Tutorial 8, Statistical Mechanics: Concepts and applications
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Maurizio Fagotti, Olga Petrova, Werner Krauth
Tutorial exercises

I. WORKSHEET: THE ROUGHENING TRANSITION

Source: S. T. Chui and J. D. Weeks, Phys. Rev. B **23**, *2438 (1981)*
J. M. Yeomans, Statistical Mechanics of Phase Transitions (Oxford, 1992), chapter 5

1. The Chui-Weeks model

The model: Here we consider a version of the Chui-Weeks model, which describes solid-on-solid surface growth. The surface is parametrized by the height h_i above site i of the lattice, which we are going to assume to be discrete and non-negative $h_i \in \mathbb{N}$ (there is an impenetrable substrate). The Hamiltonian is given by

$$H = J \sum_{i=1}^{N} |h_i - h_{i+1}| - K \sum_{i=1}^{N} \delta_{h_i 0} \,, \tag{1}$$

where the first term represents the contribution of surface tension to the total energy, and K parametrizes an energy binding the surface to the substrate. Assume a one dimensional model with periodic boundary conditions.

(a) Write down the transfer matrix of this model in terms of $\omega = e^{-\beta J}$ and $\kappa = e^{\beta K}$.

(b) Consider a family of eigenvectors of the form

$$\vec{v}^t = \begin{pmatrix} \Psi_0 & \cos(q+\theta) & \cos(2q+\theta) & \dots \end{pmatrix}. \tag{2}$$

Find the corresponding eigenvalues.

(c) Now consider a different eigenvector:

$$\vec{w}^t = \begin{pmatrix} \Phi_0 & e^{-\mu} & e^{-2\mu} & \dots \end{pmatrix}. \tag{3}$$

Find the corresponding eigenvalue and, if necessary, specify in what temperature regime it exists.

(d) Find the eigenvalue that dominates the thermodynamics below the critical temperature (temperature at which the *roughening transition* occurs), and discuss what this means for the two phases.

(e) Set up, but do not evaluate, an expression that gives a measure of the interface width.