Tutorial 9, Advanced MCMC
2021/22 ICFP Master (second year)

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(Dated: 23 March 2022)

1. **Bayesian statistics in the children's game**

   In this week's lecture, we studied the Bayes approach to statistics.

   (a) Suppose now that a game of 4000 pebbles has produced 3156 "hits". By simulation, produce the a posteriori probability distribution of π, if the test values are drawn uniformly in $[0, 4]$, if the squares of the test values are drawn in $[0, 16]$, if the square roots of the test values are drawn uniformly in $[0, 2]$, and if, following Archimedes, the test values for π are drawn uniformly in $[3\frac{10}{71}, 3\frac{1}{7}]$.

   (b) Do this same calculation analytically.

2. **Dvoretzky–Kieffer–Wolfowitz inequality**

   The DKW inequality gives the probability for the cumulative distribution function being fully englobed in a corridor aroun the empirical CDF.

   (a) By simulation, check the DKW inequality for the uniform distribution in $[0, 1]$, for $n = 10, 20, 40$.