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The random anisotropy axis model in the infinite-range limit

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Abstract. We study the thermodynamic properties of the random-anisotropy-axis model of amorphous magnets when the interaction range becomes infinite. This model is exactly soluble and has a ferromagnetic second-order phase transition at the same temperature as the pure n component spin system. There is no spin glass phase and mean-field theory is rigorously justified in this case. Explicit expressions are given for the magnetisation, specific heat and susceptibility near the transition and at low temperature, in the limit of strong anisotropy.

1. Introduction

Amorphous intermetallic compounds often exhibit magnetic properties that differ markedly from those of their crystalline counterparts. Much work has been devoted to such materials recently, both experimentally and theoretically, and rapid progress has been made (Cochrane *et al* 1978). Harris *et al* (1973) in particular have proposed a randon anisotropy model (RAM) that is believed to describe adequately compounds of the Dy_xCu_{1-x} type (i.e. a rare earth and a non-magnetic metal).

Renormalisation-group studies (Aharony 1975, Pelcovits *et al* 1978) suggest that in this model a spin glass phase appears at low temperature, at least below six dimensions, and that ferromagnetism is unstable below four dimensions. Above six dimensions, it is possible that only the ferromagnetic phase remains at low T, independently of the anisotropy strength. After some initial controversy on the result of Monte Carlo experiments (Chi and Alben 1977, Harris and Sung 1978) more extended numerical work by Jayaprakash and Kirkpatrick (1979) has provided strong evidence for the absence of ferromagnetism in three dimensions, even in the limit of very strong anisotropy. This raises many questions on the relationships of the RAM with the usual random-interaction models of spin glasses.

In this paper we study the thermodynamic properties of the RAM when the interaction range becomes infinite. Jayaprakash and Kirkpatrick (1979) have pointed out the interest of this limit and shown that the system is ferromagnetic at zero temperature. We extend their work to all temperatures and all values of the anisotropy ratio. One usually expects mean-field theory to be valid for systems with infinite-range interactions, but experience with random-bond models of spin glasses (Sherrington and Kirkpatrick 1975) has shown that 'naive' mean-field equations are incorrect. Local-field corrections must be taken into account (Thouless *et al* 1977) but the situation is not yet clear (Blandin

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1978). It is natural to ask whether similar difficulties arise in the random-anisotropy model, since a correct mean-field theory is a pre-requisite for more realistic calculations.

We obtain the free energy of the model explicitly. It exhibits a second-order phase transition from a ferromagnetic phase to a paramagnetic phase, at a temperature independent of the anisotropy ratio. This transition is governed by standard mean-field equations. We discuss the role of fluctuations in the axis distribution.

We investigate the limit of strong anisotropy in more detail, because it is expected to be the nearest to a spin glass and has interesting low-temperature properties. The specific heat, for instance, is linear in T as observed experimentally on some amorphous magnets and predicted from simple effective-field arguments (Coey and Von Molnar 1978), but the susceptibility vanishes as T^2 due to the orientational disorder.

2. Model and derivation of the free energy

2.1. General case

The random-anisotropy-axis model is defined by the Hamiltonian

$$\mathscr{H} = -\sum_{i,j} J\boldsymbol{S}_i.\boldsymbol{S}_j - D\sum_i (\boldsymbol{n}_i.\boldsymbol{S}_i)^2 \tag{1}$$

where the $\boldsymbol{S}_i$ are n component unit spins and the unit vectors $\boldsymbol{n}_i$ vary randomly from site to site. The (positive) exchange interaction J and anisotropy energy D are assumed to be the same for all spins. Like random-bond spin glasses, this model has been shown to present frustration effects (Alexander and Lubensky 1979).

For usual ferromagnets, mean-field theory becomes exact in the high-dimensionality limit and may be derived by letting the interaction range become infinite, with the interaction scaled as $1/N$, where N is the number of spins (Kac 1966). Extension of this approach to disordered systems is attracting considerable attention (Kirkpatrick and Sherrington 1978) and it is natural to investigate its application to the RAM. Extending the summation in equation (1) to all pairs of spins and letting $J = J_0/N$ to insure a well behaved limit, we obtain, apart from an unimportant constant term,

$$\mathscr{H}' = -\frac{J_0}{2N}(\sum_i \boldsymbol{S}_i)^2 - D\sum_i (\boldsymbol{n}_i.\boldsymbol{S}_i)^2. \tag{2}$$

The case $D = 0$ is just the pure n component classical spin system, to which we shall often refer in the following.

Let us specialise for the moment to the two-component case, with the $\boldsymbol{n}_i$ distributed over a circle. The partition function Z is for one configuration of $\{\boldsymbol{n}_i\}$:

$$Z = C^N \int \cdots \int_{S_i^2=1} \left(\prod_i \mathrm{d}\boldsymbol{S}_i\right) \exp\left\{\frac{K}{2N}\left[(\Sigma S_i^x)^2 + (\Sigma S_i^y)^2\right] + \tilde{D}\,\Sigma(\boldsymbol{n}_i.\boldsymbol{S}_i)^2\right\}$$

where $K = J_0/T$, $\tilde{D} = D/T$ and C is a normalisation constant. This can be written, using a classical trick, as

$$Z = C^N \iint \exp(-Nr^2/2)\,\mathrm{d}\boldsymbol{r} \prod_i \left\{\int_{S_i^2=1} \mathrm{d}\boldsymbol{S}_i \exp[K^{1/2}\boldsymbol{r}.\boldsymbol{S}_i + \tilde{D}(\boldsymbol{n}_i.\boldsymbol{S}_i)^2]\right\}. \tag{3}$$

Denoting by ψ and θ_i the polar angles of $\boldsymbol{r}$ and $\boldsymbol{n}_i$ respectively, the integral between the braces is

$$f(r, \psi - \theta_i) = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi} \exp\left[K^{1/2} r\cos(\varphi - \psi + \theta_i) + \tilde{D}\cos^2\varphi\right]. \tag{4}$$

For a given configuration $\{\boldsymbol{n}_i\}$ of random axes, one can compute the value of $\ln Z$ by a saddle-point method:

$$(1/N)\ln Z = \lg C + \max_{(r,\psi)}\left(-r^2/2 + (1/N)\sum_i \ln f(r, \psi - \theta_i)\right).$$

If we consider a uniform probability law for the axes, it is easy to show that

$$\left\langle \sum_i \ln f(r, \psi - \theta_i) \right\rangle = N \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \ln f(r, \psi - \theta)$$

and

$$\left\langle \left(\sum_i \ln f(r, \psi - \theta_i) \right)^2 \right\rangle = N \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \ln^2 f(r, \psi - \theta) + N(N-1)\left[\int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \ln f(r, \psi - \theta) \right]^2$$

where the averages are taken over all the distributions of axes. This shows that for almost all distributions one has

$$\sum_i \ln f(r, \psi - \theta_i) = N \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \ln f(r, \psi - \theta) + \mathrm{O}(N^{1/2}) \tag{5}$$

which is independent of the variable ψ in leading order. Therefore, for almost all distributions of axes, the free energy F per spin is given in the thermodynamic limit by

$$-\frac{F}{T} = \frac{1}{N}\ln Z = \ln C + \max_{(r)}\left[-\frac{r^2}{2} + \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \ln f(r, \theta) \right]. \tag{6}$$

In particular, equation (6) gives the averaged free energy per spin, which is the usual quantity of interest in disordered systems.

For high temperatures (small K) the maximum of $\ln Z$ is realised for $r = 0$. Expanding $f(r, \theta)$ for small r, one finds that this maximum turns into a minimum for a critical value $K_c = 2$, and the system has a second-order transition at a temperature T_c independent of D:

$$T_c = J_0/2. \tag{7}$$

It is easily shown that a similar result holds for the n component case, where T_c is equal to J_0/n independently of D.

The equation that gives the value of r at the maximum is in fact equivalent to the mean-field theory equation considered by various authors for classical spins (Harris *et al* 1973, Callen *et al* 1977, Patterson *et al* 1978). We prove this equivalence in detail for the Ising limit (infinite anisotropy) in the Appendix. As far as we know, the independence of T_c with respect to D had not been pointed out, though Patterson *et al* (1978) proved that $(\mathrm{d}T_c/\mathrm{d}D) = 0$ for $D = 0$. Presumably, this is because it is customary to compare T_c (RAM) with T_c (crystal), defined as the transition temperature of the model with all $\boldsymbol{n}_i$ fixed in the same direction, which itself depends on D. This comparison has obvious experimental importance, but it obscures the simple and striking behaviour of T_c (RAM).

The reason why the present theory is simple is that the fluctuations in the distribution of axes become unimportant in the thermodynamic limit. It is then allowed to replace a random configuration of axes by the uniform distribution.

2.2. Ising limit

We now consider the case of very strong anisotropy ($D \gg J$), where some simplification occurs and which is the least favourable for ferromagnetism, so that it represents a very useful limit. The spins are forced to align along the directions $\boldsymbol{n}_i$ and may be written as $\boldsymbol{n}_i\sigma_i$, with $\sigma_i = \pm 1$ an Ising variable. Apart from a constant term, the Hamiltonian reduces to

$$\mathscr{H}'' = -(J_0/2N)\Big(\sum_i \boldsymbol{n}_i\,\sigma_i\Big)^2 \tag{8}$$

and the total energy is just proportional to the square of the magnetisation.

The free energy takes the form derived in the Appendix (for $n = 2$):

$$F = -\frac{T}{N}\ln Z = \frac{J_0}{2}\left[\left(\int_0^{2\pi} m(\theta)\cos\theta\,\frac{\mathrm{d}\theta}{2\pi}\right)^2 + \left(\int_0^{2\pi} m(\theta)\sin\theta\,\frac{\mathrm{d}\theta}{2\pi}\right)^2\right] + T\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\left[\left(\frac{1+m(\theta)}{2}\right)\ln\left(\frac{1+m(\theta)}{2}\right) + \left(\frac{1-m(\theta)}{2}\right)\ln\left(\frac{1-m(\theta)}{2}\right)\right] \tag{9}$$

where the magnetisation profile $m(\theta)$ is the solution of

$$m(\theta) = \tanh\left[\frac{J_0}{T}\int_0^{2\pi} m(\varphi)\cos(\theta-\varphi)\,\frac{\mathrm{d}\varphi}{2\pi}\right]. \tag{10}$$

This equation immediately shows that the profile is always of the simple form

$$m(\theta) = \tanh[A\cos(\theta - \theta_0)]. \tag{11}$$

The amplitude A depends on the temperature, but θ_0 may take any value and there is a continuous degeneracy if A is non-zero. The symmetry of the order parameter is the same as for the pure system ($D = 0$) and one parameter is sufficient to describe the temperature variation. It is convenient to use the average magnetisation per spin B ($= TA/J_0$), which verifies

$$B = \int_0^{2\pi}\cos\theta\tanh\left[\frac{J_0 B}{T}\cos\theta\right]\frac{\mathrm{d}\theta}{2\pi}. \tag{12}$$

It was conceivable *a priori* that the order parameter would be a complicated function $m(\theta)$. This is not the case and the simple mean-field expression (equation (12)) is exact, which reduces the complexity of the problem but also its richness.

The calculations are readily generalised to the case where the anisotropy axes are uniformly distributed over the unit sphere. The magnetisation per spin $\boldsymbol{B}$ and the magnetisation profile are given by the set of equations

$$\boldsymbol{B} = \int_{\boldsymbol{u}^2=1} \boldsymbol{u}m(\boldsymbol{u})\,\frac{\mathrm{d}^n\boldsymbol{u}}{\Sigma_n} \tag{13a}$$

$$m(\boldsymbol{u}) = \tanh[(J_0/T)\boldsymbol{u}.\boldsymbol{B}] \tag{13b}$$

where $\boldsymbol{u}$ is an n-dimensional unit vector and Σ_n is the surface of the unit sphere $\Sigma_n = n\pi^{n/2}/\Gamma(n/2+1)$.

3. Thermodynamic properties in the Ising limit

The Hamiltonian for the strong anisotropy case (equation (8)) is invariant under all transformations that leave the magnitude of the total magnetisation unchanged, and it possesses in particular a global rotation symmetry. We have seen that this symmetry is broken at the same temperature and with the same order parameter as for the pure spin system. Below T_c the equivalence is not complete, however, and the underlying disorder modifies the thermodynamic properties. In particular, the Ising character of the effective spins has important consequences near $T = 0$. We study this region in detail.

3.1. Phase transition region

The magnetisation per spin B is non-zero below a critical temperature T_c given by

$$T_c = J_0/n. \tag{14}$$

The transition is second order and the order parameter vanishes with the usual mean-field behaviour:

$$B^2 \sim [(n+2)/n^2](T_c - T)/T_c \qquad (T_c - T \ll T_c) \tag{15}$$

The specific heat C is simply

$$C = (\mathrm{d}/\mathrm{d}T)\langle \mathscr{H} \rangle = (-J_0/2)(\mathrm{d}(B^2)/\mathrm{d}T).$$

It vanishes above T_c and has a jump ΔC at T_c:

$$\Delta C = (n+2)/2n. \tag{16}$$

For comparison, the results for the pure system are

$$B_0^2 \sim [(n+2)/n](T_c - T)/T_c).$$

$$\Delta C_0 = (n+2)/2$$

so there is a reduction by a factor n in the specific heat for the random system. In the presence of an external magnetic field $\boldsymbol{H}$ the calculations are modified in a simple way and the magnetisation satisfies:

$$\boldsymbol{B} = \int_{\boldsymbol{u}^2=1} \boldsymbol{u} \tanh\left[\frac{(J_0\boldsymbol{B} + \boldsymbol{H})}{T}.\boldsymbol{u}\right] \frac{\mathrm{d}^n\boldsymbol{u}}{\Sigma_n}. \tag{17}$$

Above T_c the susceptibility is

$$\chi = \boldsymbol{B}/\boldsymbol{H} = 1/n(T - T_c). \tag{18}$$

Below T_c the transverse susceptibility becomes infinite but the longitudinal susceptibility in a field parallel to the spontaneous magnetisation remains finite:

$$\chi_{\parallel} \sim 1/2n(T_c - T). \tag{19}$$

Both results are identical to the expressions for the pure case.

3.2. Low-temperature region

In the ground states the magnetisation $\boldsymbol{B}$ is maximum and all spins point in the same hemisphere, with the direction of $\boldsymbol{B}$ as the polar axis. There is a large ground state degeneracy due to all the possible choices of the hemisphere. This large degeneracy is a general feature of models exhibiting frustration effects and it is useful to study its implications on a soluble case.

The zero-temperature moment $B(0)$ is given by:

$$\begin{aligned} B(0) = [2/(n-1)]\,\Sigma_{n-1}/\Sigma_n = 2/\pi \qquad & \text{(planar case)} \\ \tfrac{1}{2} \qquad & \text{(Heisenberg)} \\ \sim (2/n\pi)^{1/2} \qquad & (n \gg 1). \end{aligned} \tag{20}$$

This moment is strongly reduced with respect to the saturated moment of the pure system, the reduction factor is comparable to the one observed near T_c (equation (15)). The ground state energy per spin E_0 is just $-\frac{1}{2}J_0B^2(0)$. At low temperatures the moment has a quadratic variation

$$B(T) \sim B(0) - \frac{(n-1)\pi^2}{24B(0)}\frac{T^2}{J_0^2} \qquad (T \ll J_0) \tag{21}$$

and the specific heat is linear:

$$C(T) \sim (n-1)(\pi^2/12)(T/J_0). \tag{22}$$

The entropy also vanishes linearly in T, as may be checked directly from equation (9). The discrete nature of the spin variables reappears here, ensuring that the $T = 0$ limit remains physical while for the pure system the specific heat goes to a constant and the entropy diverges as expected for classical continuous spins.

The behaviour of the specific heat is particularly interesting, since such a linear term has been observed experimentally in a Dy–Cu alloy and interpreted as being due to the existence of a finite density of spins in zero effective field (Coey and Von Molnar 1978). The effective field h_i on a spin i in a given state of the system is

$$h_i = (J_0/N)\sum_j \boldsymbol{n}_i \cdot \boldsymbol{n}_j \sigma_j = J_0 \boldsymbol{B} \cdot \boldsymbol{n}_i. \tag{23}$$

The probability distribution $P(h, T)$ is then related to the density of $\cos\theta$, where θ is the angle between the local axis and $\boldsymbol{B}$. One has explicitly

$$P(h, T) = (\Sigma_{n-1}/\Sigma_n)(1 - h^2/J_0^2B^2(T))^{(n-3)/2}/J_0B(T) \tag{24}$$

for $|h| \leqslant J_0B(T)$, and $P(h, T) = 0$ otherwise. The internal energy may be written, using equation (13), as

$$U(T) = -(J_0/2)B^2(T) = -\tfrac{1}{2}\int \tanh(h/T)P(h, T)h\,\mathrm{d}h. \tag{25}$$

This is just the expression used in the 'mean-random-field' approximation (Klein 1976), which is thus exact in the present model. It is important to note that $P(h, T)$ depends on T through $B(T)$ (equation (21)), and that the linear term in the specific heat contains a contribution from $\mathrm{d}P/\mathrm{d}T$. This contribution accounts for half the total result, so the often-made assumption that $\mathrm{d}P/\mathrm{d}T$ is negligible near $T = 0$ is invalid here.

Finally, the longitudinal susceptibility is obtained by developing equation (17) at low T. It depends quadratically on T:

$$\chi_{\parallel} \sim (n-1)\pi^2 T^2 / 12 B^2(0) J_0^3 \tag{26}$$

whereas the result for the pure system is linear in temperature. Naively, one might expect a constant susceptibility near $T = 0$ due to the non-vanishing density of spins in zero effective field. The spins in low effective field lie near the equatorial plane. This reduces the susceptibility for two reasons: first, the external field is not efficient in orientating them along the global magnetisation B; second, even when orientated they contribute very little to B. Each effect accounts for a factor (T/J_0) in the reduced susceptibility.

4. Conclusion

The interest of the random-axis model comes from its intermediate character between conventional ferromagnets and spin glasses. In the infinite-range limit studied here it clearly lies on the ferromagnetic side. Naive mean-field theory is exact, in contradistinction with the situation for spin glasses. Above the transition temperature the random anisotropy plays no role and the system behaves exactly as a pure spin system. Below T_c the disorder reduces the spontaneous magnetisation but it is not strong enough to modify the nature of the low-temperature phase. Only the low-temperature properties are affected enough to share some features with random-bond spin glasses. This suggests than an investigation at large dimensionalities may clarify the mechanism by which a spin glass state appears.

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Appendix

We present here the derivation of the mean-field equations (9) and (10), for the case of very strong anisotropy and two-component spins.

The spins are aligned on local axes $\boldsymbol{n}_i$ distributed over a circle. When N is large, a number $N(\theta_j) = N_j$ of these axes points in directions between angles θ_j and $\theta_j + \mathrm{d}\theta$ (angles θ and $\theta + \pi$ are equivalent, but it is convenient to keep the distinction). For a fixed distribution of axes, let us consider one spin configuration, i.e. one set $\{\sigma_i\}$. If the number of up spins ($\sigma_i = +1$) in direction θ_j is $N_j x_j$, the energy U of the configuration is

$$U = -\frac{J_0}{2N}\left\{\left[\sum_j N_j(2x_j - 1)\cos\theta_j\right]^2 + \left[\sum_j N_j(2x_j - 1)\sin\theta_j\right]^2\right\}. \tag{A.1}$$

The number of configurations with the same value of x_i for all θ_j is

$$W = \prod_j C_{N_j}^{N_j x_j} \sim \exp\left\{ - \sum_j N_j [x_j \ln x_j + (1 - x_j) \ln (1 - x_j)] \right\}. \tag{A2}$$

The partition function Z is given by the functional integral

$$Z = \int_0^1 .. \int_0^1 \left(\prod_j \mathrm{d}x_j \right) \exp\left[- \frac{1}{T} U(\{x_j\}) \right] W(\{x_j\})$$

which may be evaluated using a saddle-point method. The result is

$$\ln Z \sim - (1/T) U[\{(1 + m_i)/2\}] + \ln W[\{(1 + m_i)/2\}] \tag{A3}$$

where the magnetisation m_i in direction θ_i is the value of $(2x_i - 1)$ at the saddle point and is the solution of

$$m_i = \tanh\left[(J_0/NT) \sum_j m_j N_j \cos(\theta_i - \theta_j) \right]. \tag{A4}$$

To justify this calculation, the N_i must be large enough while the θ_i must be well defined. This implies

$$N \mathrm{d}\theta \gg 1 \gg \mathrm{d}\theta$$

If the axes $\boldsymbol{n}_i$ are distributed according to a given density $n(\theta)$, the same arguments that lead to equation (6) give for almost all the distributions of axes:

$$(1/N) \ln Z = \frac{J_0}{2T} \left[\left(\int_0^{2\pi} m(\theta) n(\theta) \cos\theta \frac{\mathrm{d}\theta}{2\pi} \right)^2 + \left(\int_0^{2\pi} m(\theta) n(\theta) \sin\theta \frac{\mathrm{d}\theta}{2\pi} \right)^2 \right] - \int_0^{2\pi} n(\theta) \left[\left(\frac{1 + m(\theta)}{2} \right) \lg \left(\frac{1 + m(\theta)}{2} \right) + \left(\frac{1 - m(\theta)}{2} \right) \ln \left(\frac{1 - m(\theta)}{2} \right) \right] \frac{\mathrm{d}\theta}{2\pi} \tag{A.5}$$

$$m(\theta) = \tanh\left[\frac{J_0}{T} \int_0^{2\pi} n(\varphi) m(\varphi) \cos(\theta - \phi) \frac{\mathrm{d}\varphi}{2\pi} \right]. \tag{A.6}$$

The last equation shows that for any given density $n(\theta)$ the magnetisation profile is of the form

$$m(\theta) = \tanh\left[A \cos(\theta - \theta_0) \right]. \tag{A7}$$

In the case of uniformly distributed axes, $n(\theta)$ has to be replaced by 1 to obtain equations (9) and (10) from equations (A5) and (A6).

An advantage of the present derivation is to give directly the magnetisation profile and to be readily generalisable to other distributions of axes.

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