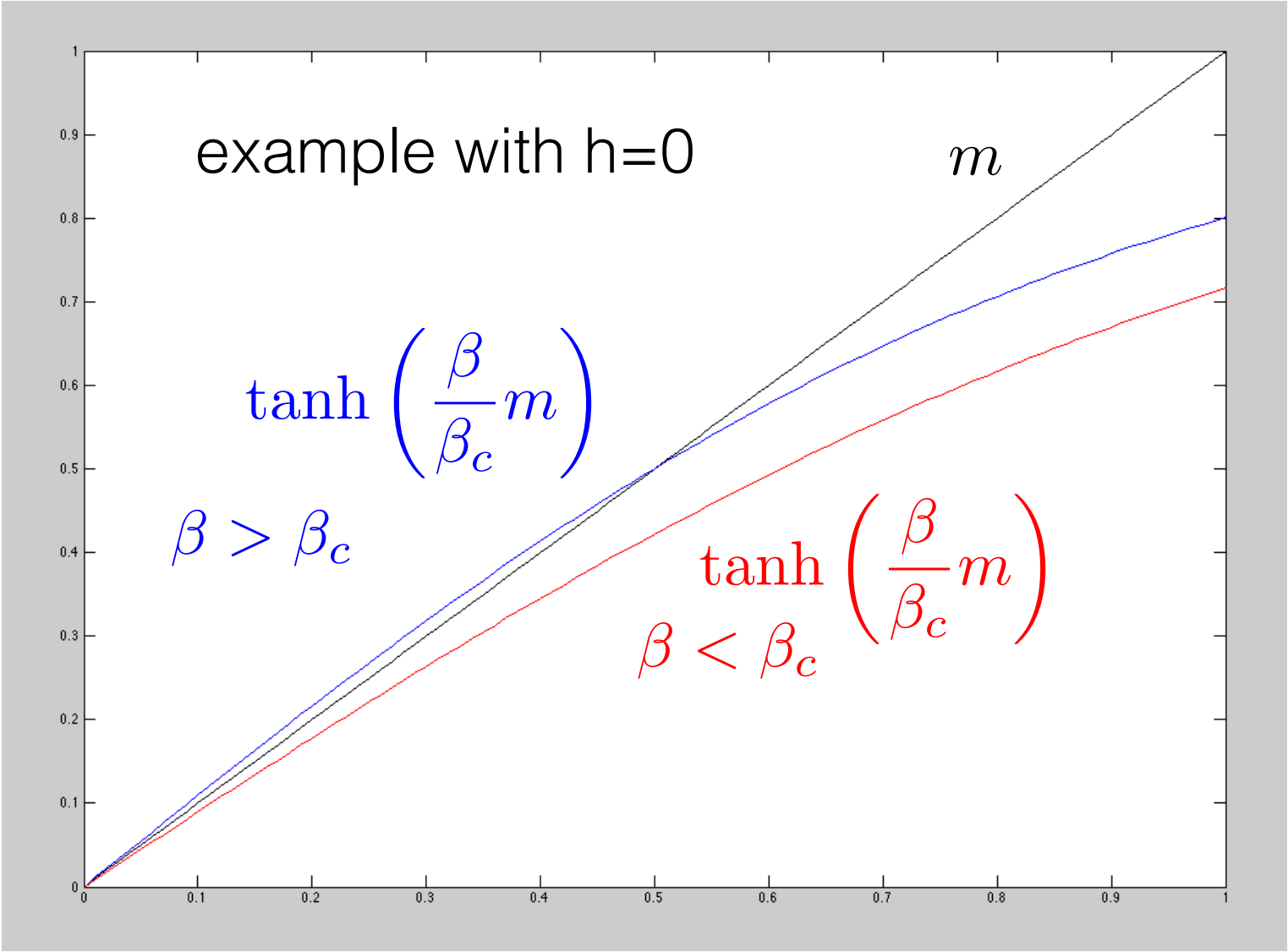
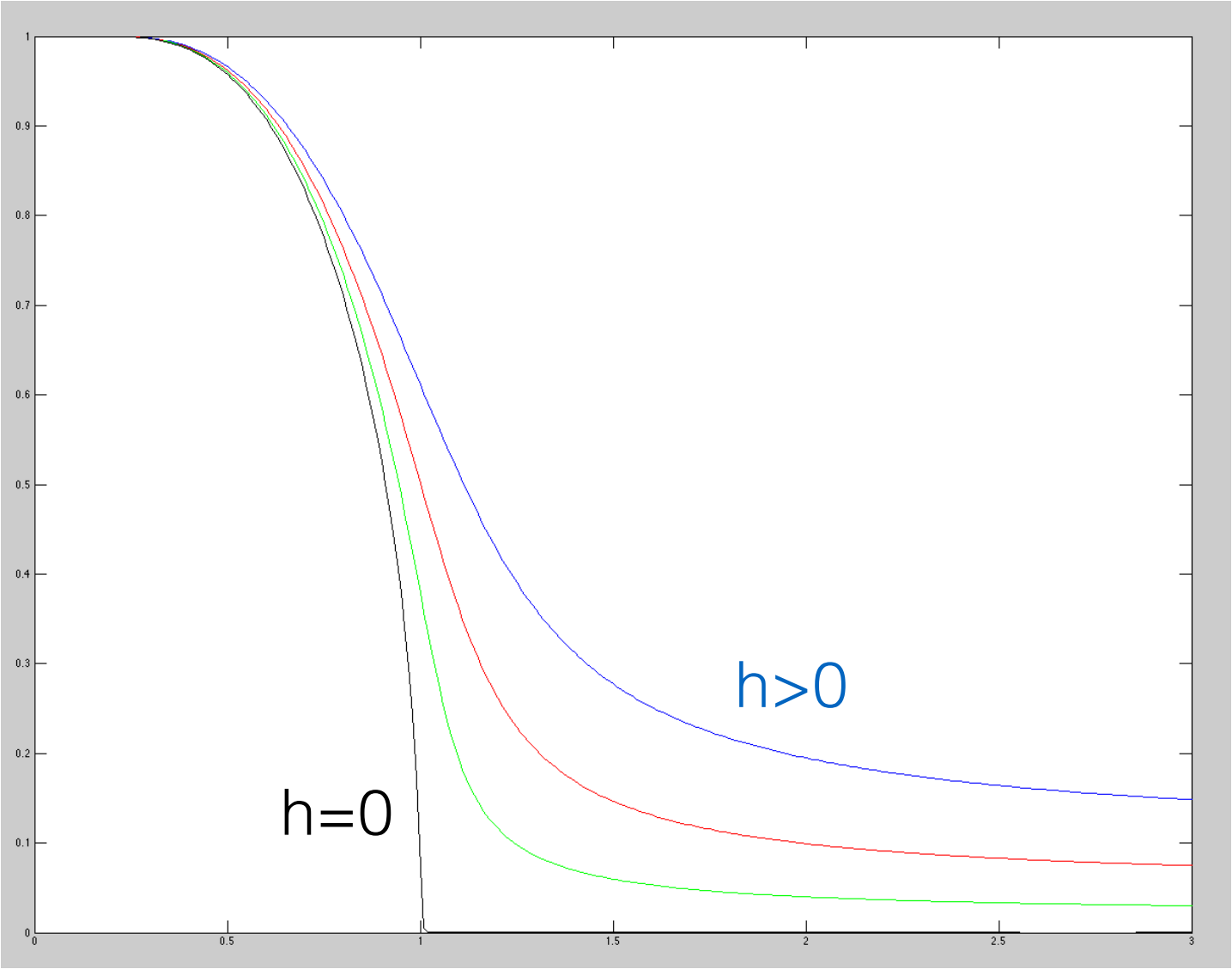
Solving $m = \tanh\left(\frac{\beta}{\beta_c} m + \beta h\right)$



Solving $m = \tanh\left(\frac{\beta}{\beta_c} m + \beta h\right)$

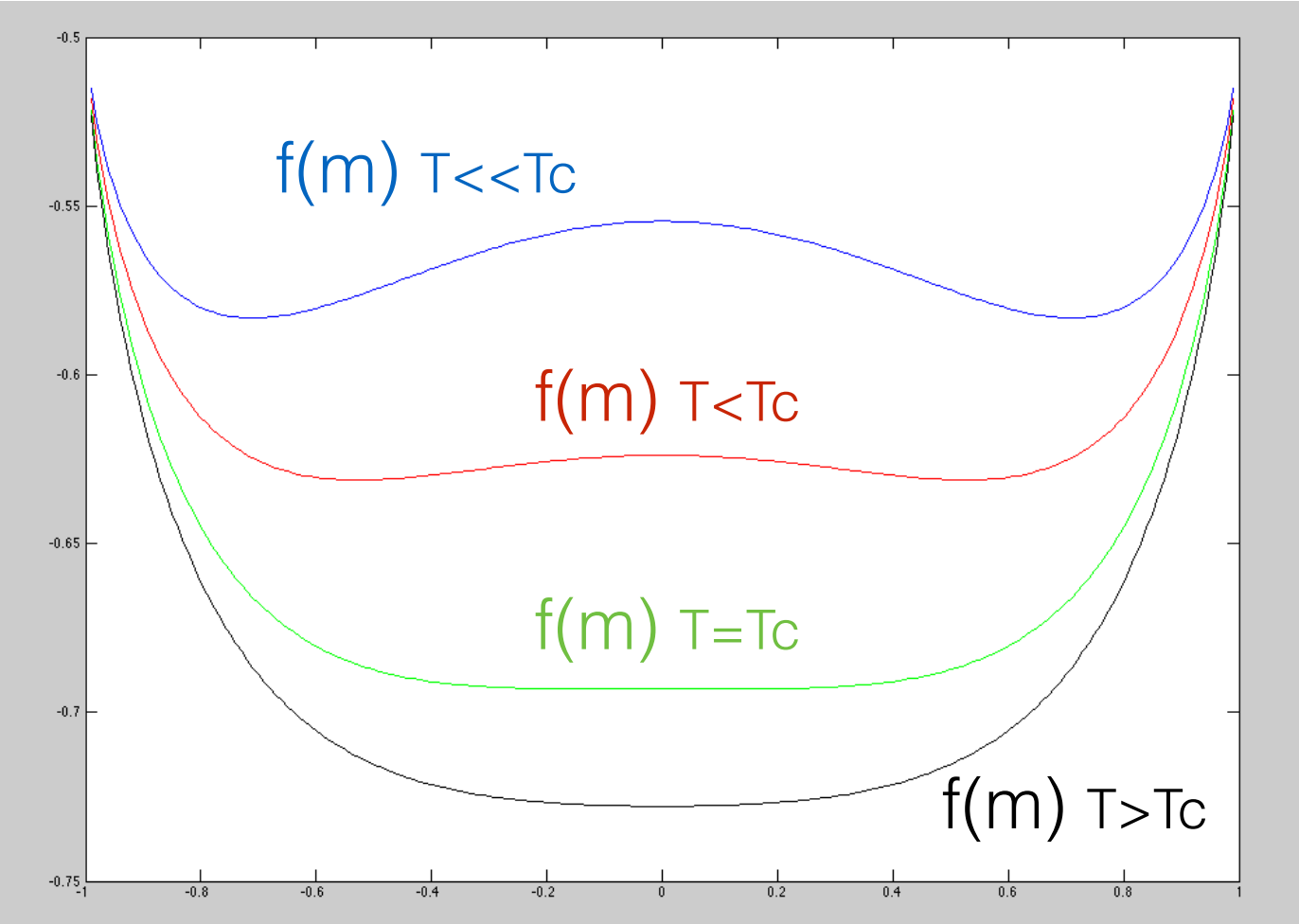
Transition du second ordre (continue) pour h=0!

m



$\frac{T}{T_c}$

The (reduced) free energy at zero external field

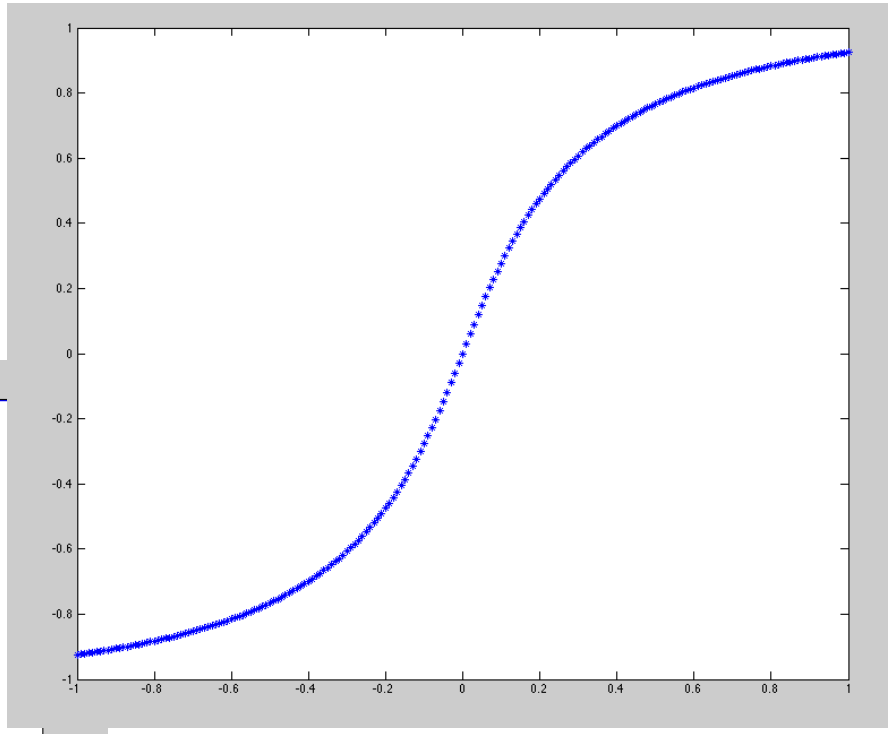


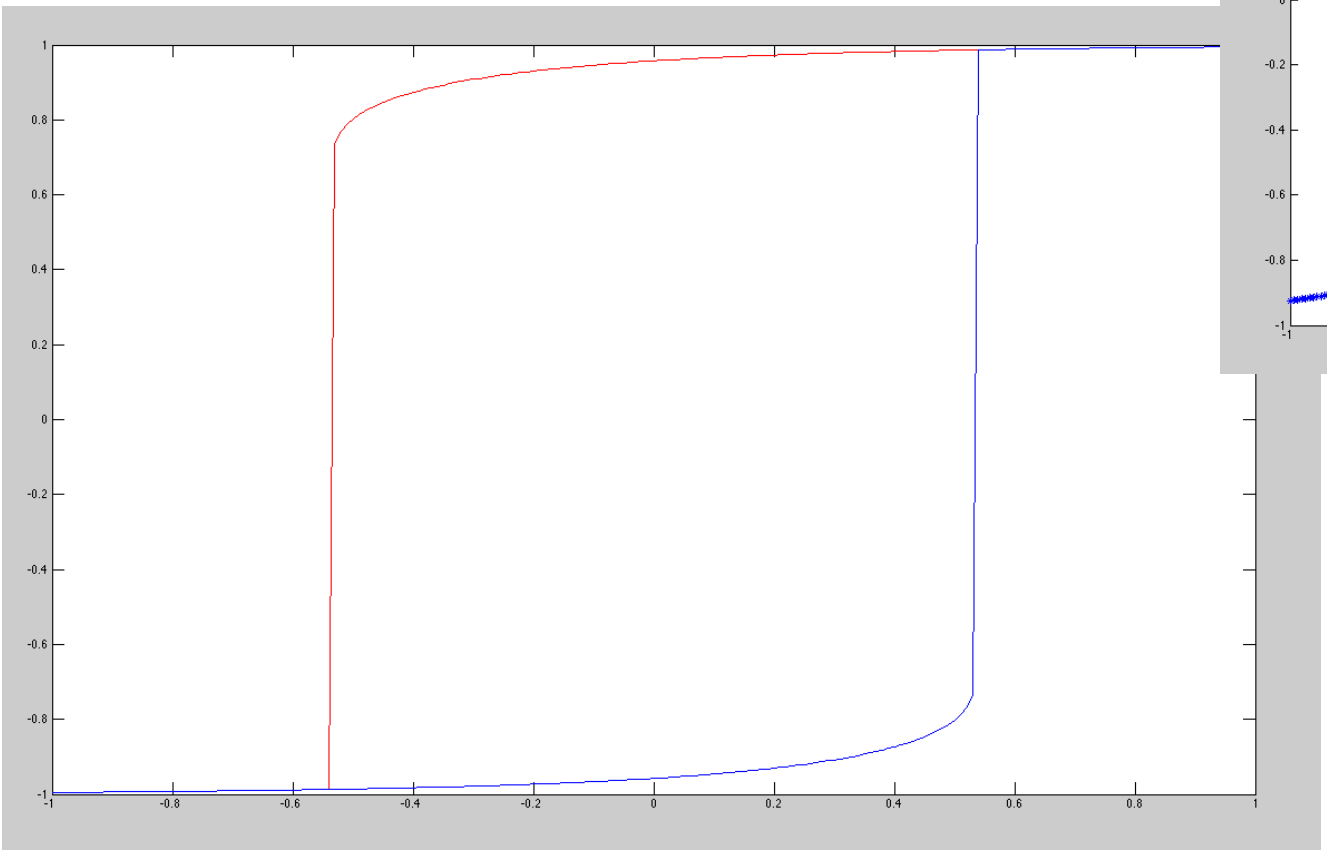
$$\frac{f(m)}{T_c} = -\frac{m^2}{2} + \frac{T}{T_c}\left(\frac{1+m}{2}\log\left(\frac{1+m}{2}\right) + \frac{1-m}{2}\log\frac{1-m}{2}\right)$$

$$m = \tanh\left(\frac{\beta}{\beta_c} m + \beta h\right)$$

h non zero!

T>Tc

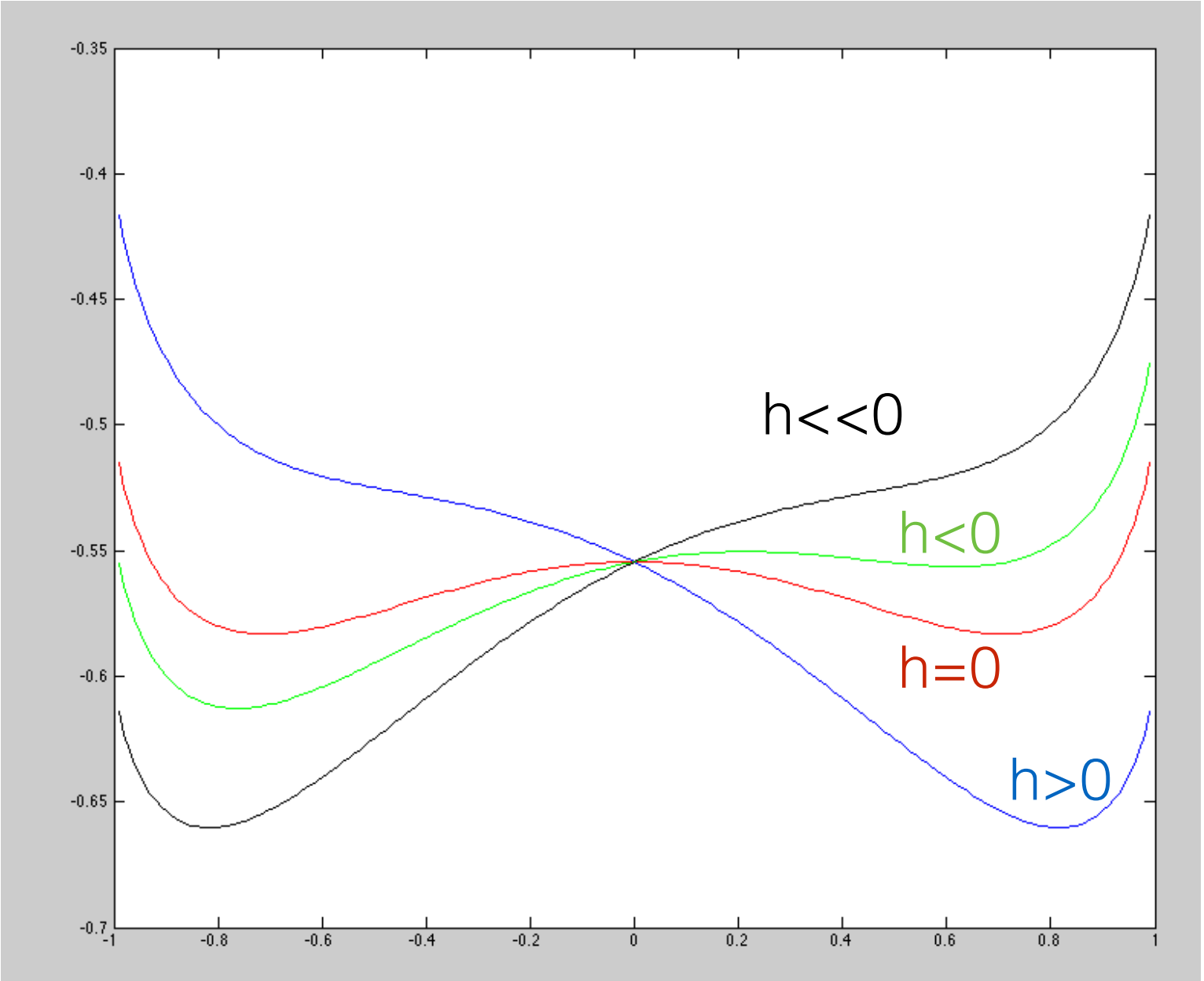




T<Tc

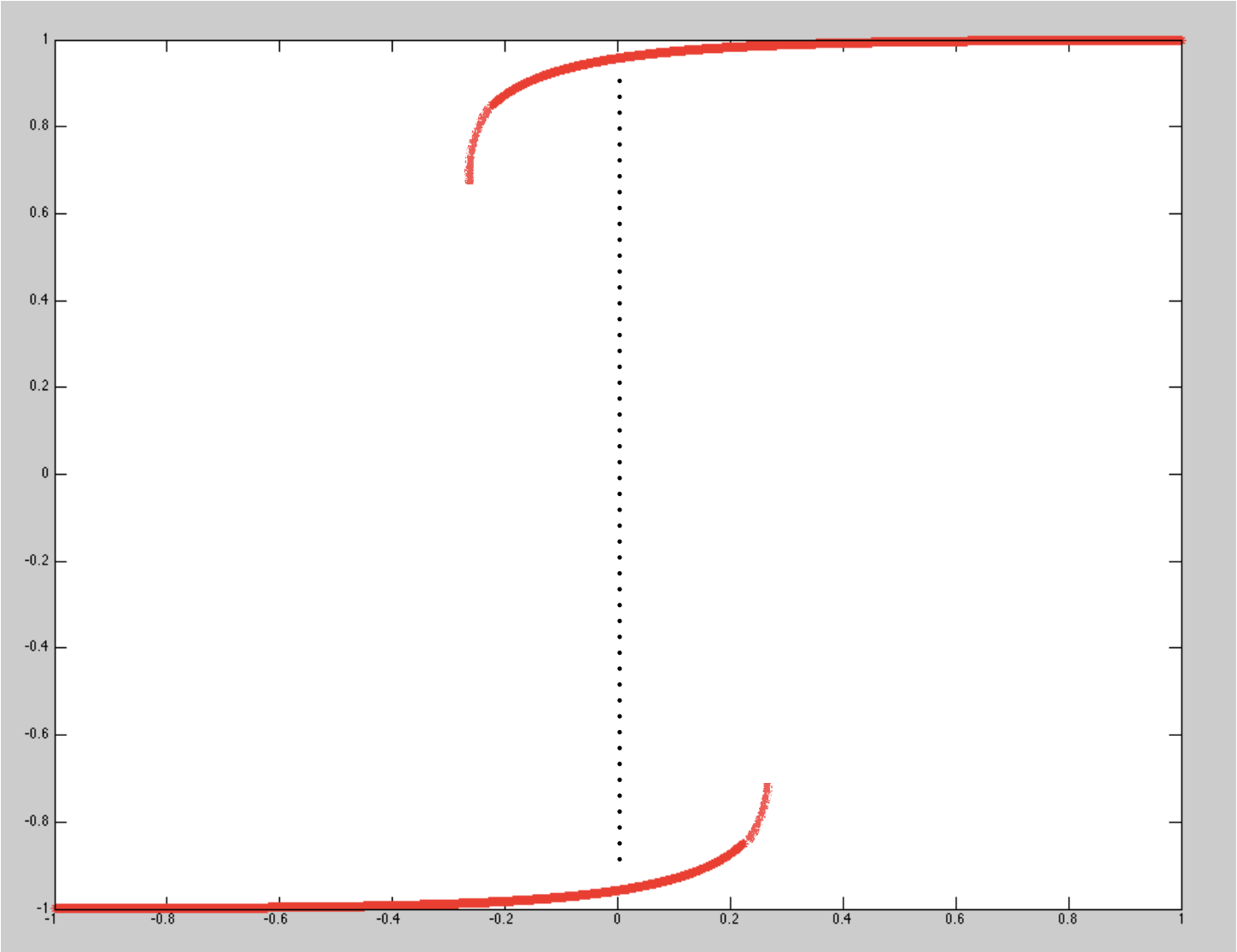
First order (discontinuous) transition!

The (reduced) free energy at zero external field



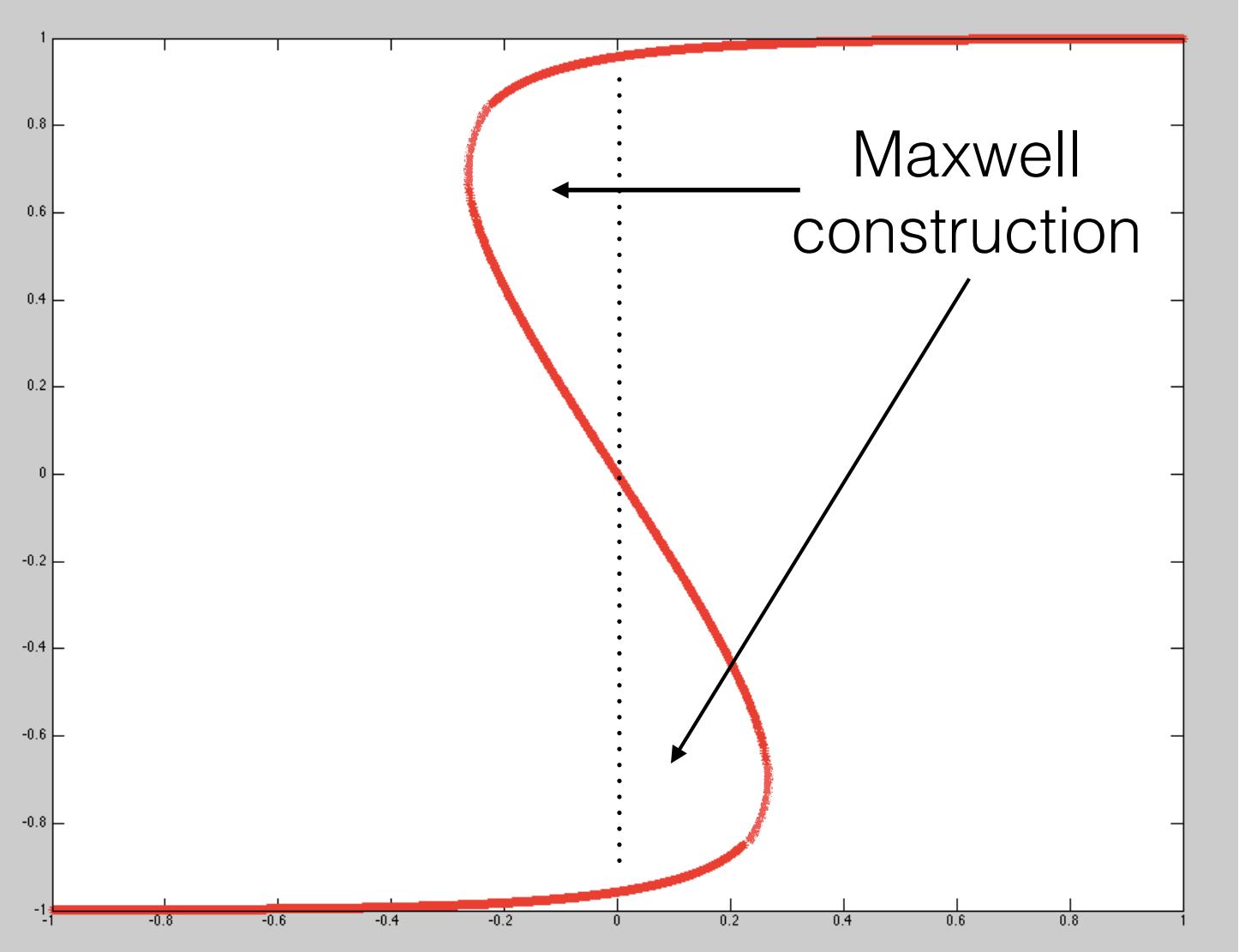
$$\frac{f(m)}{T_c} = -\frac{m^2}{2} + \frac{T}{T_c}\left(\frac{1+m}{2}\log\left(\frac{1+m}{2}\right) + \frac{1-m}{2}\log\frac{1-m}{2}\right) + \frac{h}{T_c}m$$

Phase transition at h=0!
Metastable solutions



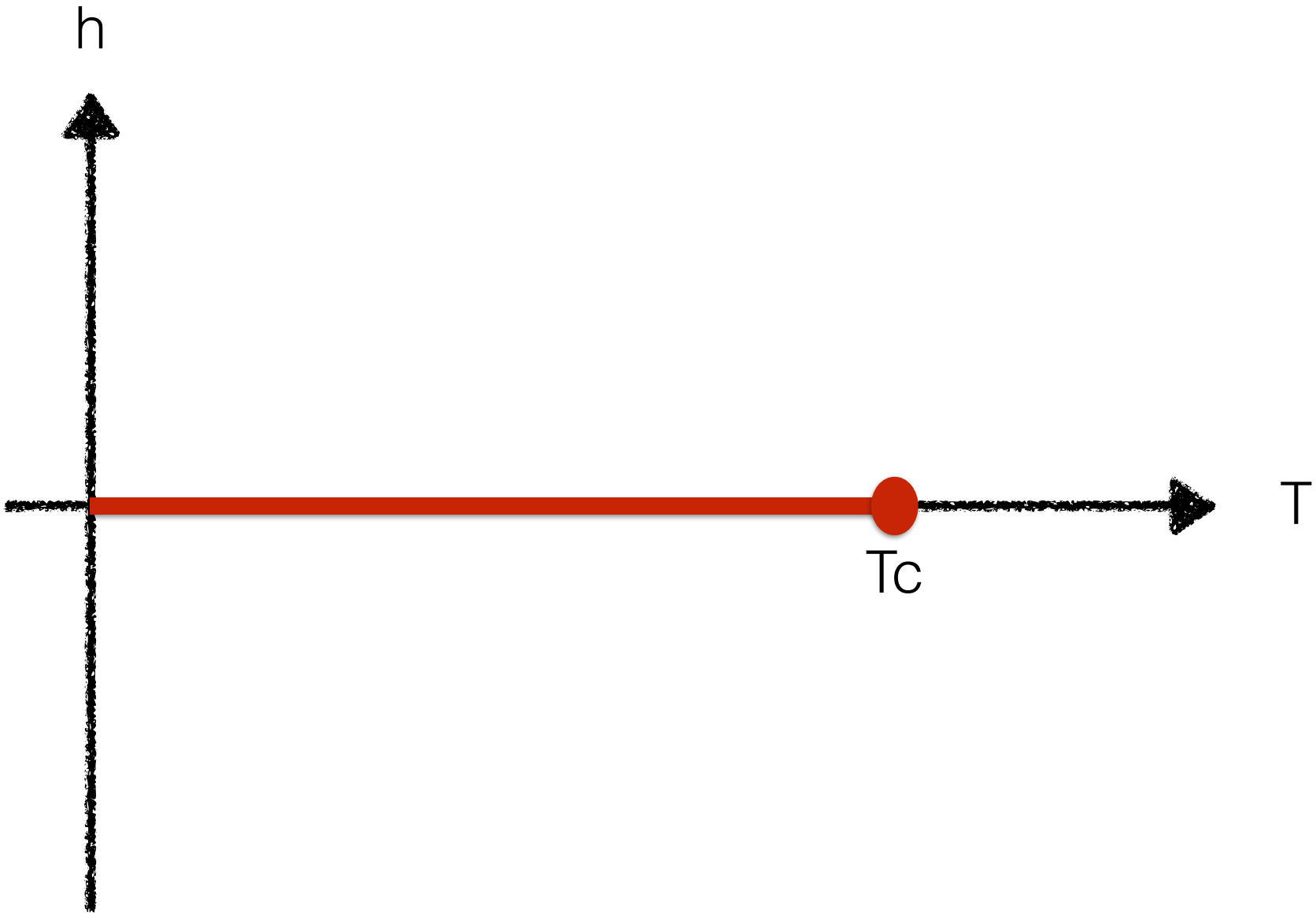
m(h)

In fact, one more (instable) solution
(corresponding to the maxima)

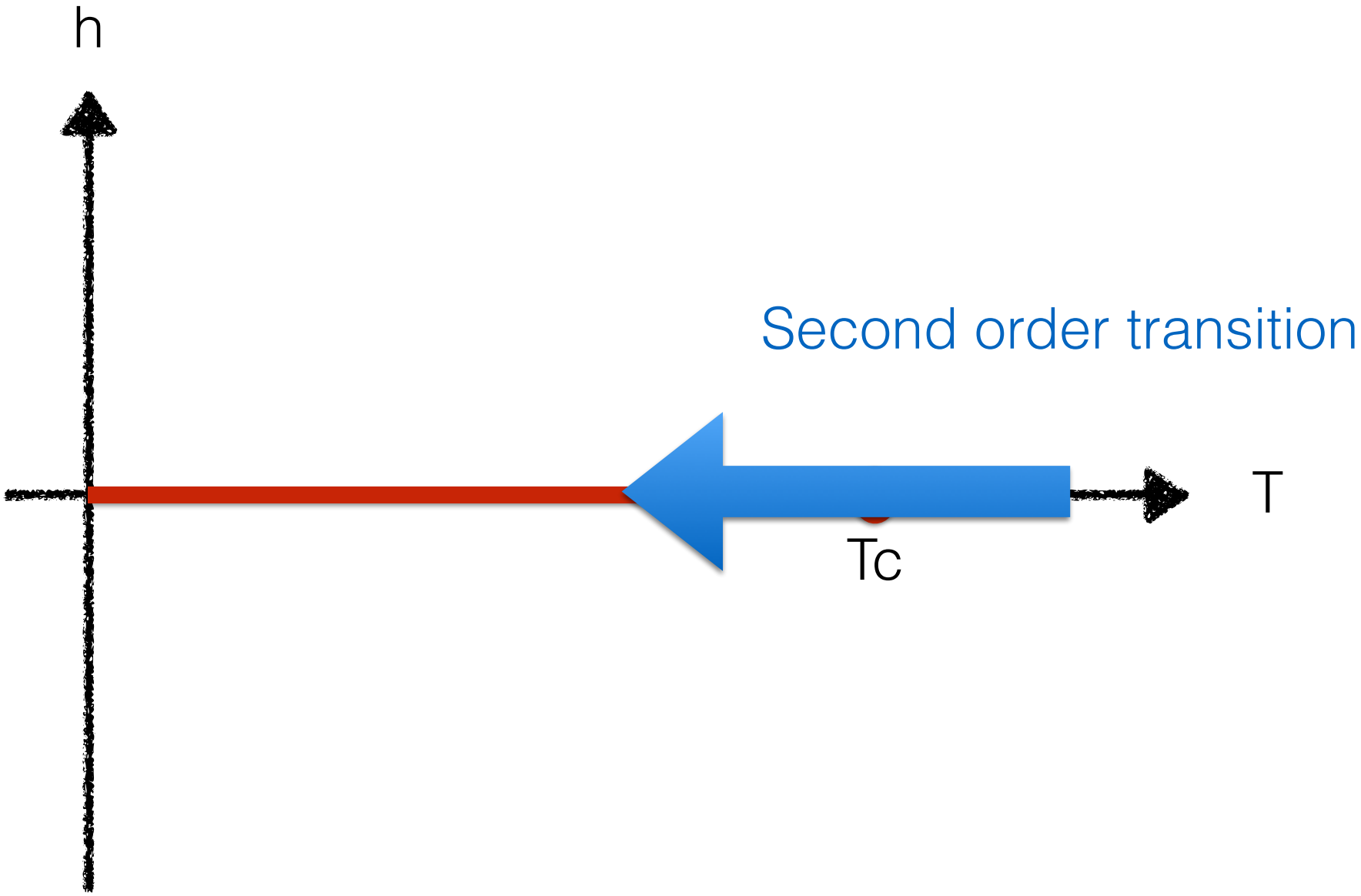


m(h)

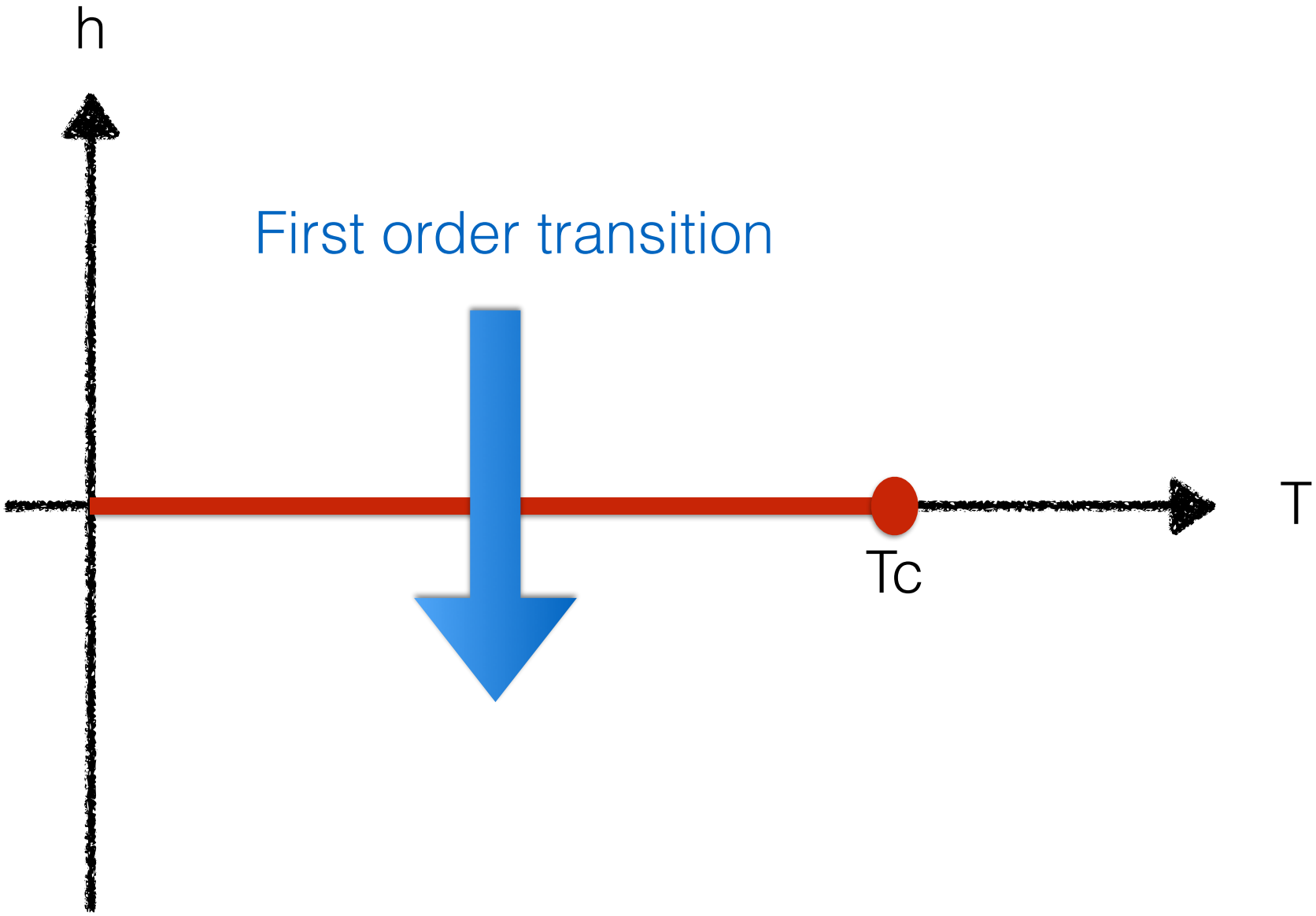
(h-T) Phase Diagram



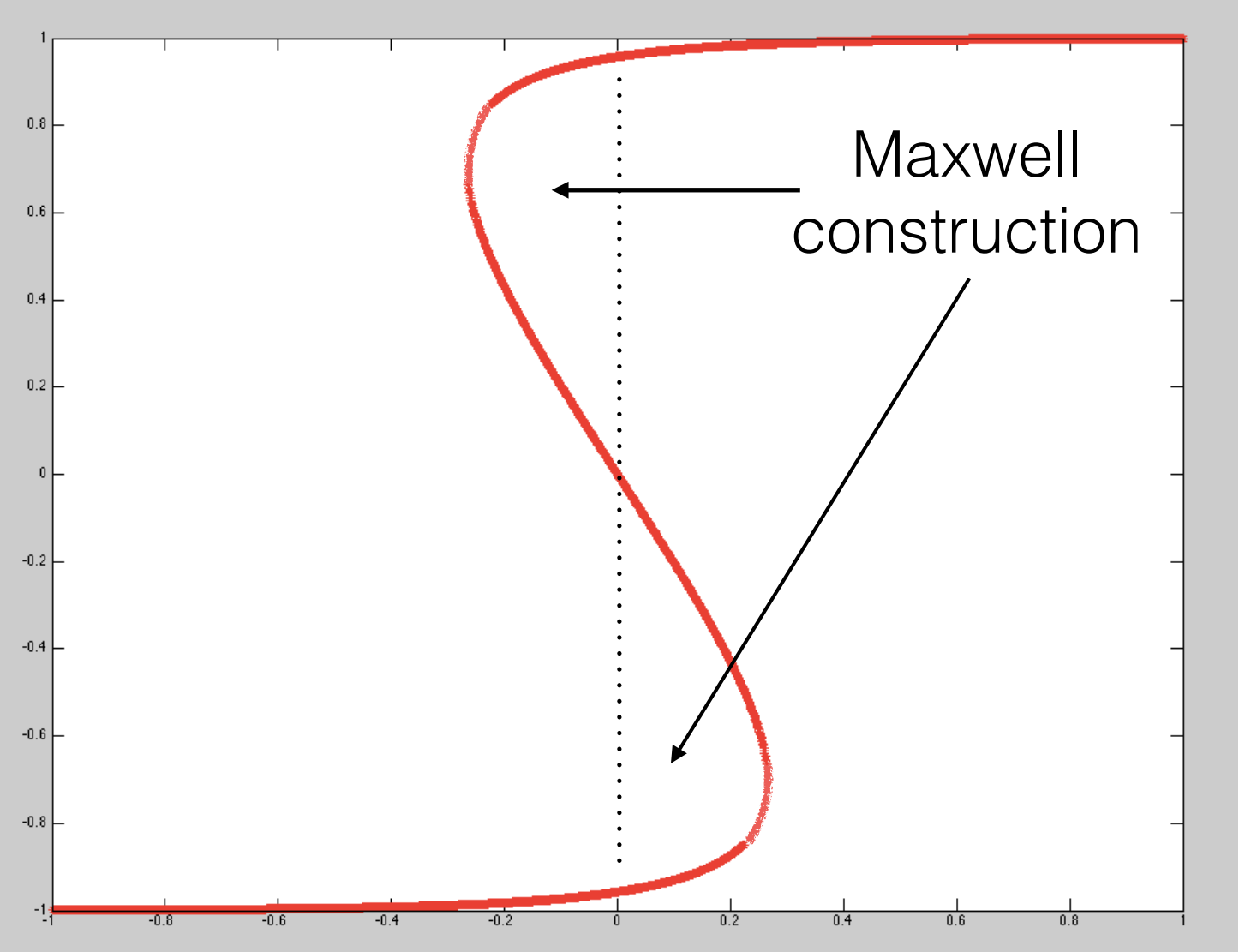
(h-T) Phase Diagram



(h-T) Phase Diagram



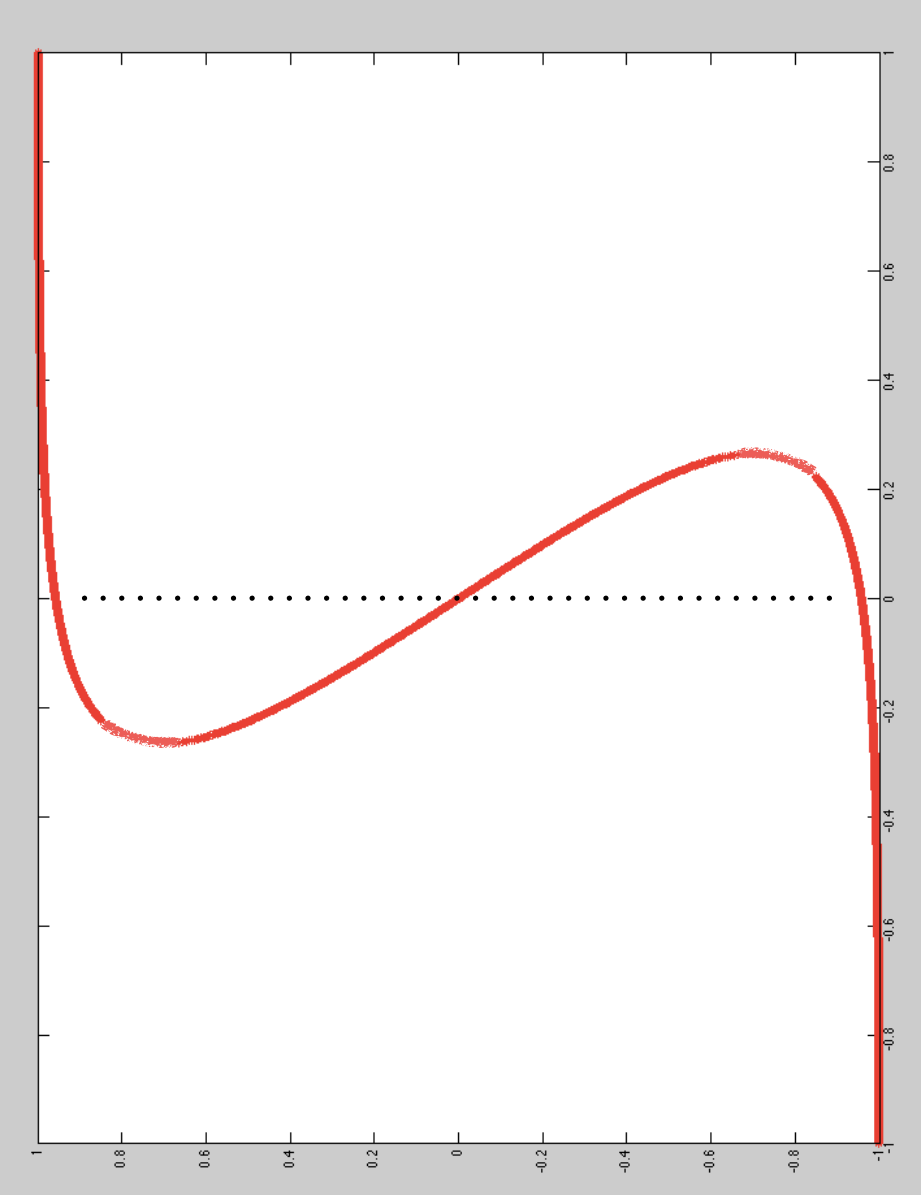
Now let us look to h(m) instead



m(h)

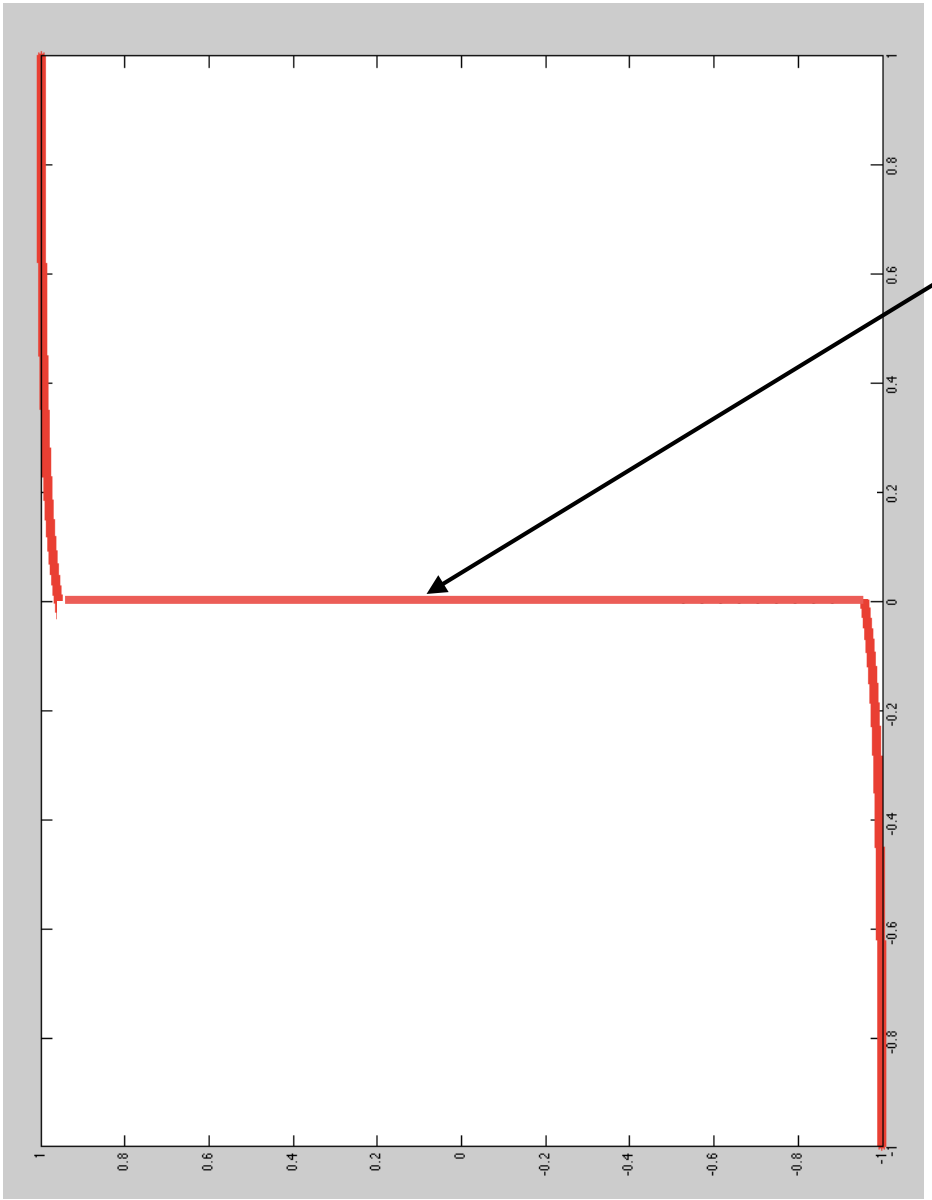
Now let us look to h(m) instead

h(m)



Now let us look to h(m) instead

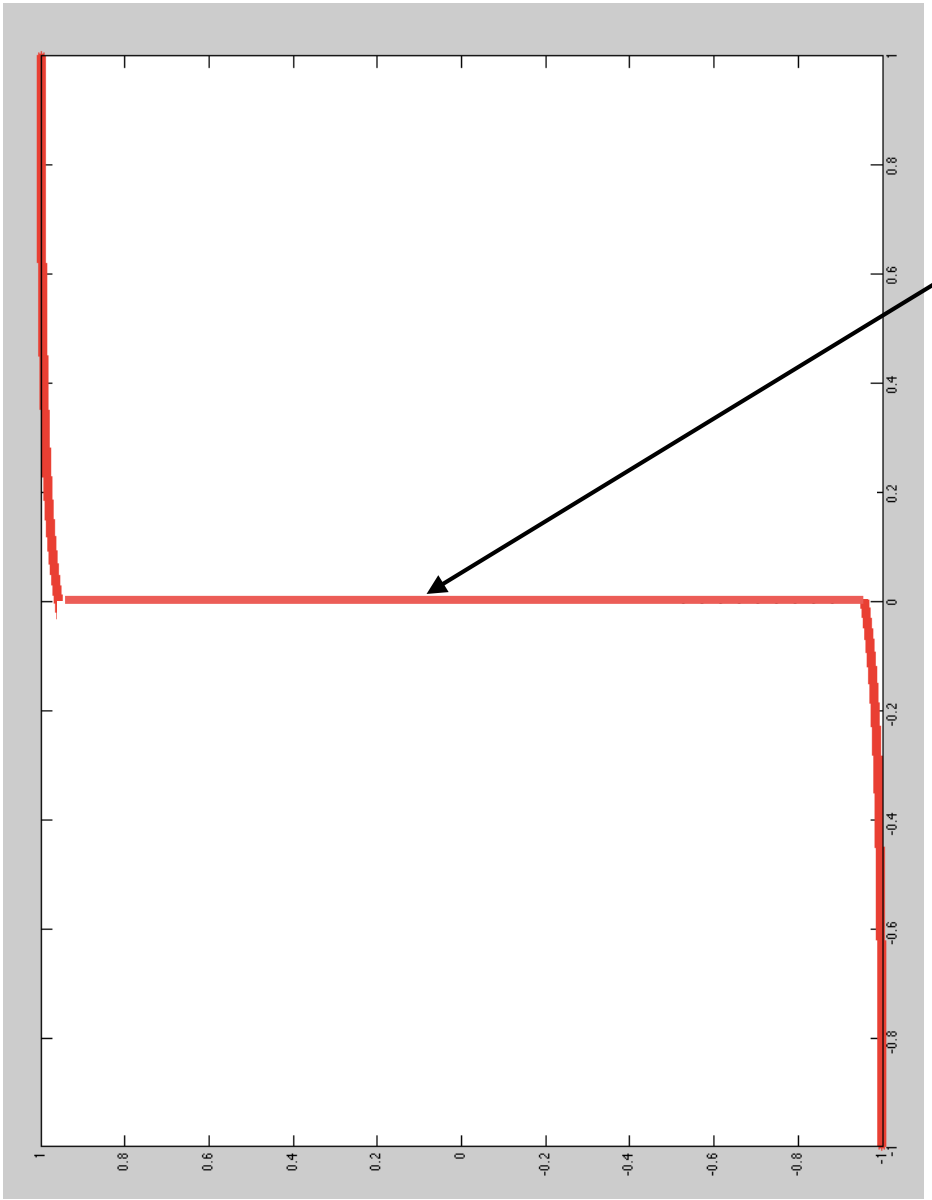
h(m)



?

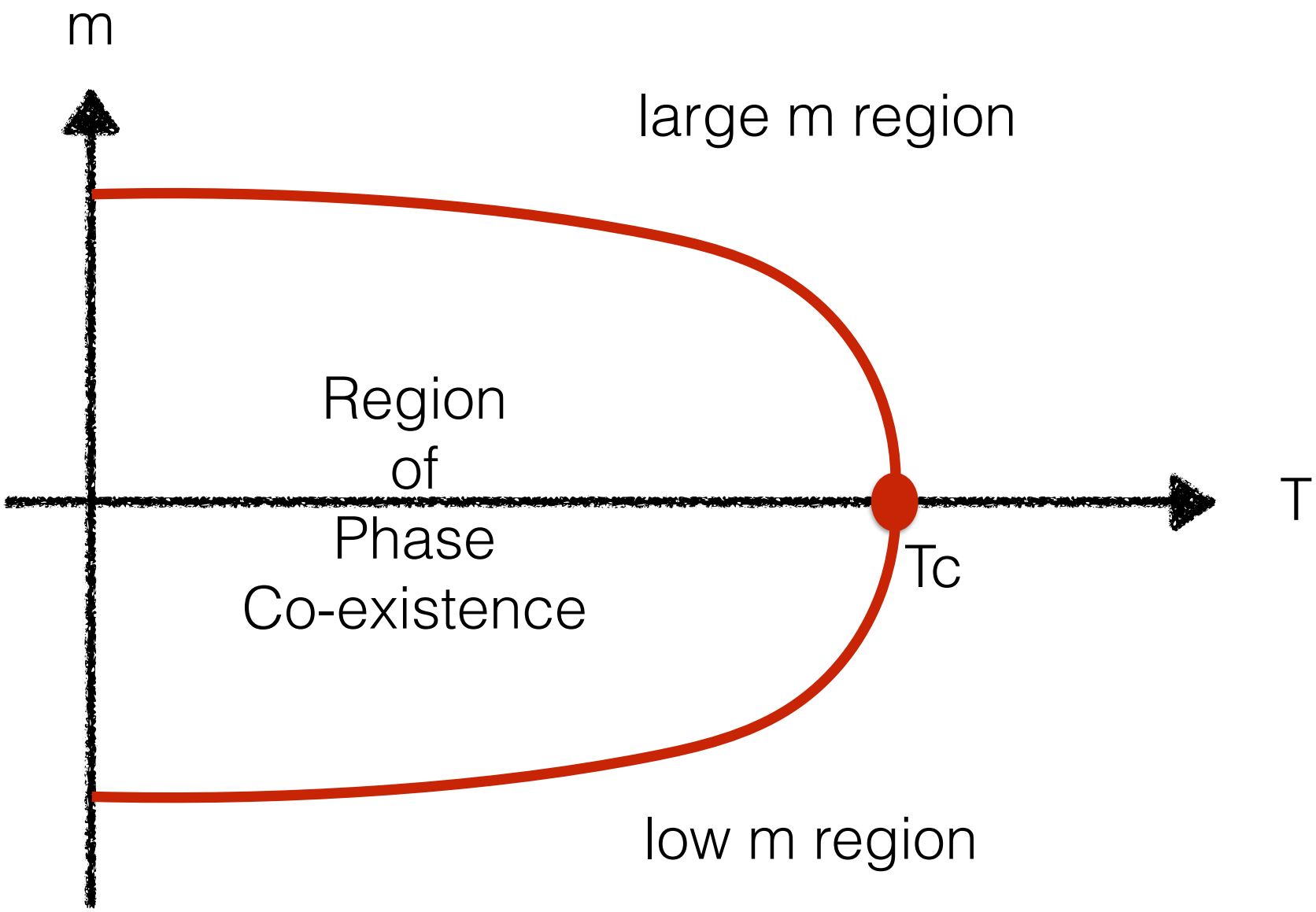
Now let us look to h(m) instead

h(m)



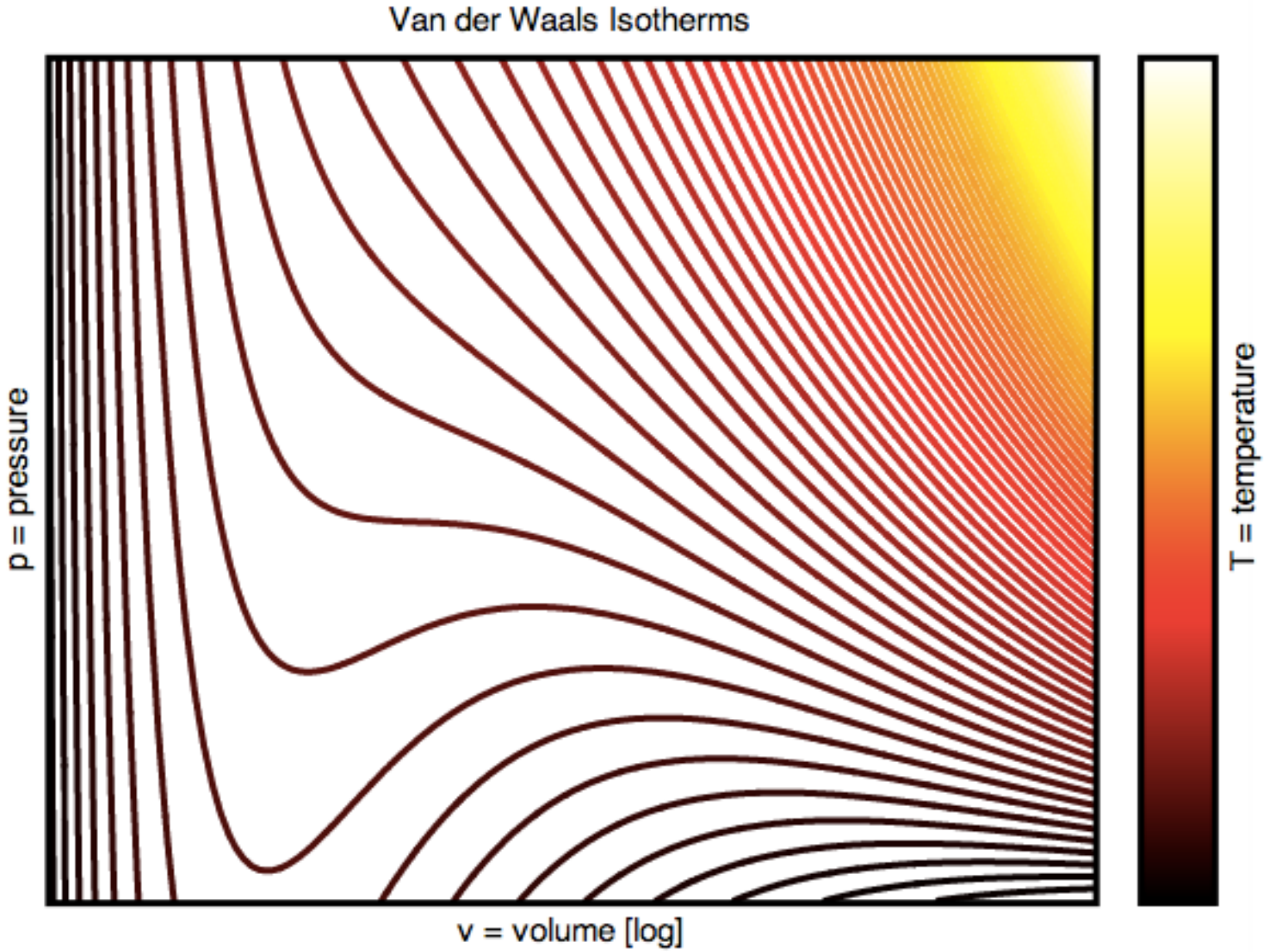
Phase co-existence!

(m-T) Phase Diagram



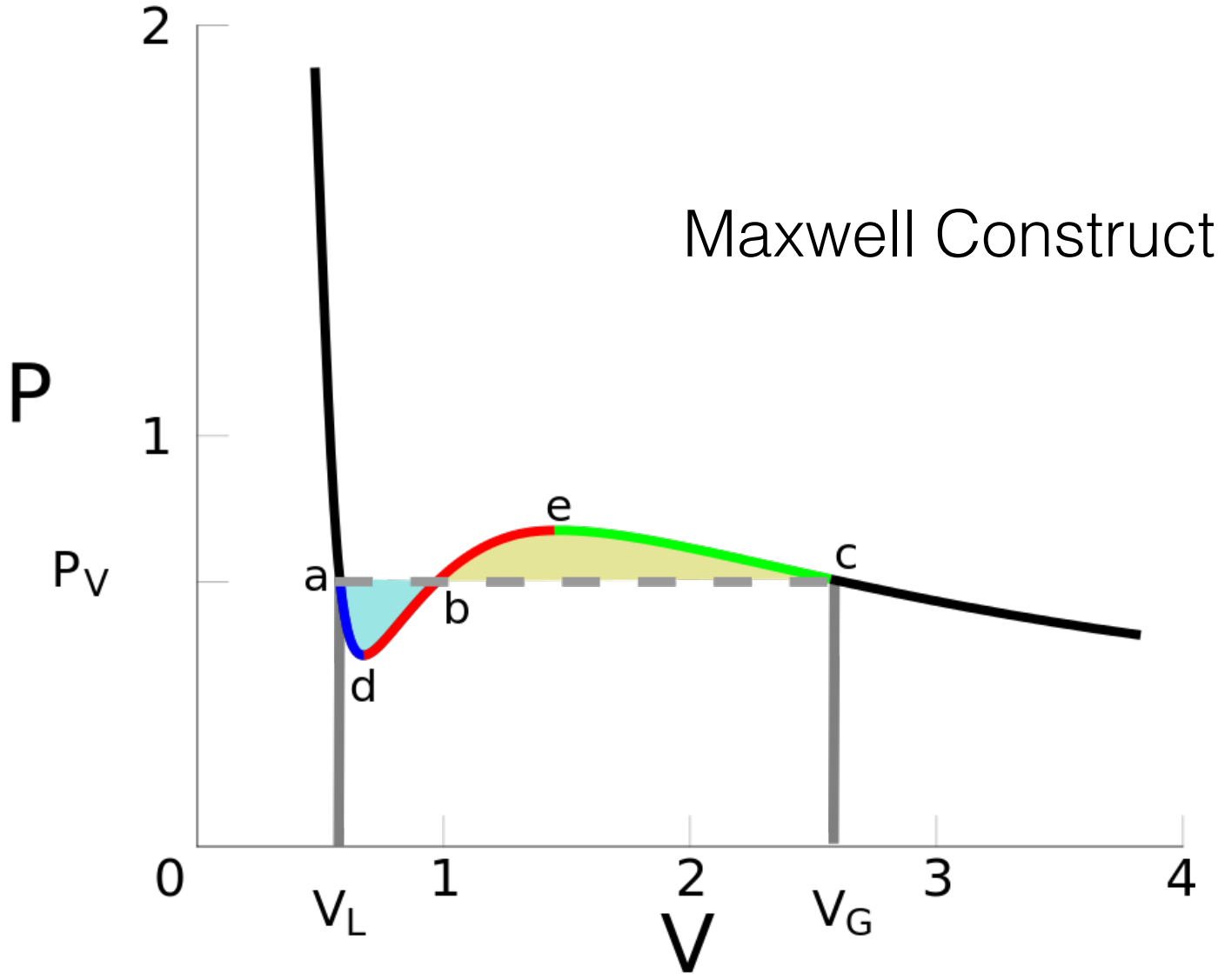
Van der Waals

$$\left(p + \frac{a'}{v^2}\right)(v - b') = kT$$



Van der Waals

$$\left(p + \frac{a'}{v^2}\right)(v - b') = kT$$

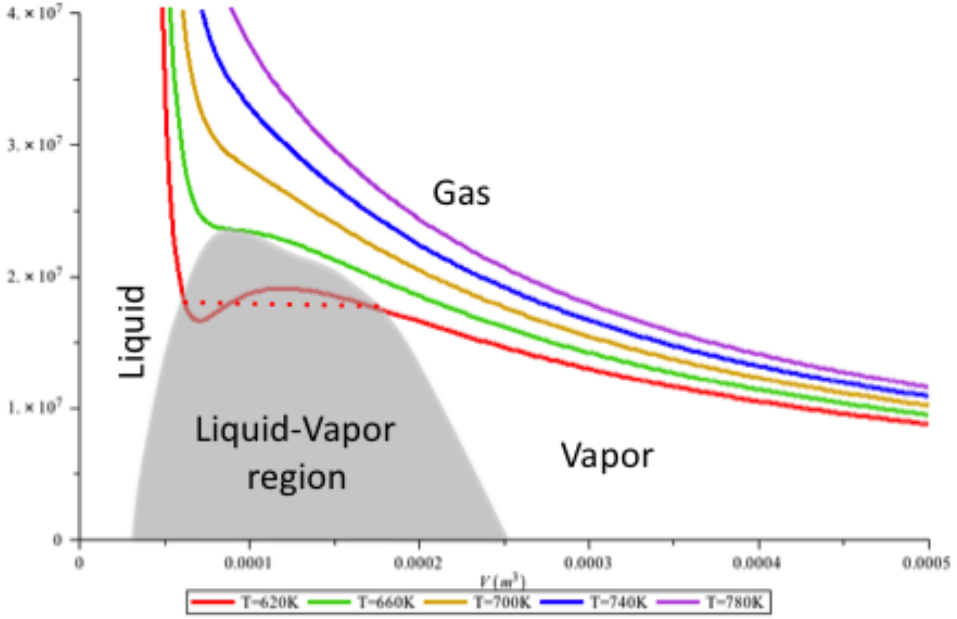


Maxwell Construction again

Van der Waals

$$\left(p + \frac{a'}{v^2}\right)(v - b') = kT$$

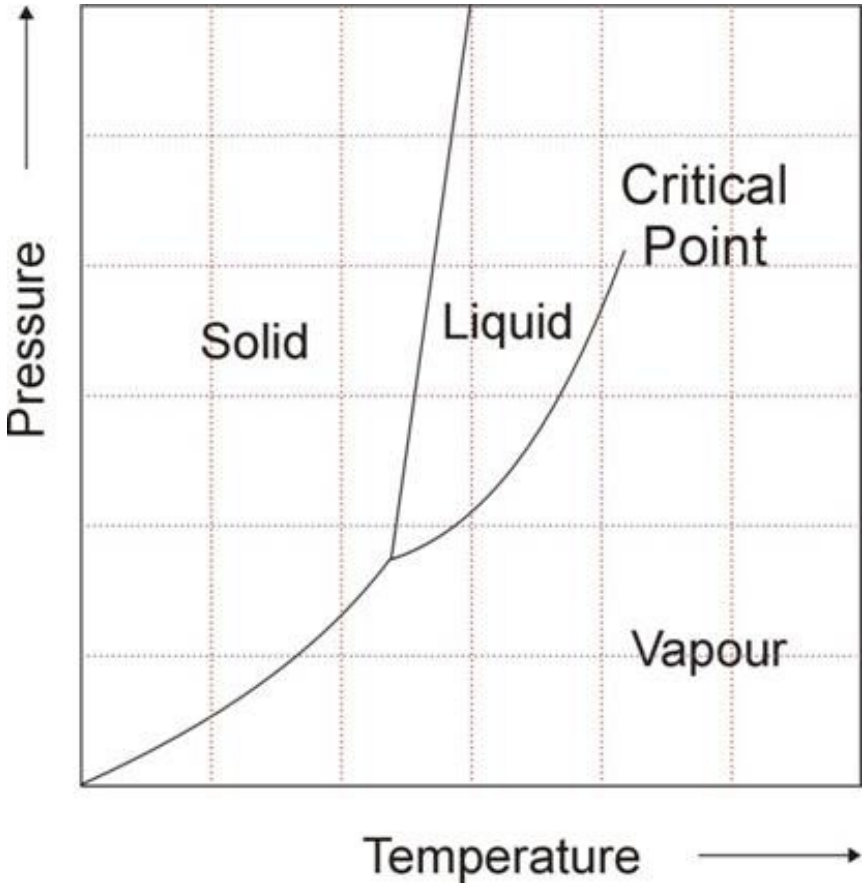
P

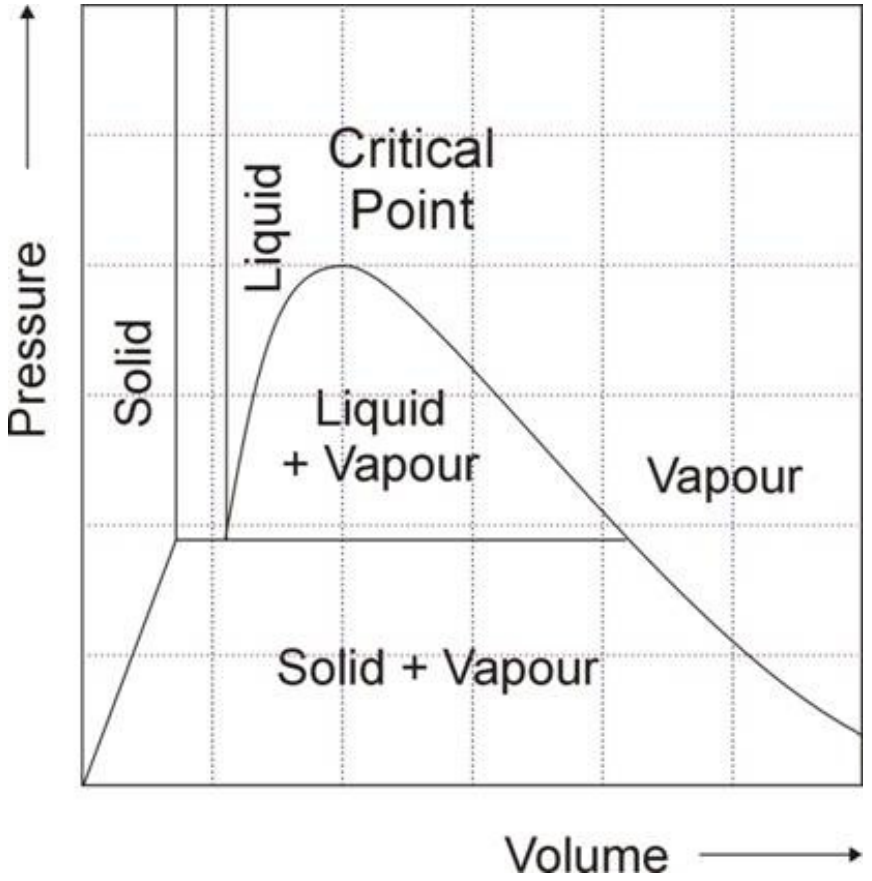


V

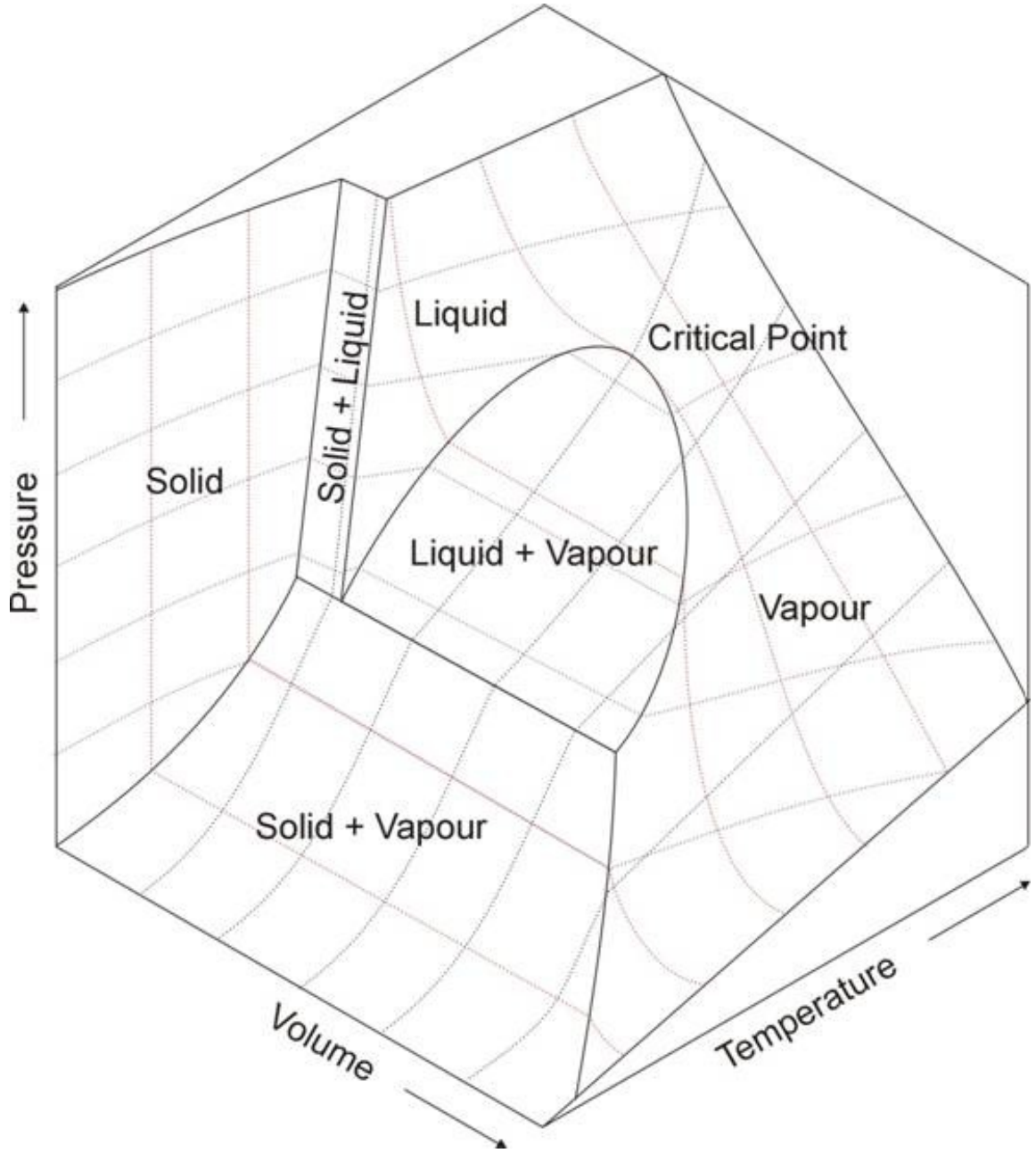
Phase co-existence

A more realistic situation

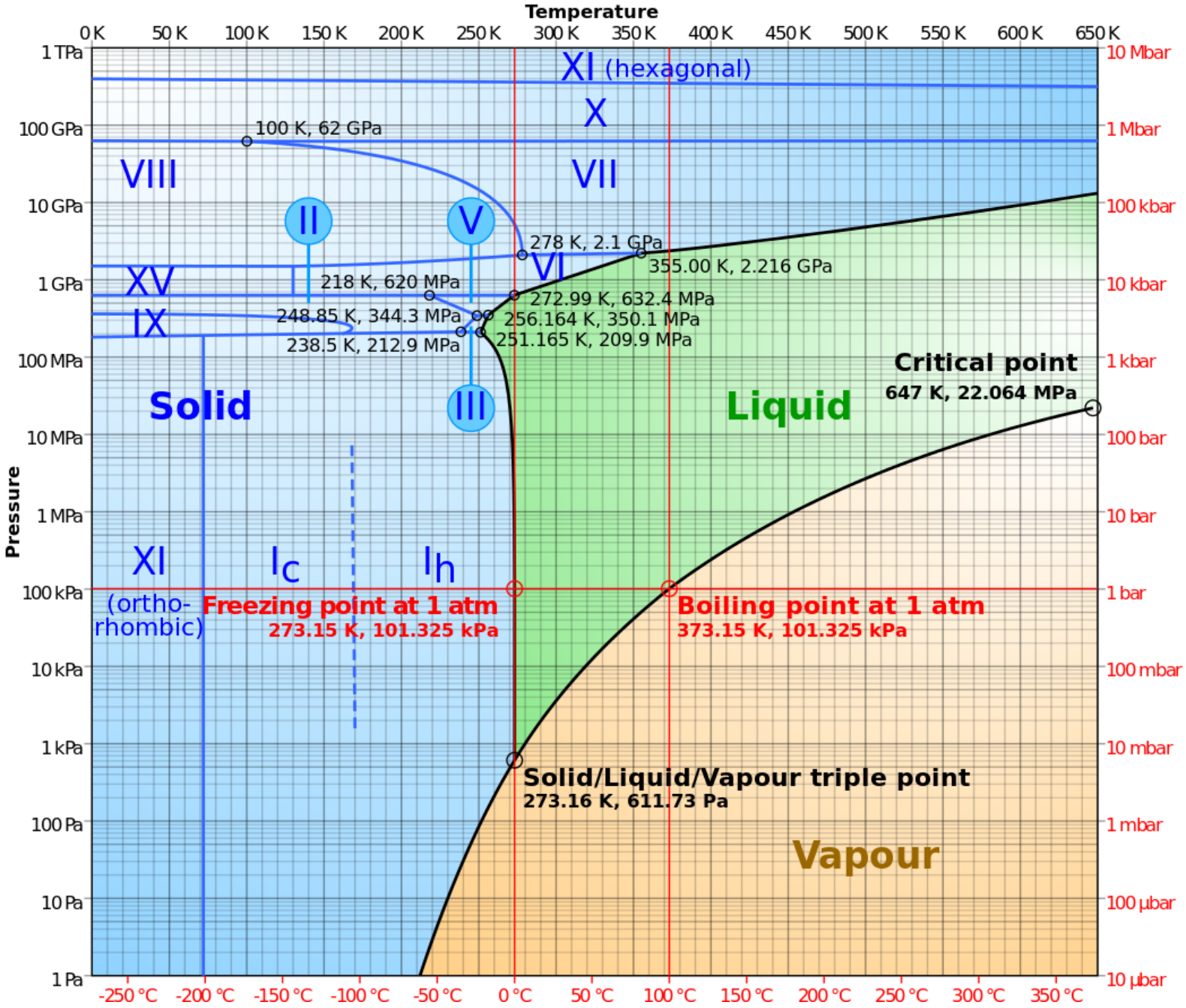




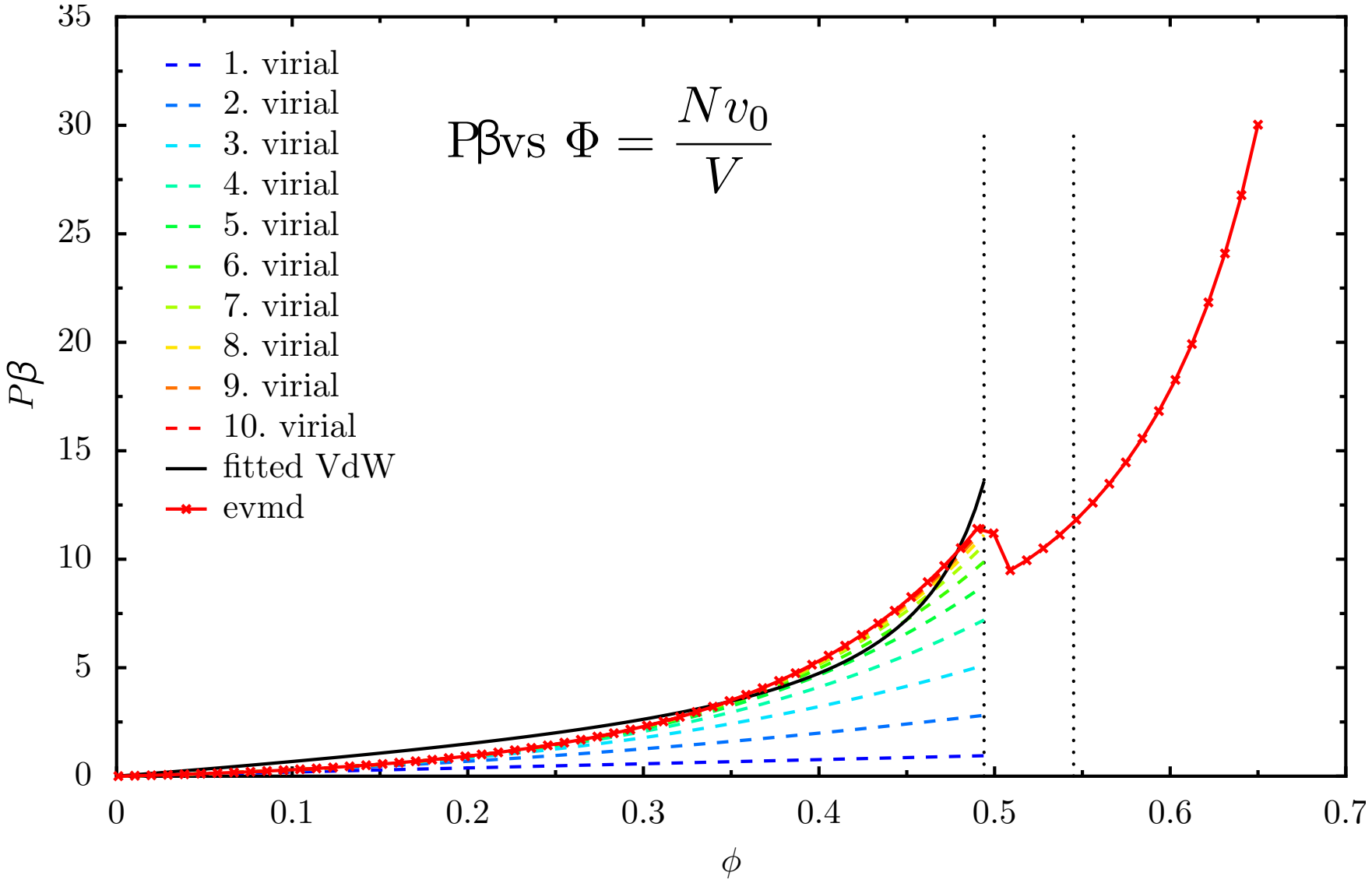
A more realistic situation



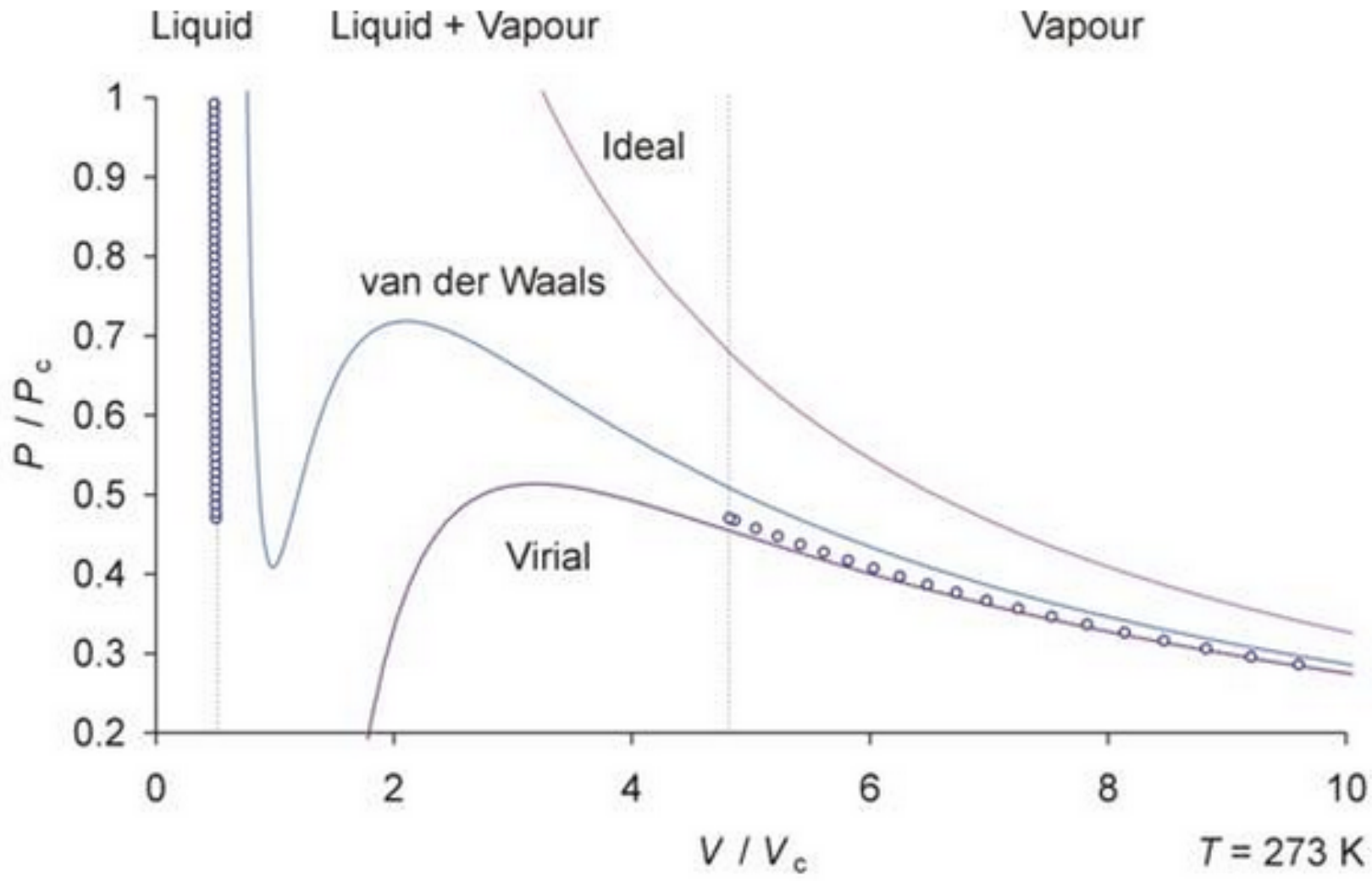
In reality, it is even more complex: WATER

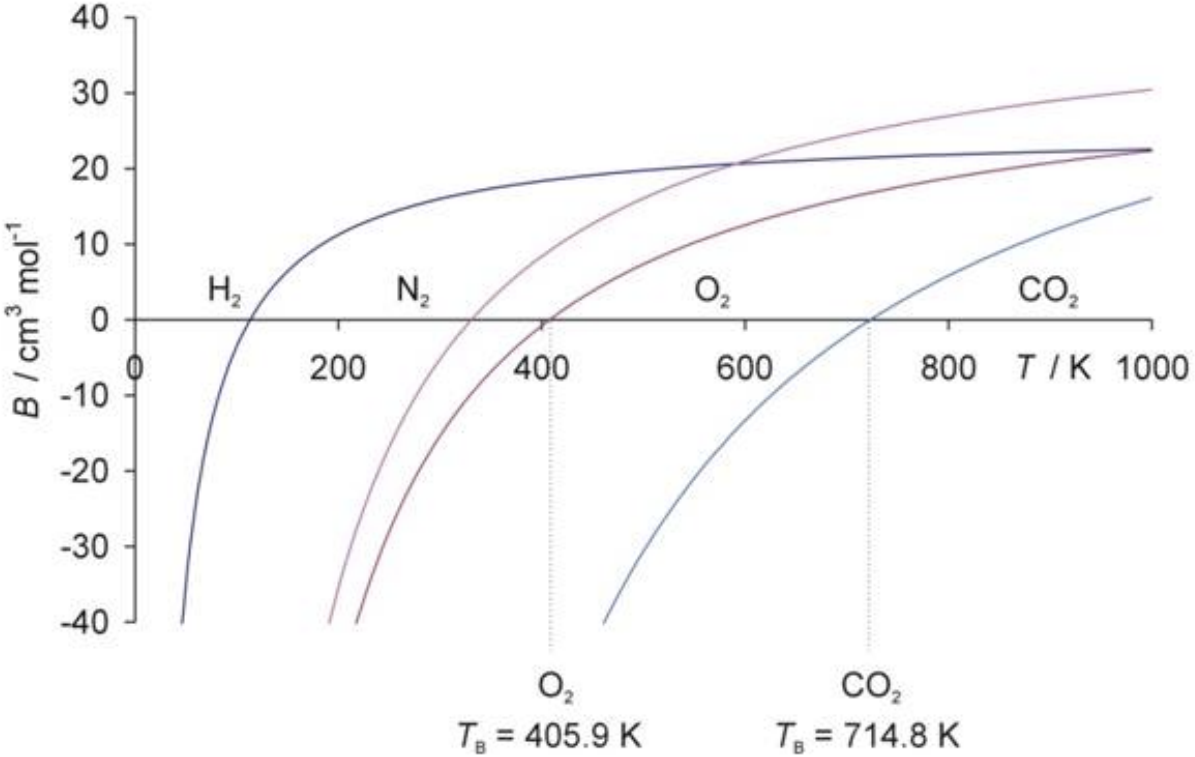


Spheres dures en trois dimensions



CO2 at 273K


Liquid
Liquid + Vapour
Vapour
Ideal
van der Waals
Virial
P / P_c
V / V_c
T = 273 K
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
2
4
6
8
10



Variation of second virial coefficient, *B*, with temperature for various gases, showing the Boyle temperature at which *B* = 0