Mathematica notebook for the Kac-Ward solution of the two-dimensional Ising model without periodic boundary conditions. Here we present the matrix for the 2x2 model and check that it yields the correct solution.

In[6]:=
```
β = 1.2
```
Out[6]= 1.2

In[7]:=
```
ν = Tanh[β]
```
Out[7]= 0.833655

In[8]:=
```
α = Exp[I Pi / 4] ν
```
Out[8]= 0.589483 + 0.589483 i

In[9]:=
```
α' = Conjugate[α]
```
Out[9]= 0.589483 - 0.589483 i

In[5]:=
```
null = {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```
Out[5]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

In[14]:=
```
one  = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```
Out[14]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

In[10]:=
```
right = {{ν, α, 0, α'}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```
Out[10]= {{0.833655, 0.589483 + 0.589483 i, 0, 0.589483 - 0.589483 i},
{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

In[11]:=
```
up = {{0, 0, 0, 0}, {α', ν, α, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```
Out[11]= {{0, 0, 0, 0}, {0.589483 - 0.589483 i, 0.833655, 0.589483 + 0.589483 i, 0},
{0, 0, 0, 0}, {0, 0, 0, 0}}

In[12]:=
```
left = {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, α', ν , α}, {0, 0, 0, 0}}
```
Out[12]= {{0, 0, 0, 0}, {0, 0, 0, 0},
{0, 0.589483 - 0.589483 i, 0.833655, 0.589483 + 0.589483 i}, {0, 0, 0, 0}}

In[13]:=
```
down = {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {α, 0, α', ν}}
```
Out[13]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0},
{0.589483 + 0.589483 i, 0, 0.589483 - 0.589483 i, 0.833655}}

In[16]:=
```
U2x2 = ArrayFlatten[ {{one, right, up, null},
    {left, one, null , up}, {down, null, one, right}, {null, down, left, one}}]
```

Out[16]=
```
{{1, 0, 0, 0, 0.833655, 0.589483 + 0.589483 i, 0, 0.589483 - 0.589483 i, 0, 0,
  0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0.589483 - 0.589483 i, 0.833655,
  0.589483 + 0.589483 i, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0,
  0, 0, 0, 0, 0.589483 - 0.589483 i, 0.833655, 0.589483 + 0.589483 i, 0},
 {0, 0.589483 - 0.589483 i, 0.833655, 0.589483 + 0.589483 i, 0, 0, 1, 0,
  0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0.833655, 0.589483 + 0.589483 i, 0,
  0.589483 - 0.589483 i}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
 {0.589483 + 0.589483 i, 0, 0.589483 - 0.589483 i, 0.833655, 0, 0, 0, 0,
  0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
  0.589483 - 0.589483 i, 0.833655, 0.589483 + 0.589483 i, 0, 0, 1, 0}, {0, 0, 0, 0,
  0.589483 + 0.589483 i, 0, 0.589483 - 0.589483 i, 0.833655, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[17]:=
```
Det[U2x2]
```

Out[17]=
$$2.19928 + 1.19473 \times 10^{-17}\, i$$

Comparison with the analytic solution (high-temperature solution with just two terms.

In[18]:=
```
(1 + Tanh[β]^4) ^2
```

Out[18]=
```
2.19928
```