Effective temperature and non-equilibrium dynamics in quantum gases

Laura Foini
DQMP, Université de Genève

in collaboration with T. Giamarchi

“Spin Glasses: an old tool for new problems”
Cargèse
August & September 2014
Non-equilibrium in quantum systems

- Non stationary dynamics
- Problem of equilibration
- Effective temperatures
- Generalized FDRs
A protocol of non-equilibrium: a quantum quench

1. Prepare the system in a state which is not an eigenstate of the Hamiltonian $H$
2. Evolve unitarily with $H$
3. Measure correlation functions
A protocol of non-equilibrium: a quantum quench

Cold atoms experiments

- No dissipation
- Many-body dynamics
- Large intrinsic time scales
- System parameters easily changed, high tunability
A protocol of non-equilibrium: a quantum quench

Cold atoms experiments

Group of J. Schmiedmayer
Vienna

Hofferberth et al., Nature 448 (2007)
Gring et al., Science 337 (2012)
Langen et al., Nature Physics (2013)
Our quench

\[ \psi_x = e^{i\theta x} \sqrt{\rho_x} \]

1. Coherent splitting \( t=0 \) \( \theta_1(x) \approx \theta_2(x) \)

System 1

System 2
Our quench

\[ \psi_x = e^{i\theta_x} \sqrt{\rho_x} \]

1. Coherent splitting \( t=0 \) \( \theta_1(x) \approx \theta_2(x) \)

2. \( t>0 \) \( H = H_1 + H_2 + H_{\text{tunneling}} \)
Interference as a probe

Release the trap ➔
Expansion of the cloud

\[ \theta_1(x) \]

\[ \theta_2(x) \]
Interference as a probe

$\theta_1(x)$

$\theta_2(x)$

Release the trap $\rightarrow$

Expansion of the cloud
Interference as a probe

Amplitude of the fringes (contrast)

Release the trap
Expansion of the cloud

\[
\langle |A|^2 \rangle \leftrightarrow \langle e^{i(\theta_1(z) - \theta_2(z))} e^{-i(\theta_1(z') - \theta_2(z'))} \rangle
\]

Polkovnikov et al., PNAS 103 (2006)
What one can learn

In general...

△ How do local observables evolve in time?

△ Do they approach a steady value?

△ Are they compatible with thermal equilibrium?
What one can learn

... and more specifically here

- Effect of **tunneling** on correlations?
- “**Prethermalization**” found in absence of tunneling

```
T_{in} = 120 \text{ nK} \quad T = 30 \text{ nK} \quad T_{\text{best fit}} = 14 \text{ nK}
```

Observables well described by thermal equilibrium, but with $T \ll T_{in}$

Gring et al., Science 337 (2012)
What one can learn

... and more specifically here

- Accessing temperature always subtle in cold atoms!
- Other studies in 1d not compatible with a single temperature scenario?!

see e.g. Kinoshita et al., Nature 440 (2006)
Theoretical description

Luttinger liquid theory

\[ H_{LL} = \frac{\hbar u}{2\pi} \int \, dx \left[ K (\nabla \theta)^2 + \frac{1}{K} (\nabla \phi)^2 \right] \]

\[ \psi^\dagger (x) = \sqrt{\rho(x)} e^{-i\theta(x)} \]

\[ \rho(x) = \rho - \frac{1}{\pi} \nabla \phi(x) \]

\[ [\theta(y), \frac{1}{\pi} \nabla \phi(x)] = i \delta(x - y) \]
Luttinger liquid theory

\[ H_{LL} = \frac{\hbar u}{2\pi} \int dx \left[ K (\nabla \theta)^2 + \frac{1}{K} (\nabla \phi)^2 \right] \]

\[ \psi^\dagger (x) = \sqrt{\rho(x)} e^{-i\theta(x)} \]

\[ \rho(x) = \rho - \frac{1}{\pi} \nabla \phi(x) \]

Theoretical description

LR repulsion  HC bosons  K=1  Experiment  K~30  K=∞  Free
Theoretical description

Luttinger liquid theory

Phonon-like excitations

\[ H_{LL} = \sum_{k\neq 0} \hbar u|k| b_k^\dagger b_k \]

\[ \phi(x) = -i \frac{\pi}{L} \sum_{p \neq 0} \frac{1}{p} \sqrt{\frac{|p|KL}{2\pi}} \ e^{-ipx-\alpha|p|/2} (b_p^\dagger + b_{-p}) \]

\[ \theta(x) = \frac{i\pi}{L} \sum_{p \neq 0} \frac{1}{|p|} \sqrt{\frac{L|p|}{2\pi K}} \ e^{-ipx-\alpha|p|/2} (b_p^\dagger - b_{-p}) \]
Coupled systems

\[ H = H_{LL}[\theta_1, \phi_1] + H_{LL}[\theta_2, \phi_2] - \frac{J}{2\pi} \int dx \, \psi_1^\dagger(x) \psi_2(x) + h.h. \]

\[ \rho \, e^{-i(\theta_1(x) - \theta_2(x))} \]
Coupled systems

\[ H = H_{LL}[\theta_1, \phi_1] + H_{LL}[\theta_2, \phi_2] - \frac{J}{2\pi} \int dx \psi_1^\dagger(x) \psi_2(x) + \text{h.h.} \]

\[ \rho e^{-i(\theta_1(x) - \theta_2(x))} \]

Symmetric and antisymmetric modes

\[ \theta_S = \theta_1 + \theta_2 \quad \theta_A = \theta_1 - \theta_2 \]

\[ \phi_S = \frac{\phi_1 + \phi_2}{2} \quad \phi_A = \frac{\phi_1 - \phi_2}{2} \]

Semiclassical approximation

\[ 2 \cos \theta_A(x) \rightarrow [\theta_A(x)]^2 \]
Initial state

\[ |\psi_0\rangle = \frac{1}{\mathcal{N}} \exp \left[ \int dk \ W_k b_k^\dagger b_{-k}^\dagger \right] |0\rangle \]

\[ W_k = \frac{1}{2} \frac{1 - \alpha_k}{1 + \alpha_k} \quad \alpha_k = \frac{|k| K}{\pi \rho} \]

\[ \langle \rho(x) \rho(x') \rangle_c = \frac{\rho}{2} \delta(x - x') \]

\[ \langle \theta(x) \theta(x') \rangle = \frac{1}{2\rho} \delta(x - x') \]

Bistritzer & Altman, PNAS 104 (2007)
Dynamics modes

\[ \langle |\theta_k|^2 \rangle(t) = \left( \frac{\pi u \rho}{\omega_k K} \right)^2 \sin^2(\omega_k t) \frac{2\rho}{2\rho} + \cos^2(\omega_k t) \frac{2\rho}{2\rho} \]

\[ \omega_k = \sqrt{m^2 + (uk)^2} \]

\[ m = \sqrt{\frac{uJ\rho}{K}} \]

Phase velocity depend on \( k \)
Correlation function

Two-point phase

\[ C(z, t) = \langle e^{i(\theta_A(z, t) - \theta_A(0, t))} \rangle \]

Ballistic expansion

\[ \langle |\hat{A}|^2 \rangle = \int_0^L dz \, dz' \, \langle \psi_1^\dagger(z) \psi_2(z) \psi_2^\dagger(z') \psi_1(z') \rangle \]

Polkovnikov et al., PNAS 103 (2006)
Correlation functions

(a) $t=3\text{ ms}$

(b) $t=6\text{ ms}$

(c) $t=9\text{ ms}$

(d) $t=12\text{ ms}$

$J=0$

$J=8\text{ Hz}$

$J=20\text{ Hz}$
Correlation functions

(a) $t=3\,\text{ms}$

(b) $t=6\,\text{ms}$

(c) $t=9\,\text{ms}$

(d) $t=12\,\text{ms}$

$C(z,t)$

$\propto \text{time}$
Correlation functions

Light-cone

Calabrese & Cardy, PRL 96 (2006)
Rieger & Iglói, PRB 84 (2011)
Cheneau et al., Nature 481 (2012)
Correlation functions

Thermal at

\[ T_{\text{eff}} = \frac{\pi \rho u}{4K} \]
Effective temperature

- Slow approach to steady value
- Asymptotic curve indistinguishable from thermic one
- \( T_{\text{eff}} \) not related to \( T_{\text{in}} \) and \( J \)

\[
T_{\text{eff}} = \frac{\pi \rho u}{4K}
\]
Effective temperature

\[ T_{\text{eff}} = \frac{\pi \rho u}{4K} \ll T_{\text{in}} \]

Without tunneling
- \( T_{\text{eff}} \) not related to \( T_{\text{in}} \)
- \( T_{\text{eff}} \) linear in \( \rho \)

Gring et al., Science 337 (2012)

With tunneling
- \( T_{\text{eff}} \) independent of \( J \)?
Effective temperature

Energy modes

\[ E_k(t) = E_k(0) = \frac{u\pi}{2K} \left[ \langle \rho_k \rho_{-k} \rangle(0) + \left( \frac{K\omega_k}{\pi u} \right)^2 \langle \theta_k \theta_{-k} \rangle(0) \right] \]
Effective temperature

Energy modes

\[ E_k(t) = E_k(0) = \frac{u\pi}{2K} \left[ \langle \rho_k \rho_{-k} \rangle(0) + \left( \frac{K\omega_k}{\pi u} \right)^2 \langle \theta_k \theta_{-k} \rangle(0) \right] \]

“Classical approximations”

Equipartition

\[ E_k = \frac{\pi \rho u}{4K} = T_{\text{eff}} \]

Zero
Effective temperature

**Equilibrium**

\[
\langle [\theta(x) - \theta(0)]^2 \rangle_T = \frac{u}{K} \int dp \ e^{-\alpha |p|} \frac{1}{\omega_p} (1 - \cos px) \coth \left( \frac{\omega_p}{2T} \right)
\]

**Non-equilibrium**

\[
\langle [\theta(x) - \theta(0)]^2 \rangle \sim \frac{u \pi^2 \rho}{4mK^2}
\]

\[
E_k = \frac{\pi \rho u}{4K} = T_{\text{eff}}
\]

```
 Classical approximation
```

\[
\omega_p = \sqrt{m^2 + (up)^2} \quad m = \sqrt{\frac{uJ\rho}{K}}
\]
Two-time correlations

\[ R(x, t; x', t') = i \theta(t - t') \langle [\theta(x, t), \theta(x', t')] \rangle = \]

\[ = \theta(t - t') \frac{1}{2\pi} \int dk \ e^{-ik(x-x')} e^{-\alpha^2 k^2} \frac{\pi u}{\omega_k K} \sin(\omega_k(t - t')) \]

\[ C(x, t; x', t') = \frac{1}{2} \langle \{\theta(x, t), \theta(x', t')\} \rangle = \]

\[ = \frac{1}{4\pi} \int dk \ e^{-ik(x-x')} e^{-\alpha^2 k^2} \left( (\cos \omega_k(t - t') - \cos \omega_k(t + t')) \left( \frac{u\pi}{\omega_k K} \right)^2 \langle |n_k|^2 \rangle(0) \right. \]

\[ + (\cos \omega_k(t - t') + \cos \omega_k(t + t')) \left. \langle |\theta_k|^2 \rangle(0) \right) \]
Two-time correlations

\[ R(x, t; x', t') = i\theta(t - t') \langle [\theta(x, t), \theta(x', t')] \rangle = \]

\[ = \theta(t - t') \frac{1}{2\pi} \int dk \ e^{-ik(x-x')} e^{-\alpha^2 k^2} \frac{\pi u}{\omega_k K} \sin(\omega_k(t - t')) \]

\[ C(x, t; x', t') = \frac{1}{2} \langle \{\theta(x, t), \theta(x', t')\} \rangle = \]

\[ = \frac{1}{4\pi} \int dk \ e^{-ik(x-x')} e^{-\alpha^2 k^2} \left( (\cos \omega_k(t - t') - \cos \omega_k(t + t')) \left( \frac{u\pi}{\omega_k K} \right)^2 \langle \|n_k\|^2 \rangle(0) \right) + (\cos \omega_k(t - t') + \cos \omega_k(t + t')) \langle \|\theta(t)\|^2 \rangle(0) \]

The classical FDT is recovered

\[ R(t) = -\frac{1}{T_{\text{eff}}} \theta(t) \frac{d}{dt} C(t) \]

\[ T_{\text{eff}} = \frac{\pi u \rho}{4K} \]
Conclusions

- Relaxation dynamics accessible in cold atoms
- One and two-time correlation functions
- Effect of tunneling on correlation functions
- Effective thermal behavior unaffected
- Classical FDT recovered in some limit
What about coupling between symmetric and antisymmetric modes?

And strong interactions/fermions?

Go beyond semiclassical approximation

Different splitting processes
Perspectives

△ What about coupling between symmetric and antisymmetric modes?

△ And strong interactions/fermions?

△ Go beyond semiclassical approximation

△ Different splitting processes

Thank you!