

**Tutorial 5, Advanced MCMC**  
**2020/21 ICFP Master (second year)**

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**1. Birth-and-death algorithm for hard spheres**

In lecture 5 (part 5.2), we discussed the birth-and-death formulation of hard-sphere MCMC.

- (a) Implement this algorithm for hard disks in a 2D box without periodic boundary conditions.
- (b) Compute an observable, for example the probability for a sphere to be at a position  $x$ , for  $N = 4$ , and check that it comes out the same as the regular MCMC algorithm for 4 disks (see fig. 2.9 p. 89 SMAC book).

**2. Global-move MCMC** In lecture 5 (part 5.2), we discussed a global-move MCMC, where the displacement went anywhere in the box.

- (a) Show that this algorithm satisfies detailed balance.
- (b) Program a coupling for this algorithm (consisting in making the same move for all configurations) and show that, empirically, it couples any two initial configurations for small enough densities.