Tutorial 7, Advanced MCMC
2021/22 ICFP Master (second year)

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1. Simulated tempering for the V-shaped stationary distribution

In lecture 7 (part 7.2), we discussed the simulated tempering algorithm. Here, we study this algorithm for the V-shaped stationary distribution. In all the below versions, one move corresponds to one displacement followed (with probability ϵ) a replica move.

(a) Set up the simulated tempering algorithm for the case $k = 2$ (flat distribution and V-shaped distribution) on the path graph P_n as a function of n and of the switching probability ϵ.

(b) Compute the conductance of this algorithm for the case $k = 2$ (flat distribution and V-shaped distribution) on the path graph P_n as a function of n and of the switching probability ϵ. Locate the bottleneck as a function of n and ϵ.

(c) Set up the direction-lifted simulated tempering algorithm for this same case.

(d) Set up the transition matrices and compute the dominant and sub-dominant (left) eigenvalues both for the simulated tempering and for the direction-lifted simulated tempering.

(e) Compute the time-dependent TDV, and from there the mixing time of the reversible Metropolis algorithm on the path graph with the V-shaped stationary distribution as a function of n and of ϵ. Generalize to the direction-lifted Metropolis algorithm, if possible.