UMR Gulliver

Effets Collectifs & Matière Molle

Jamming, (8 years of) a granular media experiment

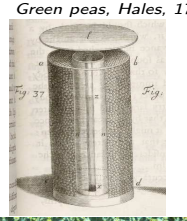
Frédéric Léchenault, Raphael Candelier
Corentin Coulais, Antoine Seguin

Coll.: Giulio Biroli, Jean-Philippe Bouchaud
Ludovic Berthier, Francesco Zamponi

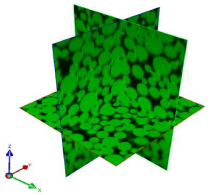
Cargèse 28/08/2014

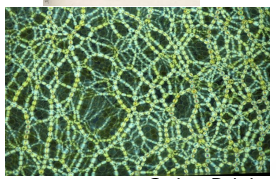
Experimental realizations

Green peas, Hales, 1727

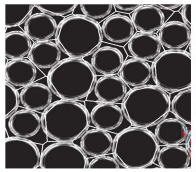


Emulsion, Jorjadze et al., 2011





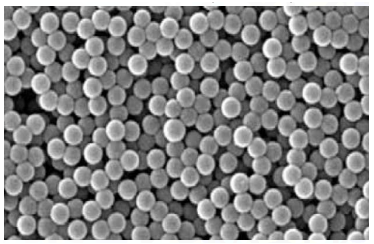
Grains, Behringer



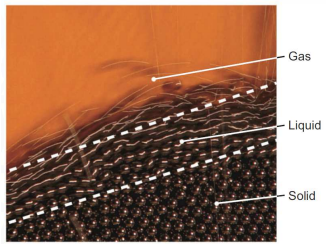
Foam, Katgert et van Hecke, 2010

Gulliver

What about *these* situations?



Colloidal suspensions
=> thermal agitation



Dense granular flows
=> mechanical excitation

Control of Dynamics by Jamming scalings?

Effect of Dynamics on jammed systems?

Gulliver

Initial goal!

Consider a (very) gently vibrated system of grains and study:

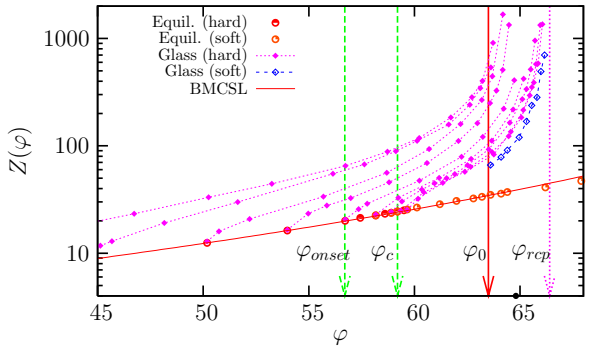
The glass and the jamming transitions

- Spontaneous fluctuations : vibrational dynamics vs. relaxation
- Shear Modulus : linear vs. non-linear regime
- Yield stress : thermal vs. mechanical origin
- Flows : rheology

Gulliver

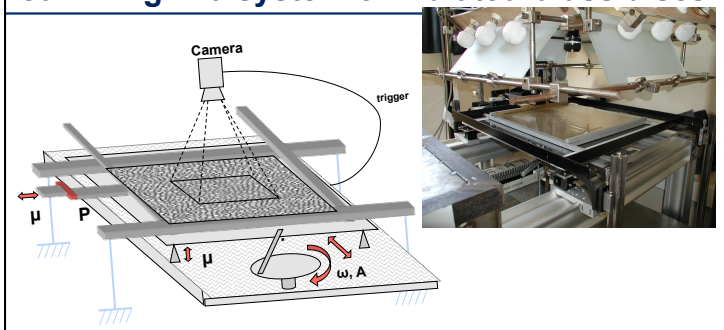
What is the plan?

- Reach jamming, i.e. enter deep into the glass phase



[Chaudhuri *et al.*, PRL '11]

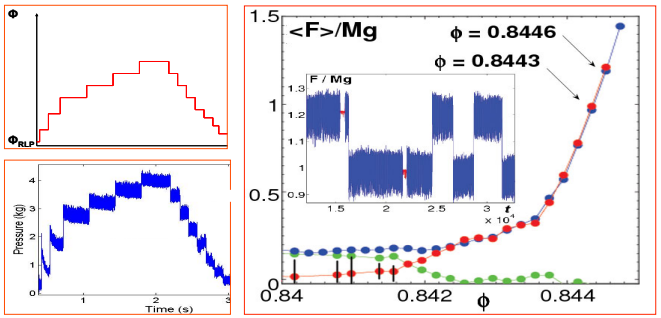
Jamming in a system of vibrated brass discs



- Horizontal vibration (ω=10 Hz, a=1cm)
- Bi-disperse : d_s = 4mm d_l = 5mm
- 8000 brass discs in the system (1500 tracked)
- Vibration-trigged camera
- Tunable volume
- Pressure measured on the side

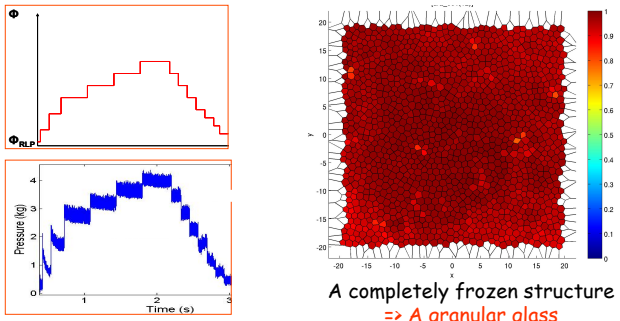
Experimental protocol

- Increase packing fraction stepwise:
  - Allow for the slow relaxation of pressure
- Then decrease packing fraction and record dynamics



Experimental protocol

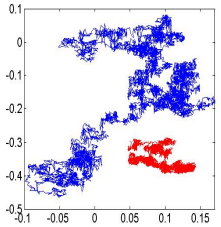
- Increase packing fraction stepwise:
  - Allow for the slow relaxation of pressure
- Then decrease packing fraction and record dynamics

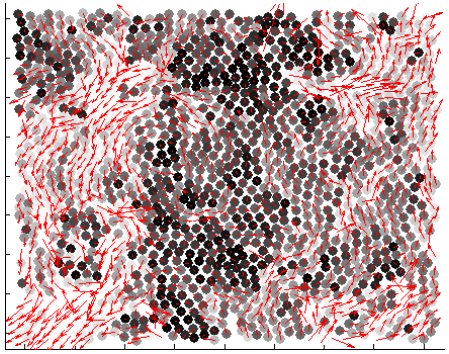


A completely frozen structure
=> A granular glass

Dynamics: Heterogeneous tiny displacements

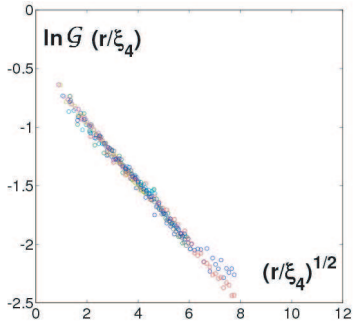
- Particles trajectories : $\vec{r}_i(t)$
- Displacement : $\Delta\vec{r}_i(t,\tau) \equiv \vec{r}_i(t+\tau) - \vec{r}_i(t)$

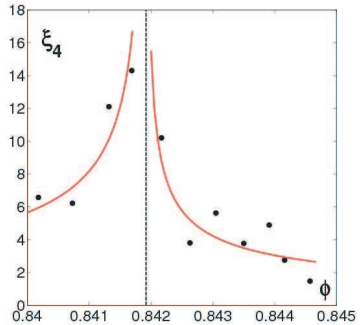




Gulliver

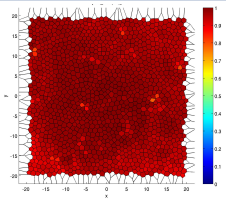
Dynamical heterogeneities

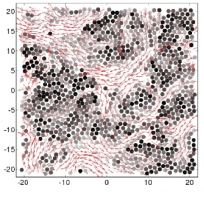




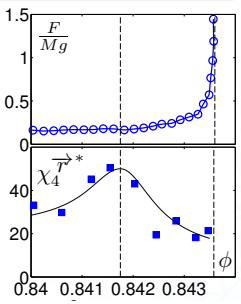
Gulliver

Altogether…





a frozen structure WITH dynamical heterogeneities

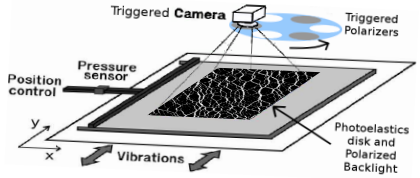


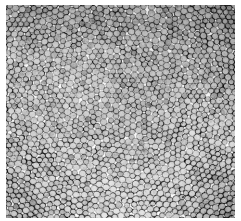
Albeit of a very different kind :
- no cage jumps
- no change of neighbours
- $a^* = 5.10^{-3}$ d
- large correlation length

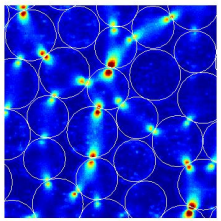
- What is the mechanism responsible for such heterogeneities?
- Why is there a maximum and not just a divergence, with the pressure?

Gulliver

Redo the experiment with soft photoelastic discs => access to contacts



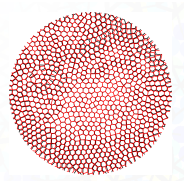


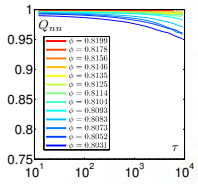


Gulliver

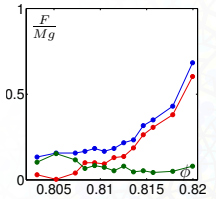
Same protocole: again a granular glass

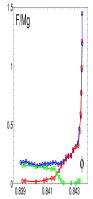
- A frozen structure





- But this time a glass of soft discs



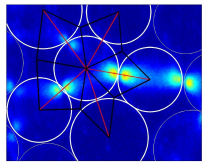


Gulliver

EC2M Effets Collectifs & Matière Molle 13

Signature of jamming within contacts

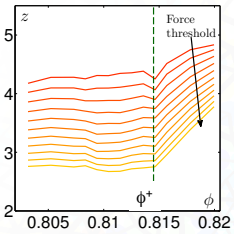
Interparticle force measurement

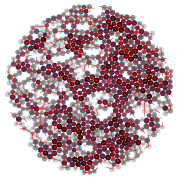


thresholding gap $< \epsilon$

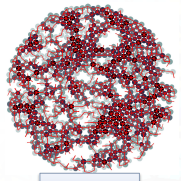
$\rightarrow$

thresholding force $> f_0$

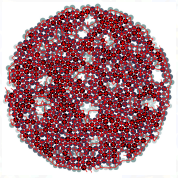




$\phi < \phi^+$



$\phi^+ = 0.814$



$\phi > \phi^+$

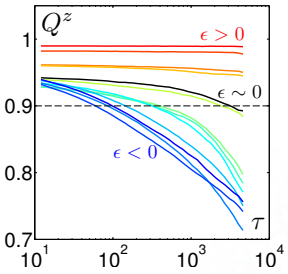
Gulliver

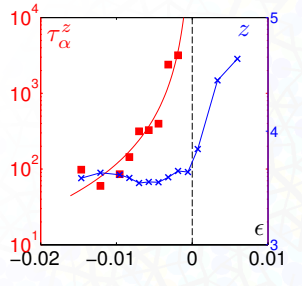
EC2M Effets Collectifs & Matière Molle 14

Dynamics of the contact network…

$$Q^z(t,\tau) = \frac{1}{N}\sum_i Q_i^z(t,\tau) \quad \text{where } Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases}$$

$$Q_z(\tau) = \langle Q^z(t,\tau)\rangle_t$$





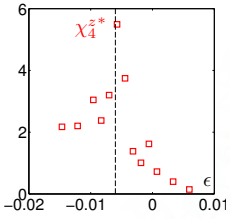
Dynamical arrest of the contact dynamics…

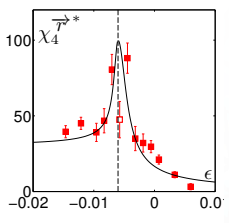
Gulliver

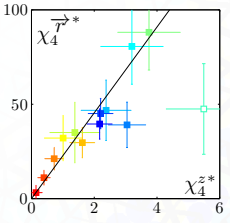
EC2M Effets Collectifs & Matière Molle 15

…is heterogeneous and governs the grains motion

$$\chi_4^{z,r}(\tau) \equiv NVar\left(\left\langle Q_i^{z,r}\right\rangle_i\right)_t \quad \begin{cases} Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases} \\ Q_i^r(t,\tau) \equiv \exp\left(-\frac{\Delta r_i^2}{2\langle \Delta r_i^2\rangle}\right) \end{cases}$$



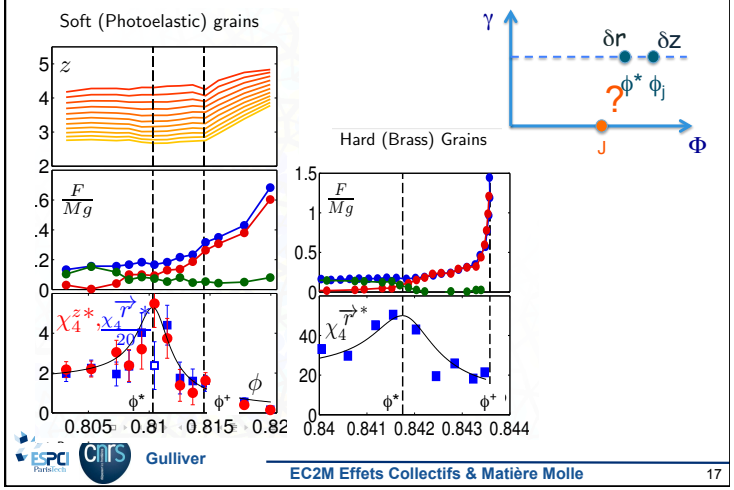




Gulliver

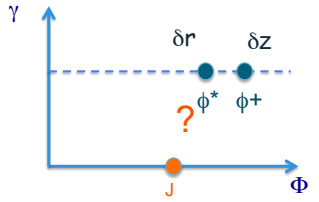
EC2M Effets Collectifs & Matière Molle 16

Summary: two distinct signatures



Gulliver

Reducing the vibration



Three sets of experiments

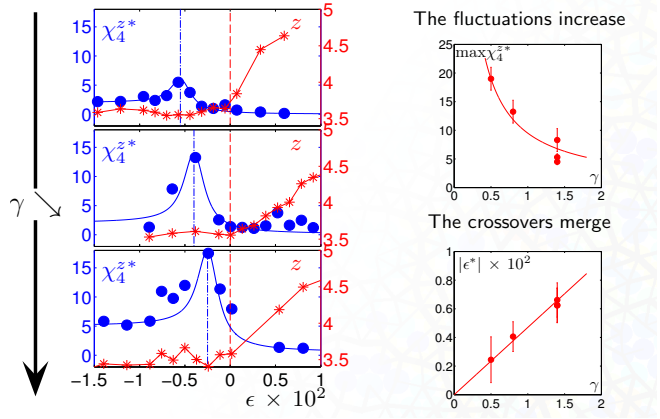
$f = 6.25, 7.50, 10.00$ Hz

If $f < f_0 = 4.17$ Hz, no motion.

$$\gamma = \frac{f - f_0}{f_0}$$

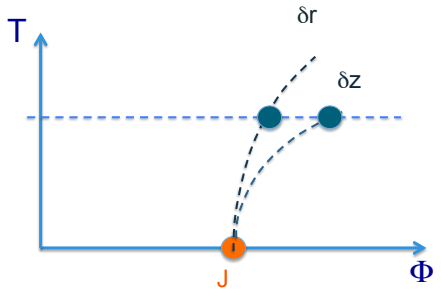
Gulliver

Decreasing the vibration



Gulliver

Hence two crossover lines



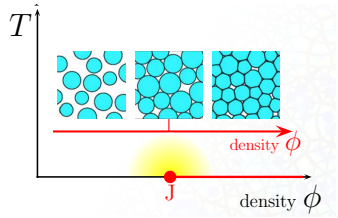
How far from the critical point ?

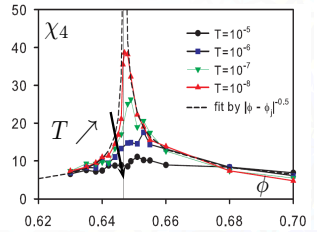
Gulliver

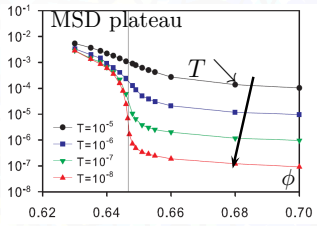
Comparison with thermal soft spheres…

Simulation of thermal soft-spheres

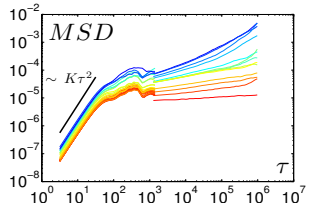
Ikeda et al, 2012

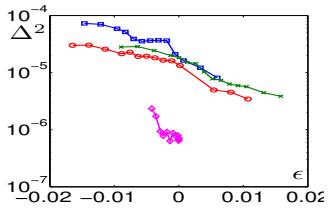


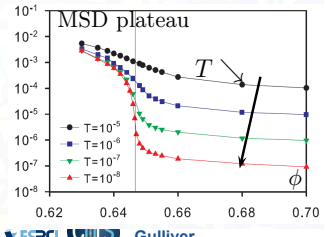


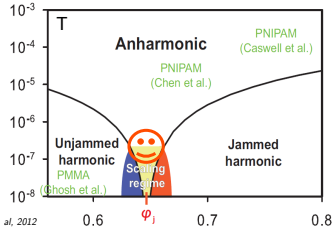


Comparison with soft spheres







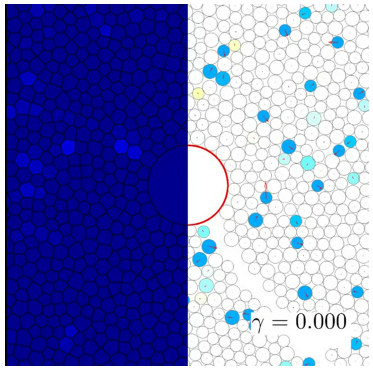


Probing elasticity

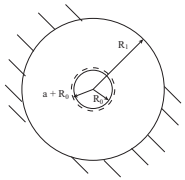
- Prepare the system at large packing fraction under vibration
- Inflate an intruder in the center (the vibration is stopped)
- Decrease the packing fraction while vibrating
- iterate

R0 - > R0 + a

γ = a/R0



Probing elasticity : the linear elastic framework



$$div(\underline{\underline{\sigma}}) = 0$$

$$\underline{\underline{\sigma}} = \frac{1}{2} Tr(\underline{\underline{\sigma}})\underline{\underline{1}} + \underline{\underline{\tau}}$$

$$\underline{\underline{\sigma}} = KTr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + 2G\underline{\underline{\gamma}}$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left[\underline{\underline{\nabla U}} + {}^{t}\underline{\underline{\nabla U}}\right] = \frac{1}{2} Tr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + \underline{\underline{\gamma}}$$

$$U(R_0) = a \qquad U(R_1) = 0$$

$$\delta = Tr(\underline{\underline{\varepsilon}}) = -2\frac{a}{R_0} A$$

$$\gamma = J_2(\underline{\underline{\gamma}}) = \sqrt{\frac{1}{2}\underline{\underline{\gamma}} \circ \underline{\underline{\gamma}}} = \frac{a}{R_0} B \left(\frac{R_0}{r}\right)^2$$

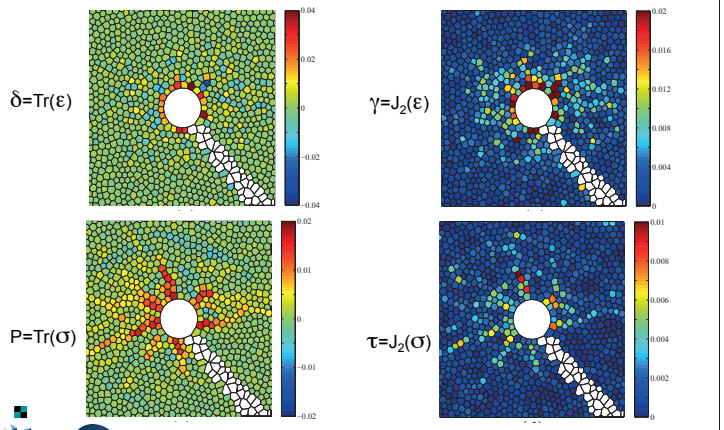
$$P = Tr(\underline{\underline{\sigma}}) = KTr(\underline{\underline{\varepsilon}})$$

$$\tau = J_2(\underline{\underline{\tau}}) = \sqrt{\frac{1}{2}\underline{\underline{\tau}} \circ \underline{\underline{\tau}}} = 2GJ_2(\underline{\underline{\gamma}})$$

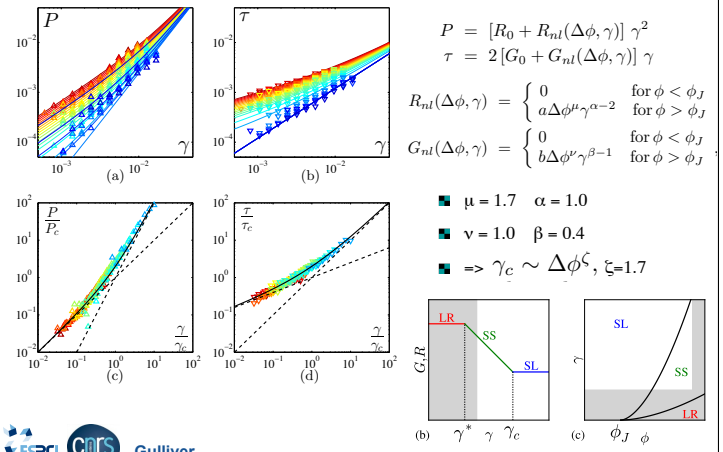
$$A = R_0^2 \Big/ \left(R_1^2 - R_0^2\right) ; B = R_1^2 \Big/ \left(R_1^2 - R_0^2\right)$$

- Nota Bene
  - In the limit of large R_1, A->0, B->1 : this is a shear test!
  - G and K are simply obtained by the ratio of the stress and strain tensor invariants

For each packing fraction and each a/R_0



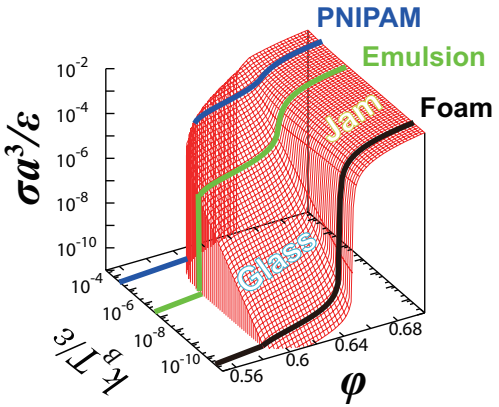
Parametric plot of stress vs strain



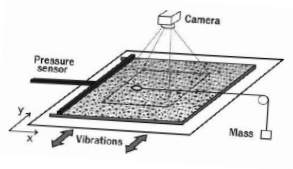
$$P = [R_0 + R_{nl}(\Delta\phi,\gamma)]\,\gamma^2$$
$$\tau = 2\,[G_0 + G_{nl}(\Delta\phi,\gamma)]\,\gamma$$
$$R_{nl}(\Delta\phi,\gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ a\Delta\phi^{\mu}\gamma^{\alpha-2} & \text{for } \phi > \phi_J \end{cases}$$
$$G_{nl}(\Delta\phi,\gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ b\Delta\phi^{\nu}\gamma^{\beta-1} & \text{for } \phi > \phi_J \end{cases}$$

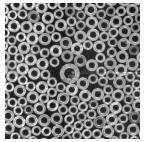
- $\mu = 1.7 \quad \alpha = 1.0$
- $\nu = 1.0 \quad \beta = 0.4$
- => $\gamma_c \sim \Delta\phi^{\zeta}$, ζ=1.7

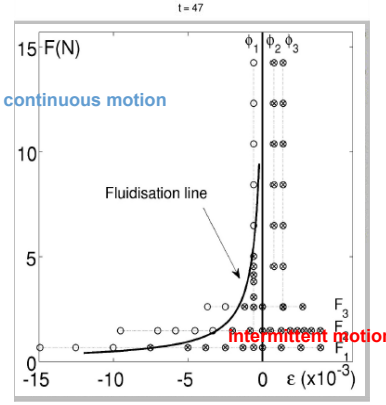
Yielding close to jamming …



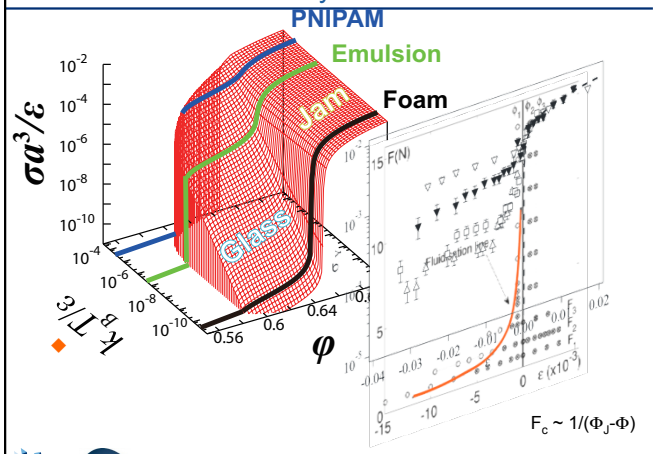
Yielding close to jamming : the motion of an intruder …





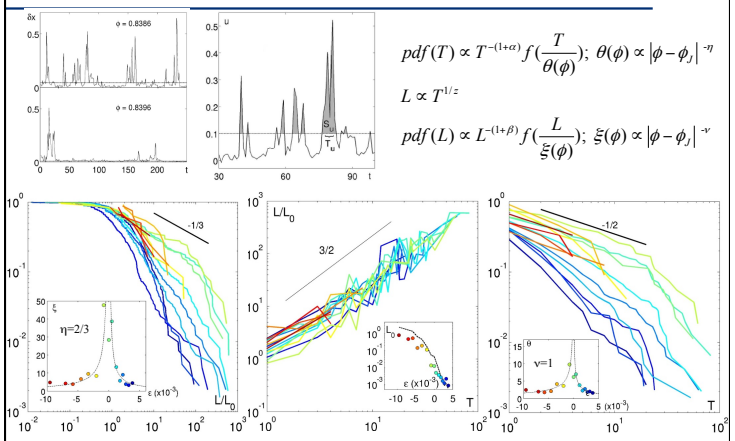


Critical force : “thermal” yield stress



Gulliver

Pinning-depinning like dynamics => Crackling noise signals



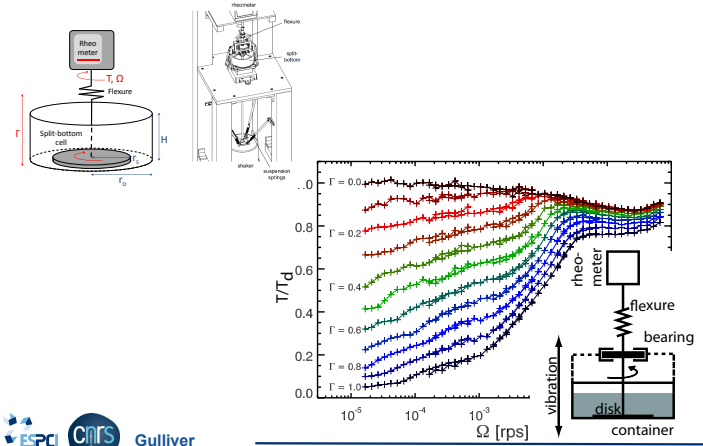
$$pdf(T) \propto T^{-(1+\alpha)} f(\frac{T}{\theta(\phi)});\ \theta(\phi) \propto |\phi - \phi_J|^{-\eta}$$

$$L \propto T^{1/z}$$

$$pdf(L) \propto L^{-(1+\beta)} f(\frac{L}{\xi(\phi)});\ \xi(\phi) \propto |\phi - \phi_J|^{-\nu}$$

Gulliver

Flows : vibro-rheology (with M. van Hecke)



Gulliver

Conclusion

- Vibrated granular media are suitable tools for probing the vicinity of jamming, (in particular low enough T_eff)
- Two distinct crossovers (one dynamical, one structural) converge toward J-point in the limit of low vibration
- Inflating an intruder in soft photo-elastic discs => Non linear rheology
- Pulling an intruder in vibrated hard discs => the yield stress of “thermal origin” and reveals complex pinning – depinning like dynamics
- Vibro-rheology : flow curves close to jamming
- Thank you!

Further readings :

- **Europhysics Letters, 83, 46003, (2008).**
- **Soft Matter, 6 (13), 3059–3064, (2010).**
- **Phys Rev Lett 103 12800 (2009).**
- **Europhysics Letters, 100, 44005 (2012).**
- **Soft Matter 10 (10), 1519-1536 (2014).**
- **Phys Rev Lett (2014) to appear**

Gulliver