Universal dynamics in Many-body localized states and the many-body localization transition

Ehud Altman – Weizmann Institute of Science

Collaborators:

Ronen Vosk (Weizmann), David Huse (Princeton),

A. Polkovnikov (BU), G. Refael (Caltech)

Y. Bahri, A. Vishwanath (Berkeley), E. Demler (Harvard),
V. Oganesyan (CUNY), D. Pekker (Caltech)

Minerva foundation

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Dynamics of closed quantum systems

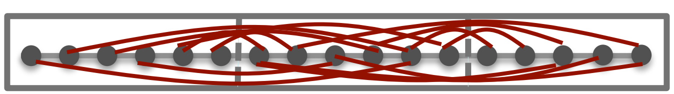
Thermalization

Quantum information stored in local objects is rapidly lost

↓

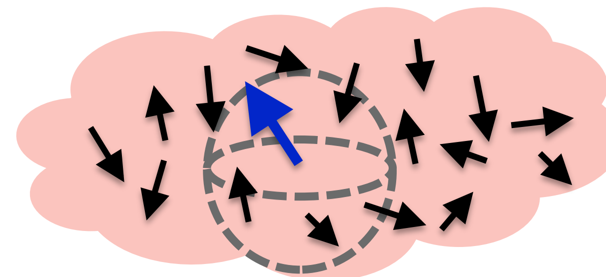
Classical hydro description of remaining slow modes (e.g. diffusion)

Thermal eigenstates (highly entangled):



$S_A \sim L^d$

Many-body localization

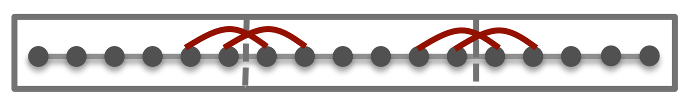


Local quantum information persists indefinitely

↓

Need quantum description of long time dynamics.

Ground-state-like high energy eigenstates (low entanglement):



$S_A \sim L^{d-1}$

?

Outline

- Thermalization in closed quantum systems
  Eigenstate thermalization hypothesis and its breaking

- What we understand about MBL dynamics
  RG, distinct phases, dynamical critical points.

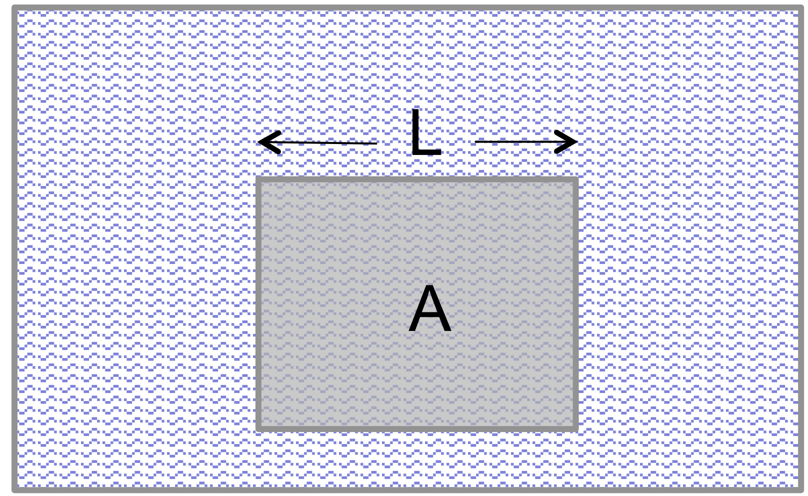
- The many-body localization phase transition
  RG approach: transport, entanglement scaling and a surprise!

Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

In a high energy eigenstate:

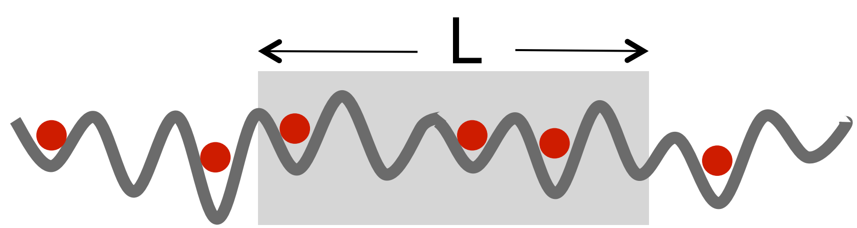
$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$



Extensive Von-Neuman entropy:

$$S_A \propto L^d$$

Example where ETH fails:
Anderson localization



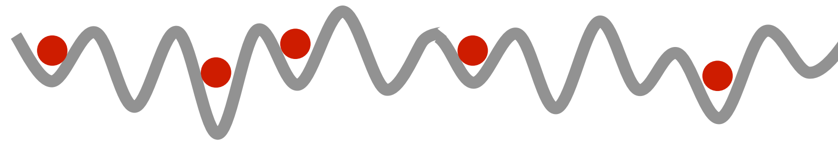
"Area law" entropy as in ground state also holds in high energy eigenstates

$$S_A \propto L^{d-1}$$

MBL = stability of the area-law to adding interactions

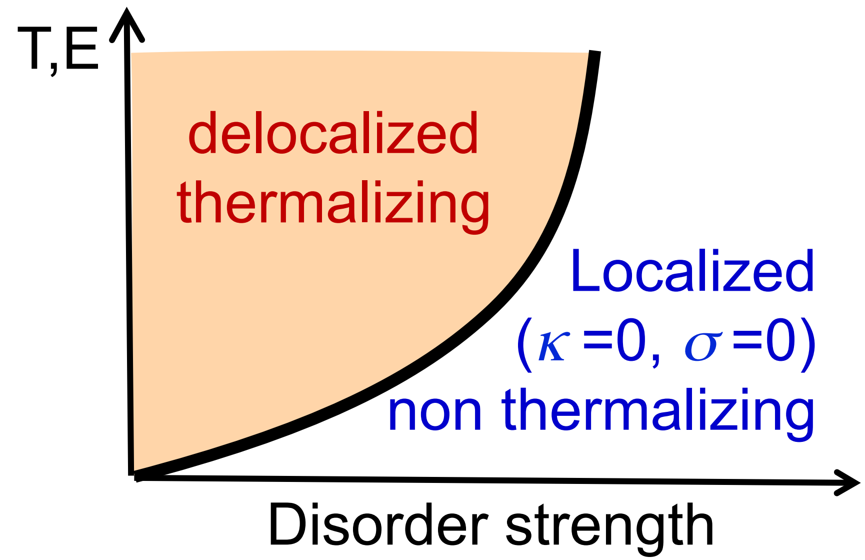
Generic exception to ETH: Many body localization

Anderson localization of
non interacting particles:



Perturbative stability to interactions
(Basko, Aleiner, Altshuler 2005)

Delocalization transition at a critical energy density, disorder or interaction strength.



Stability of MBL supported by other approaches:

Numerics – Oganesyan & Huse 2010, Pal & Huse, Bardarson et. al 2012 …

RG – Vosk an EA 2012, Vosk and EA 2013, Pekker et. al. 2013 .

Mathematical proof – Imbrie 2014

A lot of insight into the nature of the MBL phase

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  RG, local integrals of motion, slow entanglement.

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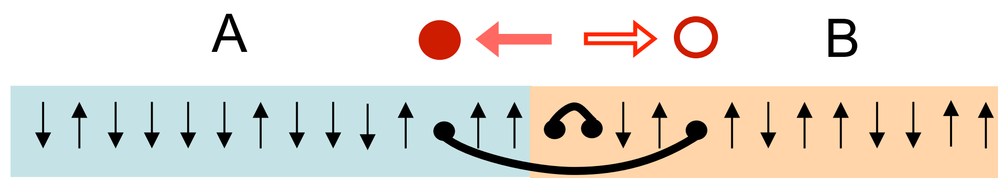
Ultra slow growth of the entanglement entropy

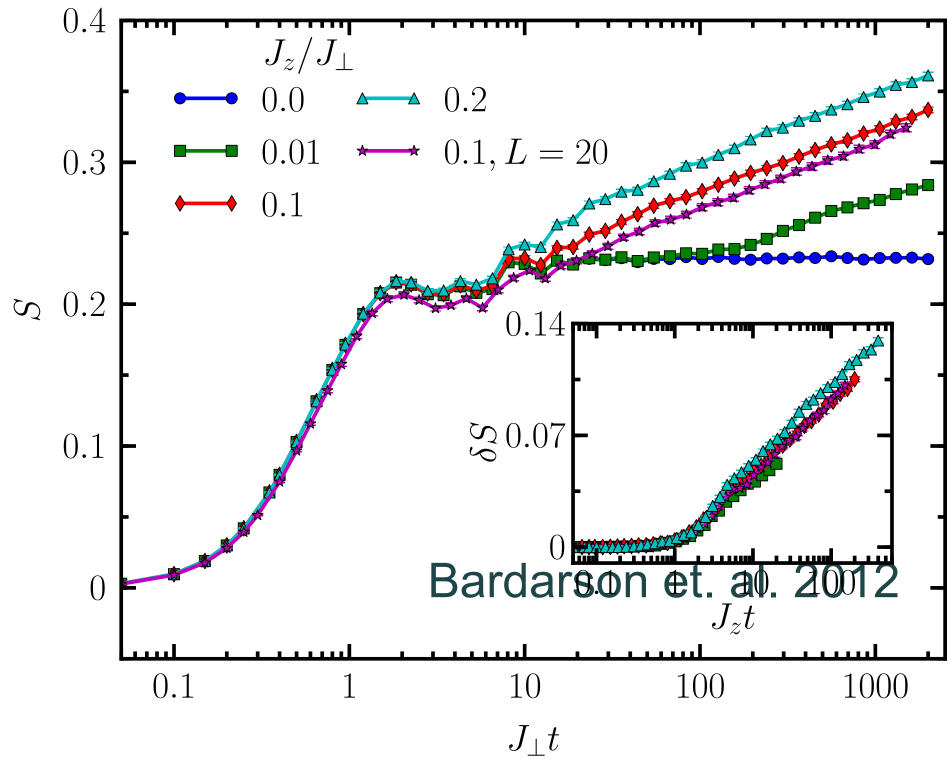
Zindaric et. al. 2008; Bardarson, Pollmann & Moore. 2012

$$H_0 = J_\perp \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \sum_i h_i S_i^z \qquad H_{\text{int}} = J_z \sum_i S_i^z S_{i+1}^z$$

$$e^{-iHt} \,|\, \Psi_0 \,\rangle$$



$$S_A(t) \sim \log t$$



Bardarson et. al. 2012

RG Solution of time evolution

R. Vosk and EA, PRL (2013); R. Vosk and EA, arXiv:1307.3256

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] \qquad e^{-iHt} \,|\, \Psi_0 \,\rangle$$

$$|\Psi_0\rangle = \downarrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow$$

H_{fast}

Pick out largest couplings $\quad \Omega = \max\left(J_i^z, h_i\right)$

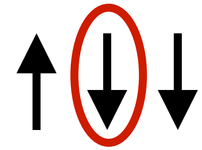
Short times ($t \approx 1/\Omega$): System evolves according to H_{fast}
Other spins essentially frozen on this timescale.

Longer times ($t >> 1/\Omega$): Eliminate fast modes (order Ω) perturbatively to obtain effective evolution for longer timescales.

Related RSRG-X: Pekker, Refael, EA, Demler & Oganesyan arXiv:1307.3253

Outcome of RG: integrals of motion = (frozen spins)

Example: strong transverse field h_i



$$H = \sum_i \left[J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + V_i \sigma_i^x \sigma_{i+1}^x \right]$$

$$H_{\text{eff}} = e^{-iS} H e^{iS}$$

$$H_{\text{eff}} = h_i \tilde{\sigma}_i^x + V_L \tilde{\sigma}_i^x \sigma_L^x + V_R \tilde{\sigma}_i^x \sigma_R^x + \frac{J_L J_R}{h_i} \sigma_L^z \tilde{\sigma}_i^x \sigma_R^z + \ldots$$

The transformation generated a quasi-local integral of motion

$$\tilde{\sigma}_i^x = Z \sigma_i^x + \text{exponential tail}$$

Fixed point Hamiltonian

$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \dots$$

$$V_{ij} \sim V e^{-|x_i - x_j|/\xi}$$

Note the analogy with Fermi-liquid theory!

Independently of the RG, the fixed-point theory may serve as a useful phenomenological description of the phase.

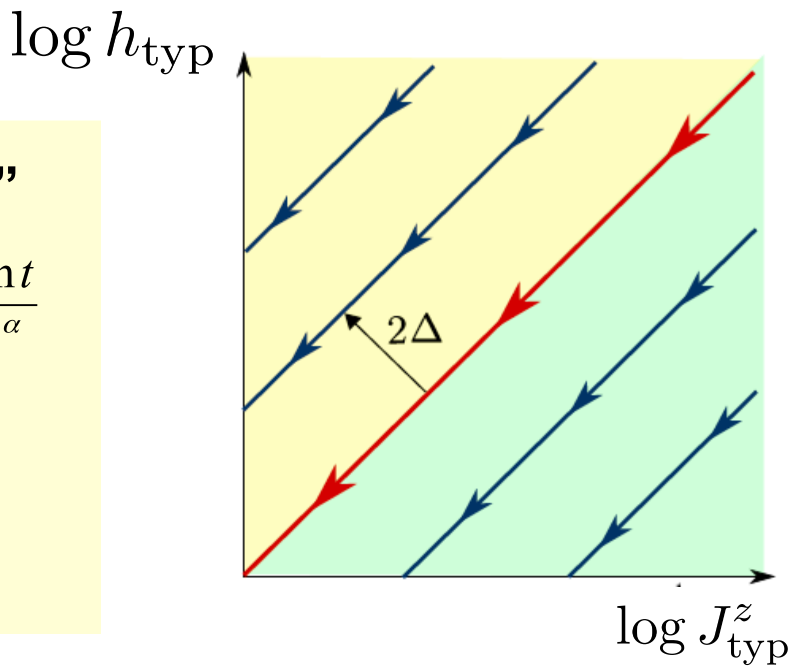
Oganesyan & Huse (2013); Serbyn, Papic & Abanin (2013)

Phase transitions between distinct localized phases:

Paramagnetic eigenstates $\quad \tilde{\sigma}_i^x \longleftrightarrow \tilde{\sigma}_i^z \quad$ Broken symmetry in eigenstates ("Eigenstate glass")

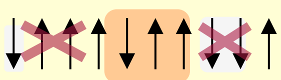
Huse et. al. 2013; Vosk and EA 2013; Pecker et. al. 2013;

Result from the RG flow



"Paramagnet"

$$\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} \frac{\ln t}{t^{\alpha}}$$



Critical point:

$$\langle \sigma_i^z(t) \rangle \sim \frac{1}{\ln^{2-\Phi} t}$$

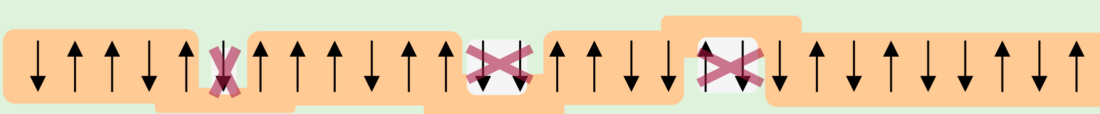
$\phi = (1+\sqrt{5})/2 \approx 1.618$

(golden ratio)

$$S_A(t) \sim \log^{2/\phi} t$$

$$S_A(t) \sim \log t$$

"glass": $\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} const \sim \Delta^{2-\Phi}$



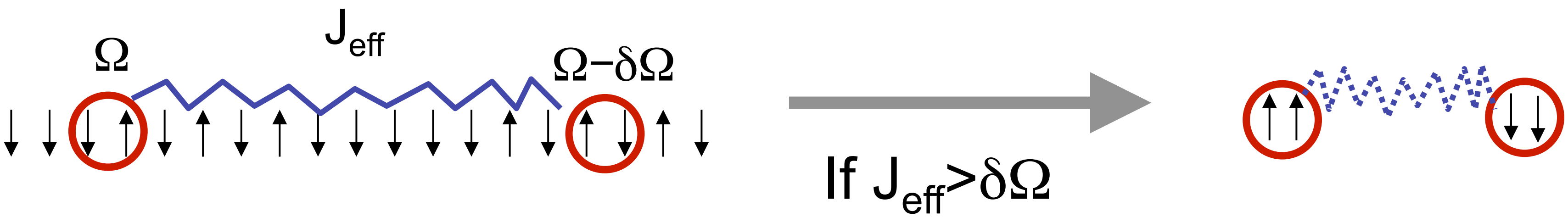
σ_i^z is an emergent integral of motion in the glass

Glass order parameter! $\overline{\langle \sigma^z(\infty) \rangle^2}$

Dynamical transition between distinct localized states

Limitation of the RG scheme: resonances

Resonances between decimated sites can generate a slow mode that is not accounted for by the RG



Resonances do not proliferate in MBL phase! (Irrelevant in RG sense).

(Vosk and EA 2013)

This RG scheme is limited to the MBL phase!

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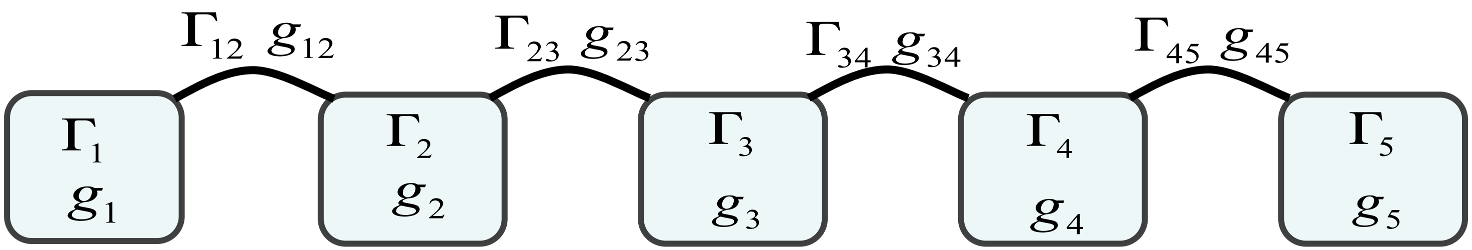
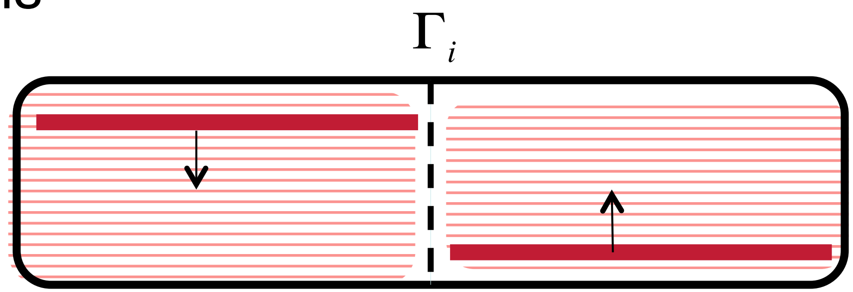
Coarse Grained Model of coupled blocks



Block = chain of l microscopic spins



The block parameters:

Δ_i Single block level spacing

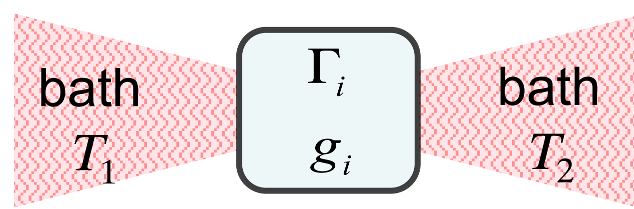
Γ_i Relaxation rate of intra block product states

$\tau = \Gamma^{-1}$ time for entangling the two sides

$g_i = \Gamma_i / \Delta_i$ Number of coupled levels

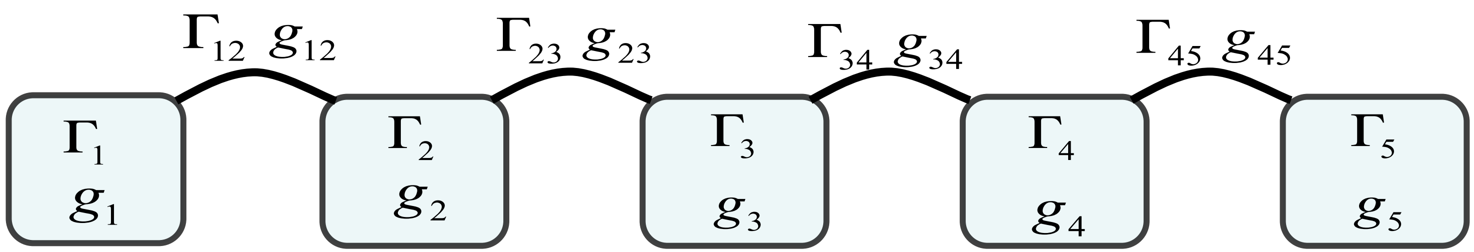
$g_i \ll 1$ "insulating block"

$g_i \gg 1$ "thermalizing block"

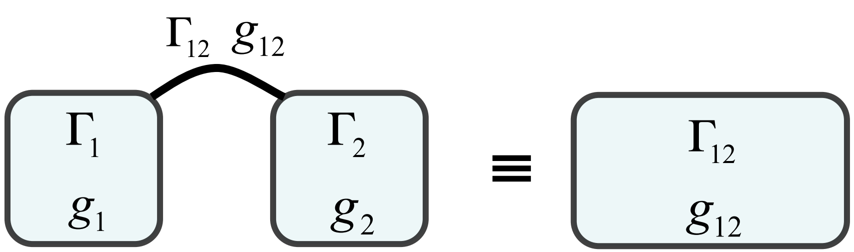


Relation to thermal conductance: $\Gamma = G$ $\tau_{tr} = l\tau$

Coarse Grained Model of coupled blocks



<u>Link parameters:</u> Parameters of new block if blocks 1 and 2 were joined

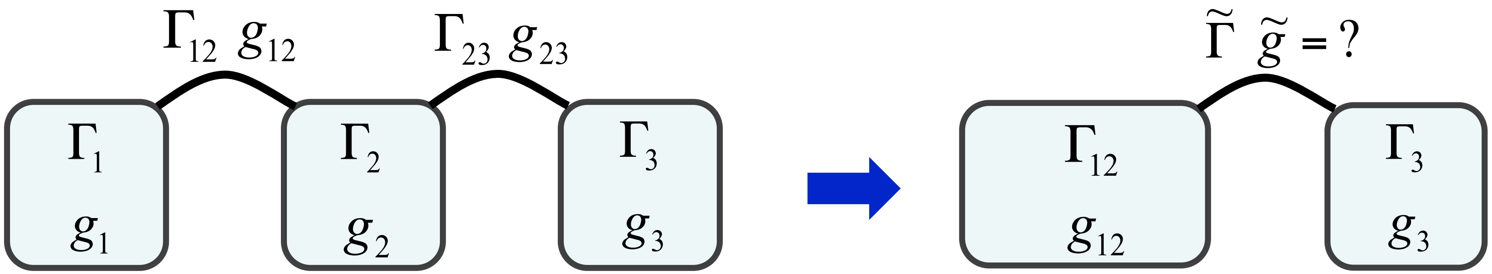


$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$ ~ Effectiveness of coupling

Requirement: $\{\Gamma_{ii+1}\} < \{\Gamma_i\}$

RG scheme

1. Join blocks coupled by the fastest rate Γ_{12}



2. Renormalize couplings to left and right blocks

Two cases:

(i) If $g_{12} \ll 1$ or $g_{23} \ll 1$ then we show

$$\tilde{\Gamma} = \frac{\Gamma_{12}\Gamma_{23}}{\Gamma_2} \qquad \tilde{g} = \frac{g_{12}g_{23}}{g_2}$$

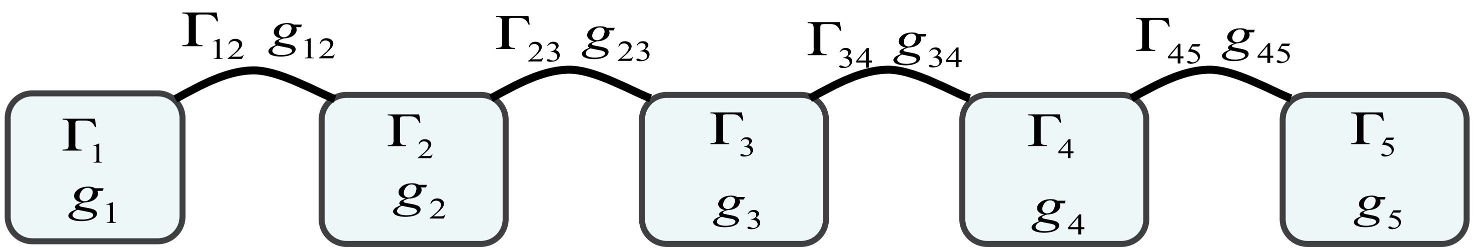
(ii) If $g_{12}, g_{23} \gg 1$

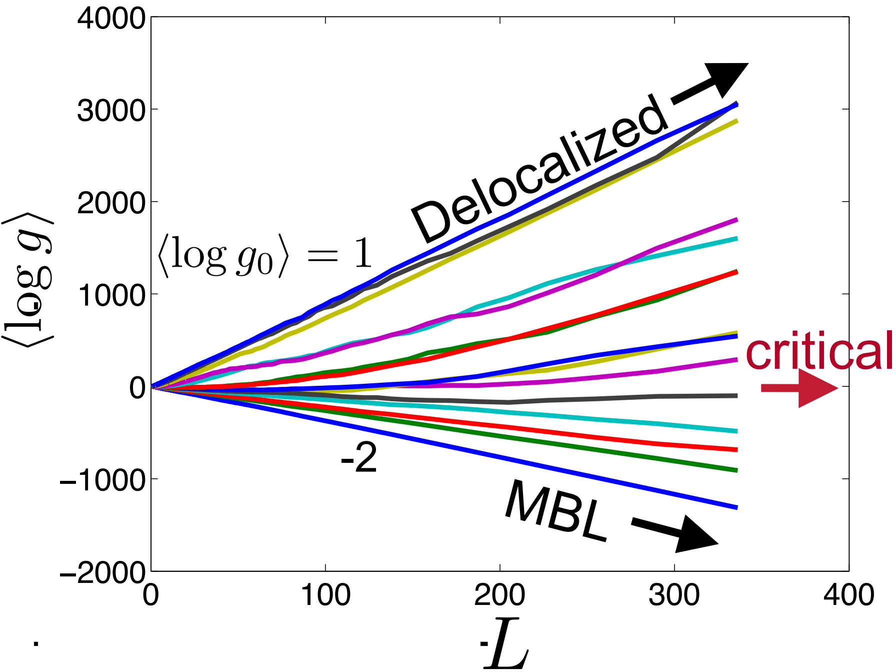
then assume ohmic transport

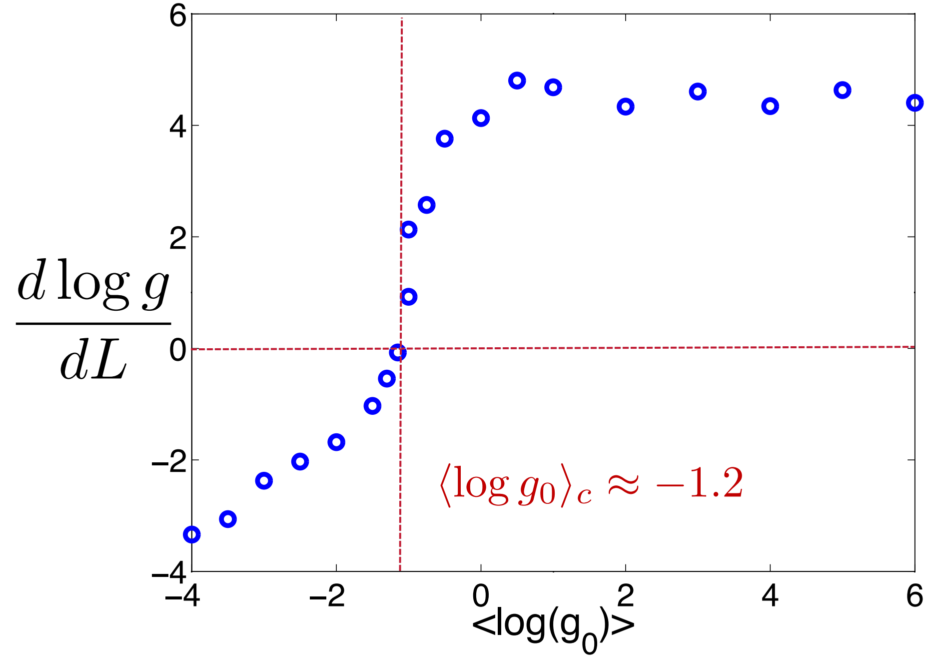
$$\frac{1}{\Gamma_R} = \frac{1}{\Gamma_{12}} + \frac{1}{\Gamma_{23}}$$

Note: the scheme is controlled if the distribution of g_{ij} is wide

Outcome of the RG flow



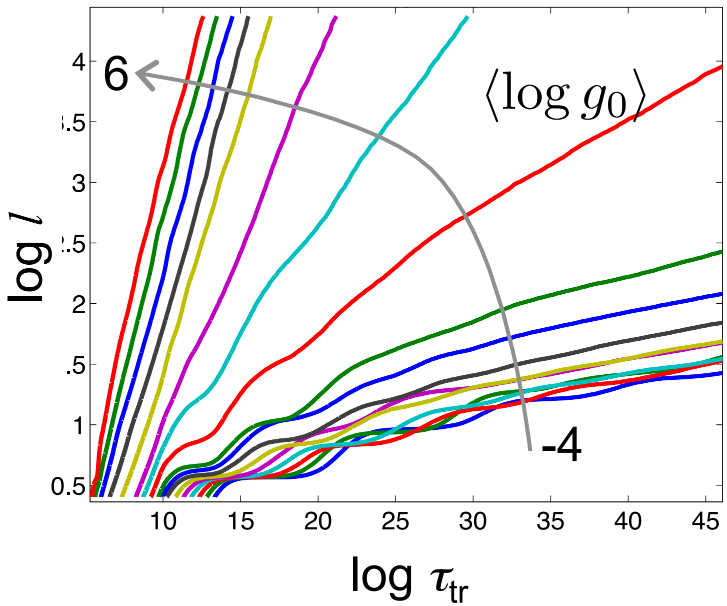


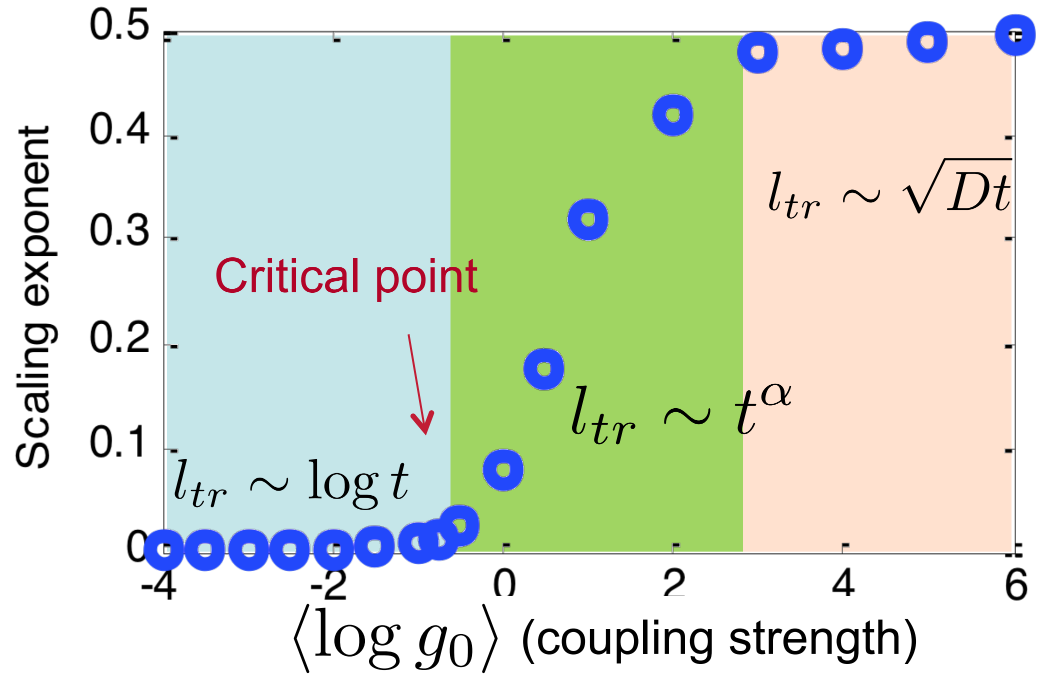


How does diffusion disappear ?

RG results – dynamical scaling exponent for transport

Relation between transport time τ_{tr} and length l of blocks:





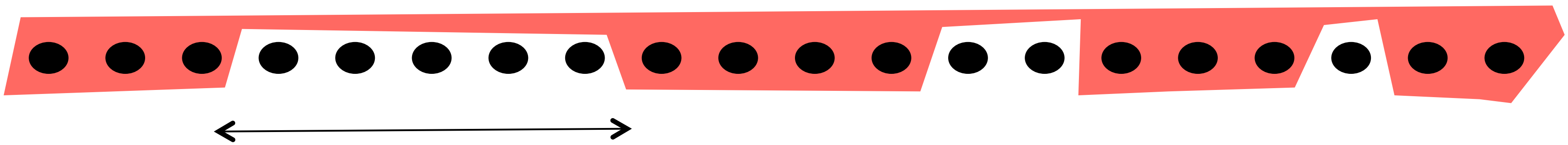
Surprise! The transition is from localized to anomalous diffusion.

Seen also in recent ED studies: Bar-Lev et al 2014 ; Agarwal et al 2014

Scaling relation between transport and entanglement spreading:

$$\tau = \tau_{tr}/l \quad \Rightarrow \quad S_E \sim t^{\frac{\alpha}{1-\alpha}}$$

Anomalous diffusion = Griffith phase



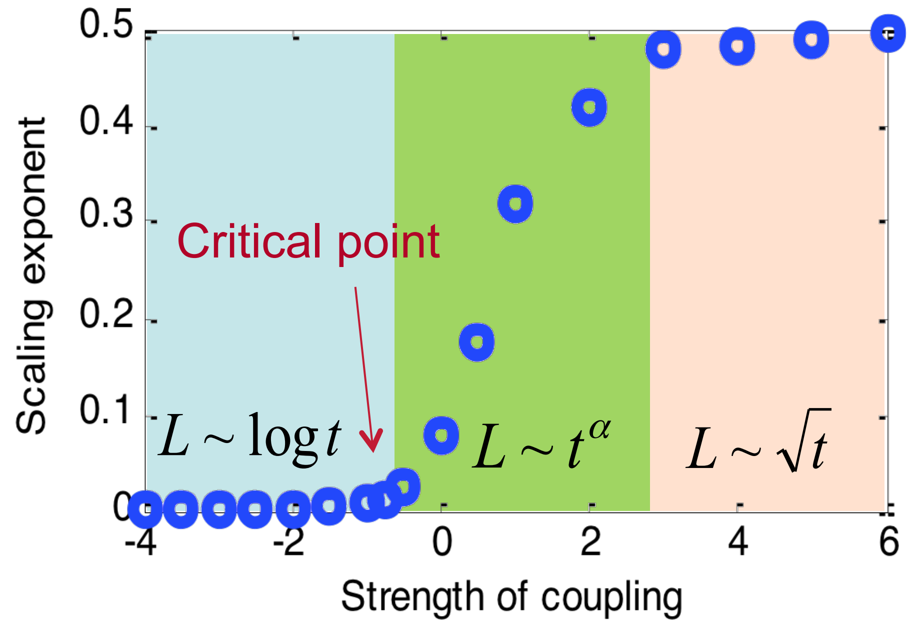
$$l \gg \xi \geq l_o$$

Exponentially rare insulating puddles in the metal

$$P(l) \sim l_0^{-1} e^{-l/\xi}$$

$$\tau(l) = \tau_0 e^{l/l_0}$$

Exponentially long delay



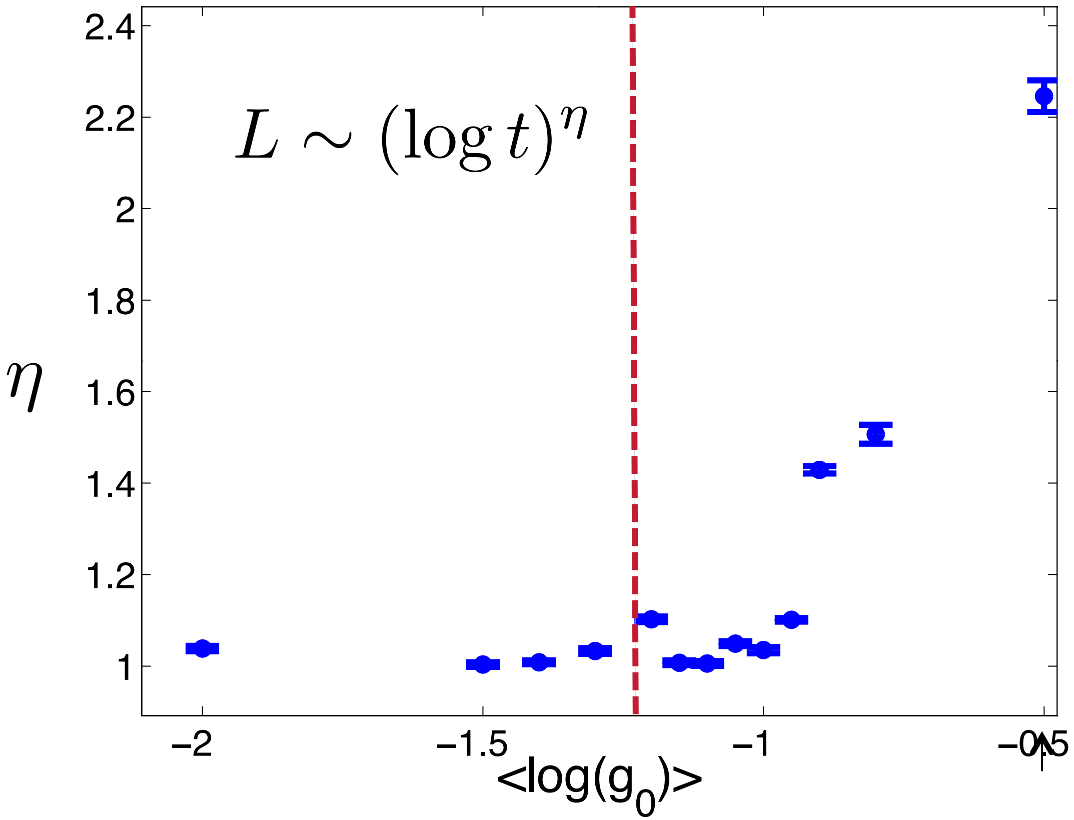
Broad distribution of times:

$$P(\tau) = \tau_0^{-1} \left(\frac{\tau_0}{\tau}\right)^{1+\frac{l_0}{\xi}}$$

All “insulating” puddles ultimately thermalize but at broadly distributed times!

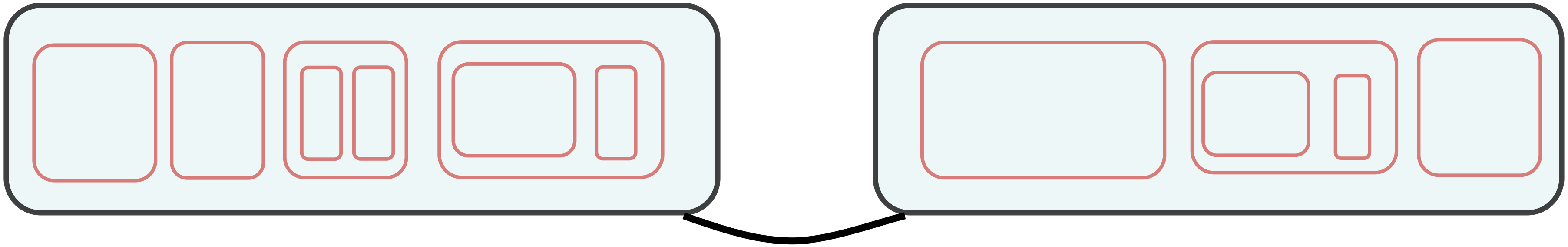
Infinite randomness but thermal critical point at $\xi \to \infty$

Scaling in the localized phase



Suggests also: $S_A \sim \log t$

Entanglement scaling in eigenstates



g_{12} ~ # of 2-block product states in an eigenstate of the coupled system

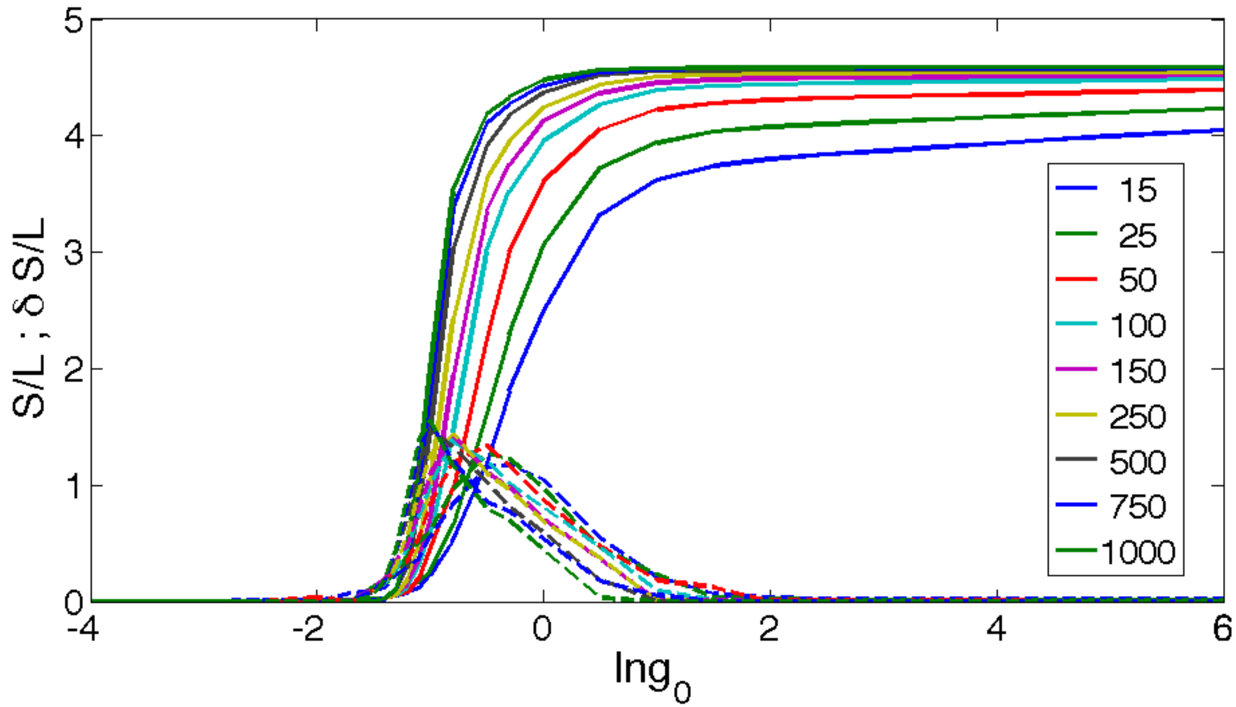
$$S_E(L/2) \sim \log_2 [g(L) + 1]$$

Near critical point expect distribution of S to scale:

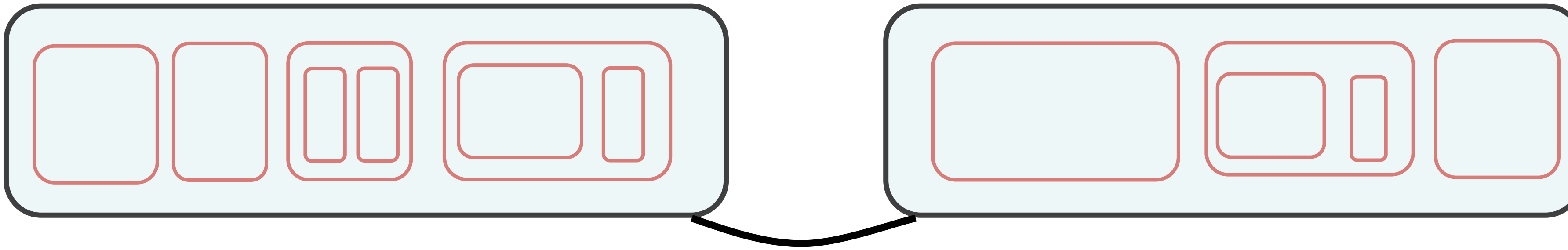
$$P(S, L, g_0) = \frac{1}{L} \tilde{P} \left[\frac{S}{L}, \frac{L}{\xi(g_0)} \right]$$

In particular all moments:

$$\mu S(L, g_0) = L f_\mu \left[\frac{L}{\xi(g_0)} \right]$$



Entanglement scaling in eigenstates



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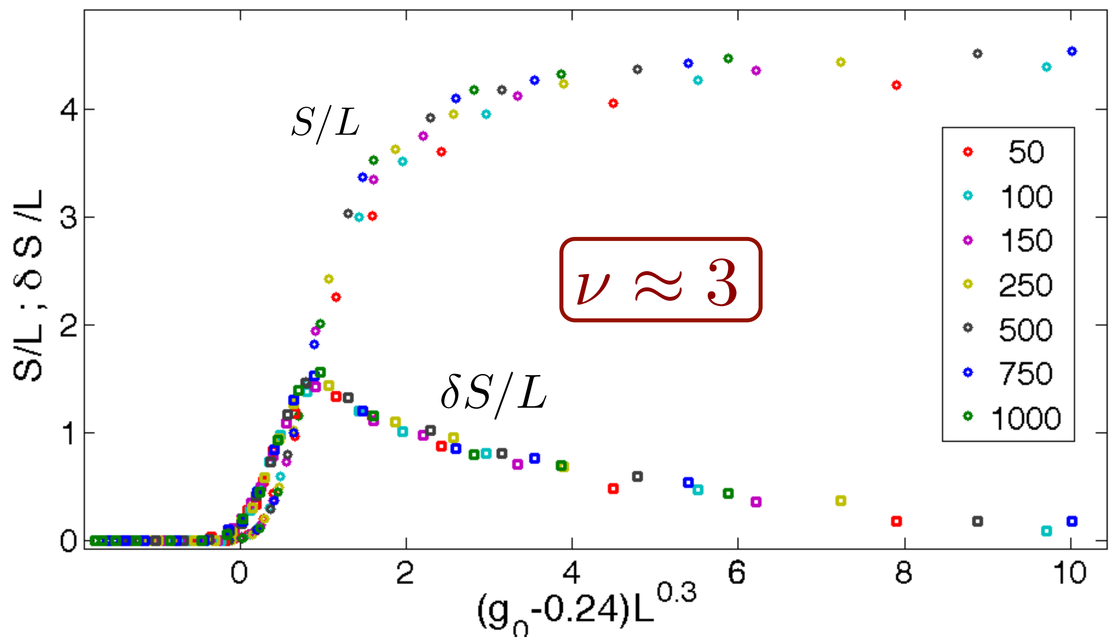
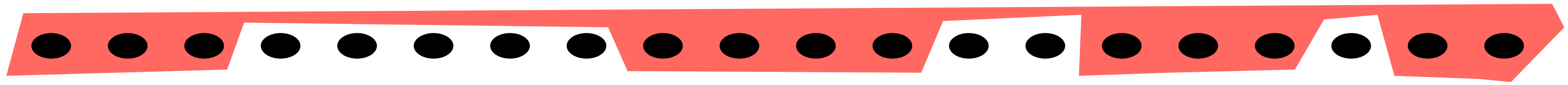


Kjall et. al. (2014) – scaling of entanglement fluctuations in ED of small systems. Found $\nu \sim 0.7$, which is inconsistent with the Harris inequality (but L<14)

Eigenstate entanglement from Griffith model



$$P(\tau) = \tau_0^{-1}\left(\frac{\tau_0}{\tau}\right)^{1+\frac{l_0}{\xi}} \qquad \Delta = \tau_0^{-1}2^{-L}$$

$$S(L/2) = \log(1+g) = \log\left[1+(\tau\Delta)^{-1}\right]$$

Scaling function for the distribution:

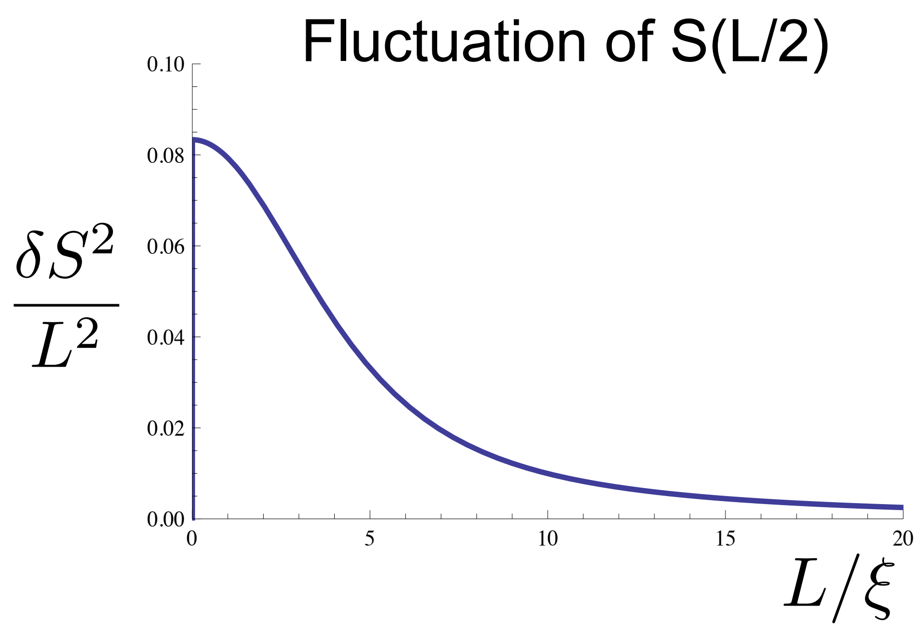
$$\tilde{P}\left(\frac{S}{L},\frac{L}{\xi}\right) = \tilde{P}(s,\lambda) \approx \frac{\lambda}{e^{\lambda}-1}e^{\lambda\, s}$$

~ flat distribution at the critical point

$$\delta S_c/S_c = \mathrm{const}$$

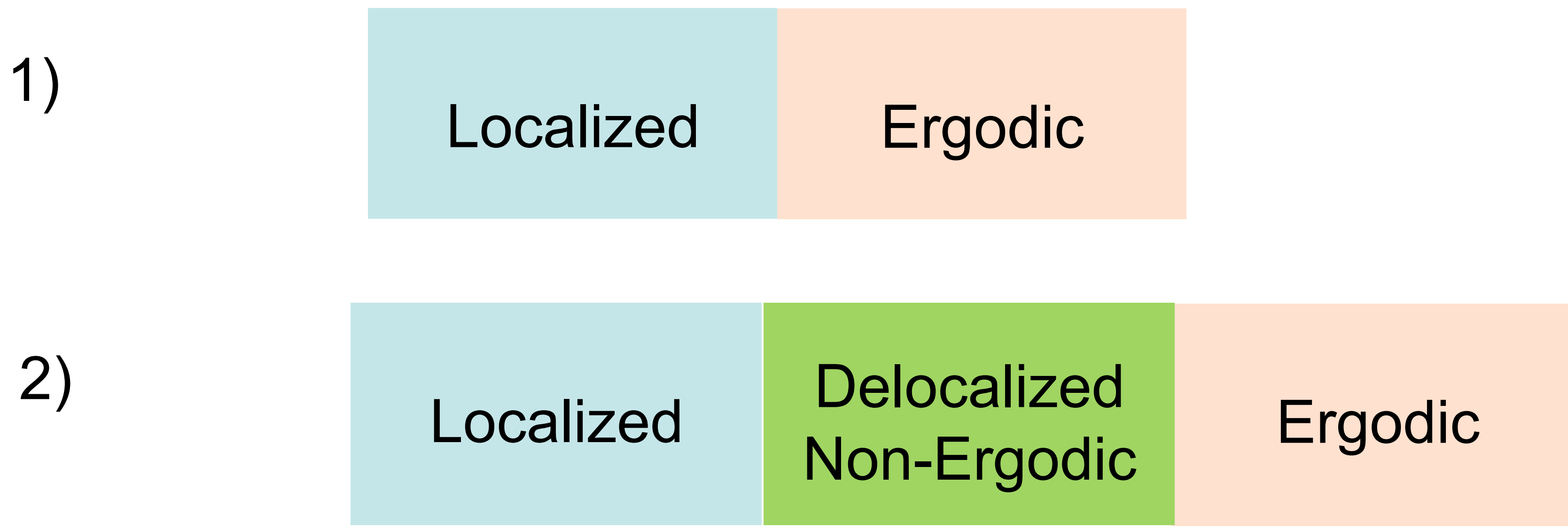
In the Griffith phase:

$$\delta S_\xi/S_\xi \sim \xi/L$$



The Many-Body Localization Transition

Two possible transitions are consistent with entanglement entropy strong subadditivity (T. Grover arXiv:1405.1471)

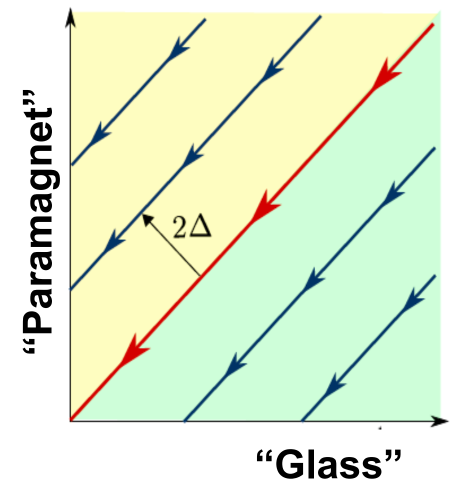


Our scheme gives case 1: the Griffith phase is ergodic!

$S_A(l)$ at the critical point is thermal for a subsystem of an infinite system.
But fluctuations are maximal (~L) when $l=L/2$

Summary

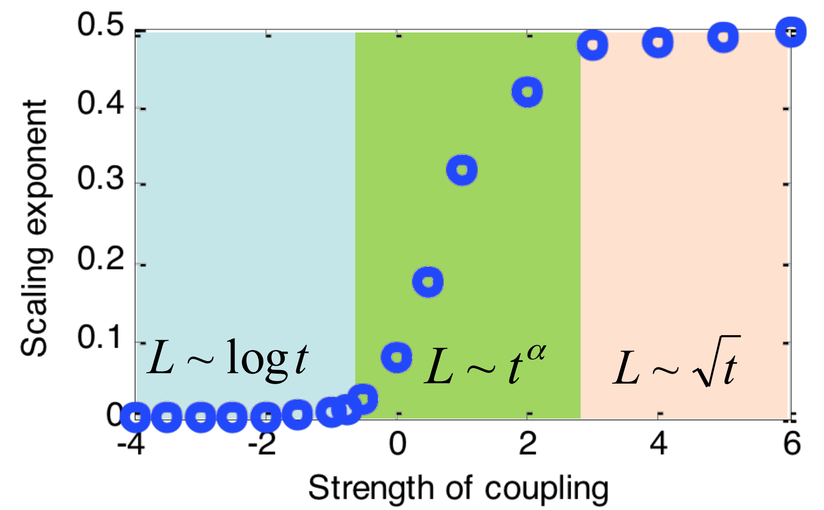
1. RG approach in the MBL state:
   - Dynamical phases and phase transitions.
   - Emergent integrals of motion.



2. RG theory of the MBL transition.

   Found intermediate phase!
   Thermal but anomalous diffusion.
   “Griffiths phase”

   Infinite randomness at the critical point shows up in entanglement



Many open questions

1. Generalization to 2d and 3d ? Does the Griffiths phase survive?
2. How to see MBL physics in experiments? Cold atoms?