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Interfacial wetting in the *q*-state Potts model

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Abstract. We consider an interface between two ordered phases of a q-state Potts model below its bulk first-order transition temperature. The mean-field equations are solved analytically for small $q-2$ and the solution shows that the interface is wetted by the disordered phase as the transition is approached. The excess absorption and surface entropy diverge logarithmically. Numerical calculations indicate this wetting also occurs at larger q.

1. Introduction

The behaviour of an interface between two phases as coexistence with a third phase is approached is the subject of current experimental and theoretical investigation (Sullivan and Telo Da Gama 1985). Attention centres on the question of whether a film of the third phase intervenes between the first two phases and, if so, whether its thickness is microscopic or macroscopic. In the latter case, the third phase is said to wet the interface. Examples include the wetting of the vapour/A-rich liquid interface by B-rich liquid in binary-liquid mixtures (Sullivan and Telo Da Gama 1985, Tarazona *et al* 1983), and the surface melting of a solid in coexistence with its vapour (Frenken and van der Veen 1985). The same question can be posed in systems in which there are more than three possible phases and has recently been studied in two-dimensional systems (Selke 1984, Yeomans and Derrida 1985). In particular, interfaces in the Potts model, the chiral Potts model and the Blume-Emery-Griffiths model were investigated. While the results of the last two were clear and understood (Selke 1984, Yeomans and Derrida 1985), those of the first in which the bulk transitions were first order were neither (Selke 1984). The interface between two ordered phases A and B in the Potts model might be thought to be simply due to the symmetry which exists between its ordered phases. In particular, this symmetry implies that the surface tension between any two ordered phases must be the same. An immediate consequence is that the A/B interface cannot be wetted by another ordered phase C, but only by the disordered phase, if at all. This is in contrast with the chiral Potts model in which the Potts symmetry is broken so that there is more than one surface tension between ordered phases (Huse *et al* 1983). Further, as there is only one parameter in the model, the interaction strength J, the ordered and disordered phases can coexist only at a single temperature, the bulk transition temperature T_0. Thus, at this temperature, either the disordered phase wets the A/B interface or it does not. The parameter space is not large enough to permit a line of three-phase coexistence, as occurs in the Blume-Emery-Griffiths model or in binary-liquid mixtures, or the possibility of a wetting transition. It is the purpose of this paper to determine, within the context of mean-field theory,

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which of these two behaviours actually occurs. As emphasised by Widom (1977), this can only be done by solving the full set of $q-1$ coupled equations for the $q-1$ independent densities. In particular, one cannot appeal to effective one-density theories because such theories presume one behaviour or the other at the outset. As shown below, symmetry considerations reduce the $q-1$ equations to two. Even so, it is not at all easy to solve these equations, even numerically (Dickinson 1984).

Our approach is an analytic one which is valid for the q-state Potts model in the limit in which $q-2$ is small. Our principal result is that there is, in fact, a wetting of the interface between ordered phases by the disordered phase as the temperature approaches the bulk transition temperature T_0. Given that such an interfacial wetting occurs, it is believed that effective one-density theories should yield the correct thermodynamic singularities associated with the interfacial wetting transition (Lipowsky 1984). These theories yield a width of the disordered region which diverges like $\ln(T_0-T)$ (Widom 1978) and an excess surface entropy with the same divergence. Our explicit solution of the coupled equations which depend on all densities explicitly confirms this behaviour.

2. The equations for the profile

We consider a q-state Potts model on a d-dimensional hypercubic lattice governed by the Hamiltonian

$$H=-J\sum_{\lambda=1}^{q}\sum_{\langle ij\rangle}c_i^\lambda c_j^\lambda \tag{1}$$

where λ can take on q values and the second sum is over nearest-neighbour pairs. The variable c_i^λ on the ith site is equal to unity if the Potts spin on that site is in the state λ and is zero otherwise. Let the ensemble average of c_i^λ be denoted by n_i^λ which must satisfy the constraint

$$\sum_\lambda n_i^\lambda=1 \qquad \forall i. \tag{2}$$

Then, within mean-field theory, the n_i^λ minimise the free energy functional (Mittag and Stephen 1974)

$$\beta F=-\beta J\sum_\lambda\sum_{\langle ij\rangle}n_i^\lambda n_j^\lambda+\sum_\lambda\sum_i n_i^\lambda\log n_i^\lambda \tag{3}$$

where β is the inverse temperature. In the situation of interest to us, a $(d-1)$-dimensional planar interface between two ordered phases, the n_i^λ vary in one direction only so that (2) may be simplified to

$$\beta F/\mathscr{A}\equiv-\beta\sum_\lambda\sum_k n_k^\lambda(dn_k^\lambda+\tfrac{1}{2}\Delta n_k^\lambda)+\sum_\lambda\sum_k n_k^\lambda\log n_k^\lambda \tag{4}$$

where $\mathscr{A}$ is the area of a hyperplane, k the index of such planes, and Δ the second difference operator:

$$\Delta X_k\equiv(X_{k+1}-2X_k+X_{k-1}). \tag{5}$$

Of the q densities n_k^λ on each plane k, only two are independent. They can be taken to be n_k^{A} and n_k^{B}. By symmetry, all the other n_k^λ are equal and can be obtained from the constraint (2) as

$$n_k^\lambda=(1-n_k^{\mathrm{A}}-n_k^{\mathrm{B}})/(q-2) \qquad \lambda\neq\mathrm{A},\mathrm{B}. \tag{6}$$

Instead of n_k^{A} and n_k^{B} we use the two independent variables

$$S_k = \tfrac{1}{2}(n_k^{\mathrm{A}} + n_k^{\mathrm{B}}) \tag{7}$$

and

$$D_k = n_k^{\mathrm{A}} - n_k^{\mathrm{B}}$$

so that

$$n_k^{\lambda} = (1-2S_k)/(q-2) \qquad \lambda \neq \mathrm{A, B}. \tag{8}$$

In terms of these variables, the free energy functional of (4) can be written as

$$\beta F/\mathscr{A} = \sum_k f(\{S_k\}, \{D_k\}) \tag{9}$$

$$\begin{aligned} f = {} & -d\beta J\left(2S_k^2 + \tfrac{1}{2}D_k^2 + \frac{(1-2S_k)^2}{(q-2)}\right) - \beta J\left(S_k\Delta S_k + \tfrac{1}{4}D_k\Delta D_k\right) + S_k \log\left(S_k^2 - \frac{D_k^2}{4}\right) \\ & + \frac{D_k}{2}\log\left(\frac{S_k + \frac{1}{2}D_k}{S_k - \frac{1}{2}D_k}\right) + (1-2S_k)\log\left(\frac{1-2S_k}{q-2}\right) + \beta J\frac{1}{q-2}(1-2S_k)\Delta S_k. \end{aligned} \tag{10}$$

Variation with respect to the S_k and D_k leads to the equations

$$q\beta J\Delta S_k = -2d\beta J(S_k q - 1) - (q-2)\log\left(\frac{1-2S_k}{q-2}\right) + \frac{q-2}{2}\log(S_k^2 - D_k^2/4) \tag{11}$$

and

$$\beta J\Delta D_k = -2d\beta J D_k + \log\left(\frac{S_k + D_k/2}{S_k - D_k/2}\right). \tag{12}$$

These equations always admit the uniform disordered solution

$$S_k = 1/q \qquad \forall k \tag{13}$$

$$D_k = 0 \qquad \forall k \tag{14}$$

corresponding to $n_k^{\lambda} = 1/q$ for all k and λ. Below the transition temperature given by

$$2dJ\beta_0 = 2\frac{(q-1)}{(q-2)}\log(q-1) \tag{15}$$

there is another uniform solution (which minimises f) in which only one colour, A say, is different from the others so that n_k^{B} is the same as n_k^{λ} for λ unequal to A. From the definition of (7) and the value of n_k^{λ} given in (8) we find that the uniform values of S_k and D_k, which for this solution we denote S_{B} and D_{B}, are related by

$$S_{\mathrm{B}} - \frac{D_{\mathrm{B}}}{2} = \frac{1-2S_{\mathrm{B}}}{q-2}. \tag{16}$$

Knowing this, one obtains the solution of (11) and (12) from the solution of

$$\frac{2(S_{\mathrm{B}}q - 1)}{q-2} = \frac{1}{2d\beta J}\log\left(\frac{2S_{\mathrm{B}}q - 2S_{\mathrm{B}} - 1}{1-2S_{\mathrm{B}}}\right) \tag{17}$$

and

$$\frac{D_{\mathrm{B}}}{2} = \frac{S_{\mathrm{B}}q - 1}{q-2}. \tag{18}$$

At the transition temperature, the explicit solution is

$$S_B = (q^2 - 2q + 2)/2q(q-1) \qquad T = T_0 \tag{19}$$

$$D_B = (q-2)/(q-1) \qquad T = T_0 \tag{20}$$

corresponding to

$$n_k^A = (q-1)/q \tag{21}$$

$$n_k^\lambda = 1/q(q-1) \qquad \lambda \neq A. \tag{22}$$

Comparison with (13) and (14) shows the first-order nature of the transition at T_0.

To determine the surface tension $\sigma_{AB}(T)$, we need to solve equations (11) and (12) subject to the boundary conditions

$$S_k \to S_B \qquad k \to \pm\infty \tag{23}$$

$$D_k \to \pm D_B \qquad k \to \pm\infty. \tag{24}$$

We denote these solutions $\hat{S}_k$ and $\hat{D}_k$. Then σ_{AB} is obtained from

$$\beta\sigma_{AB} = \sum_k f(\{\hat{S}_k\}, \{\hat{D}_k\}) - f(\{S_B\}, \{D_B\}) \tag{25}$$

with f given by (10). A measure of the amount of the disordered phase absorbed at the interface is provided by

$$W \equiv \sum_{\lambda \neq A,B} \sum_k (n_k^\lambda(\hat{S}_k, \hat{D}_k) - n_k^\lambda(S_B, D_B))$$

$$= 2\sum_k (S_B - \hat{S}_k). \tag{26}$$

It is not an easy task to solve (11) and (12) with the boundary conditions (23) and (24), even numerically. There is almost certainly no unique solution to the equations with such boundary conditions but rather a set of solutions corresponding to a series of local minima (and maxima) of (25) and one global minimum. If several of the local minima are almost degenerate with the global one, the problem of determining the latter is considerable. We avoid this problem in the next section by solving the equations analytically in the neighbourhood of q equal to 2 and for temperatures T close to T_0 (in particular for $T_0 - T \sim (q-2)^2$). This will allow us to give a complete description of the profile of the interface.

3. The profile for q close to 2

For q close to 2, and T close to T_0, one can simplify equations (11) and (12) which give the profile. To do so, let us define ψ by

$$\psi = q - 2 \tag{27}$$

and ε by

$$2d\beta J - 2d\beta_0 J = \varepsilon. \tag{28}$$

Thus ε gives a measure of the difference $T_0 - T$. If we define μ as

$$\mu = \varepsilon/\psi^2 \tag{29}$$

then we shall consider μ to be of order unity. One can expand $2d\beta_0J$ given by (15) in powers of ψ and one finds

$$2d\beta J=2d\beta_0J+\mu\psi^2=2+\psi-\tfrac{1}{3}\psi^2+\tfrac{1}{6}\psi^3+\mu\psi^2+\dots. \tag{30}$$

Let us assume that

$$D_k=\mathrm{O}(\psi) \tag{31}$$

$$S_k=\tfrac{1}{2}-\tfrac{1}{4}\psi+\tfrac{1}{4}u_k \tag{32}$$

where

$$u_k=\mathrm{O}(\psi^2). \tag{33}$$

The idea of our solution is a self-consistent one. First we assume (31), (32) and (33), i.e. that D_k is of order ψ along the profile and that the k dependence of S_k is of order ψ^2. Then we simplify equations (11) and (12) using this assumption and find a solution which is consistent with it.

Using (27) and (29)-(33) one can expand the right-hand side of equations (11) and (12) up to the third order in ψ. It turns out that the zeroth, first and second orders vanish and just the third order remains. The result is

$$\beta J(D_{k+1}+D_{k-1}-2D_k)=(\tfrac{5}{6}\psi^2-\mu\psi^2-u_k)D_k+\tfrac{2}{3}D_k^3 \tag{34}$$

$$\tfrac{1}{2}q\beta J(u_{k+1}+u_{k-1}-2u_k)=\tfrac{1}{4}\psi^3-\psi u_k-\psi D_k^2+u_k^2/\psi. \tag{35}$$

We seek a solution such that $u_k\rightarrow u_{\mathrm{B}}$ and $D_k\rightarrow\pm D_{\mathrm{B}}$ when $k\rightarrow\pm\infty$ where u_{B} and D_{B} are given by

$$u_{\mathrm{B}}=\tfrac{1}{4}[5+(1+24\mu)^{1/2}]\psi^2+\text{higher-order terms} \tag{36}$$

$$D_{\mathrm{B}}=\tfrac{1}{4}[3+(1+24\mu)^{1/2}]\psi+\text{higher-order terms.} \tag{37}$$

For small ψ, the right-hand sides of (34) and (35) are of order ψ^3 whereas the left-hand side of (34) is *a priori* of order ψ and of (35) of order ψ^2. The only way of solving these equations is to look for a solution which varies slowly with k. By inspection (34) and (35) have a solution of the form

$$D_k=\psi G\left(\frac{1}{(\beta J)^{1/2}}\psi k\right)+\text{higher-order terms} \tag{38}$$

$$u_k=\psi^2 H\left(\frac{1}{(\beta J)^{1/2}}\psi k\right)+\text{higher-order terms} \tag{39}$$

where the functions $G(t)$ and $H(t)$ satisfy the following two equations

$$\mathrm{d}^2G/\mathrm{d}t^2=(\tfrac{5}{6}-\mu-H)G+\tfrac{2}{3}G^3 \tag{40}$$

$$\tfrac{1}{2}\psi q\,\mathrm{d}^2H/\mathrm{d}t^2=\tfrac{1}{4}-H-G^2+H^2. \tag{41}$$

Here one should notice the fact which makes the problem soluble: the term which contains $\mathrm{d}^2H/\mathrm{d}t^2$ in (41) is a higher-order term (since it is multiplied by ψ) and therefore (41) can be replaced by

$$\tfrac{1}{4}-H-G^2+H^2=0. \tag{42}$$

Together with the boundary conditions (36) and (37), this yields

$$H=\tfrac{1}{2}+|G|. \tag{43}$$

Using (43) and (40), one sees that the problem reduces to finding the solution of the differential equation for G

$$\mathrm{d}^2G/\mathrm{d}t^2=(\tfrac{1}{3}-\mu-|G|)G+\tfrac{2}{3}G^3 \tag{44}$$

which tends to $\pm\frac{1}{4}[3+(1+24\mu)^{1/2}]$ when $t\to\pm\infty$.

One can integrate this equation (44) and obtain the constant of integration from the fact that $\mathrm{d}G/\mathrm{d}t$ vanishes as $t\to\pm\infty$. This leads to

$$(\mathrm{d}G/\mathrm{d}t)^2=\tfrac{1}{3}G^2(1-|G|)^2-\mu G^2+\tfrac{1}{96}[-1+60\mu+72\mu^2+(1+24\mu)^{3/2}]. \tag{45}$$

With the solution of (45), one knows the profile

$$D_k=\psi G\left(\frac{1}{(\beta J)^{1/2}}\,\psi k\right) \tag{46}$$

$$S_k=\tfrac{1}{2}-\tfrac{1}{4}\psi+\tfrac{1}{8}\psi^2+\tfrac{1}{4}\psi^2\left|G\left(\frac{1}{(\beta J)^{1/2}}\,\psi k\right)\right|. \tag{47}$$

For arbitrary μ, one can solve (45) numerically or try to express its solution in terms of tabulated functions. However, to study the critical behaviour $((T_0-T)\ll 1)$ one needs only to consider the limit $\mu\to 0$. For μ small, the function G varies from $-(1+3\mu)$ to $1+3\mu$ when t goes from $-\infty$ to $+\infty$. The critical behaviour will be dominated by the length L along which G remains close to 0. For $|G(t)|$ small and μ small, (45) can be simplified to

$$(\mathrm{d}G/\mathrm{d}t)^2=\tfrac{1}{3}G^2+\mu \tag{48}$$

which has the solution

$$G(t)=(3\mu)^{1/2}\sinh(t/\sqrt{3}). \tag{49}$$

Therefore the length L over which $|G(t)|$ is small diverges like

$$L\simeq-\sqrt{3}\log\mu \qquad \text{for } \mu\to 0. \tag{50}$$

We can now estimate W, the amount of disordered phase absorbed at the interface. From (26) and (47), one has for ψ small

$$W\simeq\frac{\psi^2}{2}\sum_k\left[\frac{3+(1+24\mu)^{1/2}}{4}-\left|G\left(\frac{1}{(\beta J)^{1/2}}\,\psi k\right)\right|\right]. \tag{51}$$

For μ small the sum is dominated by the region where G is close to 0:

$$W\simeq\frac{\psi^2}{2}\left(\frac{(\beta J)^{1/2}}{\psi}L\right)\simeq-\frac{\sqrt{3}\psi}{2\sqrt{d}}\log\mu \tag{52}$$

where we have used $d\beta_0J=1$ for $q=2$.

Thus in terms of the variables q and T, one finds that for $q-2$ small and $T_0-T\ll(q-2)^2$:

$$W\simeq-\frac{\sqrt{3}}{2\sqrt{d}}(q-2)\log\left(\frac{T_0-T}{(q-2)^2T_0}\right). \tag{53}$$

Thus, interfacial wetting does indeed take place as the temperature approaches T_0.

At the bulk critical temperature itself ($\mu=0$), the solution of (45) is particularly simple:

$$\tilde{G}(t)=\pm\frac{1}{\exp[\pm(t-t_0)/\sqrt{3}]+1}. \tag{54}$$

t_0 is an arbitrary constant of integration which reflects the fact that the location of the interface is arbitrary. The interface between phases A and B disassociates into two interfaces, between A and the disordered phase, and between the disordered phase and B, with profiles given by (54).

Equation (44) can also be used to describe finite-size effects. If one wants to describe an interface AB between two planes at distance $\mathscr{L}$, then the boundary condition $\mathrm{d}G/\mathrm{d}t \to 0$ for $t \to \pm\infty$ has to be replaced by boundary conditions at $t = \pm\mathscr{L}(\beta J)^{1/2}/2\psi$.

4. The surface tension for q close to 2

We have seen that the surface tension σ_{AB} is given by (25) and (10). If we expand σ_{AB} up to the fourth order in ψ (which is the leading order), and use the equations (27)-(35), we find

$$\beta\sigma_{\mathrm{AB}}(T) = \sum_k \left(-\tfrac{1}{12}(D_k^4 - D_{\mathrm{B}}^4) + \tfrac{1}{16}\psi^2(u_k - u_{\mathrm{B}}) - \tfrac{1}{8}(u_k^2 - u_{\mathrm{B}}^2) + \tfrac{1}{12}\psi^{-2}(u_k^3 - u_{\mathrm{B}}^3)\right) \tag{55}$$

where u_{B} and D_{B} are the bulk values given by (36) and (37) and u_k and D_k are the solutions of (34) and (35). At the bulk transition temperature

$$\beta\sigma_{\mathrm{AB}}(T_0) = \frac{\psi^4}{12}\left(\sum_{k=-\infty}^{\infty} |\tilde{G}^3[\psi k/(\beta J)^{1/2}]| - \tilde{G}^4[\psi k/(\beta J)^{1/2}]\right)2 \tag{56}$$

$$= \frac{\psi^3}{12}(\beta J)^{1/2}\int_{-\infty}^{\infty} [|\tilde{G}^3(t)| - \tilde{G}^4(t)]2\,\mathrm{d}t \tag{57}$$

where $\tilde{G}$ is given by (54).

The last factor of 2 comes from the fact that there are two interfaces between phases A and B which are an infinite distance apart. Clearly the surface tension σ between the ordered and disordered phases, which is defined only at T_0, is given by

$$\sigma = \tfrac{1}{2}\sigma_{\mathrm{AB}}(T_0). \tag{58}$$

For $q-2$ small, the result of the expression (57) can be written:

$$\beta\sigma_{\mathrm{AB}}(T_0) \simeq \frac{(\beta J)^{1/2}\psi^3}{6\sqrt{3}} \simeq \frac{(q-2)^3}{6(3d)^{1/2}}. \tag{59}$$

One can also calculate $\sigma_{\mathrm{AB}}(T)$ for $T < T_0$. For small μ one finds that the main contribution to the difference $\sigma_{\mathrm{AB}}(T) - \sigma_{\mathrm{AB}}(T_0)$ comes from the length L of the intervening disordered region

$$\beta[\sigma_{\mathrm{AB}}(T) - \sigma_{\mathrm{AB}}(T_0)] = (\tfrac{1}{4}\mu\psi^4)(-\sqrt{3}\log\mu)[(\beta J)^{1/2}/\psi]. \tag{60}$$

In this equation, $\frac{1}{4}\mu\psi^4$ is the difference in free energy between the ordered and disordered phases. It is linear in $T - T_0$ because the transition is first order. The remaining factor is the length L of equation (50) expressed in units of the lattice spacing. Expressing this result in terms of $T_0 - T$ and $q-2$ and again using $(\beta J)^{1/2} = d^{-1/2}$ for $q=2$ one obtains

$$\beta(\sigma_{\mathrm{AB}}(T) - \sigma_{\mathrm{AB}}(T_0)) \sim -\left(\frac{3}{4d}\right)^{1/2}(q-2)\left(\frac{T_0 - T}{T_0}\right)\log\left(\frac{(T_0 - T)}{(q-2)^2}\right). \tag{61}$$

Equations (53) and (61) are the main results of this work. They are valid for $(q-2) \ll 1$ and $0 < T_0 - T \ll (q-2)^2$. One can, of course, calculate W and $\sigma_{AB}(T)$ for $(q-2)$ small and $(T_0 - T)/(q-2)^2$ not small from the solution of (45).

5. Discussion

We have seen that, for q close to 2 and within mean-field theory, there is a wetting of the interface between two ordered phases of the q-state Potts model by the disordered phase as T approaches T_0, the temperature at which all three phases are in coexistence. The excess interfacial absorbtion W diverges, according to (53), like $\log(T_0 - T)$. In addition, equation (61) for the surface tension shows that the excess surface entropy, $-\partial\sigma_{AB}/\partial T$, also diverges logarithmically. That these divergences are intimately related is easily seen. For temperatures near T_0, the thickness L of the disordered region, which intrudes between the two ordered phases, is large and is directly proportional to W. Further, the surface tension σ_{AB} varies near T_0 as L multiplied by the difference of the free energy per unit volume of the ordered and disordered phases at the same temperature. The latter quantity is, of course, well defined in mean-field theory but, even more generally, it can be defined as the continuation of the disordered free energy in the vicinity of T_0 (we ignore the possibility of essential singularities at a first-order transition). When the bulk transition of the Potts model is first order, this free energy difference is linear in $T_0 - T$ from which follows the proportionality between the excess absorbtion and excess entropy.

In addition to solving equations (11) and (12) analytically near $q = 2$, we have also tried to solve them numerically for larger values ($q = 3, 10, 20$). This was difficult because the algorithm we used converged very slowly. Our numerical results indicated that there is an interfacial wetting for these larger values of q. We were not able to extract from our data the form of the divergence of W or the singular part of σ_{AB} as T approaches T_0. However, as we have seen, explicit solution of the many-density theory for q near 2 yields the same q-independent singularities as obtained from one-component theories which, by their construction, are independent of q and all symmetries (Widom 1977). It is reasonable to expect, therefore, that the results of one-component theories are applicable to the larger values of q as well. Further support for this view derives from the work of Kikuchi and Cahn (1980) which provides an example of an interfacial wetting in a system with more than one component which does not have Potts symmetry but which does display the same singular behaviour as found above.

Lastly we note that the existence of the interfacial wetting could have been anticipated by the use of the sharp-kink approximation in which the smooth continuous variation of all densities through the interface is ignored and the minimum of the surface tension, (25), is sought within the following subspace:

$$\begin{aligned} n_k^\lambda &= n_A^\lambda && k \leqslant 0 \\ &= n_{DIS} && 0 < k \leqslant Z \\ &= n_B^\lambda && Z < k \end{aligned} \tag{62}$$

where n_A^λ, n_B^λ and n_{DIS} are the values of n_k^λ appropriate in the A, B and disordered phases at T_0. In this approximation, σ_{AB} at T_0 depends only on the thickness of the disordered phase Z. Using this ansatz and the fact that all phases are in coexistence, we easily obtain

$$\sigma_{AB}(Z)-\sigma_{AB}(0)=-J\sum_{\lambda}(n_A^\lambda-n_{DIS})(n_{DIS}-n_B^\lambda) \qquad Z\geqslant 1. \tag{63}$$

As $n_{\lambda'}^\lambda=(q-1)/q$ for $\lambda=\lambda'$ and $1/q(q-1)$ for $\lambda\neq\lambda'$ and $n_{DIS}=1/q$, (63) yields

$$\sigma_{AB}(Z)-\sigma_{AB}(0)=-\frac{J(q-2)^2}{q(q-1)^2} \qquad Z\geqslant 1. \tag{64}$$

Thus, within this approximation, a layer of disordered phase of any non-zero thickness is favourable over no such layer but σ_{AB} is otherwise independent of Z. In the exact solution of the mean-field equations, the densities n_k^λ do not change abruptly, but exponentially (cf (54)). This causes a repulsion between the kinks which in turn causes $\sigma_{AB}(Z)$ to decrease exponentially with Z. Hence an infinite value of Z minimises σ_{AB} which corresponds to a wetted interface.

The value of the interfacial tension at T_0 in the sharp-kink approximation is

$$\frac{\sigma_{AB}}{J}=\frac{(q-2)^2}{q(q-1)} \qquad \text{(sharp kink)} \tag{65}$$

which is to be compared with the analytic result of (59) valid for $(q-2)$ small

$$\frac{\sigma_{AB}}{J}=\frac{1}{6}\left(\frac{d}{3}\right)^{1/2}(q-2)^3. \tag{66}$$

The latter is smaller than the former by a factor of $\sqrt{d}(q-2)$ which has been assumed to be small. The approximation overestimates the surface tension precisely because the kinks are not sharp but vary on the length scale $1/[\sqrt{d}(q-2)]$ which is assumed to be large. However for large q the kinks do become sharp so that the approximate expression (65) should become increasingly accurate.

Our results may have some experimental relevance. As we noted earlier, an A/B interface cannot be wetted by another ordered phase. Similarly, the interface between the disordered phase and one ordered phase cannot be wetted by another ordered phase. From this we surmise that in a binary-liquid mixture in which the two fluids are very similar, neither vapour–liquid interface will be wet by the other fluid. Rather, the vapour will wet the liquid/liquid interface. The first statement can be checked experimentally rather easily. The second cannot because gravity prevents the vapour from intervening between the liquids. It might be checked, however, in a three-dimensional realisation of the three-state Potts model (Mukamel *et al* 1977, Barbara *et al* 1978). This consists of a Heisenberg ferromagnet in a crystal with a strong anisotropy which favours the six directions in the set $\leqslant 100 \geqslant$. A magnetic field along the (111) direction favours the three directions (100), (010) and (001) resulting in a three-state Potts symmetry. Quite generally, there will be domains of the different directions of magnetisation. On heating to T_0, the paramagnetic phase is predicted to wet the interfaces between these domains, an effect which should be readily observable.

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