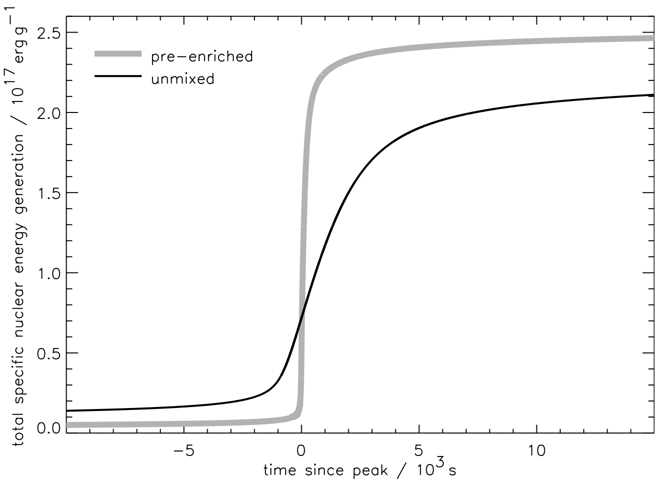

FIG. 6.— Time-integtated average specific nuclear energy generation of the accreted enevelope. The curves are shifted so that the zero in time corresponds to the peak energy generation rate. The gray curve indicates the model with the highest wave-induced pre-enrichment (see Tab. 1 and Fig. 5); the black curve indicates the model without prior enrichemnt (Fig. 4). The slope of a curve is proportional to the average specific energy generation rate in the envelope.

In none of these cases did the convection reach the WD interface. Eventually, for the case of an enriched mass of $\lesssim 10^{-9}\,M_\odot$, the runaway behaves as in the non-enriched case: the energy release rate in the mixed layers is too small to drive convection above and heat conduction cools it efficiently so that the runaway does not occur in its vicinity. The slightly higher envelope masses obtained in the limit of small enrichment is likely to be an artifact of our suspension of nuclear energy generation for the first fraction of the accretion phase.

3. In a third scenario we assume accretion of material enriched to a mass fraction of 50 % in C/O. Allowing for nuclear energy generation instantaneously in this material leads to a rapid runaway after only a thin layer has been accumulated at the surface of the star. If this magnitude of enrichment was due to instantaneous mixing with a colder WD substrate, then a lower temperature in the enriched material and thus a significantly later runaway could result. We include this run to demonstrate how qualitatively different the accretion of pre-enriched material behaves as compared to first accreting unenriched material and then, only later after it has settled, mixing the accreted layers with the C/O substrate.

4. IMPLICATIONS AND SUMMARY

Using a constant wind profile that blows across the surface of a white dwarf, we have performed a two-dimensional parameter study of the mechanism proposed by Rosner et al. (2001). Our primary results of the mixing rate and the maximum mixed mass, equation (2), suggest that this process can mix about 10^{-6}–$10^{-5}\,M_\odot$ of the underlying C/O into the hydrogen-rich envelope. From this, we investigated two scenarios: that the wind is the result of convection, and that the wind originates during the accretion phase.

We find that if no enrichment occurs prior to the onset of convection, then the convective zone does not reach the C/O interface, and no additional mixing occurs (in the one-dimensional model) in the absence of convective overshoot. The result in this case will be a slow nova, with little enrichment of the ejecta. In contrast, an envelope with a mixed layer at the C/O interface, consistent with the scalings from high-resolution numerical studies, provides a more violent runaway and the ejecta are enriched by $\sim 25\%$ in C/O, consistent the enrichment observed in some nova ejecta. Such an event does require a strong shear velocity, but the saturation amount of mass mixed is roughly independent of the shape of the shear profile, so long as its thickness is much less than a pressure scale height. Our runs, outlined in Table 1, show that when the metals are concentrated at the base of the accreted envelope, the amount of mass accreted prior to runaway is *larger* than if the C/O were uniformly distributed over the envelope, i.e., if the white dwarf were to accrete material with a supersolar metallicity. The reason is that the opacity of the envelope is larger than when the C/O are concentrated at the base.

There are a number of issues that we have not yet addressed or investigated. First, our results might depend on the dimensionality of the system. We would therefore expect that although the generation of the gravity waves is captured by two-dimensional dynamics, the energy cascade of the waves and the advection of spray and vorticity will be different in a three dimensions. Second, this work investigates only the density ratio for an accreted envelope in thermal equilibrium. During the early phase of accretion and during the runaway the density ratio will likely be different. Third, although our parameter study covers more than one order of magnitude in δ, it is important to know if our results still hold at even larger values. As δ becomes a respectable fraction of H, we expect that the stratification of the envelope, and in particular the effect of a non-zero Brunt-Väisälä frequency, will become important. All of these differences can affect the mass and thickness of the mixed layer, and the investigation of these issues is the subject of ongoing work.

In our one-dimensional nova simulations, we assumed that a convective roll will only entrain the mixed layer if the base of the convective zone (as computed from a mixing-length formalism) reaches the interface. Future multidimensional studies can inform us as to how convection interacts with the interfacial gravity waves, and to whether this assumption is in fact correct. In addition, we have not self-consistently calculated the wind profile during the accretion phase. Because the amount of C/O entrained depends quadratically on the velocity in the wind profile, our calculation underscores the need for detailed calculations of the sheer profile of an accreting star. These issues are clearly important and are the next steps for future study.

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