The Astrophysical Journal, 562:L177–L179, 2001 December 1

ON THE C/O ENRICHMENT OF NOVA EJECTA

R. Rosner,[1,2] A. Alexakis,[2] Y.-N. Young,[3] J. W. Truran,[1] and W. Hillebrandt[4]
Received 2001 September 21; accepted 2001 October 29; published 2001 November 14

ABSTRACT

Using the results of recent work in shear instabilities in stratified fluids, we show that the resonant interaction between large-scale flows in the accreted H/He envelope of white dwarf stars and interfacial gravity waves can mix the star's envelope with the white dwarf's surface material, leading to the enhancement of the envelope's C/O abundance to levels required by extant models for nova outbursts.

Subject heading: novae, cataclysmic variables

The substantial enrichment of CNO nuclei in the ejecta from novae (see Truran 1985; Gehrz et al. 1998 and references therein) has been a puzzle for over two decades. Early theoretical models of nova outbursts (e.g., Starrfield, Truran, & Sparks 1978; Fujimoto 1982) clearly showed that nuclear processing during hydrogen burning during the nova flash could not account for the observed CNO abundances, which can reach 30% by mass. These early studies already recognized that the solution to the puzzle must involve "dredge-up" of C/O from the white dwarf before or during the nova outburst. This mixing was required both to meet the constraints on CNO abundances in the ejecta and to power the nova itself, since the energy production rate per unit mass depends directly on the metallicity (e.g., Wallace & Woosley 1981); thus, Starrfield et al. (1978) and Fujimoto (1982) showed explicitly with one-dimensional models that runaway in a pure H/He envelope did not release enough energy in order to eject enough matter with sufficient velocity to match observations.

Concerns that mixing may occur at the interface between the accreting matter and the underlying star (and already accreted material) followed closely upon the recognition that such mixing was essential in order to understand the elemental composition of the nova ejecta (Starrfield et al. 1972). At that time, there was already some interest in understanding mixing at the interface between a stellar surface and an accreting flow. For example, Kippenhahn & Thomas (1978) examined shear flow instability in the stratified boundary layer between a white dwarf and the infalling accretion flow associated with an accretion disk, and they established the linear stability properties (based on using the Richardson number[5] as the control parameter).

The shear instability considered by Kippenhahn & Thomas (1978; also Sung 1974) has been extensively revisited (viz., MacDonald 1983). Kippenhahn & Thomas conjectured that this instability saturates at the marginal state for stability and therefore weak mixing; MacDonald, upon revisiting this problem, showed that the shear instability would lead to rapid dispersal of the accreted matter over the entire white dwarf surface (as opposed to the relatively narrow accretion belt that emerged from Kippenhahn & Thomas' analysis) but also suggested that the radial mixing time was long (set by the thermal timescale of the envelope). These arguments lead to rather minimal mixing, and for these reasons shear mixing has not been regarded as a likely candidate for the required mixing process.[6] Indeed, until the late 1990s, the absence of a plausible mixing mechanism was considered to be a major stumbling block for understanding novae. In the mid-1990s, several authors conjectured that the convection that was known to initiate some ~1000 yr before runaway might be associated with convective undershoot and convective penetration, processes that might lead to mixing of the stellar C/O into the envelope (Shankar, Arnett, & Fryxell 1992; Shankar & Arnett 1994), but quantitative calculations were not done until the mid- and late-1990s (Glasner & Livne 1995; Glasner, Livne, & Truran 1997; Kercek, Hillebrandt, & Truran 1998a). These more recent calculations investigated the possibility that convective undershoot just before, and possibly during, nova runaway might lead to the required mixing. However, Kercek, Hillebrandt, & Truran (1998b, 1999) have shown convincingly (both by comparing two and three-dimensional simulations and by conducting resolution studies in which the extent of mixing was measured as a function of grid resolution) that convective undershoot was not likely to work as an effective mixing process. In particular, the resolution studies showed *less* mixing as grid resolution was increased. This can be readily understood if the boundary layer between the stellar surface and the accreted (convecting) envelope is laminar: in that case, since the dominant viscosity in these simulations is numerical, increased resolution leads to a thinner boundary (or mixing) layer, and whence to *less* mixing as the grid resolution is increased. Thus, it would appear that we are once again lacking an effective mixing process.

For this reason, we have recently reexamined the physics of shear flow instabilities (Alexakis, Young, & Rosner 2001; Y.-N. Young et al. 2001, in preparation). The question to answer was whether previous astrophysical studies of this subject had in fact fully explored this mixing process. As we show below, the past work in fact missed an important aspect of shear mixing in stratified media. In the following discussion, we will extract the critical aspects of our earlier results that apply to the problem at hand.

[1] Department of Astronomy and Astrophysics, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637.

[2] Department of Physics, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637.

[3] Department of Applied Mathematics, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208.

[4] Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, Postfach 1317, D-85741 Garching, Germany.

[5] The Richardson number, $\mathrm{Ri} = \alpha g A l_0/U_0^2$, is a measure of the competition between the stabilizing effect of buoyancy and the destabilizing effect of the shear flow. Here α is the coefficient of volume expansion, g is the local gravitational acceleration, l_0 is a local characteristic length scale, and U_0 is the shear flow amplitude; A is the Atwood number across the material interface separating the white dwarf surface and the H/He envelope $[\equiv(\rho_{\rm star} - \rho_{\rm envelope})/(\rho_{\rm star} + \rho_{\rm envelope})]$.

[6] However, very recently, Brüggen & Hillebrandt (2001a, 2001b) have begun to reexamine the nonlinear aspects of this problem computationally, in an attempt to place the earlier analytical calculations on a firmer quantitative footing.

One very important aspect of the shear mixing problem is that the instability that leads to turbulent mixing between fluid layers depends only on certain essential features of the shear flow. For present purposes, it suffices to consider prototypical velocity profiles of the form $U(z) = U_0 + U_1 \ln(z/\sigma + 1)$ for $z \geq 0$ [$U(z) = 0$ below the interface] or $U(z) = U_0 + U_1 \tanh(z/\sigma)$ for $z \geq 0$ [$U(z) = 0$ below the interface], where U_0 is the velocity jump (if any) at the envelope/stellar surface interface, z is the vertical coordinate (with $z = 0$ marking the initial envelope-star interface), and σ is the characteristic scale length of the shear flow in the envelope.[7] It is then well known that if $U_0 = 0$, the Kelvin-Helmholtz instability is entirely absent for a velocity profile with either of the two functional forms given above. Since (in the presence of viscosity) the relative velocity between the two layers of fluid at the interface must be zero (i.e., an attached flow boundary condition) and since σ is proportional to the viscous boundary layer thickness, Kelvin-Helmholtz instability is unlikely to be important in contributing to the mixing on relevant spatial scales and timescales for the above wind profile, which appears to be a reasonable approximation to the actual boundary layer flow (see Alexakis et al. 2001).

Now, the same argument applies to the generation of terrestrial surface water waves by winds; and as originally pointed out by Miles (1957; see also Phillips 1957), winds are nevertheless able to amplify such surface waves to finite amplitude. Hence, there must be some other instability present. One such instability (critical-layer instability) was identified and studied extensively in the linear regime by Miles (1957), Howard (1961), Lighthill (1962), and others. This instability originates from the continuum of unstable modes formed when surface gravity waves travel at the same velocity as the wind at some height above the interface. In order to treat this instability, it is essential to formulate the shear instability problem more generally; this is in part the motivation for considering velocity profiles of the forms given above; velocity profiles of this type in stratified atmospheres have been explored extensively by the geophysical fluid dynamics community. These previous geophysically motivated studies were largely confined to the parameter regime characteristic of the water/air interface; but recently, Alexakis et al. (2001) have fully explored the control parameter space governing these instabilities: these nondimensional parameters are the Atwood number A and the Richardson number Ri (see Alexakis et al. 2001); in physical terms, the key parameters are the gravitational acceleration g, the shear scale length σ, the density ratio $\rho_{\rm star}/\rho_{\rm envelope}$, and the shear amplitude U. In this Letter, we now apply the results presented by Alexakis et al. to the astrophysical context by developing a new model for the interface mixing.

First, consider the underlying physics. This is most readily done in the context of a particularly simple model for the interface, in which the shear flow has step discontinuity across the density interface between the C/O white dwarf surface and the bottom of the H/He envelope. Start with the simplest case, in which we ignore stratification. The classical Kelvin-Helmholtz instability is then based on the observation that a spatial interface perturbation can be destabilized because the flow must speed up over the "hills" of the perturbation and slow over the "valleys"; Bernoulli's law then tells us that a low-pressure region develops over the hills, and a high-pressure region over the valleys, thus pulling up the hills and pushing down the valleys, leading to a linear instability whose growth rate $\gamma \sim kU$, where k is the wavenumber of the interface perturbation and U is the shear amplitude.

If the shear flow interface is not a step but has a finite thickness (viz., given by σ, as above), and if stratification is allowed, then it is well known that the dispersion relation is no longer linear (Chandrasekhar 1962, § 102) and that both low- and high-wavenumber cutoffs appear, with $\gamma > 0$ only for $\zeta_{\rm min} < k\sigma < \zeta_{\rm max}$ [where the values of $\zeta_{\rm min}$ and $\zeta_{\rm max}$ depend on the specifics of the velocity profile (see Figs. 119 and 120 in Chandrasekhar 1962) and a maximum growth at, for example, $(k\sigma)^2 \sim 0.5$ for the $\tanh(z/\sigma)$ shear profile]. As a result, instability can only occur in a finite region of the wavenumber–Richardson number plane; for the tanh velocity profile, the stability boundary is defined by the curves $J = 0$ and $J = (k\sigma)^2[1 - (k\sigma)^2]$, with instability only in the domain bounded by these two curves. In order to apply this to the nova case, we simplify the actual case by assuming an exponentially stratified background atmosphere of the form $\rho(z) \sim \rho_0 \exp(-\beta z)$, with a horizontal shear layer of the form $U = U_0 \tanh(z/\sigma)$ located at the white dwarf surface; it is readily seen that in the event that this surface shear flow is driven by thermal convection in the overlying envelope, then the unstable modes will lie in a wavelength band defined by 6×10^4 cm $< \lambda_{\rm unstable} < 2 \times 10^5 (T/10^8\ {\rm K})^{1/2} (U_0/10^5\ {\rm cm\ s^{-1}})$ cm, where we have assumed a white dwarf of radius $\sim 10^{-2} R_\odot$, a shear layer thickness $\sigma \sim 10^4$ cm, gravitational acceleration $g_{\rm wd} \sim 2.7 \times 10^8$ cm s^{-2}, and a density scale height $\beta^{-1} \sim 3 \times 10^8 (T/10^8\ {\rm K})$ cm; inclusion of the density jump at the (C, O/H, He) interface would lower the wavelength of unstable modes yet further. The upper bound on this mixing scale is of the order of the grid resolution in the currently highest resolution calculations (viz., Glasner et al. 1997; Kercek et al. 1999), consistent with the observation by these authors that little shear mixing occurred in their computations. Since there is an upper bound on the shear flow length scale σ in order for Kelvin-Helmholtz instability to occur at all,[8] and since the mixing scale is at most of order 10 times the shear scale, this suggests that Kelvin-Helmholtz instability will not be an effective CNO mixing process under any circumstances.

In contrast, consider the interaction of the same wind with the normal modes supported by the free stellar surface between the star and the accreted envelope. These normal modes are akin to "deep water waves" seen at the surface of terrestrial oceans and are known to grow in amplitude as a result of the resonant interaction between these waves and the wind. More specifically, at any given wavenumber k, linear theory provides the wave's phase velocity $v_{\rm phase} \equiv \omega/k \sim (A/k)^{1/2}(g + \Sigma k^2/\rho^{\rm water})^{1/2} \sim (Ag/k)^{1/2}$, where Σ is the surface tension; in the case of the gaseous media characterizing stars, the surface tension term is of course absent. For any given wind profile, $U(z)$, where z is the vertical coordinate, one can then satisfy a resonance between the wind and a surface mode such that $U(z) = v_{\rm phase} \sim (Ag/k)^{1/2}$; that is, a wave with wavenumber $k \sim Ag/U(z)^2$ will be driven resonantly unstable. (For typical values of A, g, and U characteristic of a white dwarf surface, one finds that the wavelength of unstable modes lies in the range of 0.01–1 km.) The key issue is then how to determine the mixing-layer width once these unstable modes cease their growth and finally saturate: naively, one might expect the saturation process to simply limit

[7] The logarithmic velocity profile is commonly observed in the boundary layer of winds blowing over the surface of extensive bodies of water (see Miles 1957); the tanh profile has the advantage of bounded shear velocity far from the shear interface.

[8] This bound is computed from the stability criterion $J < \frac{1}{4}$ for the tanh velocity profile; thus, $\sigma < 1.67 \times 10^4 (T/10^8\ {\rm K})^{1/2} (U_0/10^5\ {\rm cm\ s^{-1}})$ cm in order for Kelvin-Helmholtz instability to occur at all.

the mode amplitude and thereby determine the width of the mixing layer. In the case of interfacial gravity modes, however, saturation is well known to occur via wave breaking (see Chen et al. 1999 and references therein); it is the resulting spray that then determines (from a statistical point of view) the effective mean width of the mixing layer—this width can be substantially larger than the mode amplitude at saturation, as is well known in the case of wind-driven spray from breaking ocean waves. In any case, let us assume for the moment that we have determined this layer width, which we shall denote as λ. Finally, we note that while one would need, in general, to take into account stratification effects (viz., molecular weight gradients) on either side of the density jump, such effects are to lowest order unimportant here because the mixing layer defined by wave breaking is likely to be much narrower than the local gravitational scale height.

We are now ready to describe our simplified model: consider first the amount of carbon and oxygen in the breaking-wave mixing layer, which we write in the form $M_{\rm C+O}^{\rm mixing\ layer} \sim \alpha\rho_0^{\rm envelope}\lambda\Lambda\xi$, where α is the coefficient for the C + O mass fraction in the mixing layer, λ is the mixing-layer width (both α and λ are to be determined from simulations; Y.-N. Young et al. 2001, in preparation), $\rho_0^{\rm envelope}$ is the density of the envelope at its base (i.e., in the breaking-wave mixing layer), Λ is the characteristic length scale of the large-scale circulation (which can be identified with the outer scale of convection in the envelope), and ξ is a length scale transverse to the wind direction (this dimension will drop out of our formulation). Note that the remaining parameters appearing in this relation can be obtained from extant (one-dimensional) nova models. Now, as mentioned earlier, the amount of C + O needed to be mixed into the envelope is roughly $\frac{1}{3}$ by mass of the ejecta mass $M_{\rm total}^{\rm envelope}$ or $M_{\rm C+O}^{\rm envelope} \sim \frac{1}{3}M_{\rm total}^{\rm envelope} \sim \frac{1}{3}(\frac{2}{3}\Lambda\rho_0^{\rm envelope}\Lambda\xi)$. The "sweep-out time," i.e., the timescale on which the boundary mixing layer is swept out by a penetrating convective roll, is just $\tau_{\rm sweep} \sim \Lambda/U$, so that the time needed to mix the necessary amount of carbon and oxygen into the envelope is just $\tau_{\rm mixing} \sim M_{\rm C+O}^{\rm envelope}/(M_{\rm C+O}^{\rm mixing\ layer}/\tau_{\rm sweep})$, or

$$\tau_{\rm mixing} \sim \frac{2}{9}\Lambda^2/\alpha\lambda U \sim 5\alpha^{-1}(\Lambda/10^8\ {\rm cm})^2(\lambda/10^2\ {\rm cm})^{-1} \times (U/10^5\ {\rm cm\ s^{-1}})^{-1}\ {\rm yr},$$

with $\alpha \sim 0.3$–1. Thus, it is evident that the evolution timescale for the envelope prior to nova runaway (which is roughly of the order of the time between onset of envelope convection and runaway or $\sim 10^3$ yr) is much longer than the mixing timescale. This confirms that resonantly driven mixing at the star-envelope boundary can be an efficient mixing process during the prenova star evolution; the clear next step is to verify these results via simulations of weakly compressible fluids subject to these mixing instabilities. We also note that this mixing timescale is much longer than the dynamical time characteristic of the nova runaway itself. For this reason, the amount of additional C + O material mixed in during the outburst itself can be regarded as a small perturbation. One remaining significant issue relates to the possible effects of magnetic fields on the C + O mixing process; that is, one might be concerned that turbulent mixing may be suppressed if local magnetic fields in the envelope become large as convection sets on ∼1000 yr before runaway. We are not currently in a position to resolve this possible problem but only note that because the conservative mixing timescale $\tau_{\rm mixing} \ll 1000$ yr, substantial mixing suppression by magnetic fields could be accommodated within this model without vitiating the main point, namely, that resonant instability of the C + O/envelope boundary can lead to effective mixing across that boundary. This is a critical point for any nova model because novae have been observed for white dwarfs with relatively strong magnetic fields (e.g., V1500 Cygni 1975; Stockman, Schmidt, & Lamb 1988). However, in the absence of a detailed calculation, this point remains to be addressed by theory.

To conclude, by using the results of linear stability theory as well as extrapolating from existing numerical simulations of nova outbursts, we have estimated the mixing-zone parameters and have shown that prenova erosion of the wave-breaking mixing layer by slow convection could mix sufficient C/O into the accreted H/He envelope to satisfy observations. We have constructed a simple mixing-length subgrid prescription to describe this mixing process and have shown that this subgrid model only needs to be used for the prenova phase. Further mixing during the outburst is no longer required. Because the C/O abundance in the envelope builds up gradually during the prenova slow convective phase, we expect that the nova envelope mass attained before outburst may be substantially larger than in standard models assuming a "preseeded" envelope.

This work has been supported by the DOE-funded ASCI/Alliances Center for Astrophysical Thermonuclear Flashes at the University of Chicago.

REFERENCES

Alexakis, A., Young, Y.-N., & Rosner, R. 2001, Phys. Rev. E, submitted
Brüggen, M., & Hillebrandt, W. 2001a, MNRAS, 320, 73
———. 2001b, MNRAS, 323, 56
Chandrasekhar, S. 1962, Hydrodynamics (New York: Dover)
Chen, G., Kharif, C., Zaleski, S., & Li, J. 1999, Phys. Fluids, 11, 121
Fujimoto, M. Y. 1982, ApJ, 257, 752
Gehrz, R. D., Truran, J. W., Williams, R. E., & Starrfield, S. 1998, PASP, 110, 3
Glasner, S. A., & Livne, E. 1995, ApJ, 445, L149
Glasner, S. A., Livne, E., & Truran, J. W. 1997, ApJ, 475, 754
Howard, L. N. 1961, J. Fluid Mech., 10, 509
Kercek, A., Hillebrandt, W., & Truran, J. W. 1998a, A&A, 337, 379
———. 1998b, in Proc. 16th Int. Conf. on Numerical Methods in Fluid Dynamics, ed. C.-S. Bruneau (Lecture Notes in Physics 515; New York: Springer), 512
Kercek, A., Hillebrandt, W., & Truran, J. W. 1999, A&A, 345, 831
Kippenhahn, R., & Thomas, H.-C. 1978, A&A, 63, 265
Lighthill, M. J. 1962, J. Fluid Mech., 14, 385
MacDonald, J. 1983, ApJ, 273, 289
Miles, J. 1957, J. Fluid Mech., 3, 185
Phillips, O. M. 1957, J. Fluid Mech., 2, 417
Shankar, A., & Arnett, W. D. 1994, ApJ, 433, 216
Shankar, A., Arnett, W. D., & Fryxell, B. A. 1992, ApJ, 394, L13
Starrfield, S., Truran, J., & Sparks, W. M. 1978, ApJ, 226, 186
Starrfield, S., Truran, J., Sparks, W. M., & Kutter, G. G. 1972, ApJ, 176, 169
Stockman, H. S., Schmidt, G. D., & Lamb, D. Q. 1988, ApJ, 332, 282
Sung, C.-H. 1974, A&A, 33, 99
Truran, J. W. 1985, in Nucleosynthesis, ed. W. D. Arnett & J. W. Truran (Chicago: Univ. Chicago Press), 292
Wallace, R. K., & Woosley, S. E. 1981, ApJS, 45, 389