

Disordered Elastic Systems

T. Giamarchi

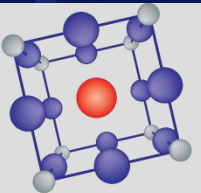
http://dqmp.unige.ch/gr_giamarchi/



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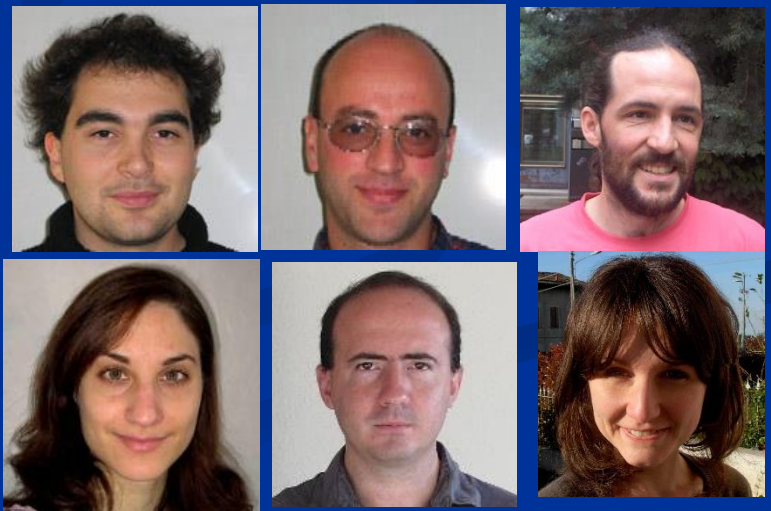


MaNEP
SWITZERLAND

E. Agoritsas (Geneva)
S. Barnes (Miami U.)
S. Bustingorry (Bariloche)
D. Carpentier (ENS Lyon)
P. Chauve (Orsay/ENS)
R. Chitra (ETHZ)
L. Cugliandolo (Jussieu)
J. P. Eckmann (Geneva)
L. Foini (Geneva)
A. Kolton (Bariloche)
W. Krauth (ENS)
V. Lecomte (Jussieu)
P. Le Doussal (ENS)
E. Orignac (ENS)
A. Rosso (LPTMS)
G. Schehr (LPTMS)

S. Lemerle (Orsay)
J. Ferré (Orsay)
J.P. Jamet (Orsay)
V. Jeudy (Orsay)

J.M. Triscone (Geneva)
P. Paruch (Geneva)
T. Tybell (Trondheim)



General References

Disordered Elastic Media:

TG, Encyclopedia of Complexity and Systems Science (Springer)

Domain walls:

E. Agoritsas, V. Lecomte, TG,

Arxiv:1111.4899, Physica B 407 1725 (2012)



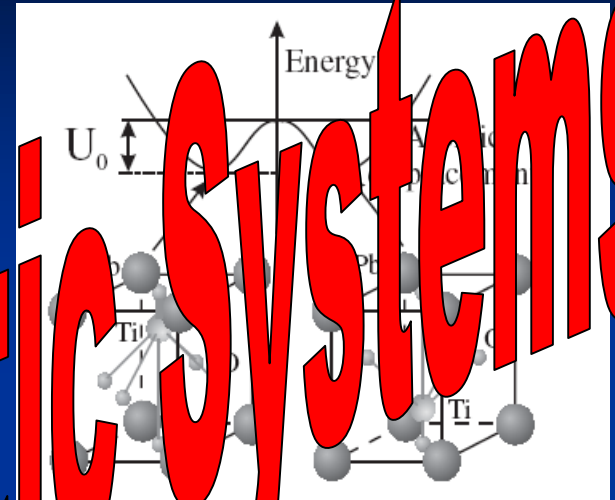
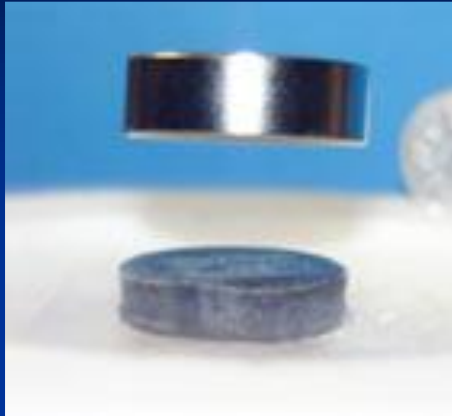
Vol 14 - N° 8 - octobre 2013

P. 637-756

Académie des sciences

Disordered systems / Systèmes désordonnés

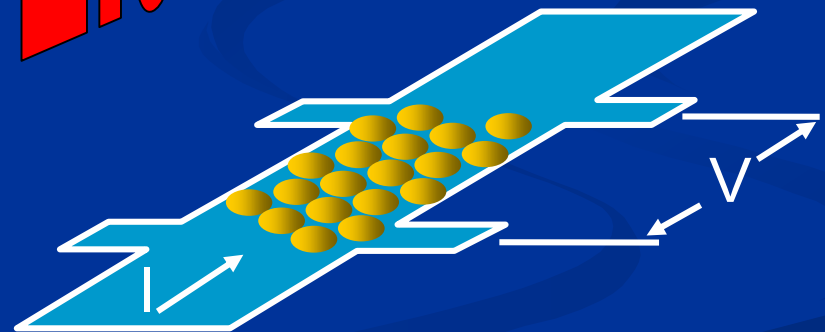
Common denominator of:



Superconductor

Magnet

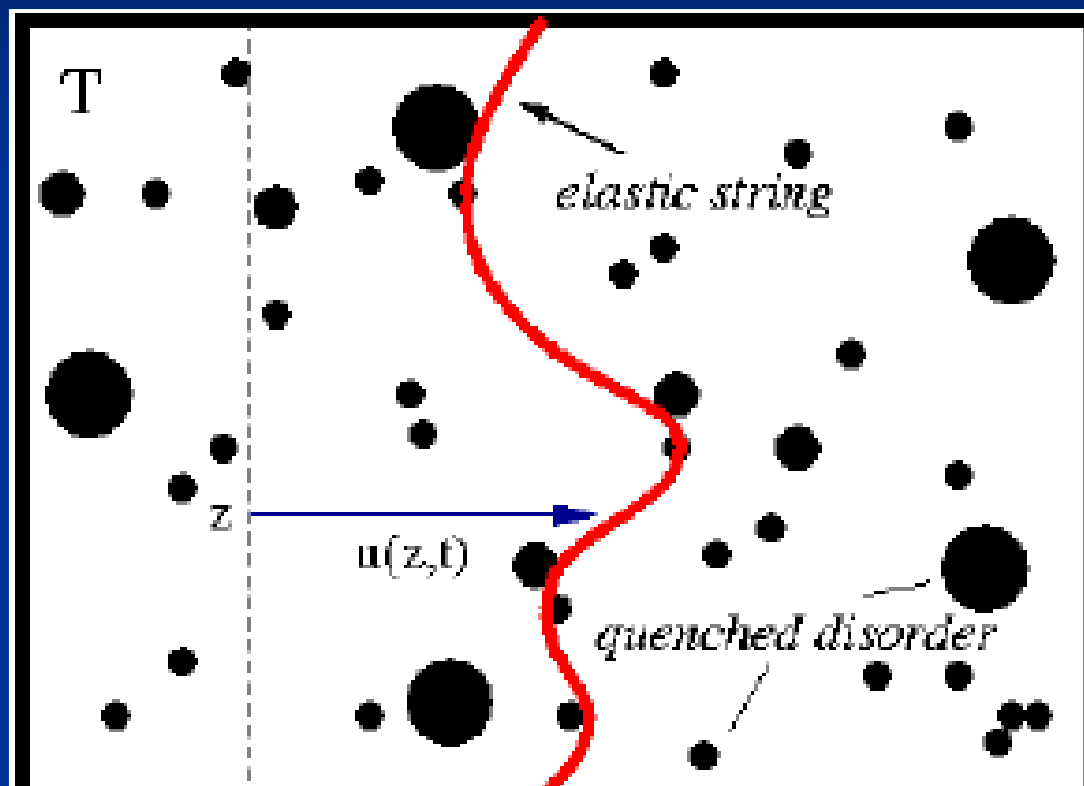
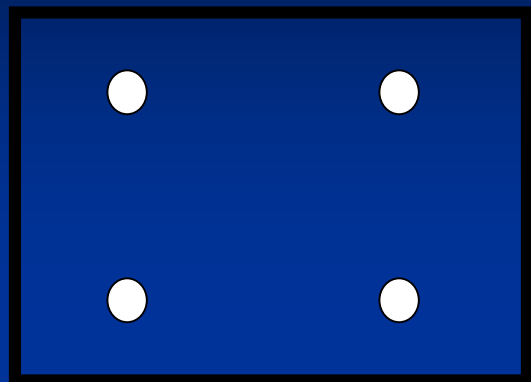
Ferroelectric



Contact line

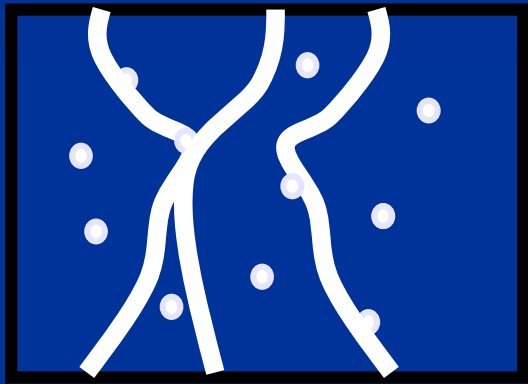
Two dimensional electron gas

Basic Features



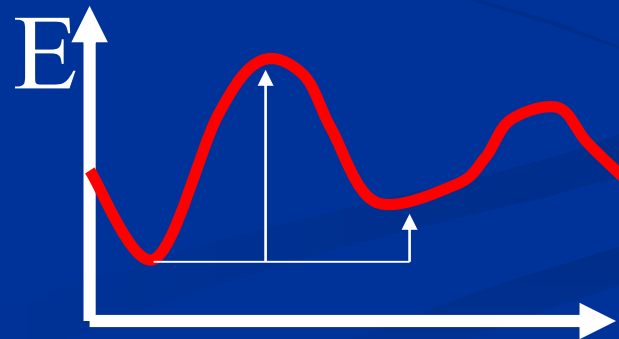
$$H = \frac{c}{2} \int dz (\nabla u(z))^2 + \int dz V(u(z), z)$$

Very difficult stat-mech problem

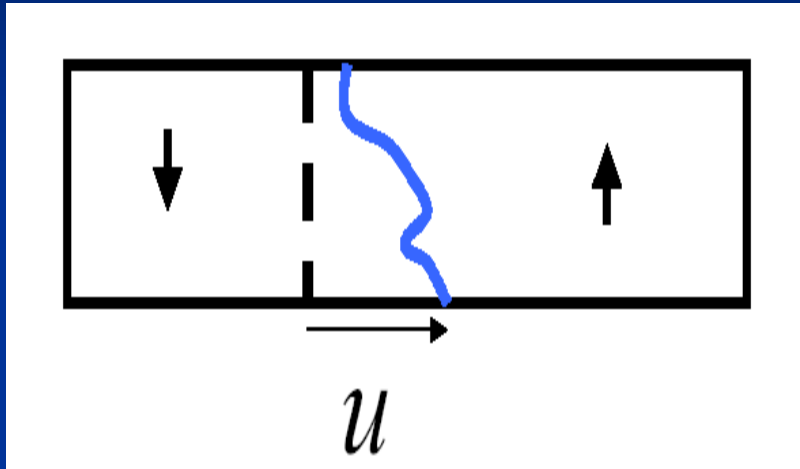


- Optimization :
many solutions

Glass !



What to measure (statics)



$$B(r) = \overline{\langle [u(r) - u(0)]^2 \rangle}$$

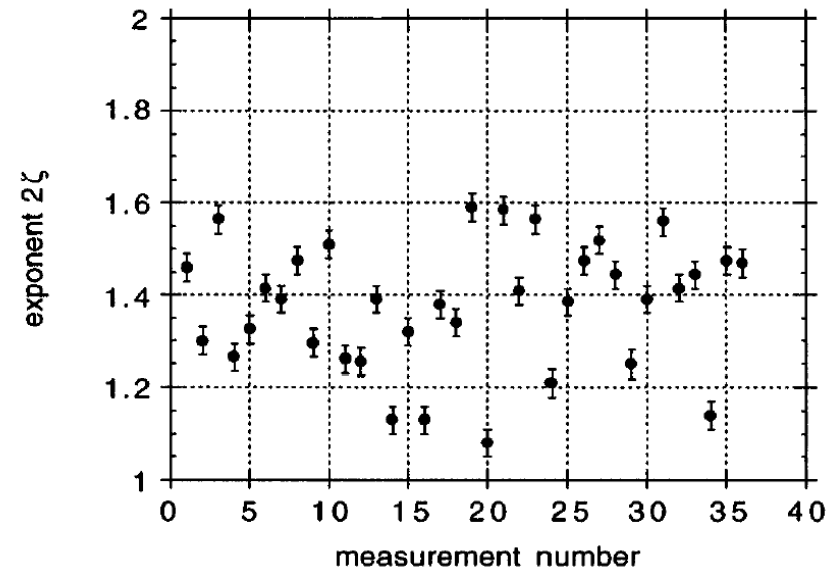
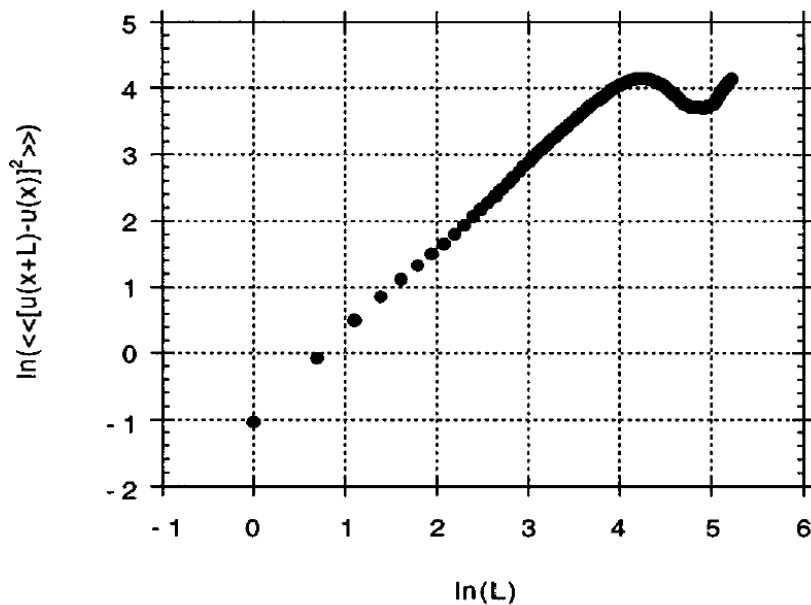
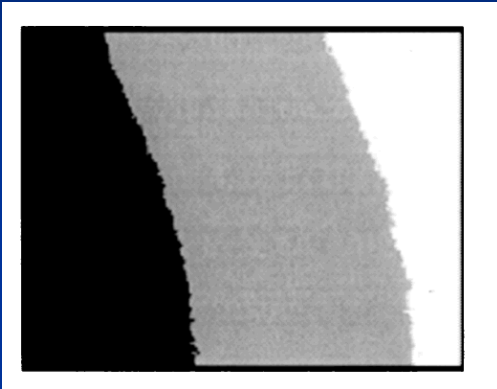
$$B(z) \sim z^{2\zeta}$$

Positional order

Amplitude: new questions (Calabrese, Le Doussal, Quastel, etc.)

E. Agoritsas et al. PRE 87 042406 (13)

Magnetic systems

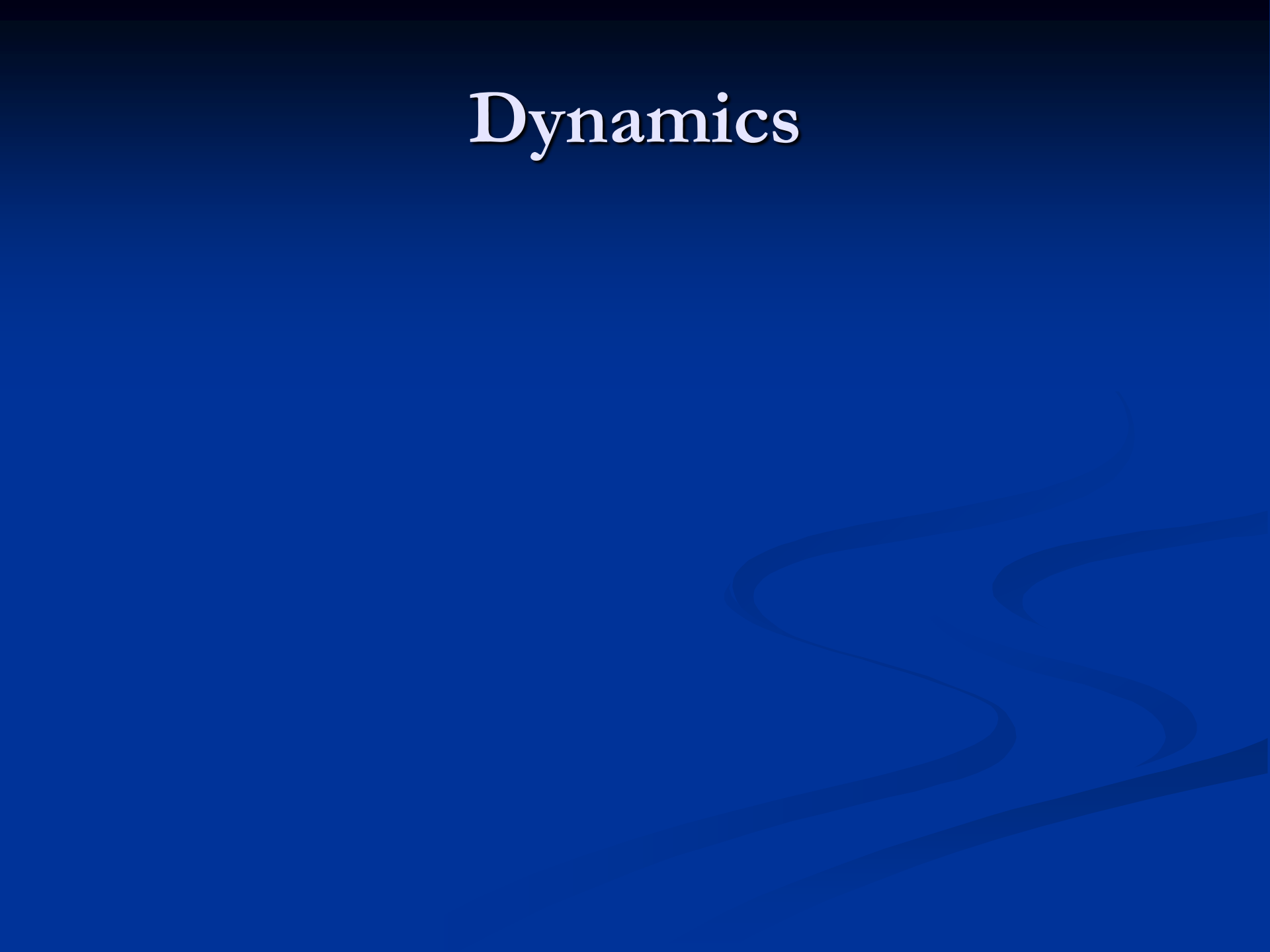


$$u \propto L^\zeta$$

Theory: $\zeta = 2/3$

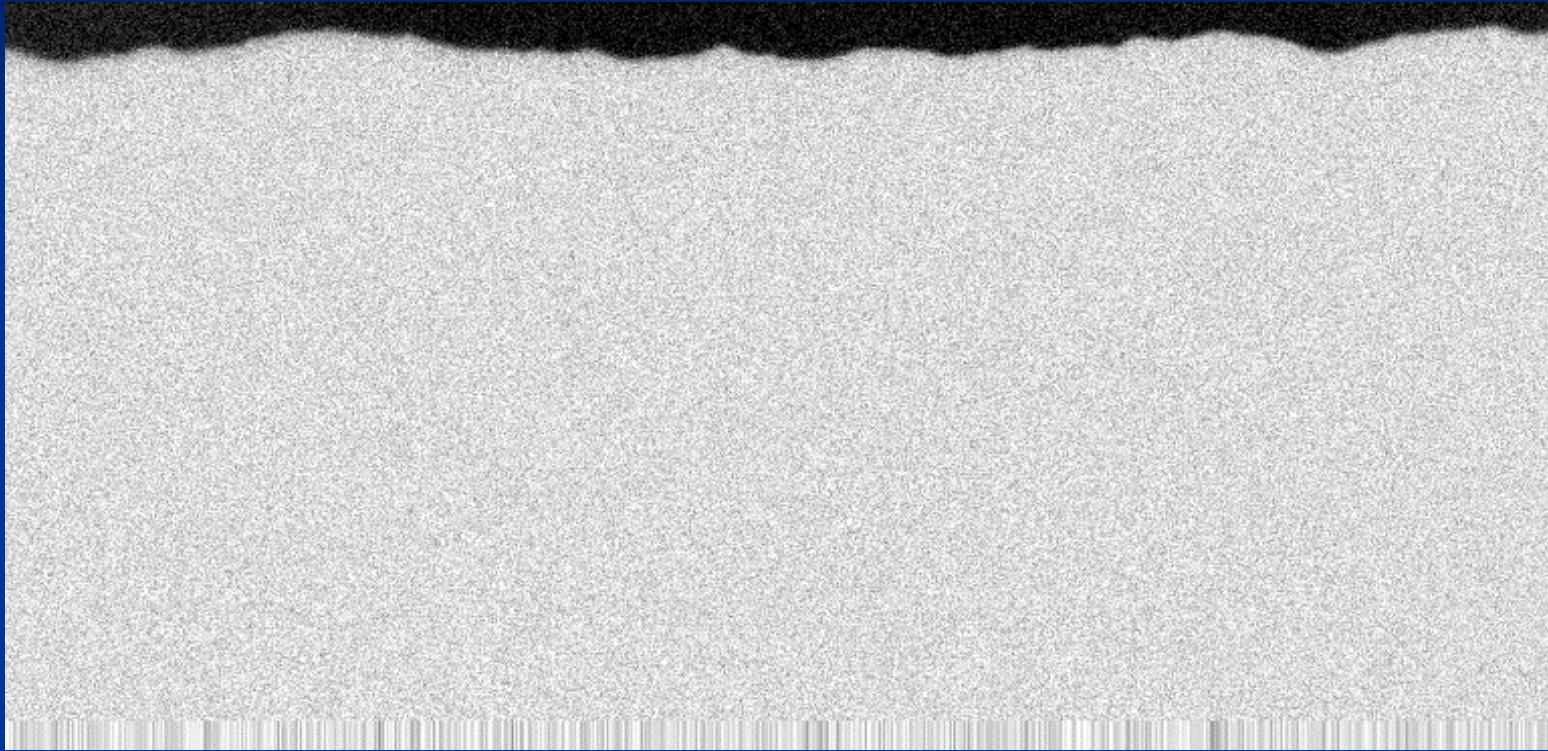
S. Lemerle et al. PRL
80 849 (98)

Dynamics

The background of the slide is a solid dark blue. In the lower right quadrant, there are several faint, white, wavy lines that resemble stylized water ripples or a dynamic pattern, adding a sense of movement to the design.

Dynamics

+



-

Groupe J. Ferre/J.P. Jamet; Exp: V. Repain



Equation of motion

$$\eta \partial_t u(z, t) = c \nabla_z^2 u(z, t) + F_{pin}[u(z, t)] + F + \zeta(z, t)$$

η : friction

ζ : thermal noise

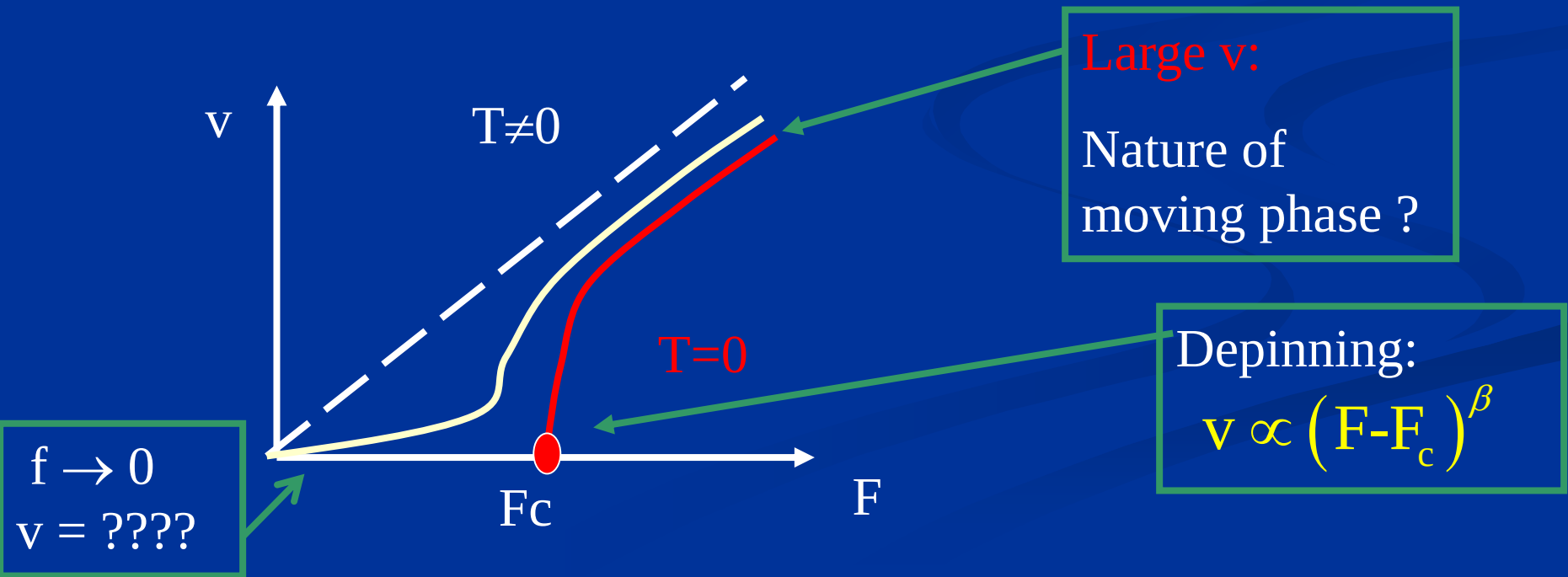
$$F_{pin} = - \left. \frac{\partial V(x, z)}{\partial x} \right|_{x=u(z, t)}$$

$$\overline{\zeta(z, t) \zeta(z', t')} = \eta T \delta(z - z') \delta(t - t')$$

$$\langle V(x, z) V(x', z') \rangle = D f(x - x') \delta(z - z')$$

Questions for dynamics

- Disorder = pinning
- Finite temperature probes barriers

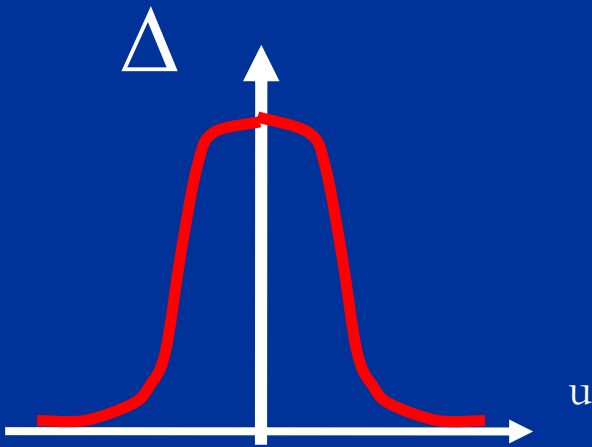


How to study microscopically

$$\eta \partial_t u = c \nabla^2 u + F_{pin}[u] + f$$

$$\int Du D\hat{u} e^{i\hat{u}(\partial_t u - c \nabla^2 u - \dots)}$$

$$\begin{aligned} S_{\text{uns}}(u, \hat{u}) = & \int_{rt} i\hat{u}_{rt} (\eta \partial_t - c \nabla^2) u_{rt} - \eta T \int_{rt} i\hat{u}_{rt} i\hat{u}_{rt} \\ & - f \int_{rt} i\hat{u}_{rt} \\ & - \frac{1}{2} \int_{rtt'} i\hat{u}_{rt} i\hat{u}_{rt'} \Delta(u_{rt} - u_{rt'}) \end{aligned} \quad (4.1)$$



Correlator of disorder

Study by RG

Usual vs Functional RG

$$\Delta(u) \approx a + bu^2 + cu^4 + \dots$$

Needs only to keep b and c (higher powers are irrelevant)

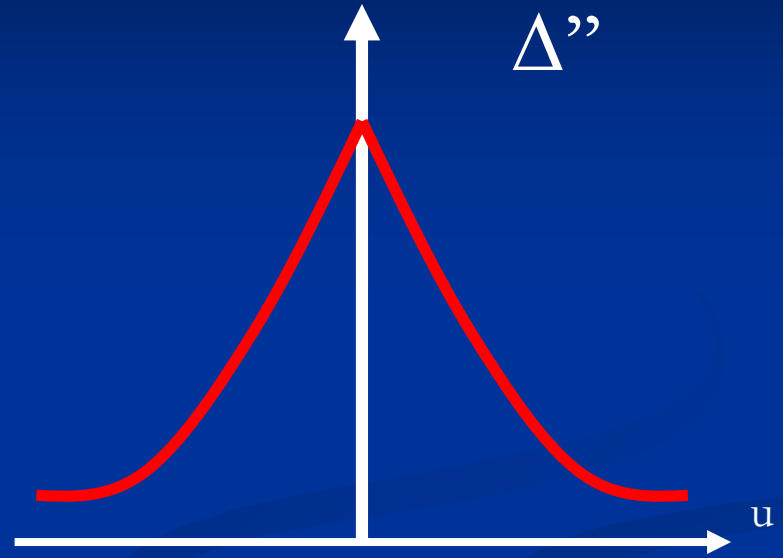
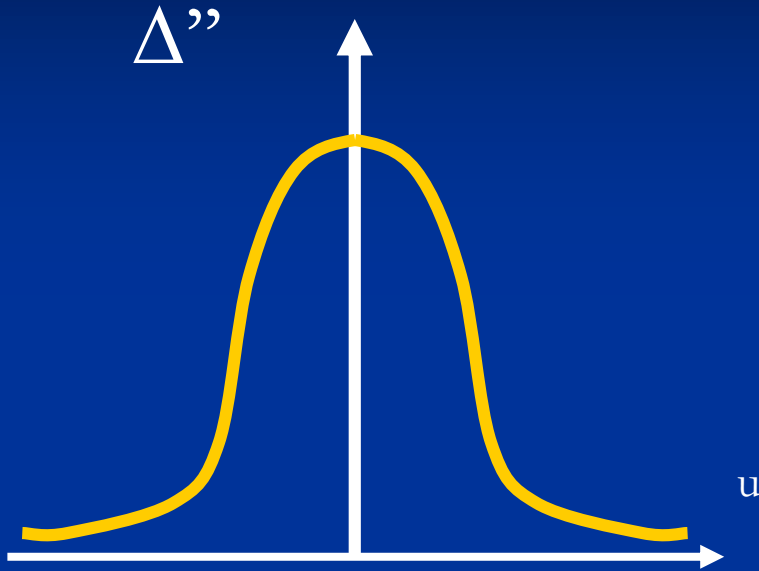
- Disorder: all powers are important

Renormalize the whole function

Review: P. Le Doussal + K. Wiese

[arXiv:cond-mat/0611346](https://arxiv.org/abs/cond-mat/0611346)

$$\partial \tilde{\Delta}''(0) = \epsilon \tilde{\Delta}''(0) - 3 \tilde{\Delta}''(0)^2$$



- Nonanalyticity at a finite lengthscale R_c such that $u(R_c) \sim l_c$
(A. Larkin, D. Fisher)

- Cusp signals metastability and glassy states

Example: static

$$\begin{aligned}\partial\tilde{\Delta}(u) &= (\epsilon - 2\zeta)\tilde{\Delta}(u) + \zeta u\tilde{\Delta}'(u) + \tilde{T}\tilde{\Delta}''(u) \\ &\quad + \tilde{\Delta}''(u)[\tilde{\Delta}(0) - \tilde{\Delta}(u)] - \tilde{\Delta}'(u)^2, \\ \partial \ln \tilde{T} &= \epsilon - 2 - 2\zeta.\end{aligned}$$

- Periodic system (crystal): $\Delta(u) = A \cos(u)$

- Fixed point: $\zeta = 0$

$$\Delta^*(ax) = \frac{\epsilon a^2}{6} \left(\frac{1}{6} - x(1-x) \right)$$

Dynamics from FRG

$$\begin{aligned}
 \partial \tilde{\Delta}(u) = & (\epsilon - 2\zeta) \tilde{\Delta}(u) + \zeta u \tilde{\Delta}'(u) + \tilde{T} \tilde{\Delta}''(u) \\
 & + \int_{s>0, s'>0} e^{-s-s'} (\tilde{\Delta}''(u) \{ \tilde{\Delta}[(s'-s)\lambda] \\
 & - \tilde{\Delta}[u+(s'-s)\lambda] \} - \tilde{\Delta}'(u-s'\lambda) \tilde{\Delta}'(u+s\lambda) \\
 & + \tilde{\Delta}'[(s'+s)\lambda] [\tilde{\Delta}'(u-s'\lambda) - \tilde{\Delta}'(u+s\lambda)]),
 \end{aligned}
 \tag{4.11}$$

$$\partial \ln \lambda = 2 - \zeta - \int_{s>0} e^{-s} s \tilde{\Delta}''(s\lambda),$$

$$\partial \ln \tilde{T} = \epsilon - 2 - 2\zeta + \int_{s>0} e^{-s} s \lambda \tilde{\Delta}'''(s\lambda),$$

$$\partial \tilde{f} = e^{-(2-\zeta)l} c \Lambda_0^2 \int_{s>0} e^{-s} \tilde{\Delta}'(s\lambda),$$



$$\tilde{\Delta}_l(u) = \frac{S_D \Lambda_l^D}{(c \Lambda_l^2 e^{\xi l})^2} \Delta_l(u e^{\xi l}),$$

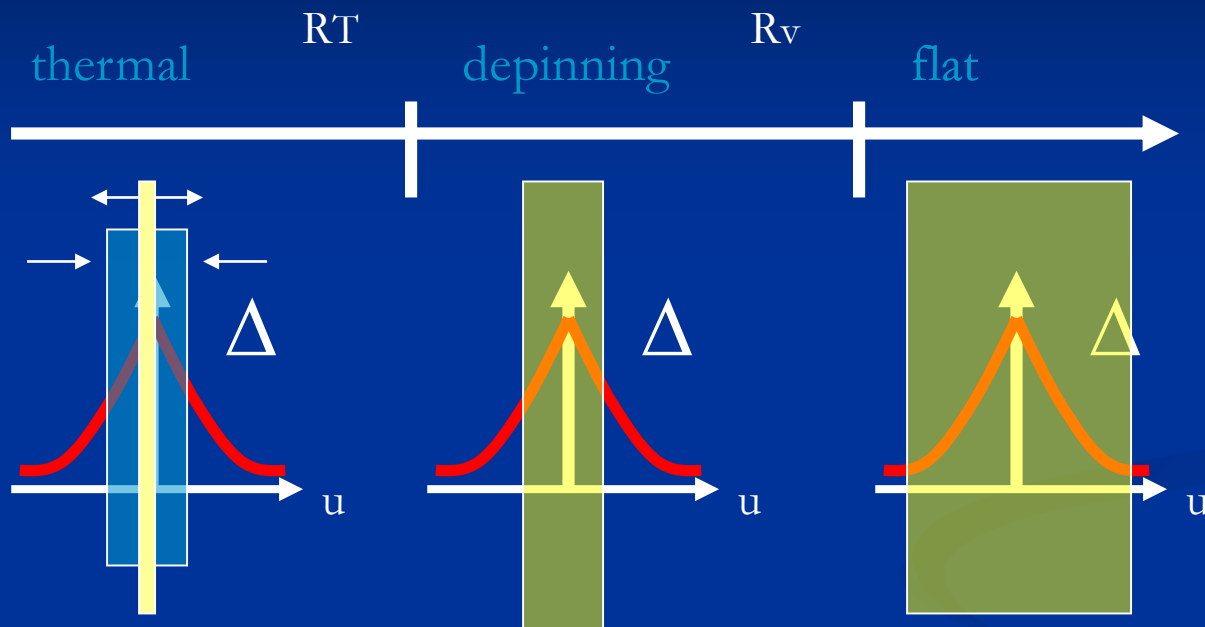
$$\tilde{T}_l = \frac{S_D \Lambda_l^D}{c \Lambda_l^2 e^{2\xi l}} T_l,$$

$$\lambda_l = \frac{\eta_l v}{c \Lambda_l^2 e^{\xi l}},$$

$$\tilde{f}_0 = f - \eta_0 v,$$

P. Chauve, T. Giamarchi, P. Le Doussal EPL 44 110 (98);
PRB 62 6241 (2000)

Small force response



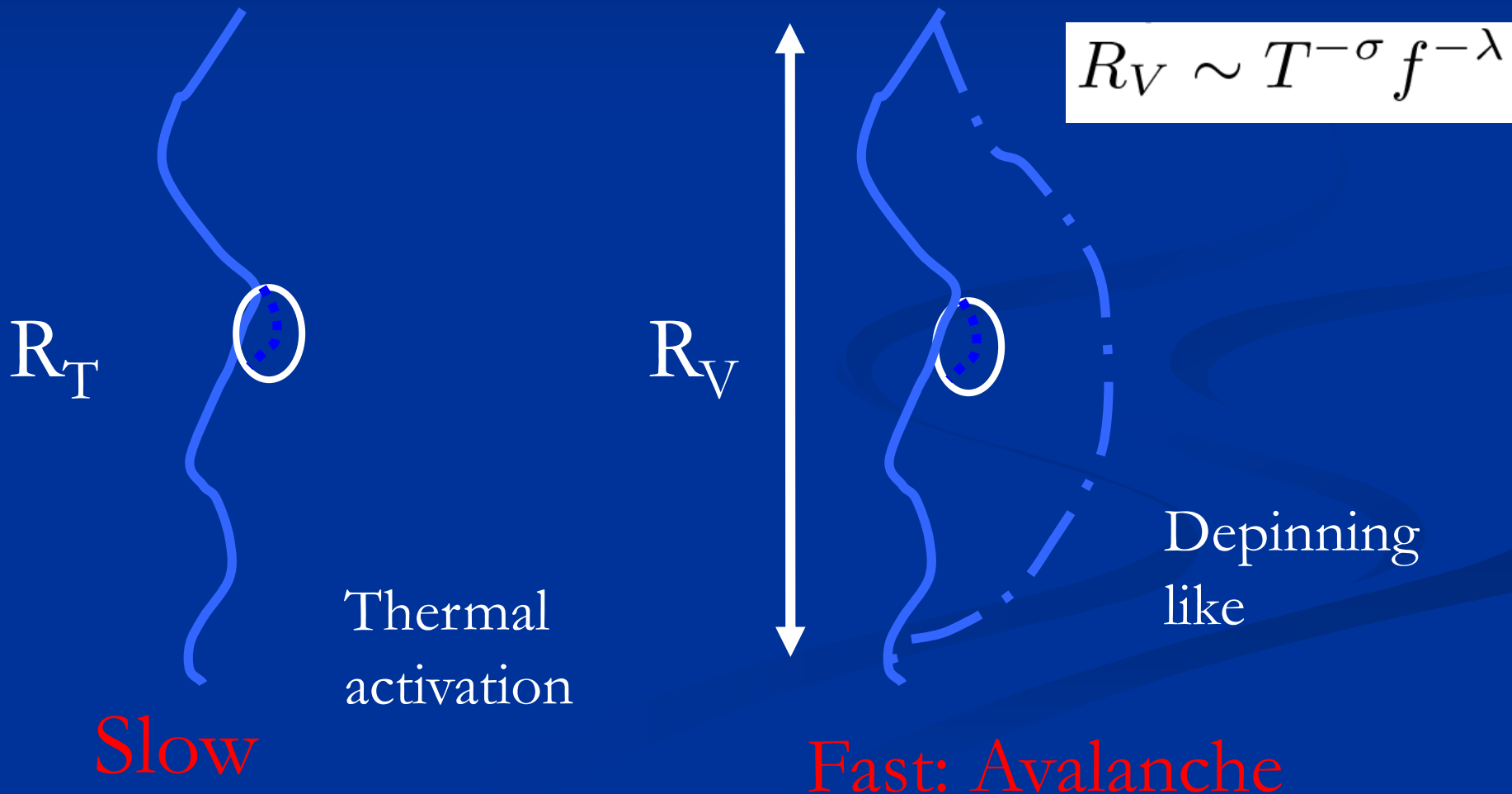
$$\frac{\eta v}{f_c} \approx \exp \left[-\frac{U_c}{T} \left(\frac{f}{f_c} \right)^{-\mu} \right]$$

$$\mu = \frac{D - 2 + 2\zeta_{eq}}{2 - \zeta_{eq}}$$

Phenomenological
derivation: Ioffe +
Vinokur;
Nattermann

New lengthscale: avalanches

Motion different from phenomenological picture
(two regimes)

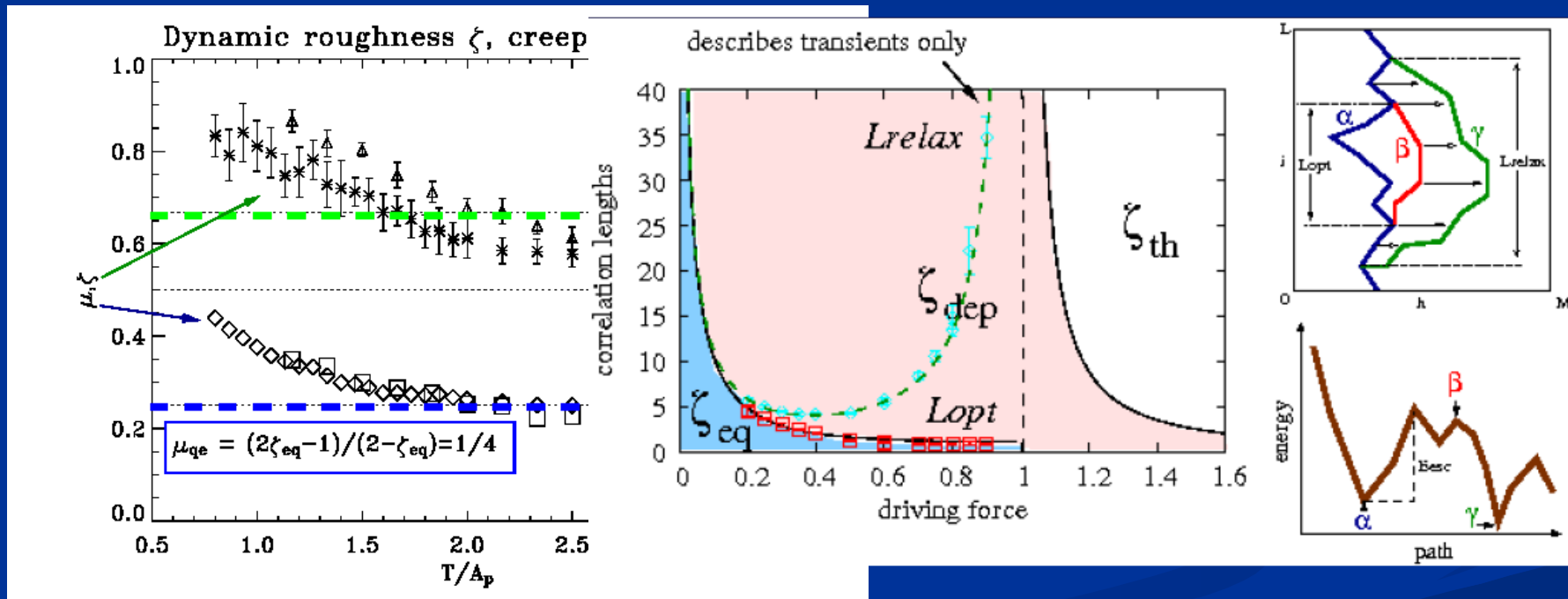


Tests

Numerical study d=1

Molecular dynamic simulations:

A. B. Kolton, A. Rosso, TG, PRL 91 056603 (03)



Exact enumeration algorithm: A.B. Kolton, A. Rosso, TG, W. Krauth PRL 97 057001 (06); PRB 79 184207 (09)

Experiments

S. Lemerle et al. PRL 80 849 (98)

$$v \propto e^{-\beta(1/B)^\mu}$$

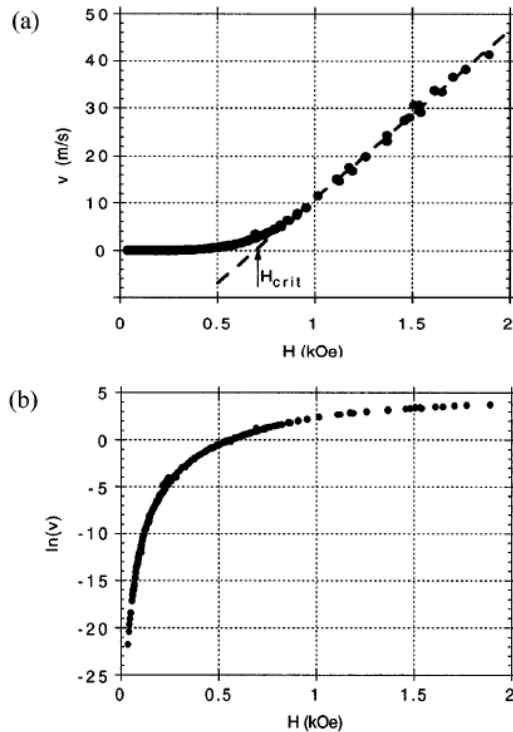
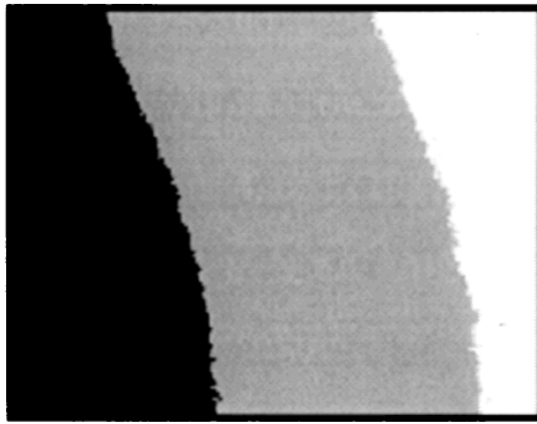
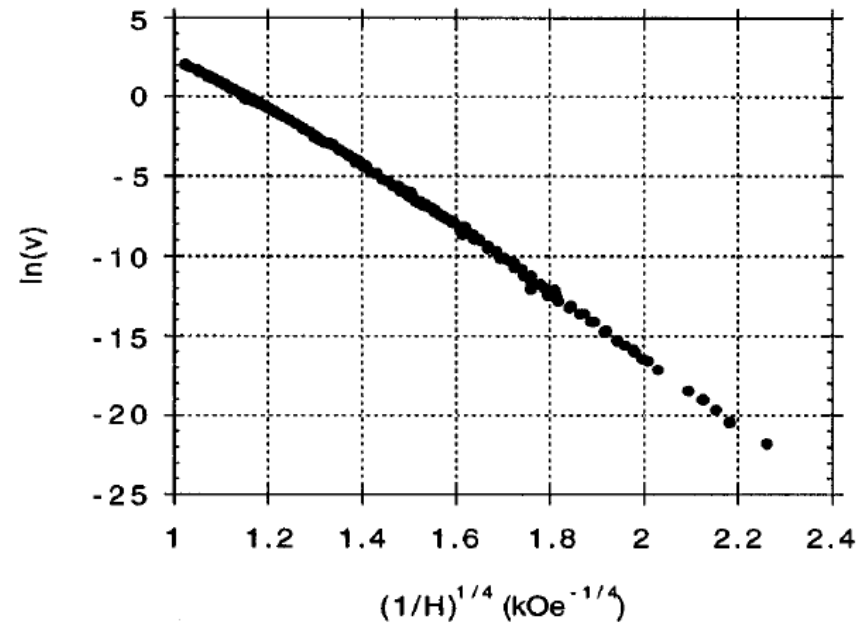
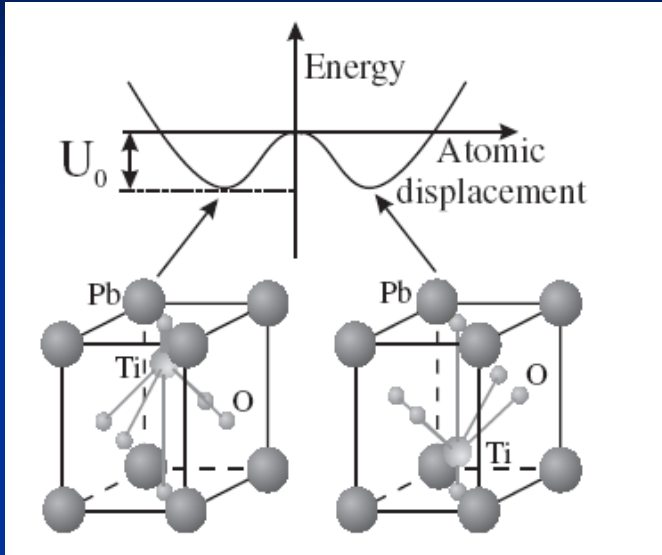


FIG. 2. (a),(b): MDW velocity versus applied magnetic field at room temperature (v in m/s). The dashed line in (a) is the linear fit of the high field part ($H > 0.86$ kOe) and the arrow marks its intersection with the line $v(H) = 0$. This is the definition of H_{crit} .

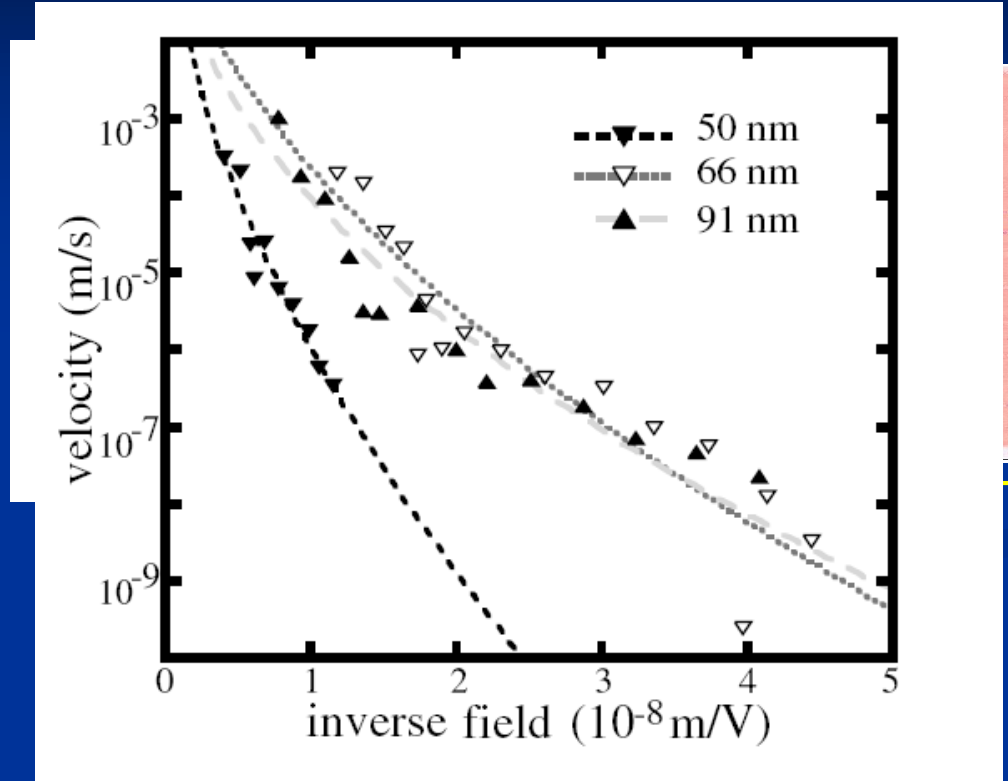


$$\zeta = 2/3 \quad \mu = \frac{d-2+2\zeta}{2-\zeta} = 1/4$$

Ferroelectrics



P. Paruch et al.
cond-mat/0411178



T. Tybell et al. PRL 89 097601 (02)

P. Paruch et al. PRL 94 197601 (05)



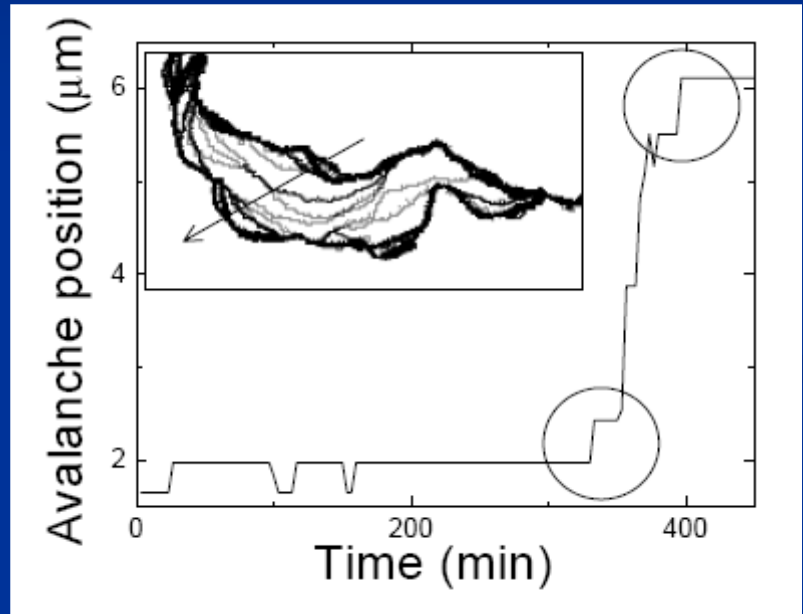
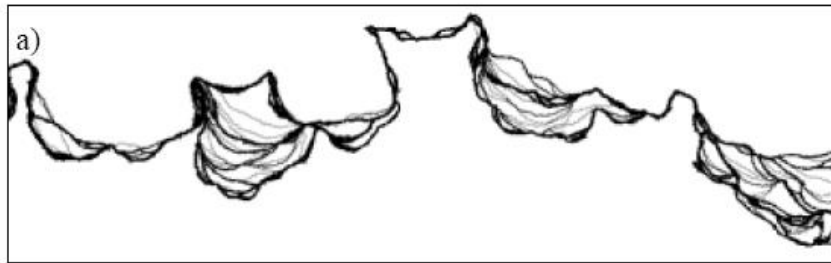
Lenth scales





Probing R_T and R_V

V. Repain et al. EPL 68 460 (04)



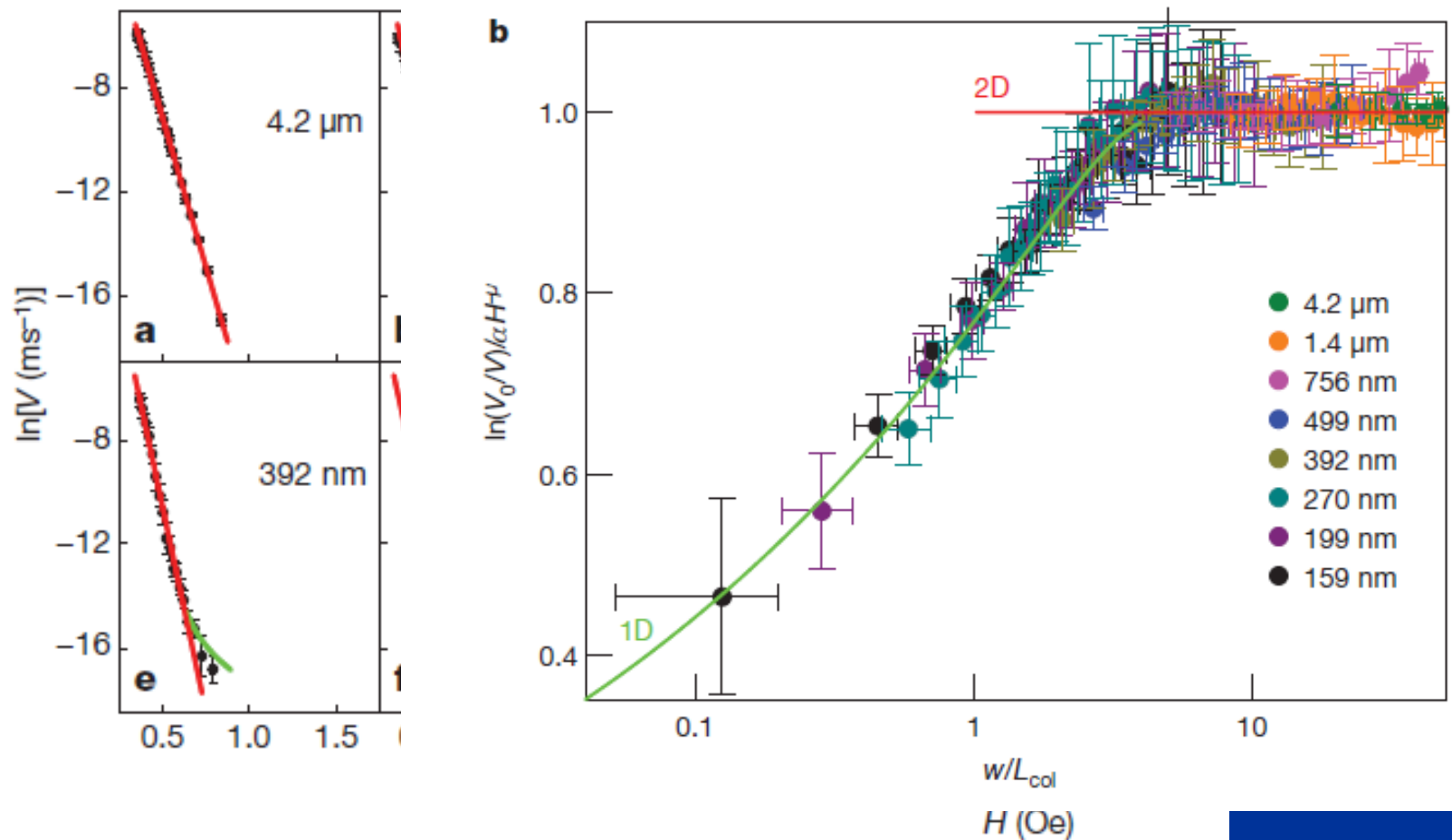
$H = 159 \text{ A m}^{-1}$ and $L_C = 40 \text{ nm}$.

$$R_T \gg 1 \text{ } \mu\text{m}$$

$$R_V \gg 17 \text{ } \mu\text{m}$$

Probe the lengthscale R_T

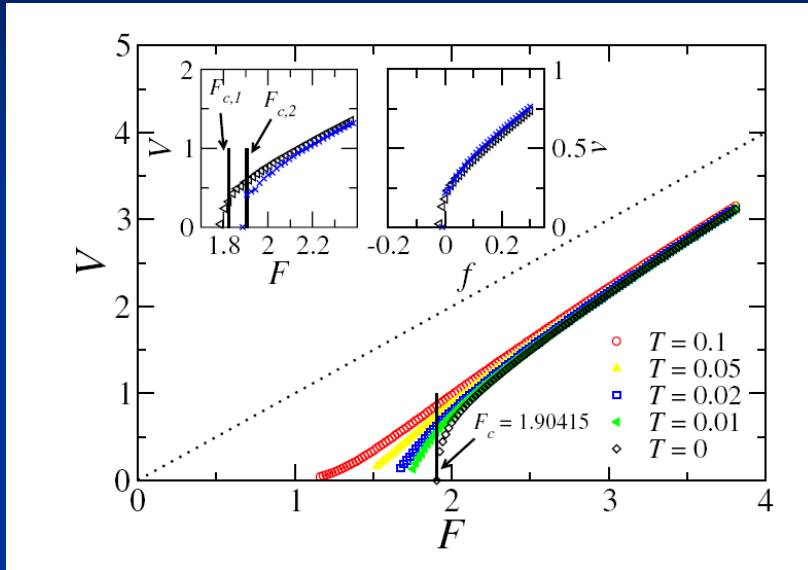
K.J. Kim et al. Nature 458 540 (09)



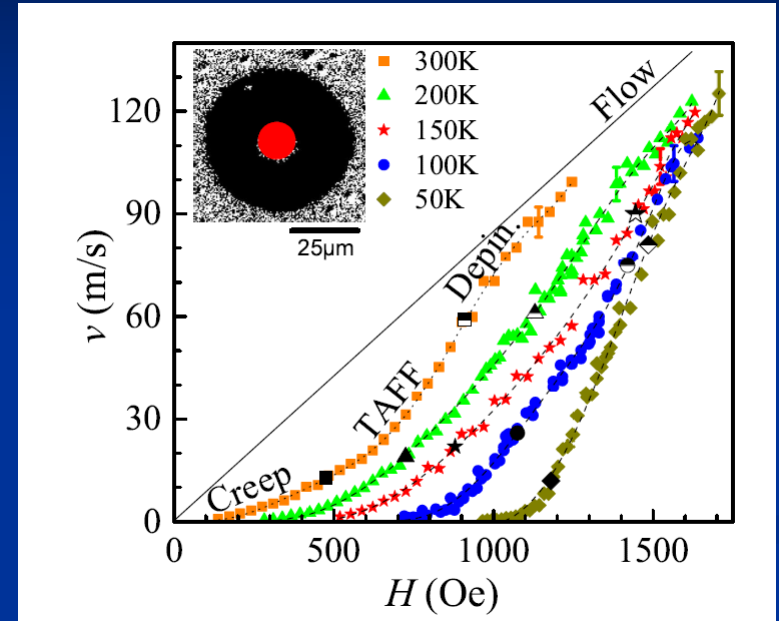
Open challenges

- Thermal rounding; Depinning
- Out of equilibrium issues (aging)
- Defects (overhangs, bubbles)
- Internal degrees of freedom (spintronic etc.)
- Quantum systems (bosons, magnets etc.)

Thermal rounding; Depinning



S. Bustingorry, A. B.
Kolton, TG, EPL 81
26005 (2008)



J. Gorchon et al. PRL (14)

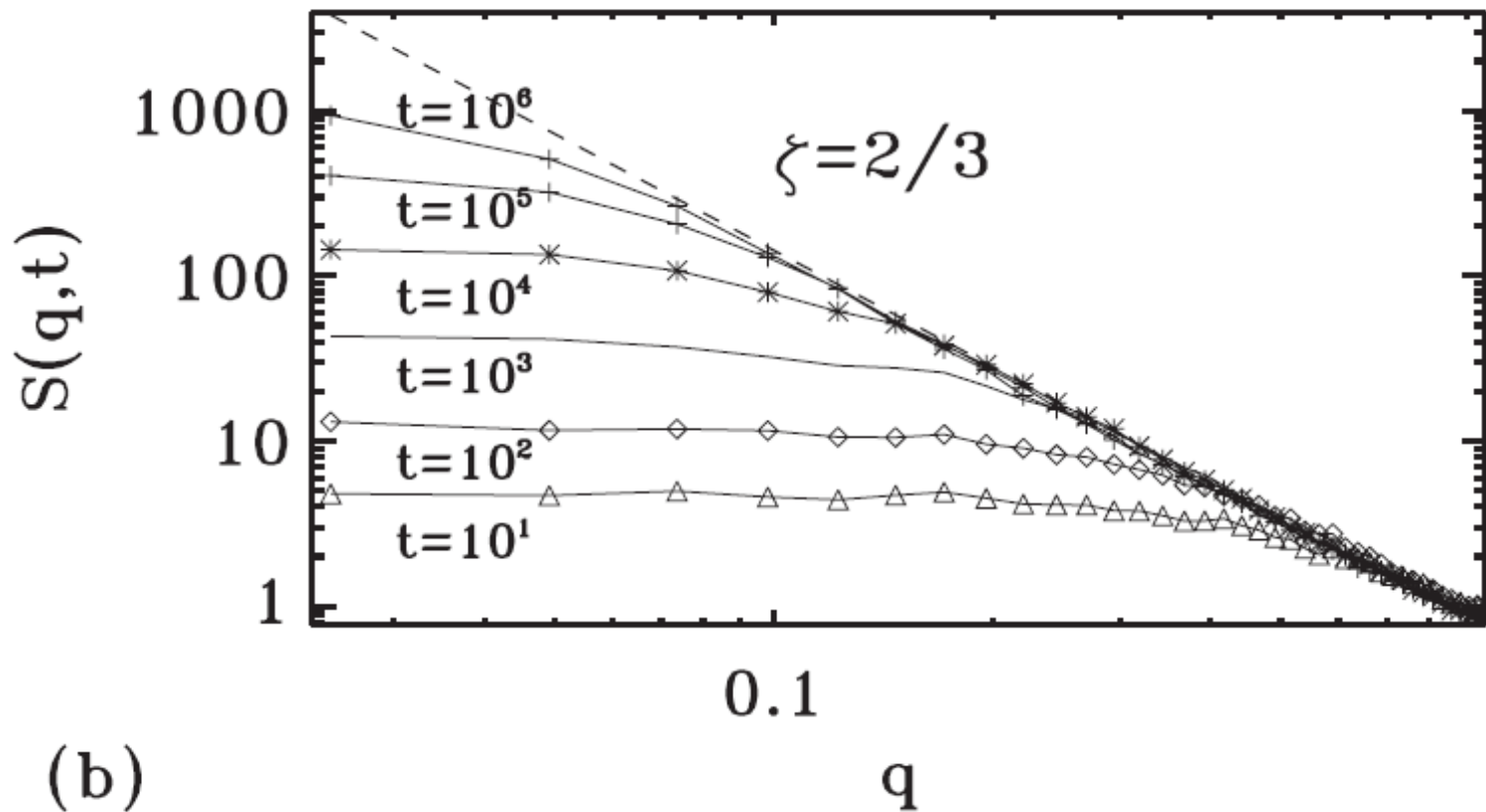
$$v(F_c) \propto T^\psi$$

$$\psi = 0.15 \pm 0.01$$

Not understood yet !!

Aging

Out of equilibrium

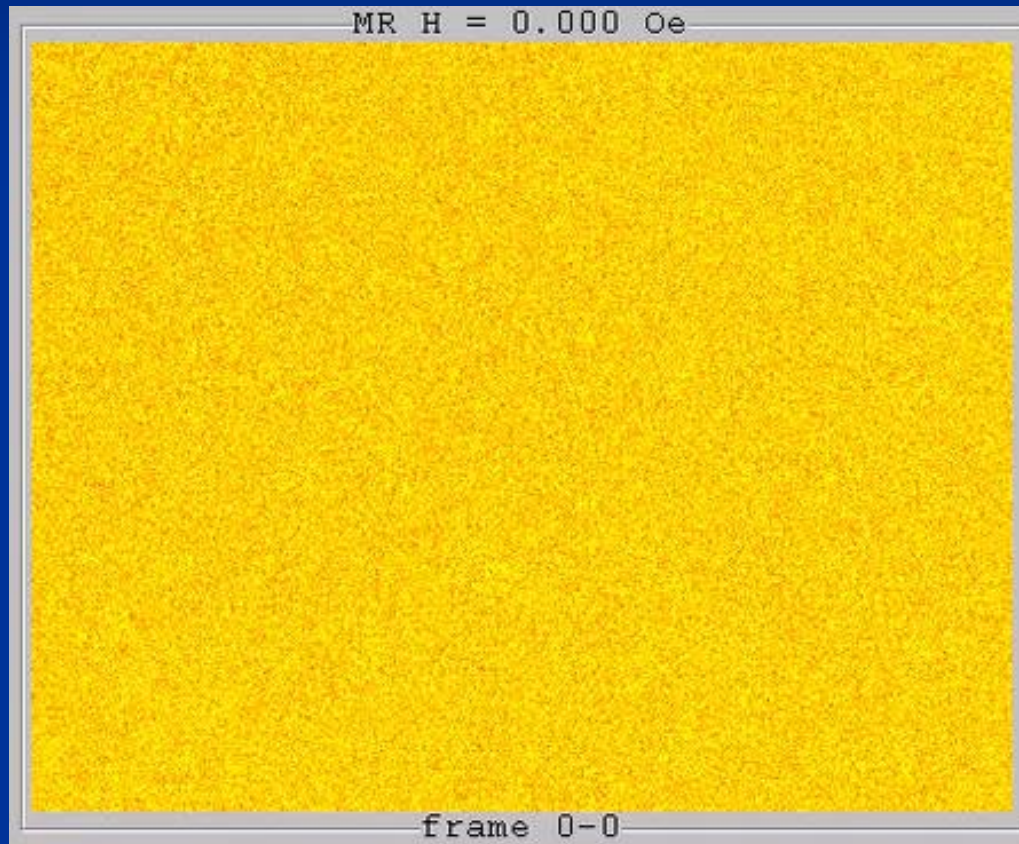


A. B. Kolton, A. Rosso, TG, PRL 95 180604 (05)

Defects



(V. Repain et al. (Orsay))



210 μm

Pt/Co(0,5 nm)/Pt/SiO₂

Internal degrees of freedom

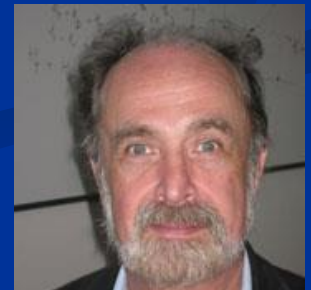
Wall with internal degree of freedom

V. Lecomte, S. Barnes, J.P. Eckmann, TG PRB 80 054413 (09)
Nonlinearity 25 1427 (12)

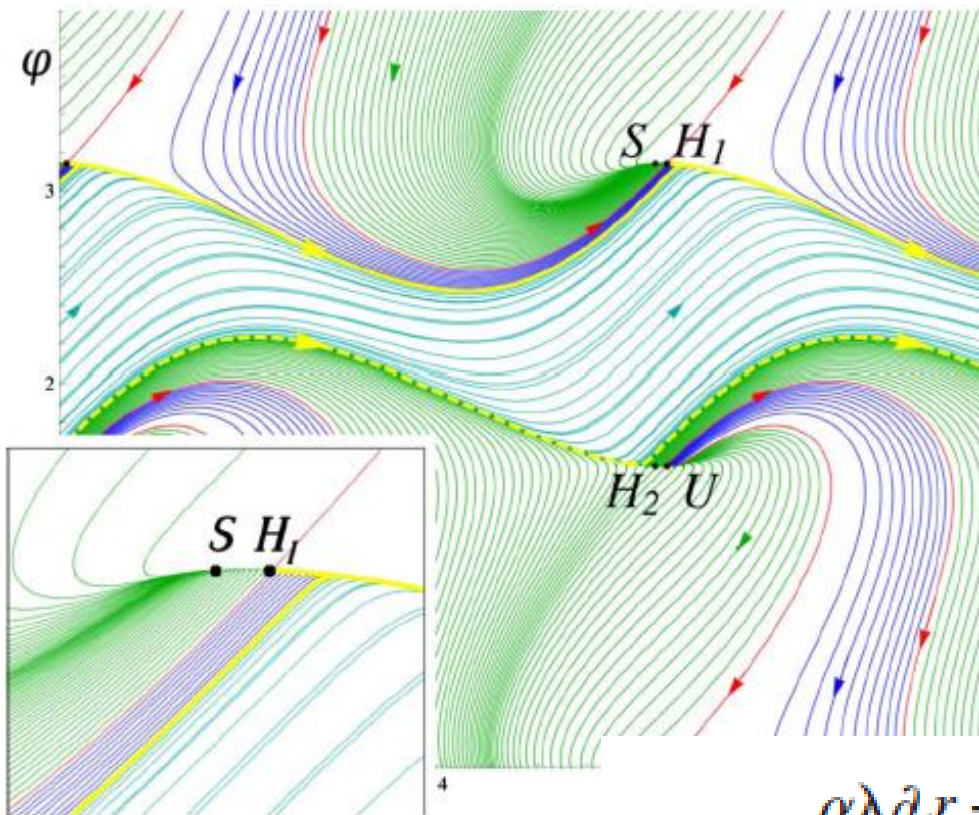
$$E = \frac{1}{2} \int d^3x \{ J [(\nabla \theta)^2 + \sin^2 \theta (\nabla \varphi)^2] + K \sin^2 \theta + K_{\perp} \sin^2 \theta \cos^2 \varphi \}.$$

$$\alpha \lambda \partial_t r - \partial_t \varphi - \beta_s v_s = f_{\text{ext}}(r) + \eta_1,$$

$$\alpha \partial_t \varphi + \lambda \partial_t r - v_s = -\frac{1}{2} K_{\perp} \sin 2\varphi + \eta_2.$$



Rigid wall approximation

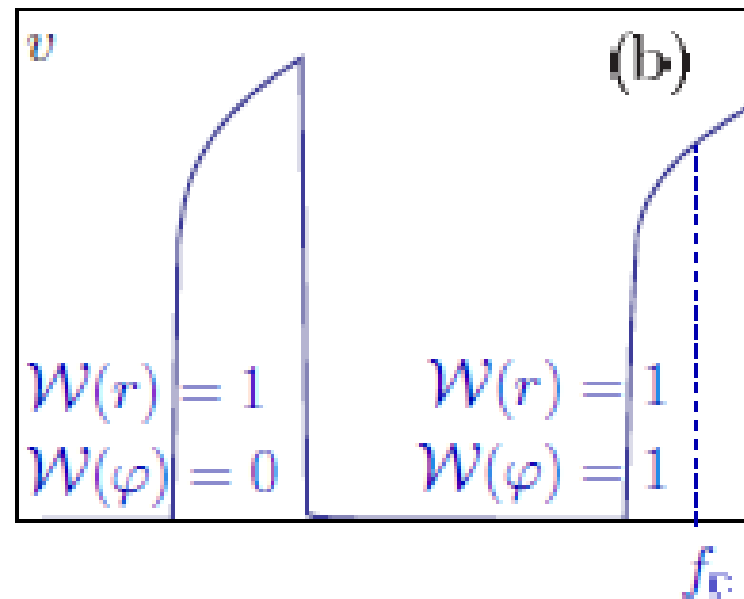
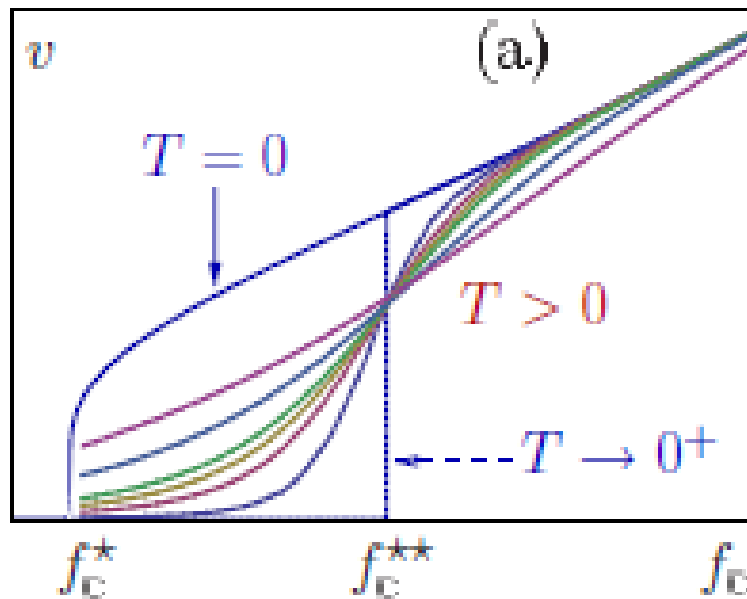


$$\alpha \lambda \partial_t r - \partial_t \varphi - \beta_s v_s = f_{\text{ext}}(r) + \eta_1,$$

$$\alpha \partial_t \varphi + \lambda \partial_t r - v_s = -\frac{1}{2} K_{\perp} \sin 2\varphi + \eta_2.$$

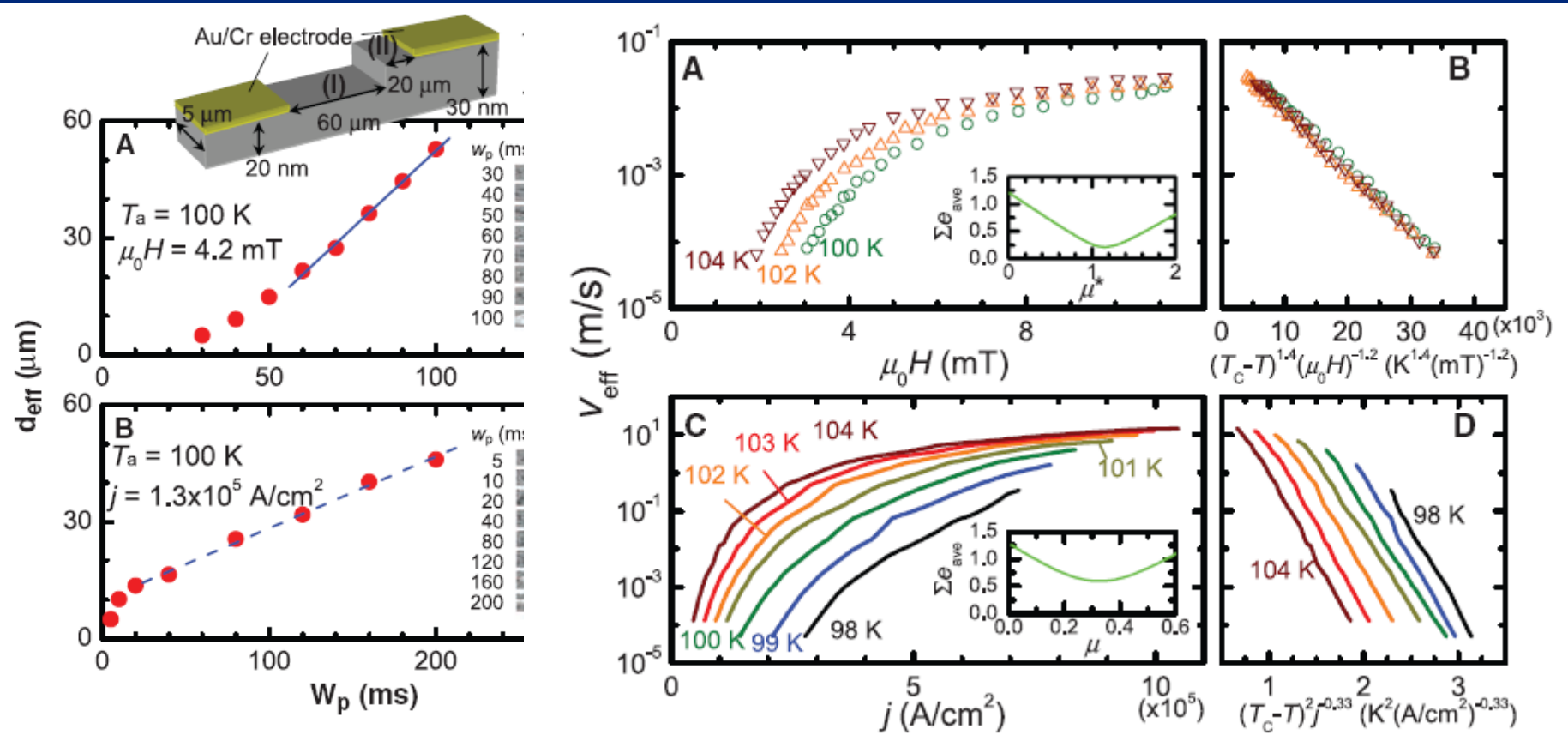
Different from standard depinning

$$\vec{S} = (-\varepsilon, 0) \quad \vec{H}_L = (0, 0)$$



force

Drive with magnetic field or current



Yamanouchi et al. Science 317 1726 (2007); H. Ohno Nature Materials 9 952 (2010)