Habilitation à Diriger des Recherches
Turbulent Limits

ALEXAKIS, Alexandros

16-03-2015

Remerciements

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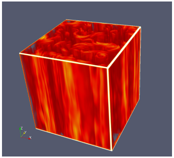
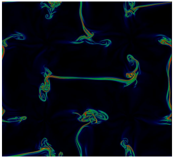
The work presented in the report

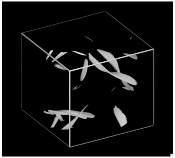


MHD turbulence

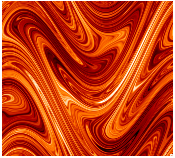


Universality

with: Vassilios DALLAS

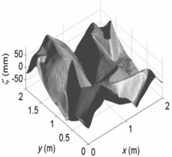


Dynamo



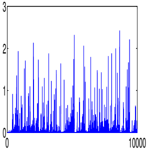
Mixing

with: Alexandra TZELLA



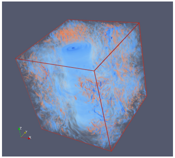
Elastic waves

with: Benjamin MIQUEL & Nicolas MORDANT

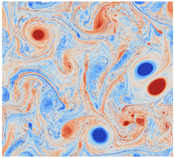


Intermittency

with: Francois PETRELIS



Rotating turbulence

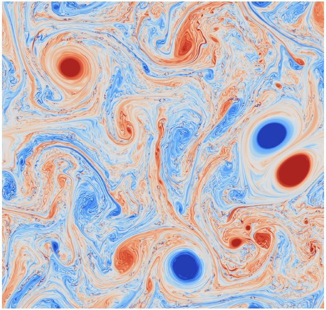


Inverse cascades

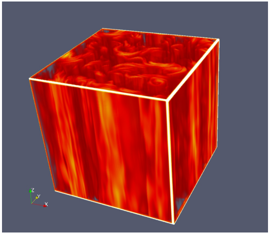
with:

Kanna SESHASAYANAN, Santiago BENAVIDES

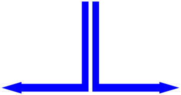
The work that will be presented today: Inverse Cascades

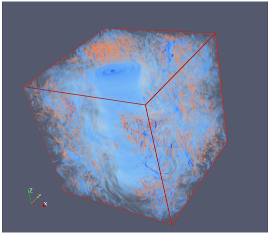


Kannabiran SESHASAYANAN, Santiago Jose BENAVIDES



MHD turbulence





Rotating turbulence

Outline

- An introduction to turbulence
- Forward and Inverse cascades
- Inverse cascades in MHD turbulence
- Inverse cascades in rotating turbulence
- A model problem: 2D MHD
  - Global Quantities
  - Spectra and Fluxes
  - Fields
- Conclusions

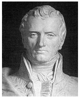
Turbulence is met everywhere...

Astrophysics, Atmospheric physics, weather prediction, geophysics, engineering, aviation, Industry, ...

The problem of (3D hydrodynamic) turbulence

Incompressible Navier-Stokes

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P + \nu\Delta\mathbf{u} + \mathbf{F}, \qquad \nabla\cdot\mathbf{u} = 0, \qquad +B.C.$$



Claude-Louis Navier (1785-1836),

Sir George Gabriel Stokes, (1819-1903)

One non dimensional control parameter: $Re = \dfrac{U_{rms}L}{\nu}$

Osborne Reynolds 1842-1912

Turbulence obtained at the $Re \to \infty$ limit.

- Chaotic system
- Many degrees of freedom
- Injection of energy at one scale by F dissipation of energy at an other scale by viscosity

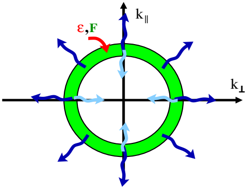
The problem of (3D hydrodynamic) turbulence

$$\text{Energy dissipation} = \epsilon \equiv \nu \langle (\nabla \mathbf{u})^2 \rangle = \langle \mathbf{u} \cdot \mathbf{F} \rangle = \text{Energy injection}$$

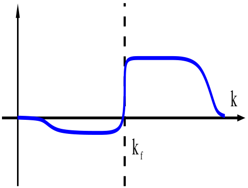
The conjecture of turbulence

$$\lim_{\nu \to 0} \nu \langle (\nabla \mathbf{u})^2 \rangle > 0$$

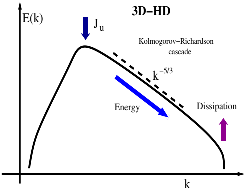
by an ever increasing amplitude of the gradients of u due to the transfer of energy to smaller scales by the nonlinearity.



Fourier space



Energy flux



Energy Spectrum

The problem of (3D hydrodynamic) turbulence

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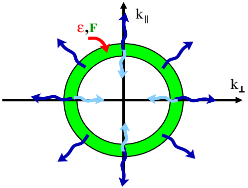
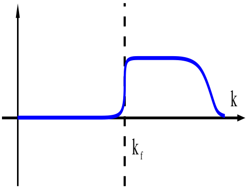
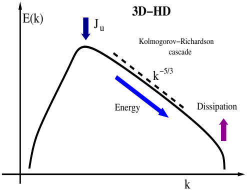


Fourier space



Energy flux (3D HD)



Energy Spectrum

2D hydrodynamic turbulence

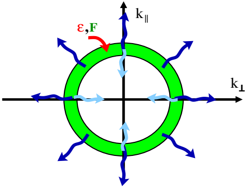
$$\text{Energy dissipation} = \epsilon \equiv \nu \langle (\nabla \mathbf{u})^2 \rangle = \langle \mathbf{u} \cdot \mathbf{F} \rangle = \text{Energy injection}$$

2D turbulence dissipation

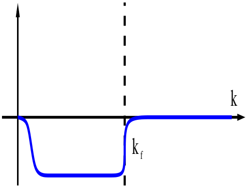
$$\lim_{\nu \to 0} \nu \langle (\nabla \mathbf{u})^2 \rangle \propto \nu u_{rms}^2 / L^2$$

No transfer of energy to smaller scales.

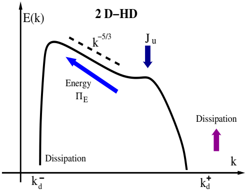
.



Fourier space



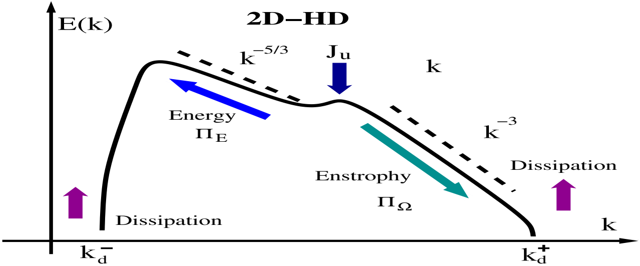
Energy flux (2D HD)



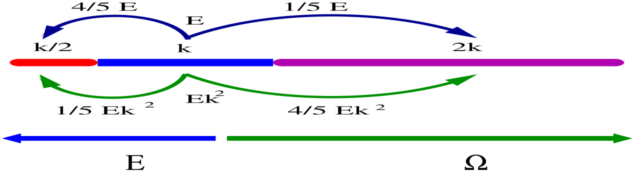
Energy Spectrum

Why 2D turbulence cascades energy inversely?

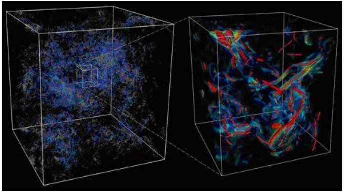
Kraichnan-Leith-Batchelor Cascade



$$\text{Energy}: \quad E = \frac{1}{2}\langle \mathbf{u}^2 \rangle \quad \& \quad \text{Enstrophy}: \quad \Omega = \frac{1}{2}\langle (\nabla \times \mathbf{u})^2 \rangle$$

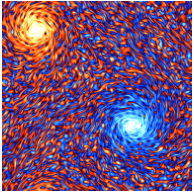


3D Turbulence and 2D Turbulence



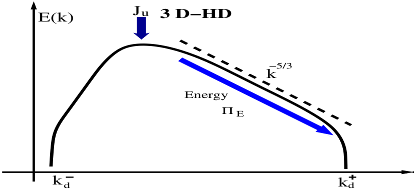
3D simulations at 4096^3

from website of P.K. Yeung

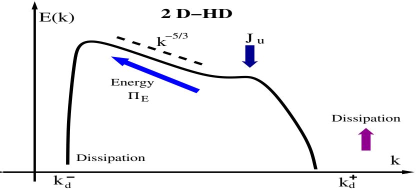


Energy condensation in 2D turbulence

From PhysRevE **85** 036315



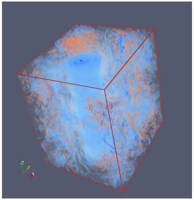
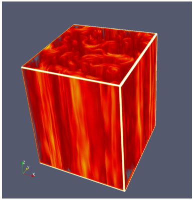
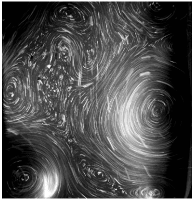
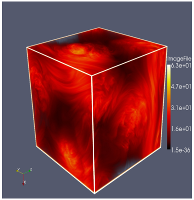
Forward cascade



Inverse cascade

Intermediate cases

There are some systems ...

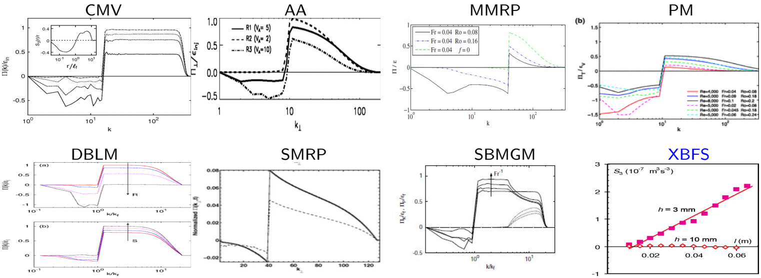
- Fast rotating flows ($Ro \equiv U/\Omega\ell \ll 1$)
- Flows in the presence of a magnetic field ($M \equiv U/B_0 \ll 1$)
- Confined flows (thin geometries) ($\Gamma \equiv h/\ell_f \ll 1$)
- Helical MHD flows ($h_M \equiv$ helicity injection/energy injection $\cdot k_f$)
- ...

for which the inverse cascade depends on a parameter

$$\mu = Ro, \Gamma, M, h_M, \ldots$$

Motivation

Fluxes in: Thin layers/Rotating/Stratified/Magnetic fields ...



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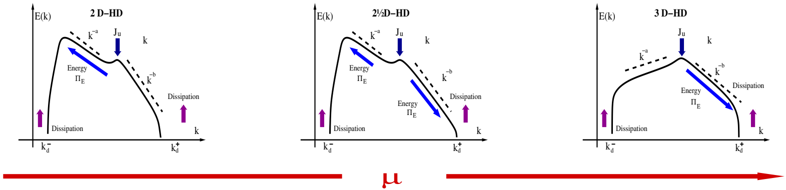
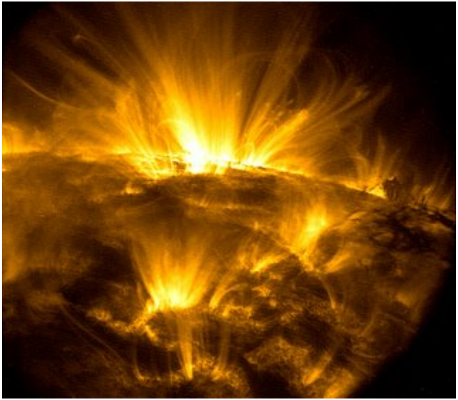
- H. Xia, D. Byrne, G. Falkovich, M. Shats Nature Physics 7, 321-324 (2011)
- M. Shats, D. Byrne, H. Xia Phys. Rev. Lett. **105**, 264501 (2010)

A turbulence to turbulence transition ...



- the system transitions from one turbulent state (inverse cascading) to an other (forward cascading) varying a parameter μ. (μ is not Re)
- the transition occurs in the presence of turbulent noise
- these transitions are not only observed as dimensional (ie 2D to 3D), but weak to strong, HD to MHD, ...
- these transitions are not only observed for the energy cascade but also for other invariants (magnetic helicity, square vector potential, wave action, ...)

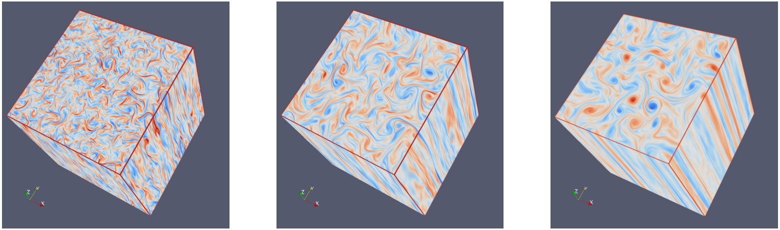
First example: MHD with a strong uniform magnetic field



MHD equations

$$\begin{aligned}\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= B\partial_z \mathbf{b} - \nabla P + \mathbf{b}\cdot\nabla\mathbf{b} + \nu\Delta\mathbf{u} + \mathbf{F_u} \\ \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{b} &= B\partial_z \mathbf{u} + \mathbf{b}\cdot\nabla\mathbf{u} + \eta\Delta\mathbf{b}\end{aligned}$$

First example: MHD with a strong uniform magnetic field



$\mathbf{B}$ →

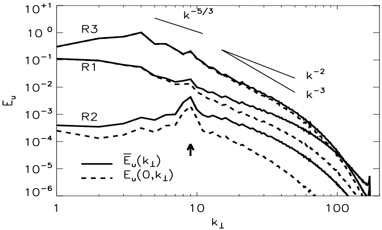
$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= B\partial_z \mathbf{b} \quad -\nabla P \quad +\mathbf{b}\cdot\nabla\mathbf{b} + \nu\Delta\mathbf{u} + \mathbf{F_u} \\
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{b} &= B\partial_z \mathbf{u} \quad +\mathbf{b}\cdot\nabla\mathbf{u} + \eta\Delta\mathbf{b}
\end{aligned}
$$

Large B_0 forces the flow to be 2-dimensional.
The transition from 2D to 3D occurs when the largest mode becomes unstable:

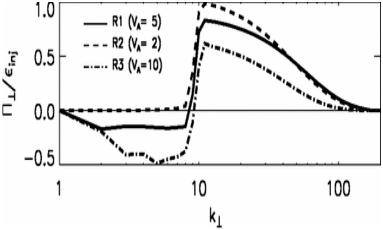
$$B/L_{box} \sim U/\ell$$

Quantifying the transition:

Energy Spectra



Energy flux



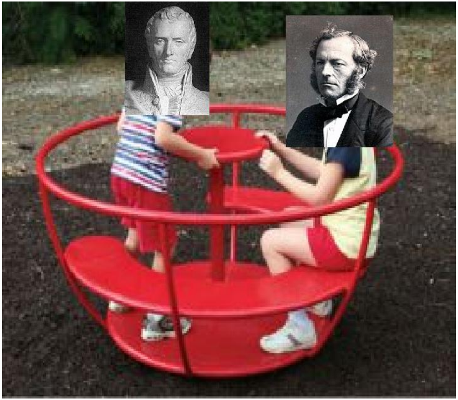
$$B = 2, \qquad B = 5, \qquad B = 10$$

As B is increased

- the flux towards the large scales increases
- the flux towards the small scales decreases
- **below a value of B there is no visible flux to the large scales**

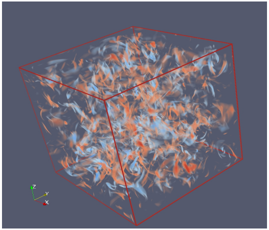
Second example: Turbulence with a twist

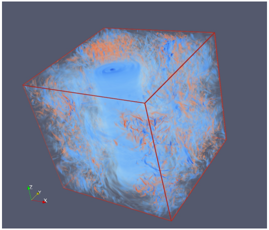
Rotating Navier-Stokes



$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \times \mathbf{u} = -\nabla P + \nu \Delta \mathbf{u} + \mathbf{F}, \qquad \nabla \cdot \mathbf{u} = 0, \qquad + B.C.$$

Rotating turbulence (Taylor Green Forcing)





$\boldsymbol{\Omega}$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \times \mathbf{u} = -\nabla P + \nu \Delta \mathbf{u} + \mathbf{F}, \qquad \nabla \cdot \mathbf{u} = 0, \qquad +B.C.$$

$$\nabla \times \cdots \Rightarrow$$

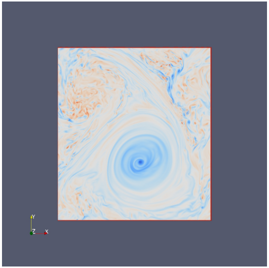
$$\partial_t \mathbf{w} + \mathbf{u} \cdot \nabla \mathbf{w} + 2\Omega \partial_z \mathbf{u} = \mathbf{w} \cdot \nabla \mathbf{u} + \nu \Delta \mathbf{w} + \nabla \times \mathbf{F}, \qquad \mathbf{w} = \nabla \times \mathbf{u},$$

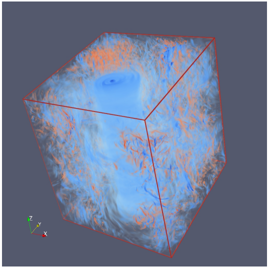
The transition from 2D to 3D occurs when the largest mode becomes unstable:

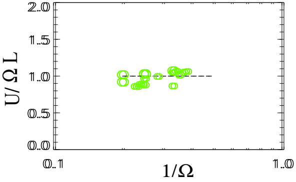
$$\Omega \sim U/\ell \quad \text{or} \quad Ro \equiv U/\Omega L \simeq 1$$

Large scale condensates

How an inverse cascade saturates
(in the absence of large scale dissipation)







at saturation the flow forms condensates
and moves Ro to the **critical** value:

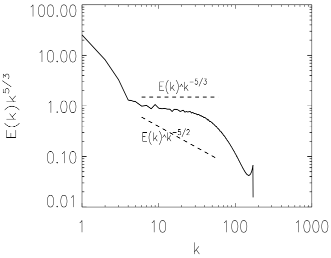
$$Ro = U/\Omega L \simeq 1$$

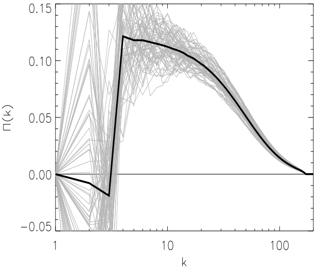
where the inverse cascade stops

The system is attracted to the marginal inverse-cascade-state!

Spectrum and fluctuations of condensates

At steady state:





- At the condensate state large & small scales co-exist
- Flux to the large scales $\simeq$0
- Large fluctuations of the flux (noise)

Attempting a detailed study of the transition.

Previous models had a varying inverse cascade of energy due to a dimensional transition from a 3D to a 2D

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Thus they require 3D high resolution numerical simulations.

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Thus they require 3D high resolution numerical simulations.

Are there computationally more tractable models that show a transition from forward to inverse cascade?

Breaking the enstrophy conservation

The inverse cascade of energy in 2D is solely due to enstrophy $\langle(\nabla \times \mathbf{u})^{\mathbf{2}}\rangle$ conservation.

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The inverse cascade of energy in 2D is solely due to enstrophy $\langle(\nabla \times \mathbf{u})^2\rangle$ conservation.

2D or not 2D?

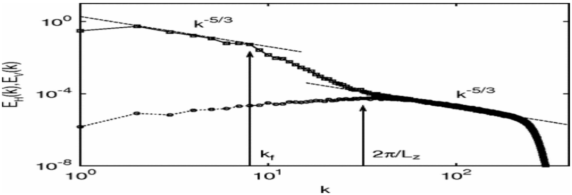


Figure from A. Celani, S. Musacchio, and D. Vincenzi,

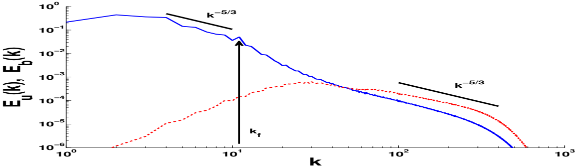
Phys. Rev. Lett. **104**, 184506 (2010)

Energy spectra of in-plane and out-of-plane velocity components

Breaking the enstrophy conservation

The inverse cascade of energy in 2D is solely due to enstrophy $\langle(\nabla \times \mathbf{u})^{\mathbf{2}}\rangle$ conservation.

2D-HD or 2D-MHD?



Kinetic and magnetic energy spectra
In a 2D MHD simulation

....

2D-HD vs 2D-MHD

Equations:

$$\partial_t \omega + \mathbf{u} \cdot \nabla \omega \quad = \qquad + [\nu_n^+ \nabla^{2n} \omega + \nu_n^- \nabla^{-2n} \omega] + F_\omega$$

where

$$\omega = \hat{\mathbf{e}}_n \cdot \nabla \times \mathbf{u}$$

Nonlinearity conserves

$$E = \frac{1}{2} \int \mathbf{u}^2 dv, \qquad \Omega = \frac{1}{2} \int \omega^2 dv$$

2D-HD vs 2D-MHD

Equations:

$$\begin{aligned}
\partial_t \omega + \mathbf{u} \cdot \nabla \omega &= \mathbf{b} \cdot \nabla j + [\nu_n^+ \nabla^{2n} \omega + \nu_n^+ \nabla^{-2n} \omega] + F_\omega \\
\partial_t a + \mathbf{u} \cdot \nabla a &= \quad + [\eta_n^+ \nabla^{2n} a + \eta_n^- \nabla^{-2n} a] + F_a
\end{aligned}$$

where

$$\omega = \hat{\mathbf{e}}_n \cdot \nabla \times \mathbf{u}, \quad \mathbf{b} = \nabla \times (\hat{\mathbf{e}}_n a), \quad j = \hat{\mathbf{e}}_n \cdot \nabla \times \mathbf{b}$$

Nonlinearity conserves

$$E = \frac{1}{2} \int \mathbf{u}^2 + \mathbf{b}^2 dv, \qquad A = \frac{1}{2} \int a^2 dv$$

2D-HD vs 2D-MHD

Equations:

$$\begin{aligned} \partial_t \omega + \mathbf{u}\cdot\nabla\omega &= \mathbf{b}\cdot\nabla j + [\nu_n^+ \nabla^{2n}\omega + \nu_n^+ \nabla^{-2n}\omega] + F_\omega \\ \partial_t a + \mathbf{u}\cdot\nabla a &= + [\eta_n^+ \nabla^{2n} a + \eta_n^- \nabla^{-2n} a] + F_a \end{aligned}$$

where

$$\omega = \hat{\mathbf{e}}_n \cdot \nabla \times \mathbf{u}, \quad \mathbf{b} = \nabla \times (\hat{\mathbf{e}}_n a), \quad j = \hat{\mathbf{e}}_n \cdot \nabla \times \mathbf{b}$$

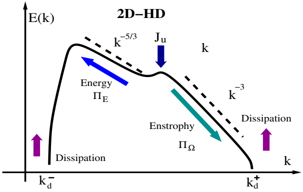
Nonlinearity conserves

$$E = \frac{1}{2}\int \mathbf{u}^2 + \mathbf{b}^2 dv, \qquad A = \frac{1}{2}\int a^2 dv$$

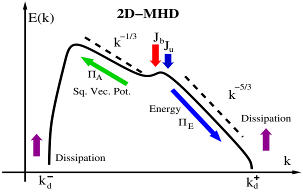
- we will use the hypodissipation $\eta_n^- \nabla^{-2n}$ to avoid condensates
- we will use the hyperdiffusion $\eta_n^- \nabla^{-2n}$ to extend the inertial range:n=2.

2D-HD vs 2D-MHD

2D-HD



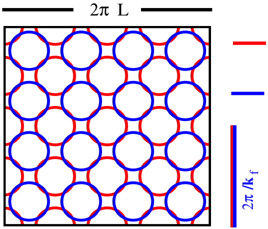
2D MHD



$\lvert\mathbf{F}_u\rvert > 0,\ \mathbf{F}_b = 0$	$\lvert\mathbf{F}_u\rvert > 0,\ \lvert\mathbf{F}_b\rvert > 0$	$\mathbf{F}_u = 0,\ \lvert\mathbf{F}_b\rvert > 0$
Inverse cascade of E	**?**	Forward cascade of E
Forward cascade of Ω	**?**	not conserved
Forward cascade of A	**?**	Inverse cascade of A

What is the fate of the forward/inverse cascade as we vary $\mathbf{F}_u, \mathbf{F}_b$?

Set-up of Numerical Experiments



2D square periodic box of side $2\pi L$

No mean magnetic field $\langle \mathbf{b} \rangle = \mathbf{0}$

$$F_\omega(x,y) = f_u k_f^{+1} \sin(k_f x)\sin(k_f x)$$
$$F_a(x,y) = f_b k_f^{-1} \cos(k_f x)\cos(k_f x)$$

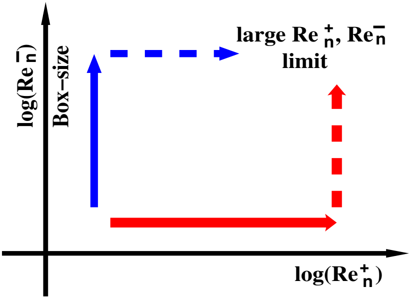
Control Parameters / Non-dimensional Numbers

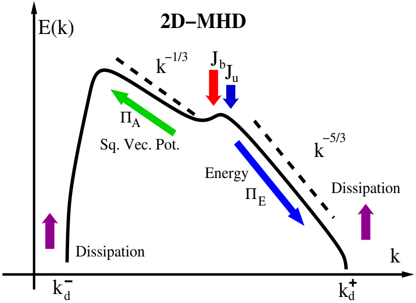
$$\mu_f \equiv \frac{f_b}{f_u}$$

$$k_f L \qquad Re_f^- = \frac{f_u^{1/2} k_f^{1/2+2n}}{\nu_n^-} \qquad Re_f^+ = \frac{f_u^{1/2} k_f^{1/2-2n}}{\nu_n^+}$$

$$P_M^- \equiv \nu_n^-/\eta_n^- = 1, \qquad P_M^+ \equiv \nu_n^+/\eta_n^+ = 1$$

Limiting procedure





- Fix Re_n^+, vary μ_f for different box sizes $(Re_n^-, k_f L)$
- Fix box size $(Re_n^-, k_f L)$ vary μ_f for different values of Re_n^+,

Global Quantities

Quantifying the cascades

Inverse and Forward cascades of energy:

$$\epsilon_{E}^{-} \equiv \nu_{n}^{-} \langle (\nabla^{-n}\mathbf{u})^{\mathbf{2}} + (\nabla^{-\mathbf{n}}\mathbf{b})^{\mathbf{2}} \rangle, \qquad \epsilon_{\mathbf{E}}^{+} \equiv \nu_{\mathbf{n}}^{+} \langle (\nabla^{+\mathbf{n}}\mathbf{u})^{\mathbf{2}} + (\nabla^{+\mathbf{n}}\mathbf{b})^{\mathbf{2}} \rangle$$

$$\epsilon_{E} \equiv \epsilon_{E}^{-} + \epsilon_{E}^{+} \qquad 0 \leq \frac{\epsilon_{E}^{-}}{\epsilon_{E}} \leq 1,$$

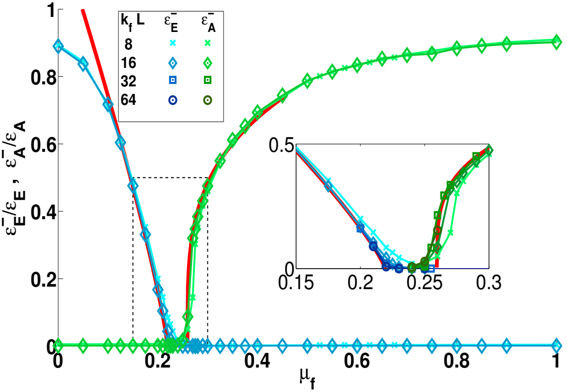
Inverse and Forward cascades of square vector potential:

$$\epsilon_{A}^{-} \equiv \nu_{n}^{-} \langle (\nabla^{-n} a)^{2} \rangle, \qquad \epsilon_{A}^{+} \equiv \nu_{n}^{+} \langle (\nabla^{+n} a)^{2} \rangle$$

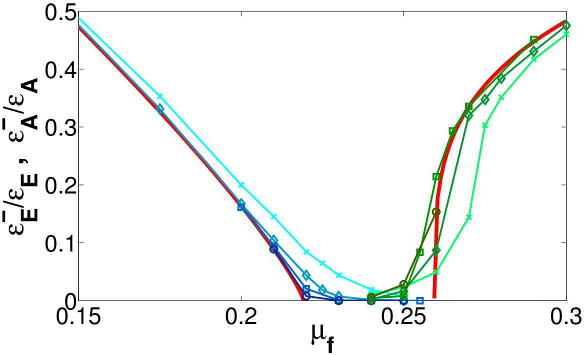
$$\epsilon_{A} \equiv \epsilon_{E}^{-} + \epsilon_{E}^{+} \qquad 0 \leq \frac{\epsilon_{A}^{-}}{\epsilon_{A}} \leq 1,$$

A Critical transition

Varying μ_f for different box-size and fixed Re_n^+.



A Critical transition



critical behavior:

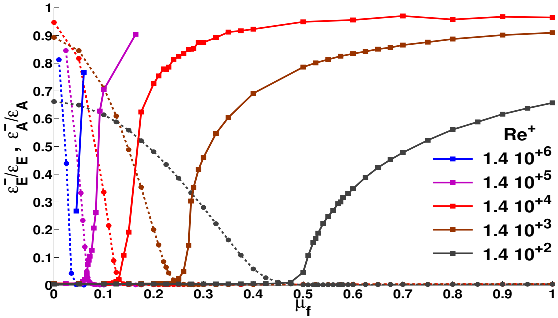
$$\epsilon^-_E \propto (\mu_{c_E} - \mu)^{\gamma_E} \quad and \quad \epsilon^-_A \propto (\mu - \mu_{c_A})^{\gamma_A}$$

a best fit leads to:

$$\mu_{c_E} \simeq 0.22\ldots, \ \gamma_E \simeq 0.82 \quad and \quad \mu_{c_A} \simeq 0.25\ldots, \ \gamma_A \simeq 0.27$$

Critical point dependence on Re_n^+

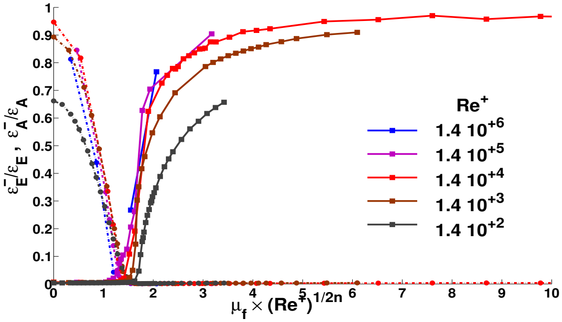
Varying μ_f for different values of Re_n^+ and fixed box-size.



- $\mu_c = \mu_c(Re_n^+)$

Critical point dependence on Re_n^+ (rescaling)

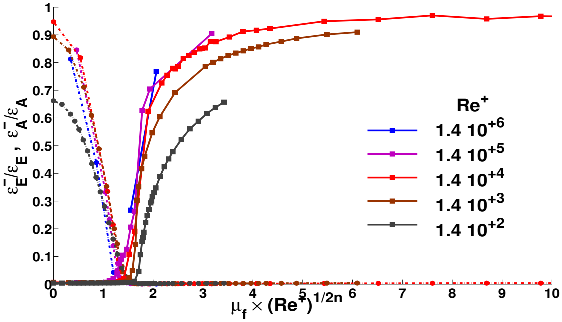
Varying μ_f for different values of Re_n^+ and fixed Re_n^-.



- $\mu_c \propto (Re_n^+)^{-1/2n}$

Critical point dependence on Re_n^+ (rescaling)

Varying μ_f for different values of Re_n^+ and fixed Re_n^-.



- $\mu_c \propto (Re_n^+)^{-1/2n}$

Magnetic tension determines the transition:

$$\mu_b \equiv \frac{b^2 k_f}{f_u} \propto \mu_f^2 \left(\frac{k_d^+}{k_f}\right)^2 \propto \mu_f^2 [Re^+]^n$$

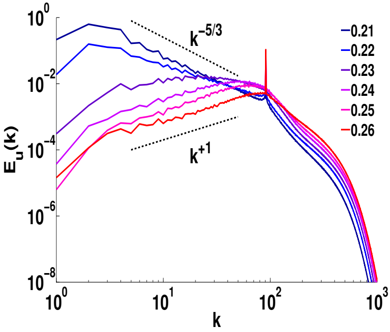
Energy distribution among scales

Large scale spectra

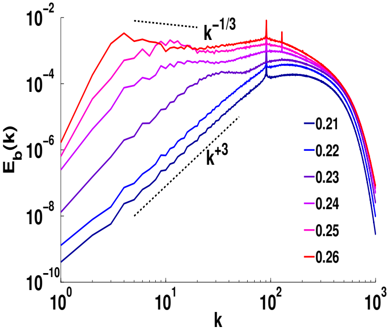
Varying μ_f for large box-sizes $k_f L \gg 1$.

$\mu_{c_E} \simeq 0.22\ldots, \qquad \& \qquad \mu_{c_A} \simeq 0.25\ldots$

Kinetic energy spectra

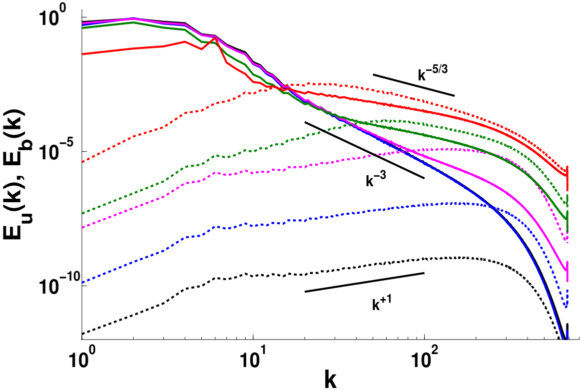


Magnetic energy spectra

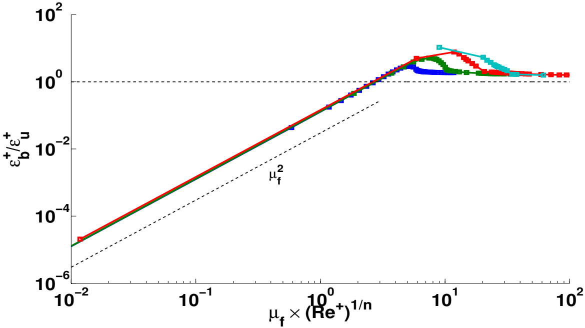


Small scale spectra

Varying μ_f for $Re_n^+ \gg 1$.



Small scale dissipations



For small μ

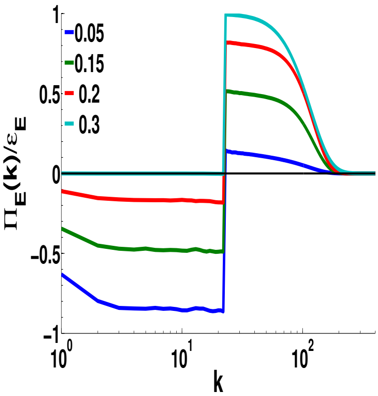
- magnetic energy at the smallest scales is $b_\ell^2 \propto \mu^2 \ell_d^{-2}$ (passive advectio)
- kinetic energy at the smallest scales is $u_\ell^2 \propto \epsilon_\Omega^{2/3} \ell_d^2$ (enstrophy cascade)

Nonlinearity starts when

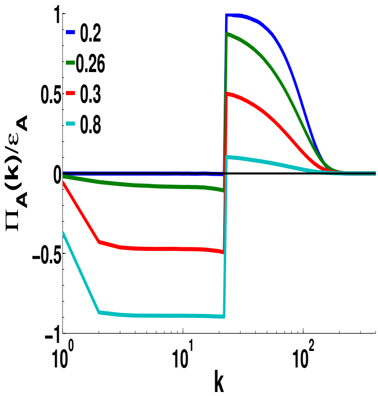
$$\mu \geq \mu_{NL} \propto \ell_d^2 \propto Re^{-1/n}$$

Variable forward and backward fluxes

Energy flux

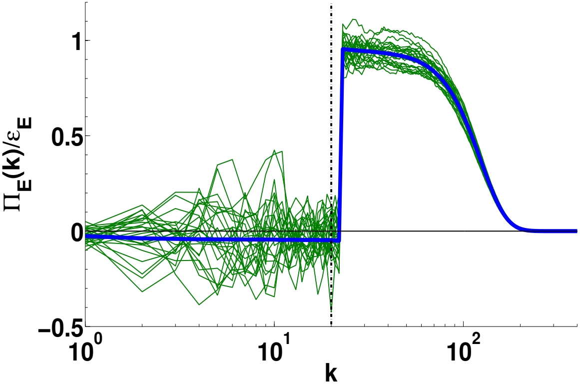


Sq.Vec.Pot. flux



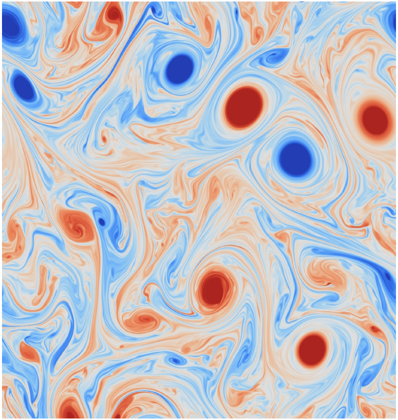
Instantaneous and time averaged fluxes

Strong fluctuations of the energy fluxes .....

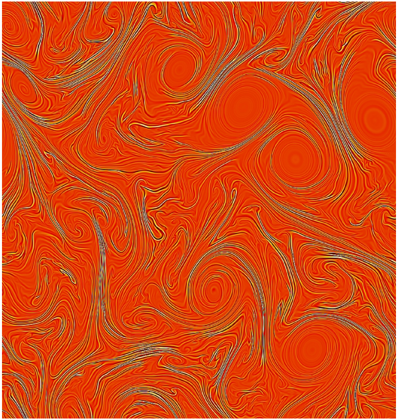


Fields

$$\mu \ll \mu_{NL}$$



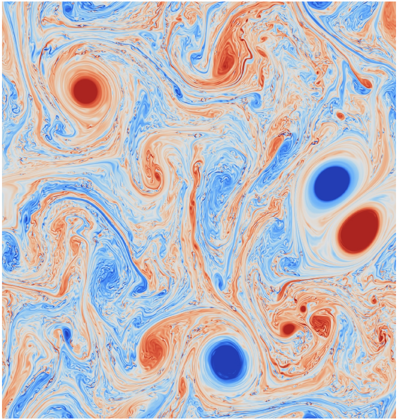
Vorticity



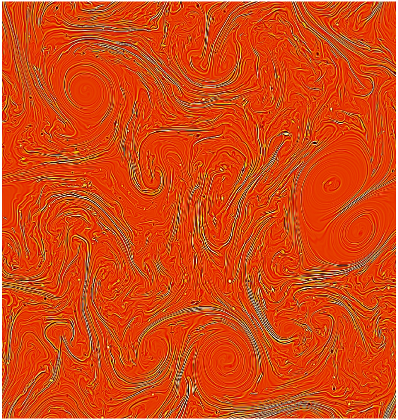
Current density

Small scale Structures — 2D-HD to 2D-MHD

$$\mu_{NL} \lesssim \mu \ll \mu_c$$



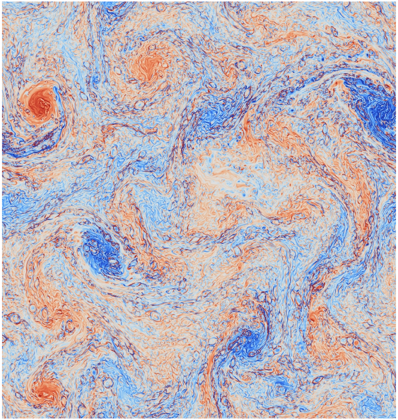
Vorticity



Current density

Small scale Structures — 2D-HD to 2D-MHD

$$\mu_{NL} \ll \mu \ll \mu_c$$

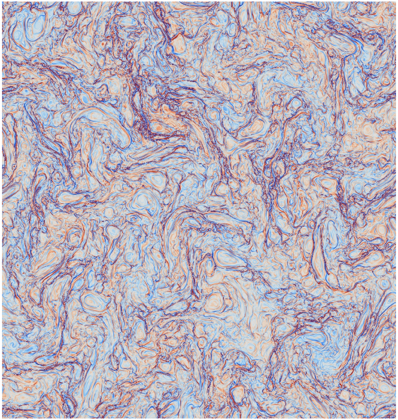


Vorticity

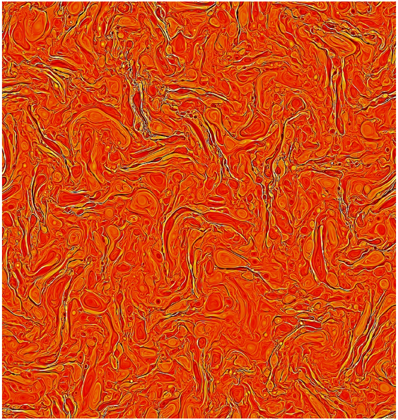
Current density

Small scale Structures — 2D-HD to 2D-MHD

$$\mu_{NL} \ll \mu \lesssim \mu_c$$



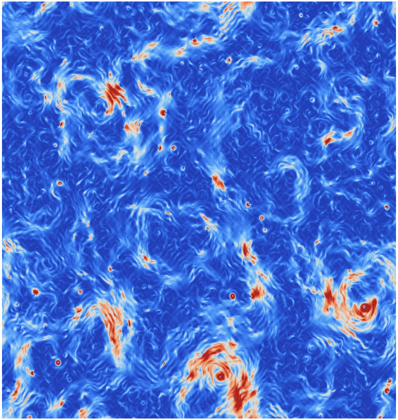
Vorticity



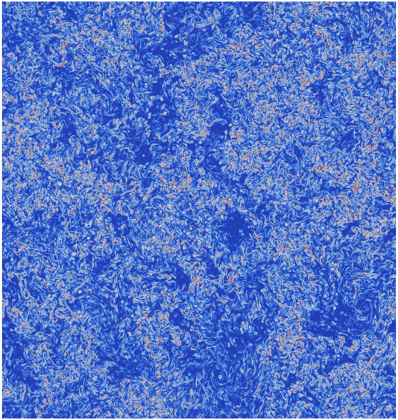
Current density

Large scale Structures — 2D-HD to 2D-MHD

$$\mu = 0.21 \cdots \lesssim \mu_c$$

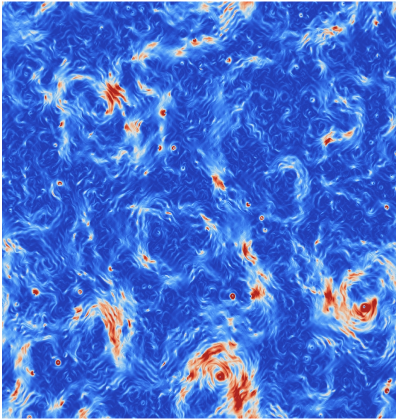


Kinetic energy

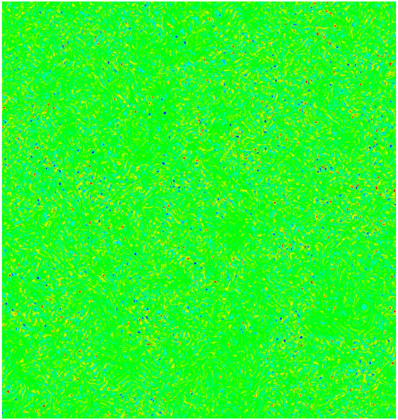


Magnetic energy

$$\mu = 0.21 \cdots \lesssim \mu_c$$



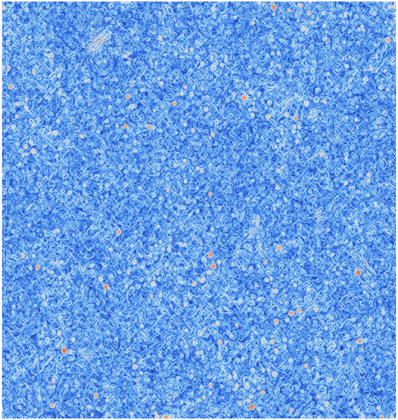
Kinetic energy



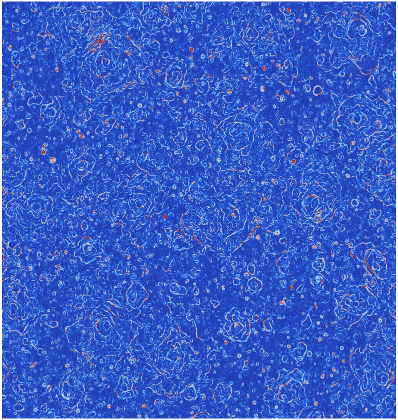
Vector Potential

Large scale Structures — 2D-HD to 2D-MHD

$$\mu = 0.26\cdots \gtrsim \mu_c$$



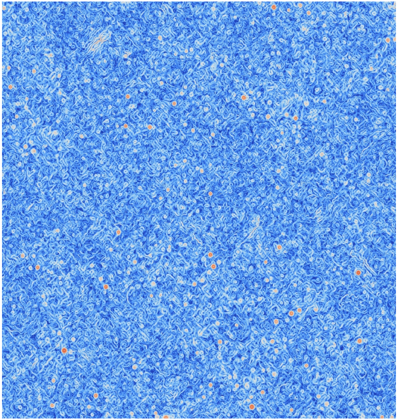
Kinetic energy



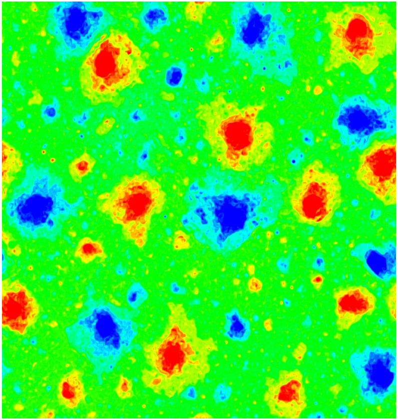
Magnetic energy

Large scale Structures — 2D-HD to 2D-MHD

$$\mu = 0.26 \cdots \gtrsim \mu_c$$



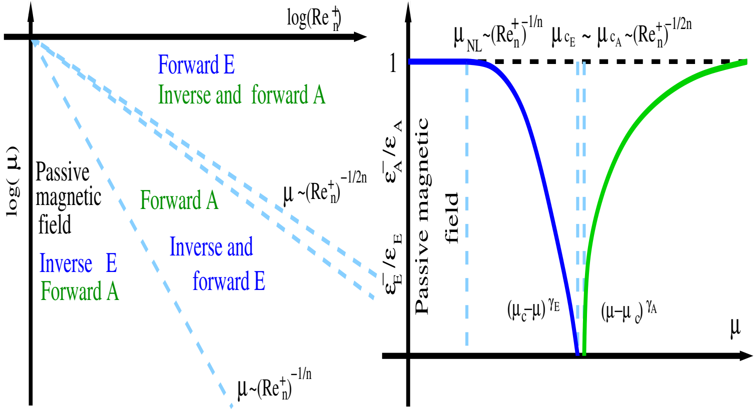
Kinetic energy



Vector Potential

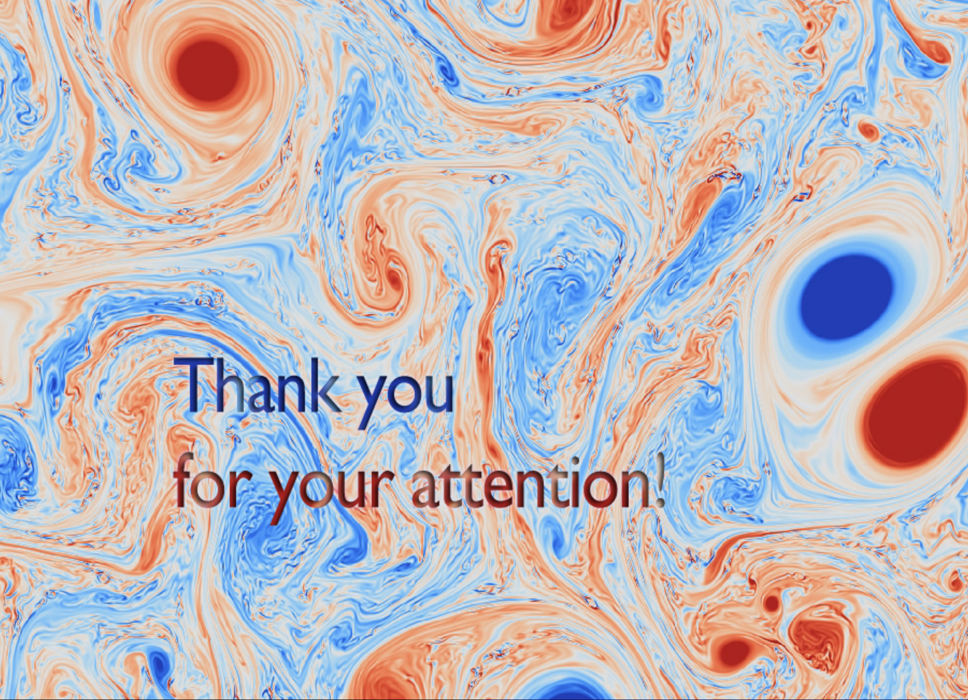
Phase diagram

In the limit $k_f L \to \infty$



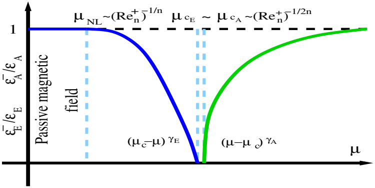
Conclusions

- Transitions from forward to inverse cascades (**IC**) are common in turbulence
- The transition to **IC** can be critical
- In the absence of large scale dissipation (some) **IC** saturate at the marginal state for an **IC**
- 2D MHD is the first model to demonstrate that this transition happens through a critical point.
- New theoretical venues open for expansions around the critical point

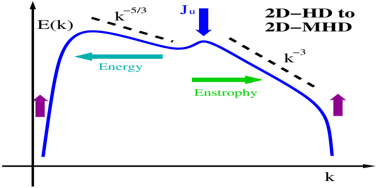

Thank you
for your attention!

A cartoon summary

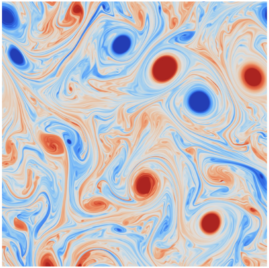
Transition Diagram



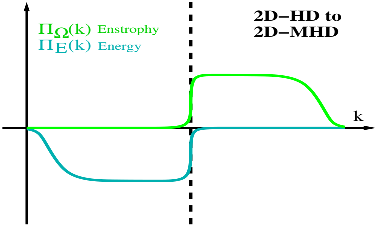
Energy Spectra



Stream Function

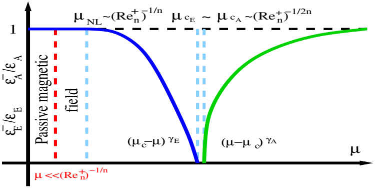


Fluxes

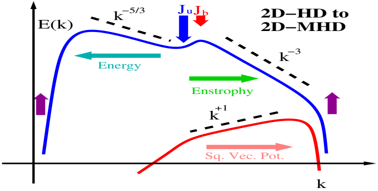


A cartoon summary I

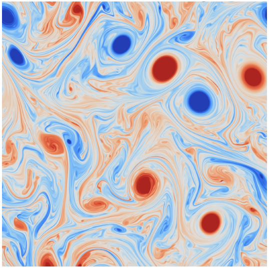
Transition Diagram



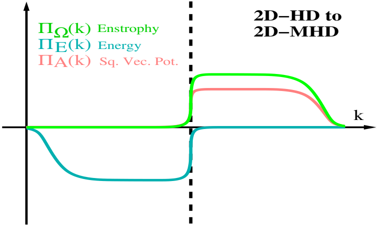
Energy Spectra



Stream Function

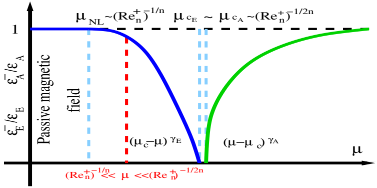


Fluxes

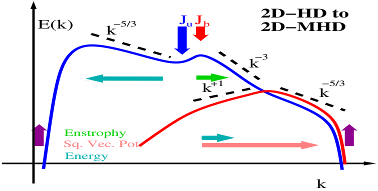


A cartoon summary II

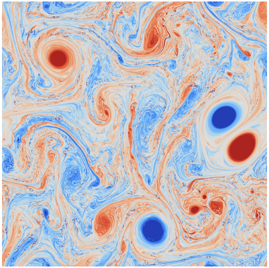
Transition Diagram



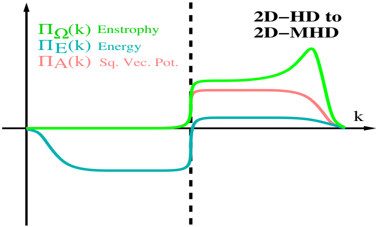
Energy Spectra



Stream Function

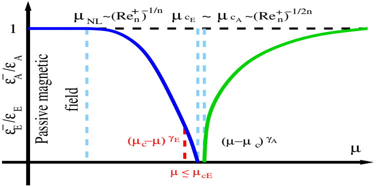


Fluxes

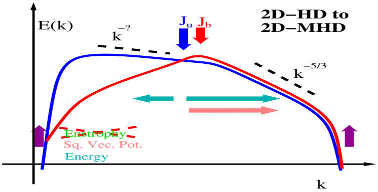


A cartoon summary III

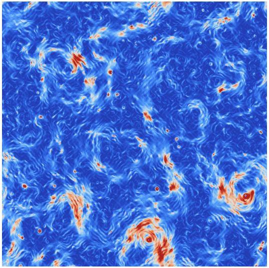
Transition Diagram



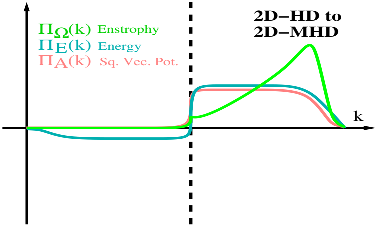
Energy Spectra



Kinetic Energy

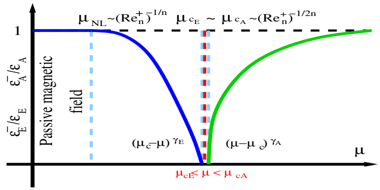


Fluxes

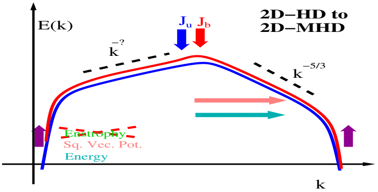


A cartoon summary IV

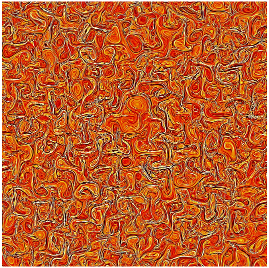
Transition Diagram



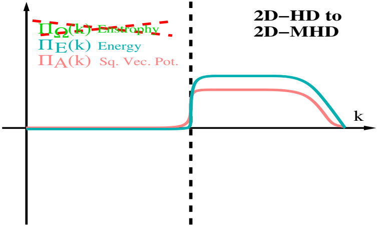
Energy Spectra



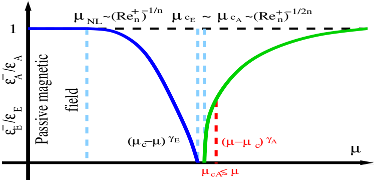
Current density



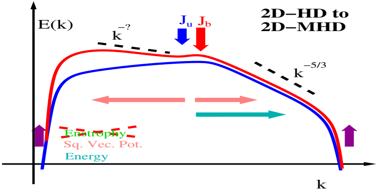
Fluxes



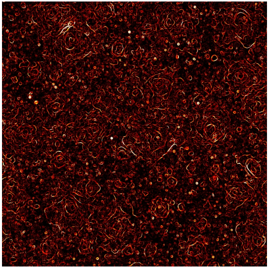
Transition Diagram



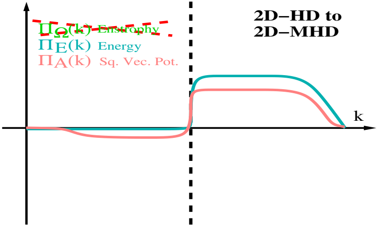
Energy Spectra



Magnetic Energy

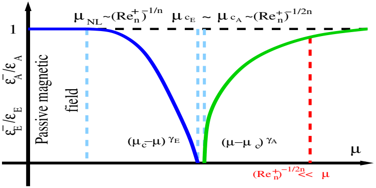


Fluxes

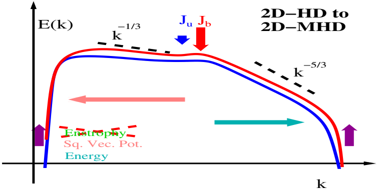


A cartoon summary VI

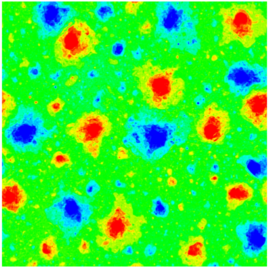
Transition Diagram



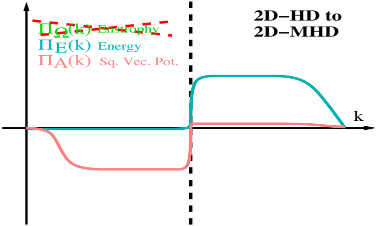
Energy Spectra



Vector Potential



Fluxes



Break down of the enstrophy conservation