Systems Out of Equilibrium: Some Exactly Solvable Models

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Abstract—Exact results have been obtained recently for two systems out of equilibrium: the asymmetric exclusion model and the problem of coarsening in one dimension.

ASYMMETRIC EXCLUSION

In the one-dimensional asymmetric exclusion model, particles hop to their right at random times on a one-dimensional lattice, provided that the site they jump to is empty. All the steady-state properties can be calculated exactly [1] using a matrix formulation [2]. It consists of writing the steady-state weights of a system of N sites as the matrix element of a product of N matrices: in this product, each matrix can take two possible forms $D$ or $E$ depending on whether the corresponding site is occupied or empty.

Several extensions of the model have also been solved exactly using this matrix method: systems of several species of particles, shocks [3], and diffusion constants [4,5].

COARSENING IN ONE DIMENSION

When a ferromagnetic system is quenched to its low temperature phase, domains grow and the size of domains increases with a universal exponent $t^{1/2}$. Monte Carlo simulations of the Glauber dynamics of the Ising and the Potts model indicate that unequal time correlations are characterized by more complex exponents. For example, at zero temperature, the fraction $r(t)$ of spins which have never flipped up to time $t$ decays like a power law

$$r(t) \sim t^{-\theta},$$

where the exponent $\theta$ is nontrivial and depends both on the number of states of the Potts model and on the dimension of space [6–8]. This exponent can be calculated exactly [9], confirming the values previously determined by numerical methods.

Similar nontrivial exponents are also present in even simpler models of coarsening, where the dynamical rule is deterministic [10,11].

REFERENCES