Tutorial 6, Advanced MCMC
2021/22 ICFP Master (second year)

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1. **Thermodynamic integration for the V-shaped stationary distribution**

   In lecture 6 (part 6.2), we discussed the thermodynamic integration for the V-shaped stationary distribution.

   (a) Implement a sequence of samplings, using the reversible MCMC algorithm, for different values of α, as discussed, to compute the partition function at $\alpha = 1$.

   (b) Provide an error analysis for your multiple (independent) computations, using the Gaussian error-propagation formula.

2. **Simulated annealing for the V-shaped stationary distribution**

   In lecture 6 (part 6.2), we also discussed simulated annealing for the V-shaped stationary distribution, modified by setting $\pi(n) = 0^+$, so that the vertex $i = 1$ has the highest weight. The aim is to find this vertex with probability 1 through Monte Carlo sampling. We define the energies $E_i = -\log(\pi_i)$, to agree with the notations of Hajek (1988)

   (a) For small n, set up different temperature schedules $T_k, k \in \{1, 2, \dots\}$ of the form $T_k = c/(\log(k+1)$ and use the Metropolis algorithm to propagate your Markov chain.

   (b) Can you confirm Hajek's theorem, that $c > d^*$ for having sure convergence to $i = 1$?

3. **Simulated tempering for the V-shaped stationary distribution**

   In lecture 6 (part 6.2), we also finally discussed simulated tempering for the V-shaped stationary distribution.

   (a) Set up simulated tempering for two values of α, with moves that either change the copy, or the position, as discussed. Use small values of n.

   (b) Check that the configurations of the target copy $\alpha = 1$ are well distributed according to the correct distribution