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The self-organization of capillary wave sources

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Abstract
A liquid drop can be kept bouncing at the surface of a bath of the same liquid for any length of time if the bath is kept oscillating vertically. Several regimes can be observed. For a liquid of moderate viscosity, the bouncing of the drop generates damped capillary waves with a wavelength corresponding to the forcing frequency. Therefore when several identical drops are placed on the oscillating surface, the interaction of their waves leads to the self-organization of the drops with a 2D triangular lattice. Another remarkable regime is observed when the forcing amplitude is increased close to the Faraday instability threshold: the drop starts moving in the horizontal plane at a constant velocity. We have studied the movement of one 'walking drop' as well as the possible interactions of several of these particular drops placed on the surface of the liquid, leading to their complex self-organization. These drops can collide via their waves and in certain situations attract each other and start orbiting.

1. Introduction

In a previous article (Couder *et al* 2005), we showed that the coalescence of a drop deposited on a bath of the same fluid could be inhibited by vertical oscillations. Normally, if a drop falls on the surface of the same liquid, gravity makes it press upon the air film between the drop itself and the surface. The attractive van der Waals force and then the surface tension bring the two surfaces together, leading to the spreading of the drop into the liquid. We can inhibit this phenomenon by oscillating the bath of liquid on which the drop is placed. When the amplitude of the forcing is large enough, the drop lifts up from the surface periodically and the intermediate air film is constantly renewed. This phenomenon, first observed in soap solutions (Walker 1978), has recently been studied in pure fluids (Couder *et al* 2005), and the major role played by the intermediate air film was characterized. This paper is devoted to the self-organization observed when several drops are simultaneously present on the fluid surface. In the limit of very viscous fluids they attract and form dense clusters. For less viscous fluids the drops' bouncing generates capillary waves. Because of the interaction of these waves, static clusters are formed.

All these experiments are performed on a plane interface. It is well known however that when a bath of liquid is oscillated vertically, there is a threshold over which the free surface becomes unstable and parametrically forced standing waves form. Faraday (1831) first characterized this instability. It has been widely studied from an experimental (e.g. Ciliberto and Gollub 1984, Douady and Fauve 1988) as well as a theoretical point of view (e.g. Benjamin and Ursell 1954, Kumar and Tuckerman 1994). In our experimental situation, for large forcing amplitudes the bouncing becomes sub-harmonic so that the drops become local sources of Faraday waves, even though we are still below the instability threshold. In this case, dynamical regimes of self-organization are observed. Preliminary results were published in Couder *et al* (2005).

2. Experiment

The liquid chosen for this experiment is silicon oil because its surface is not sensitive to surfactant effects. The bouncing regime can be observed for drops of various sizes ($D < 3$ mm) and for a large range of oil viscosities (from 5×10^{-3} to 1 Pa s).

Several parameters define the system: f_0 the forcing frequency, γ_{m} the maximum acceleration of the oscillation, D the drop's diameter, and the viscosity of the liquid μ. The bouncing of large drops of high viscosity has been studied in Couder *et al* (2005). Small drops of very viscous oil at low frequencies remain spherical during their bouncing motion and the surface changes very little. With very viscous oils, $\mu > 100 \times 10^{-3}$ Pa s, the waves emitted by the drops are damped. If the drops are close enough, they will drift towards each other, due to a classical attractive interaction of their menisci. The drops then form a compact aggregate, with only an air film between them inhibiting their coalescence (figure 1(a)). When the wavelength of the waves formed by the drops is approximately the same as the drops' diameter, the aggregate starts to rotate (figure 1(b)). In this situation, at each bounce the drops fall on the inner slope of the global trough formed in the interface by the bouncing aggregate. This gives the drops a centripetal force which balances the centrifugal forces.

We can now investigate the situation for small drops ($D < 1$ mm) of silicon oils of weaker viscosity (20×10^{-3} or 50×10^{-3} Pa s). Bouncing drops can be observed for frequencies 20 Hz $< f_0 <$ 200 Hz. For a forcing amplitude larger than the threshold $\gamma_{\mathrm{m}}^{\mathrm{C}}$, a small drop can be kept bouncing for an unlimited amount of time. In most of our experiments we will work with a fluid of given viscosity at a fixed frequency. As the amplitude of the bath's oscillations is increased, drops of various sizes will behave very differently. Consequently we can draw a phase diagram showing the different behaviour of a drop as a function of size and of the acceleration (figure 2). Two thresholds set the limit of the region for which bouncing drops exist. The lowest limit is the minimum acceleration $\gamma_{\mathrm{m}}^{\mathrm{C}}$ needed to inhibit the coalescence of the drop. During the bouncing motion, the smaller drops ($D < 1$ mm) remain spherical whereas the larger drops become oblate when pressed against the air film and return to sphericity when they lift off the surface. This threshold $\gamma_{\mathrm{m}}^{\mathrm{C}}$ grows as ω_0^2, with ω_0 being the forcing pulsation, for drops of a given size. For the smaller drops, we have the simple limit $\gamma_{\mathrm{m}}^{\mathrm{C}} \approx g$. The higher limit corresponds to the Faraday instability threshold over which the drops bounce chaotically on the wavy interface, this chaos usually leading to coalescence.

2.1. Simple bouncers

When the acceleration is increased, we have to differentiate the motion of smaller ($D <$ 0.5 mm) and larger (0.5 mm $< D <$ 1.1 mm) drops. Since the smaller drops' size is much smaller than the capillary length $\kappa^{-1} = (\sigma/\rho g)^{1/2}$, both the drop and the interface behave as rigid bodies. We have filmed the drop's vertical motion with a fast camera. Spatio-temporal diagrams of the motion of the drop as well as its reflection in the oscillating bath's surface can

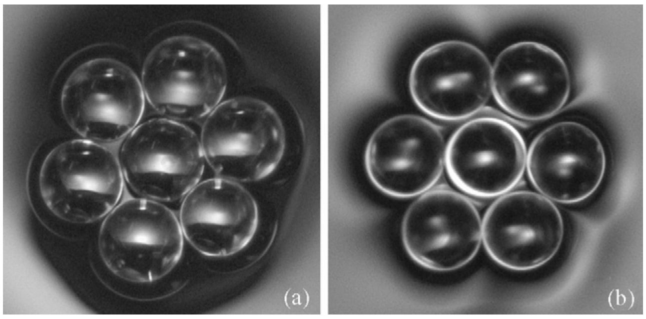

Figure 1. (a) Compact aggregate of seven drops of very viscous oil ($\mu = 500 \times 10^{-3}$ Pa s, $f_0 = 30$ Hz, $D = 2.2$ mm). (b) Aggregate of seven drops of viscous oil rotating at 2 turns s^{-1} ($\mu = 100 \times 10^{-3}$ Pa s, $f_0 = 70$ Hz, $\gamma_m/g = 5.8$, $D = 2.2$ mm).

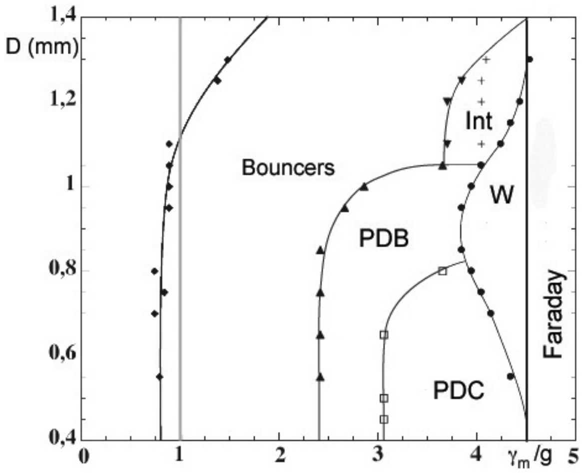

Figure 2. Phase diagram showing the drop's various behaviour as a function of its diameter D and of the forcing acceleration. The liquid used here is silicon oil of viscosity 50×10^{-3} Pa s and the forcing frequency is $f_0 = 50$ Hz. B corresponds to the simple bouncing region, PDB to period doubling bouncing, PDC to the transition to chaos, Int. to an intermittent behaviour. W is the region where the drops become walkers, just below the Faraday instability threshold.

thus be obtained. For increasing accelerations of the same drop they show successively the simple bouncing, then the period doubling and transition to chaos. The bouncing is thus very similar to that of a solid ball on a vibrating solid plate (Tufillaro and Albano 1986) and we observe the same transition to chaos with a period doubling scenario.

When several identical bouncers coexist at the surface of the liquid, their waves interact. In the regime of simple bouncing, when the drops are close enough, they slowly drift towards each other until they reach a fixed distance d. Each drop is then a source of propagative capillary waves. A steady regime is formed where each drop bounces with a forced periodicity, in the same trough of the sum of the waves emitted by the two drops. This steady regime occurs when the two drops are at a distance d slightly smaller than the wavelength λ_0 of the forcing frequency f_0. If several drops are placed on the surface, the drops then form a cluster of drops at this same distance d (figure 3(c)).

When one of the drops accidentally coalesces, the remaining drops rearrange themselves to keep this same distance between them. The drops are then organized in a cluster of triangular

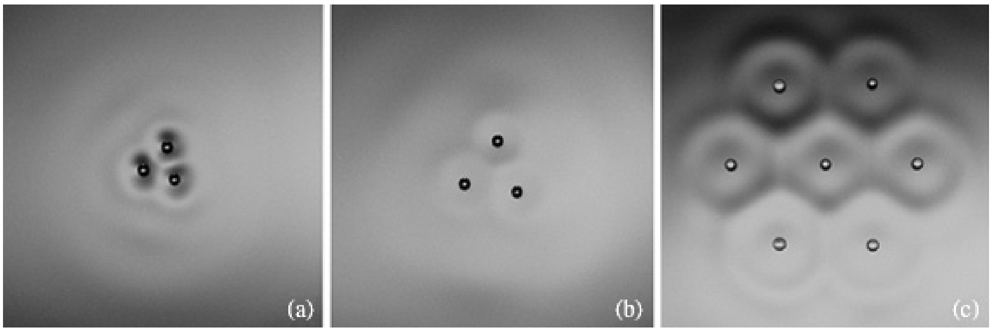

Figure 3. (a) A cluster of three identical drops will organize themselves so as to bounce at the distance λ_0 when the forcing acceleration is $\gamma_m/g = 2.5$ ($\mu = 20 \times 10^{-3}$ Pa s, $f_0 = 80$ Hz, $D = 0.7$ mm). (b) The same cluster of three drops will expend as we increase the forcing acceleration to $\gamma_m/g = 2.8$ ($\mu = 20 \times 10^{-3}$ Pa s, $f_0 = 80$ Hz, $D = 0.7$ mm). (c) Seven identical drops bouncing together and forming a cluster of triangular lattice ($\mu = 20 \times 10^{-3}$ Pa s, $f_0 = 56$ Hz, $D = 0.7$ mm).

lattice (figure 3(a)), the exception being a cluster of six drops for which we observe a pentagon with a central drop. The system is then frustrated and the distance between the drops is greater than d. Nevertheless, this situation is very unstable, and one of the drops will tend to coalesce to allow the remaining drops to recover the distance d from each other. If we increase the forcing acceleration, the cluster of drops expands brutally (figure 3(b)). The drops have then reached the period doubling regime and emit waves at both the forcing and half the forcing frequency. Therefore the distance d for which a standing wave is formed between two drops is no longer the forcing wavelength but approximately the Faraday wavelength. In this situation the drops still organize themselves so that there is a standing wave between them.

The larger drops (0.5 mm $< D < 1.1$ mm) go through the same initial period doubling when the forcing acceleration is increased. However, the drops deform during the collision and the dissipation is larger. The spatio-temporal recordings show that the motion evolves into a simple bouncing at half the forcing frequency. A remarkable transition then occurs: the drop starts moving across the surface at a constant velocity. For simplicity, we shall call these drops 'walkers'.

2.2. Walkers

These drops' motion is parametrically forced: it has the Faraday frequency $f_F = f_0/2$, therefore the emitted wave has the wavelength λ_F usually observed in Faraday standing waves. This phenomenon is only observed close to the Faraday instability threshold. When this threshold is reached, the wave emitted by a walker propagates in the whole cell rapidly, letting the well-known Faraday standing waves appear.

The spatio-temporal diagram shows that a walker is in contact with the surface during approximately a fifth of a Faraday period. This is when the drop hits the surface, shaping a trough that will grow and propagate. The wave propagates freely until the drop hits the surface again. Each time the drop reaches the surface, a wave is emitted.

We have visualized the waves' motion with a fast camera. The wave emitted after the drop hits the surface is a propagative wave of modulated amplitude. The trough formed by the drop as it evolves induces the formation of a protruding bulge at the centre. But the wave formed in this experiment has a very specific motion: each time the drop hits the surface of the liquid at the Faraday period, it modifies the wave's shape from the previous bounce. Thus the wave cannot be considered to be a simple propagative wave. The drop bounces again on the surface but on

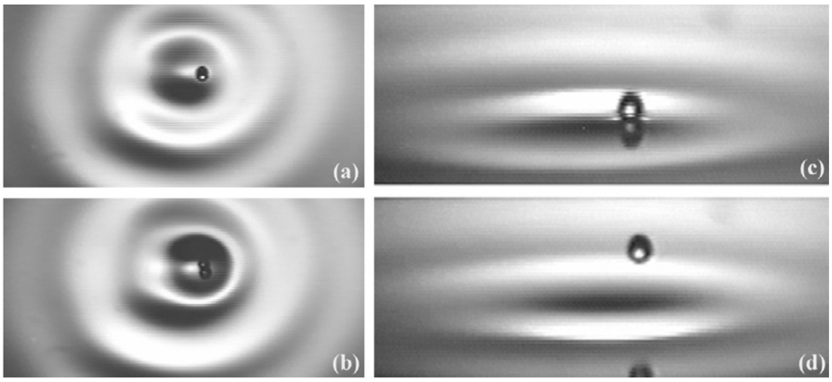

Figure 4. ((a), (b)) Walker filmed with a fast camera from top ($\mu = 20 \times 10^{-3}$ Pa s, $f_0 = 80$ Hz, $\gamma_m/g = 4.4$, $D = 0.7$ mm). The walker falls on the slope of the wave formed at its previous bounce. ((c), (d)) Walker filmed with a fast camera from side ($\mu = 50 \times 10^{-3}$ Pa s, $f_0 = 50$ Hz, $\gamma_m/g = 4.4$, $D = 0.8$ mm).

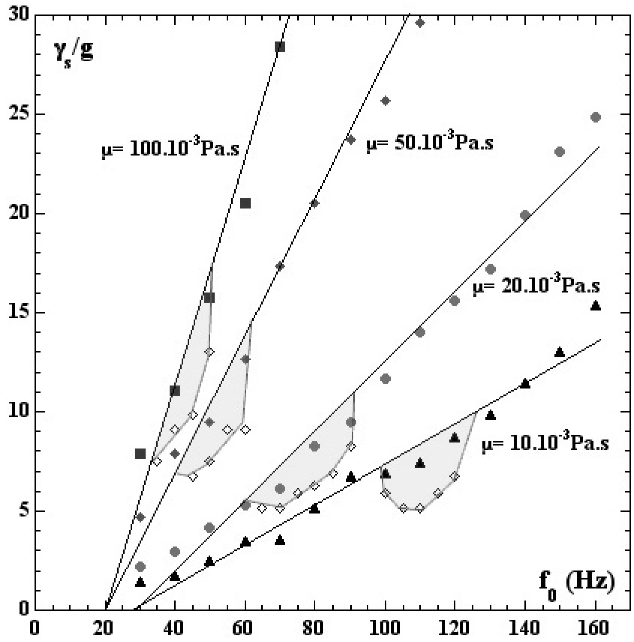

Figure 5. Acceleration threshold γ_s for the Faraday instability as a function of the forcing frequency for various oil viscosities ($\mu = 10 \times 10^{-3}$, 20×10^{-3}, 50×10^{-3} and 100×10^{-3} Pa s). Just below this threshold, the areas in grey correspond to the walker regions.

the slope of this bump giving it a horizontal velocity but also shifting the shape of the trough that should have been formed if the wave had been propagating naturally. This creates a Doppler shifted wave: the wavelength is reduced ahead of the moving drop and increased behind it. The waves on both sides of the drop have the Faraday wavelength (figure 4). The waves flatten and amplify periodically, because the forcing acceleration opposes or increases gravity.

We have measured the acceleration threshold for the Faraday instability as a function of the oil viscosity and the forcing frequency and observed that walkers only exist just below this Faraday threshold and also only for a certain range of forcing accelerations (figure 5).

In order for a drop to become a walker it has to jump high enough so that the periodicity of its contact with the fluid is twice that of the forcing. For a given forcing we checked that the walker's velocity is constant. We have measured the walker's velocity with the forcing acceleration and shown that for small drops the transition to the walking state is a supercritical bifurcation.

The drop's behaviour in the horizontal plane, once averaged over a period $2\pi/\omega$ of the vertical motion, can be modelled by the following equation:

$$m\,\mathrm{d}^2x/\mathrm{d}t^2 = a\sin((2\pi k/\omega)\,\mathrm{d}x/\mathrm{d}t) - b\,\mathrm{d}x/\mathrm{d}t. \tag{1}$$

The sine term is the impulsion which drives the horizontal motion. It is due to the bouncing of the drop on the sloping surface of the fluid. $(2\pi k/\omega)\,\mathrm{d}x/\mathrm{d}t$ is the phase shift due to the difference in velocity of the drop and the wave. The second term on the right-hand side averages the viscous effects when the drop bounces on the surface of the liquid and shears the intermediate air film. Given equation (1), the drop will start moving horizontally in any direction when a is larger than $b/(2\pi k/\omega)$. As the acceleration is increased, the experimentally observed transition from bouncing drops to walking drops is thus a supercritical transition.

A walker travels faster as the forcing acceleration is increased. Walkers of varying sizes have been studied. Since big drops, when hitting the surface, form waves of larger amplitudes, their velocity is larger. A walker's velocity can vary from 5 to 20 mm s^{-1} depending on its size and the forcing acceleration.

2.3. Interacting walkers

When several walkers coexist on the surface of the liquid, they interact. A large variety of regimes can then be observed. For simplicity, we shall first consider two identical drops moving in the cell. The emitted waves are then identical. Two fast walkers never collide directly but veer off course. Due to their waves, they can either attract or repel each other. In some of the attractive collisions there is capture and the two drops will start orbiting around their centre of mass. When the drops are of the same size, they have the same orbit. We investigated all the possible orbits and found that their diameters d_n^{orb} can take a series of discrete values. We find that these values are linked to the Faraday wavelength: they are of the type $d_n^{\mathrm{orb}} = (n - \varepsilon)\lambda_{\mathrm{F}}$ when the two drops oscillate in phase and $d_n^{\mathrm{orb}} = (n + 1/2 - \varepsilon)\lambda_{\mathrm{F}}$ when the two drops oscillate with opposite phases (the shortest possible diameter being for $n = 0$). Observation (figure 6) shows that for these distances (because of the offset ε) each of the two walkers bounces on the inner slope formed by the wave of the other. This is responsible for the existence of the attractive force necessary for the orbiting motion.

The orbiting walkers' velocities are approximately the same as the walkers'. When two walkers of different sizes start to orbit, since the angular velocities are the same and the linear velocities different, the fastest walker has the larger orbit. As a result, surprisingly the centre of rotation is closer to the smaller drop.

3. Concluding remarks

Drops can be kept bouncing at the surface of the same liquid for unlimited times. As they bounce, they can create waves at the surface of the liquid and thus become mobile wave sources. We have explored two regimes leading to the formation of static or dynamical behaviour. The latter corresponds to the emission of localized Faraday waves.

The existence of localized modes of the Faraday instability was already known without local exciters but in cases where the instability is subcritical. They were observed in vibrated sand where they were called oscillons (Umbanhowar *et al* 1996) or in very thin layers of very viscous liquids (Lioubashevski *et al* 1996). These structures belonged to the family of the

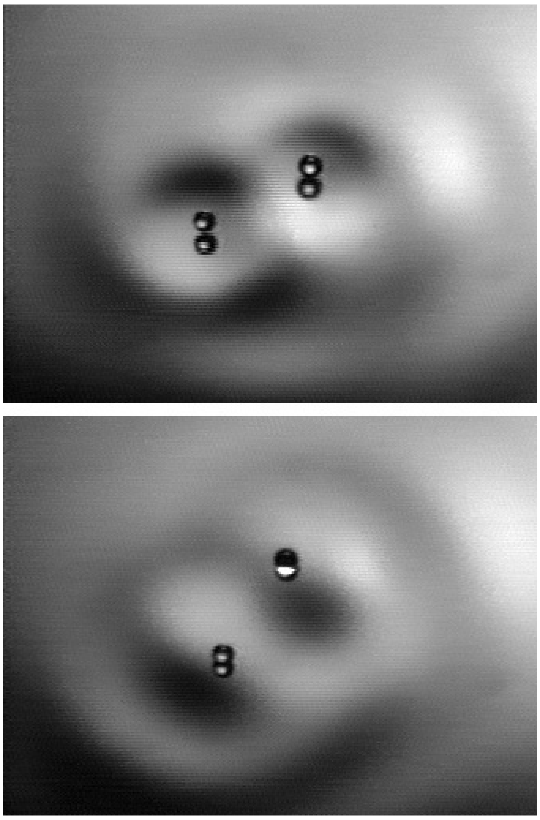

Figure 6. Identical walkers bound in the orbital mode $n = 1$ ($D = 0.7$ mm, $\mu = 20 \times 10^{-3}$ Pa s, $f_0 = 80$ Hz, $\gamma_m/g = 3.3$).

localized states observed in a large variety of experiments all having subcritical transitions in a 2D spatially extended system (e.g. Schäpers *et al* 2000, Liehr *et al* 2004). In such cases domains of the bifurcated oscillating state coexist in an otherwise stable system. In our case the situation is different: the Faraday instability is supercritical and localized waves are obtained below the instability threshold by having local and mobile exciters, the bouncing drops. This system appears as ideal for the investigation of the various modes of self-organization of interacting wave sources.

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