Effective temperature and non-equilibrium dynamics in quantum gases

Laura Foini
DQMP, Université de Genève

in collaboration with T. Giamarchi

"Spin Glasses: an old tool for new problems"
Cargèse
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UNIVERSITÉ DE GENÈVE

Non-equilibrium in quantum systems

- Non stationary dynamics
- Problem of equilibration
- Effective temperatures
- Generalized FDRs

A protocol of non-equilibrium: a quantum quench

1. Prepare the system in a state which is not an eigenstate of the Hamiltonian H
2. Evolve unitarily with H
3. Measure correlation functions

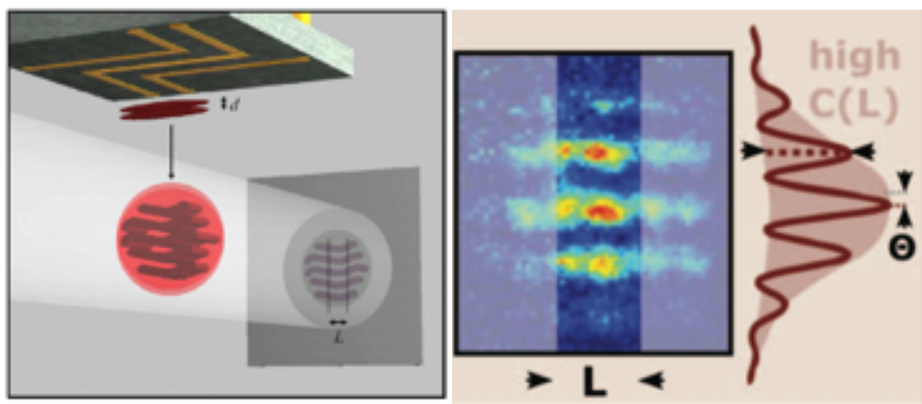
A protocol of non-equilibrium: a quantum quench

Cold atoms experiments

- No dissipation
- Many-body dynamics
- Large intrinsic time scales
- System parameters easily changed, high tunability

A protocol of non-equilibrium: a quantum quench

Cold atoms experiments



Group of J. Schmiedmayer
Vienna

Hofferberth et al., Nature 448 (2007)
Gring et al., Science 337 (2012)
Langen et al., Nature Physics (2013)

nature Vol 449|20 September 2007|doi:10.1038/nature06149

LETTERS

Non-equilibrium coherence dynamics in one-dimensional Bose gases

S. Hofferberth[1,2], I. Lesanovsky[3], B. Fischer[1], T. Schumm[2] & J. Schmiedmayer[1,2]

LETTERS
PUBLISHED ONLINE: 8 SEPTEMBER 2013 | DOI: 10.1038/NPHYS2739
nature physics

Local emergence of thermal correlations in an isolated quantum many-body system

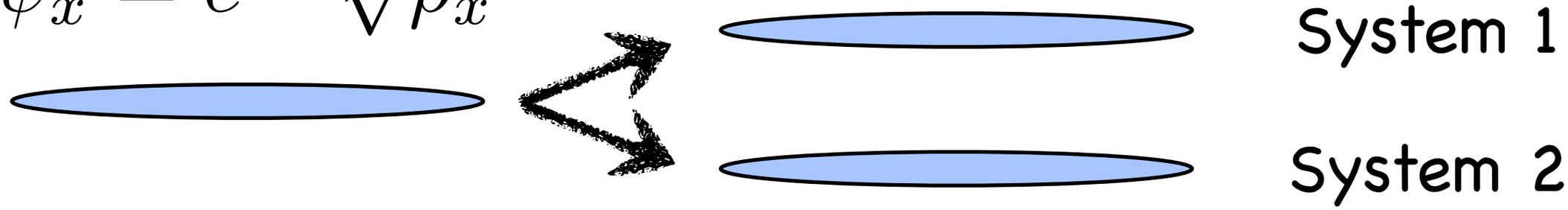
T. Langen*, R. Geiger, M. Kuhnert, B. Rauer and J. Schmiedmayer*

Science AAAS

Relaxation and Prethermalization in an Isolated Quantum System
M. Gring *et al.*
Science **337**, 1318 (2012);
DOI: 10.1126/science.1224953

Our quench

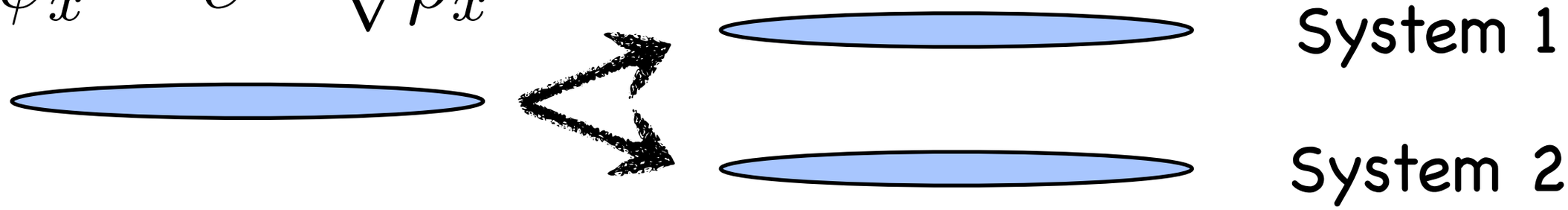
$$\psi_x = e^{i\theta_x}\sqrt{\rho_x}$$



1 Coherent splitting t=0 $\theta_1(x) \approx \theta_2(x)$

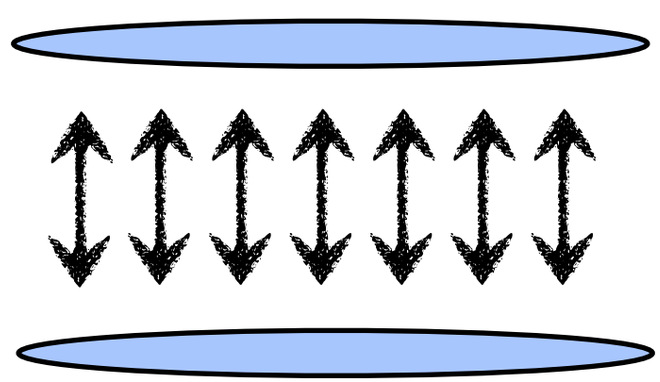
Our quench

$$\psi_x = e^{i\theta_x}\sqrt{\rho_x}$$

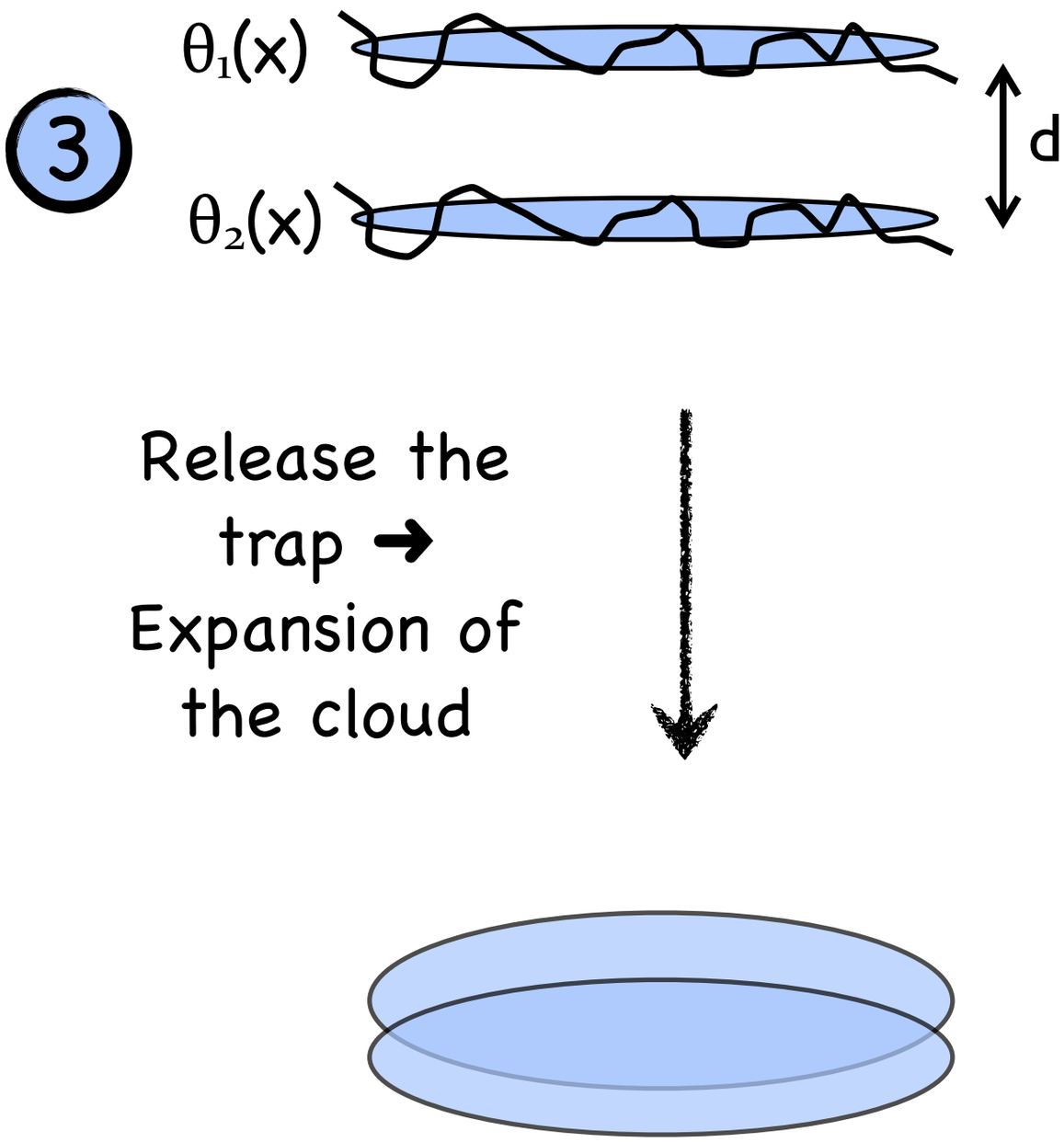


1. Coherent splitting t=0 $\theta_1(x) \approx \theta_2(x)$

2. t>0 $H = H_1 + H_2 + H_{tunneling}$

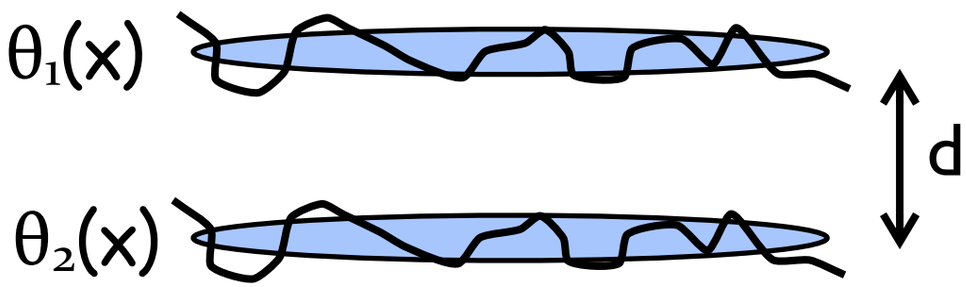


Interference as a probe

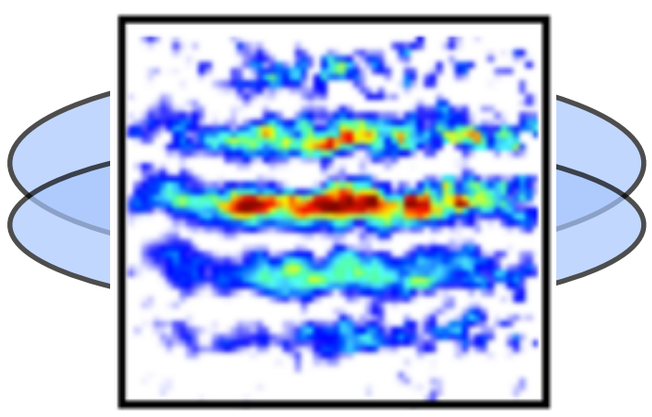


Interference as a probe

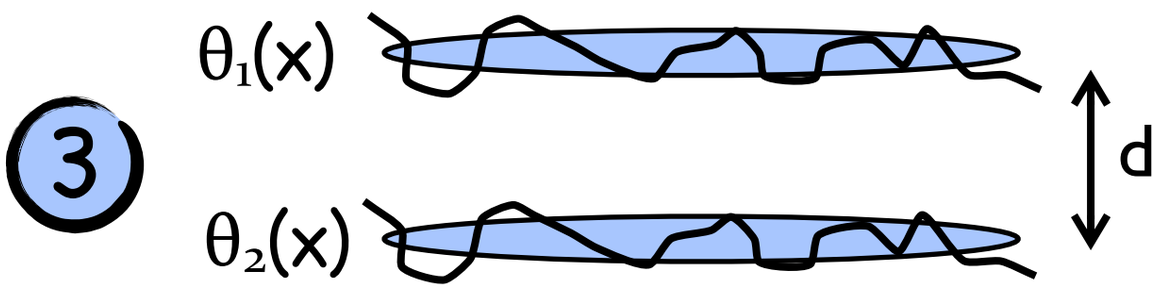
3



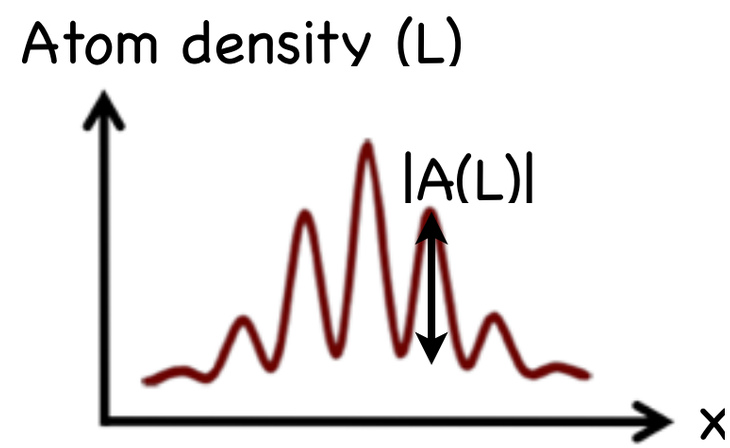
Release the trap ➜ Expansion of the cloud



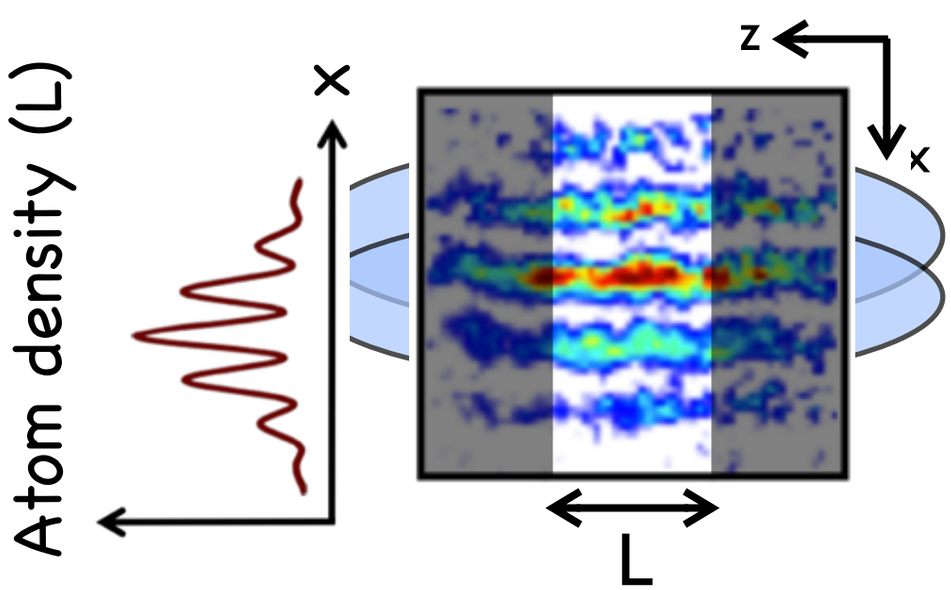
Interference as a probe



Amplitude of the fringes (contrast)



Release the trap ➜ Expansion of the cloud



$$\langle |A|^2 \rangle \leftrightarrow \langle e^{i(\theta_1(z)-\theta_2(z))} e^{-i(\theta_1(z')-\theta_2(z'))} \rangle$$

Polkovnikov et al., PNAS 103 (2006)

What one can learn

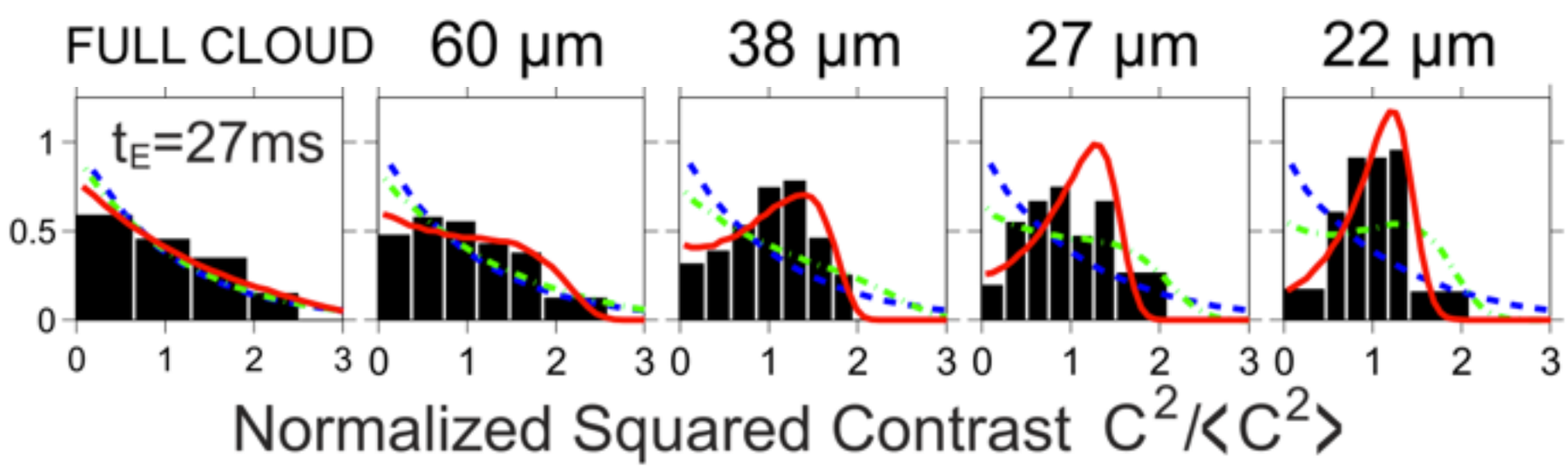
In general...

- How do local observables evolve in time?
- Do they approach a steady value?
- Are they compatible with thermal equilibrium?

What one can learn

… and more specifically here

- Effect of **tunneling** on correlations?
- "**Prethermalization**" found in absence of tunneling



T_{in}=120 nK T=30 nK $T_{best\ fit}$=14 nK

Observables well described by thermal equilibrium, but with $T \ll T_{in}$

Gring et al., Science 337 (2012)

What one can learn

... and more specifically here

- Accessing temperature always subtle in cold atoms!
- Other studies in 1d not compatible with a single temperature scenario?!

see e.g. Kinoshita et al., Nature 440 (2006)

Theoretical description

Luttinger liquid theory

$$H_{LL} = \frac{\hbar u}{2\pi} \int \mathrm{d}x \left[\mathcal{K} (\nabla \theta)^2 + \frac{1}{\mathcal{K}} (\nabla \phi)^2 \right]$$

$$\psi^\dagger(x) = \sqrt{\rho(x)} e^{-i\theta(x)}$$

$$\rho(x) = \rho - \frac{1}{\pi} \nabla \phi(x)$$

$$[\theta(y), \frac{1}{\pi} \nabla \phi(x)] = i\, \delta(x - y)$$

Theoretical description

Luttinger liquid theory

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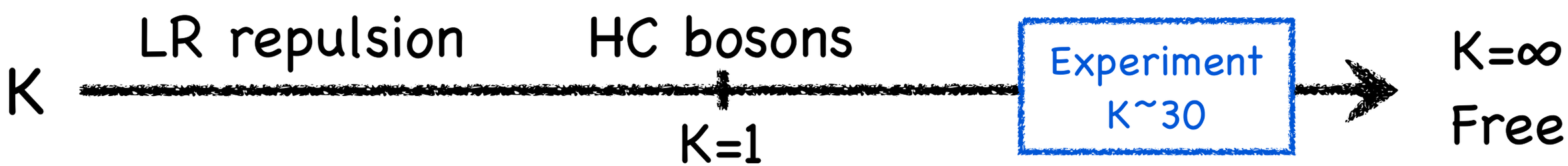


Theoretical description

Luttinger liquid theory

Phonon-like excitations

$$H_{LL} = \sum_{k \neq 0} \hbar u |k| \, b_k^\dagger b_k$$

$$\phi(x) = -i \, \frac{\pi}{L} \sum_{p \neq 0} \frac{1}{p} \sqrt{\frac{|p| \mathcal{K} L}{2\pi}} \, e^{-ipx - \alpha |p|/2} \, (b_p^\dagger + b_{-p})$$

$$\theta(x) = \frac{i\pi}{L} \sum_{p \neq 0} \frac{1}{|p|} \sqrt{\frac{L|p|}{2\pi \mathcal{K}}} e^{-ipx - \alpha |p|/2} (b_p^\dagger - b_{-p})$$

Coupled systems

$$H = H_{LL}[\theta_1, \phi_1] + H_{LL}[\theta_2, \phi_2] - \frac{J}{2\pi} \int \mathrm{d}x\, \psi_1^\dagger(x)\psi_2(x) + h.h.$$

$$\rho\, e^{-i(\theta_1(x) - \theta_2(x))}$$

Coupled systems

$$H = H_{LL}[\theta_1, \phi_1] + H_{LL}[\theta_2, \phi_2] - \frac{J}{2\pi} \int \mathrm{d}x\, \psi_1^\dagger(x)\psi_2(x) + h.h.$$

$$\rho\, e^{-i(\theta_1(x) - \theta_2(x))}$$

Symmetric and antisymmetric modes

+

Semiclassical approximation

$$\theta_S = \theta_1 + \theta_2 \qquad \theta_A = \theta_1 - \theta_2$$

$$\phi_S = \frac{\phi_1 + \phi_2}{2} \qquad \phi_A = \frac{\phi_1 - \phi_2}{2}$$

$$2\cos\theta_A(x) \;\rightarrow\; [\theta_A(x)]^2$$

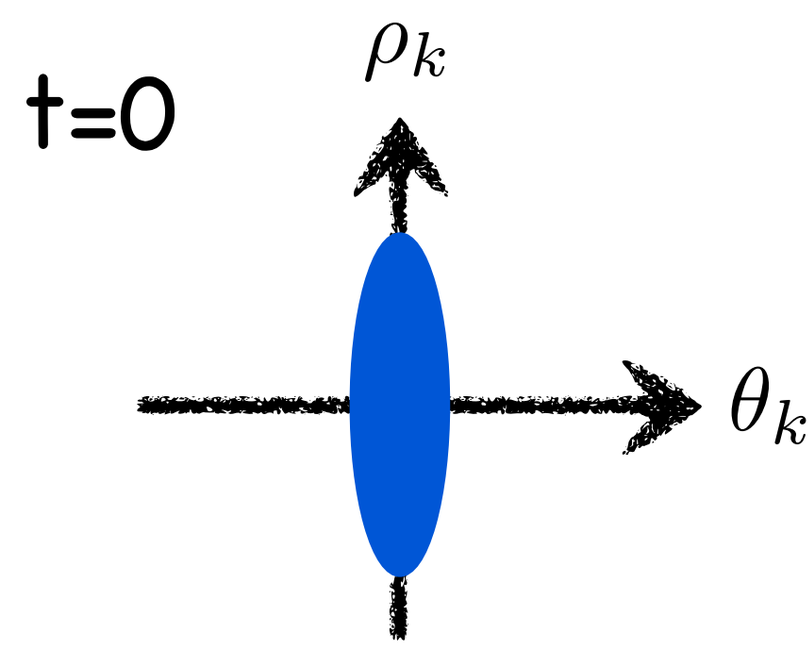
Initial state

$$|\psi_0\rangle = \frac{1}{\mathcal{N}} \exp\left[\int \mathrm{d}k\, W_k b_k^\dagger b_{-k}^\dagger\right] |0\rangle$$

$$W_k = \frac{1}{2}\frac{1-\alpha_k}{1+\alpha_k} \qquad \alpha_k = \frac{|k|K}{\pi\rho}$$

$$\langle \rho(x)\rho(x')\rangle_c = \frac{\rho}{2}\,\delta(x-x')$$

$$\langle \theta(x)\theta(x')\rangle = \frac{1}{2\rho}\delta(x-x')$$



Bistritzer & Altman, PNAS 104 (2007)
Kitagawa & al., New J. Phys. 13 (2011)

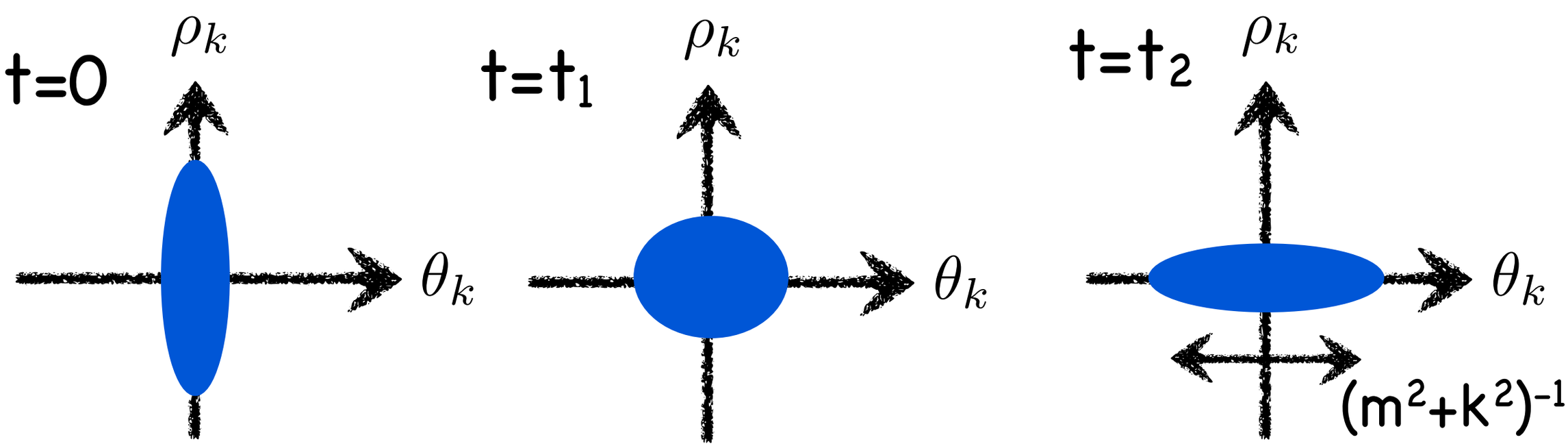
Dynamics modes

$$\langle|\theta_k|^2\rangle(t) = \left(\frac{\pi u \rho}{\omega_k K}\right)^2 \frac{\sin^2(\omega_k t)}{2\rho} + \frac{\cos^2(\omega_k t)}{2\rho}$$

$$\omega_k = \sqrt{m^2 + (uk)^2}$$

$$m = \sqrt{\frac{uJ\rho}{K}}$$

Phase velocity depend on k



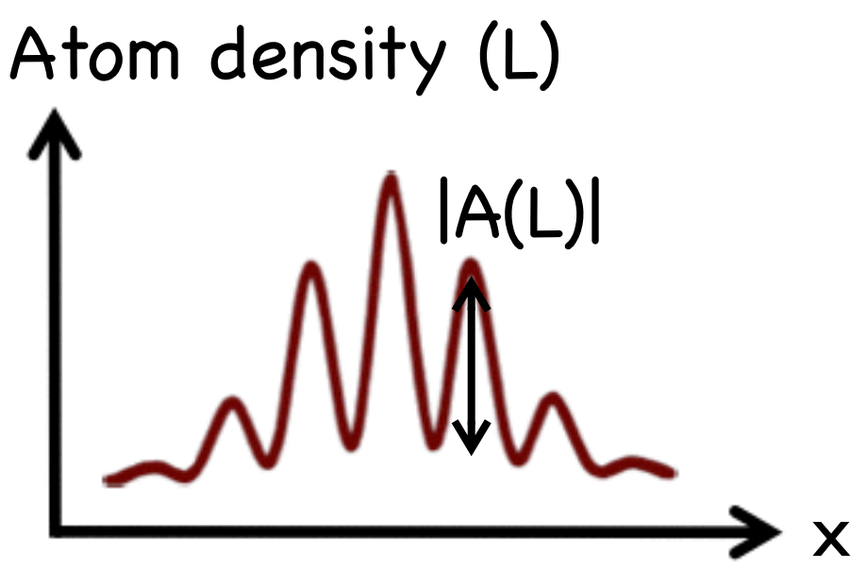
Correlation function

Two-point phase

$$C(z,t) = \langle e^{i(\theta_A(z,t)-\theta_A(0,t))} \rangle$$

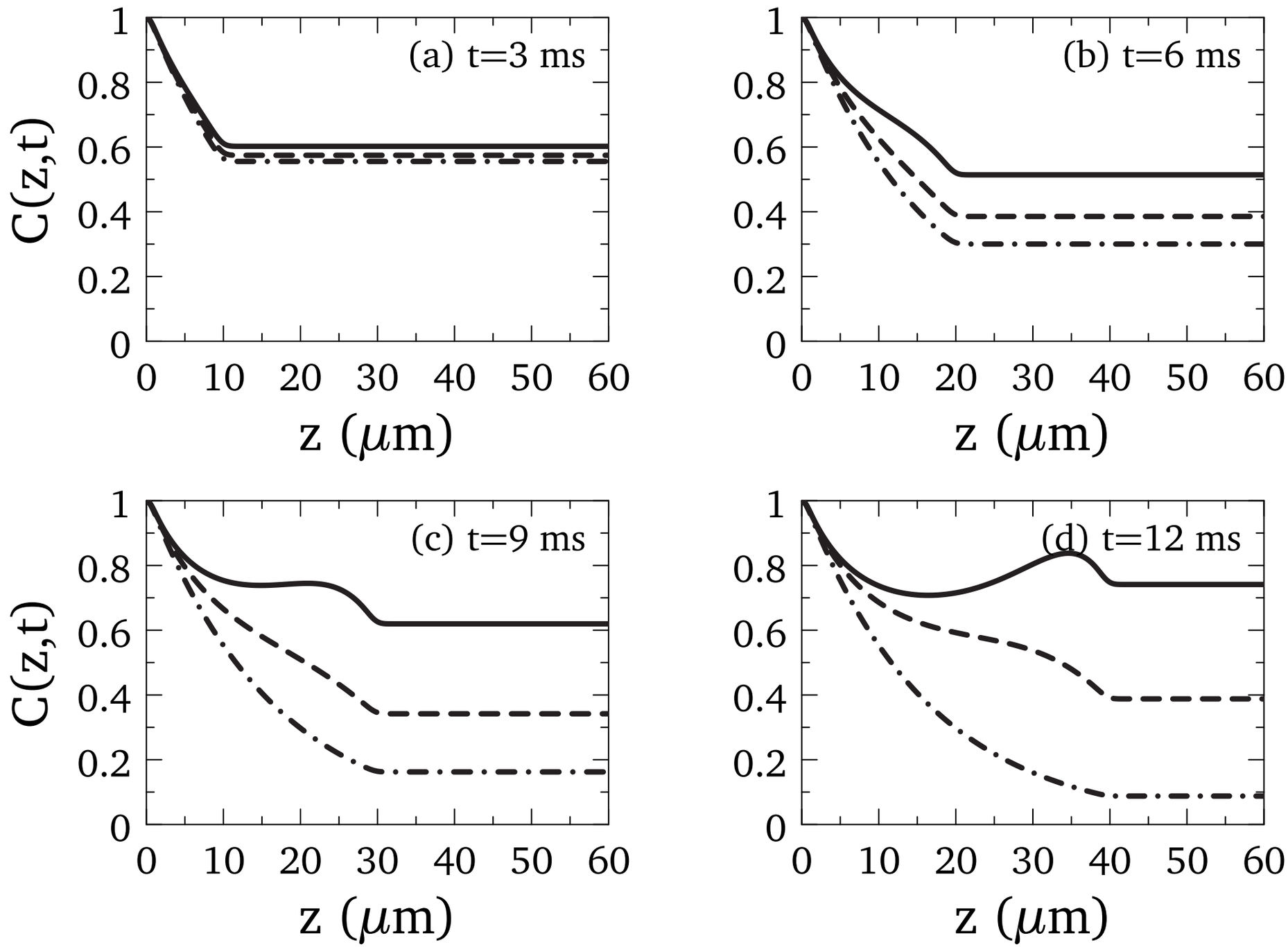
Ballistic expansion

$$\langle |\hat{A}|^2 \rangle = \int_0^L \mathrm{d}z\, \mathrm{d}z'\, \langle \psi_1^\dagger(z)\psi_2(z)\psi_2^\dagger(z')\psi_1(z') \rangle$$



Polkovnikov et al., PNAS 103 (2006)

Correlation functions

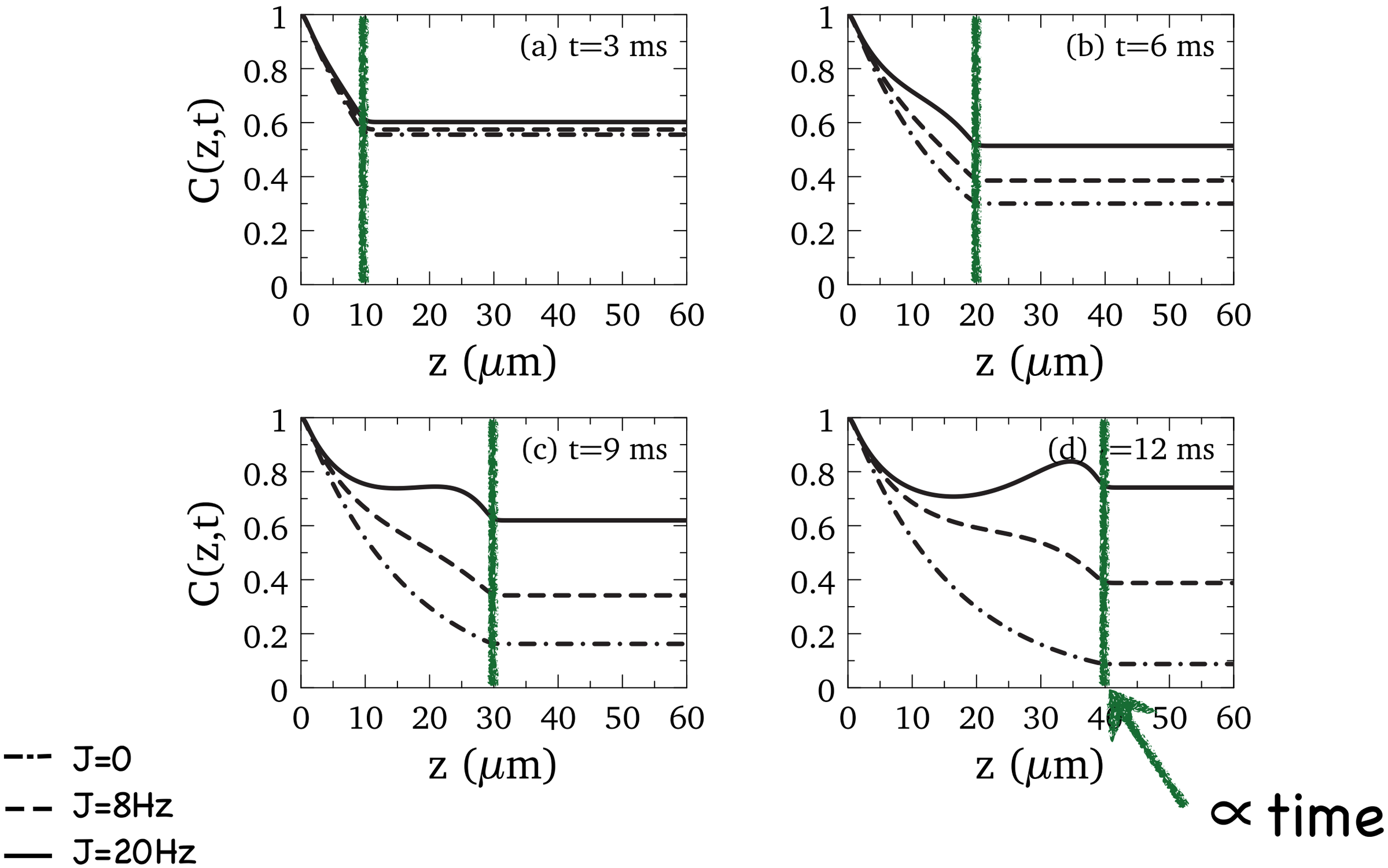


J=0

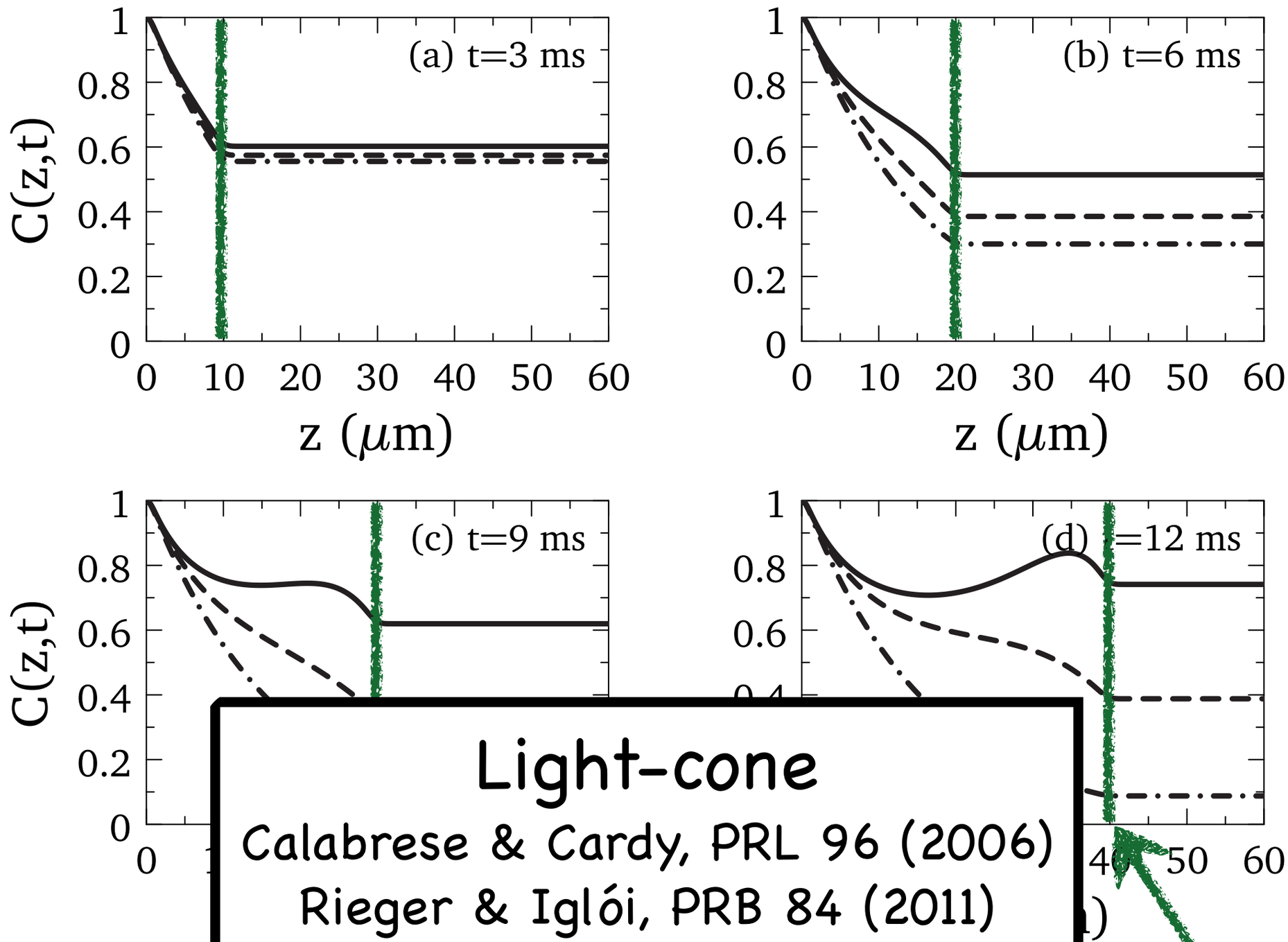
J=8Hz

J=20Hz

Correlation functions



Correlation functions



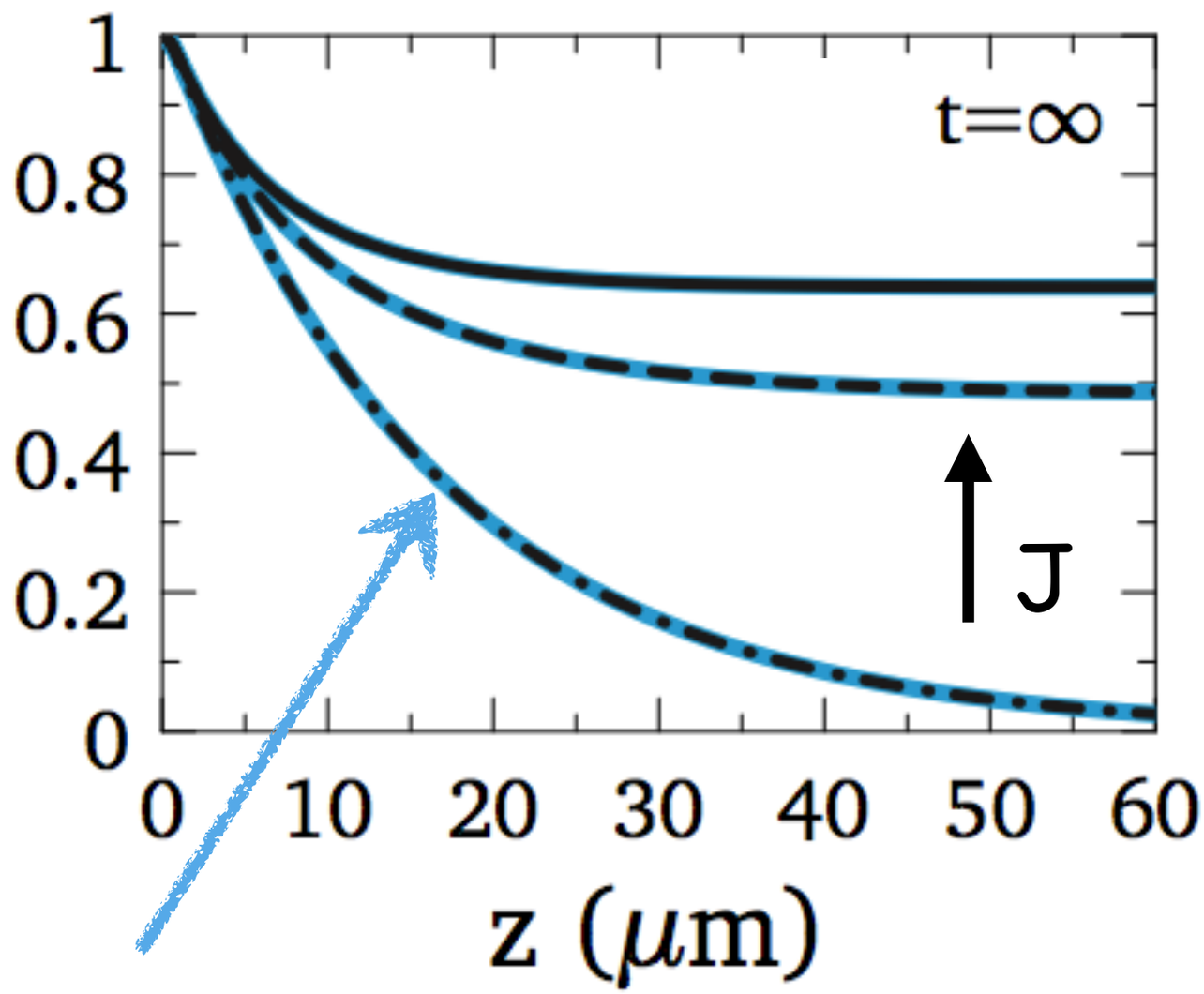
Light-cone

Calabrese & Cardy, PRL 96 (2006)
Rieger & Iglói, PRB 84 (2011)
Cheneau et al., Nature 481 (2012)

∝ time

J=0
J=8Hz
J=20Hz

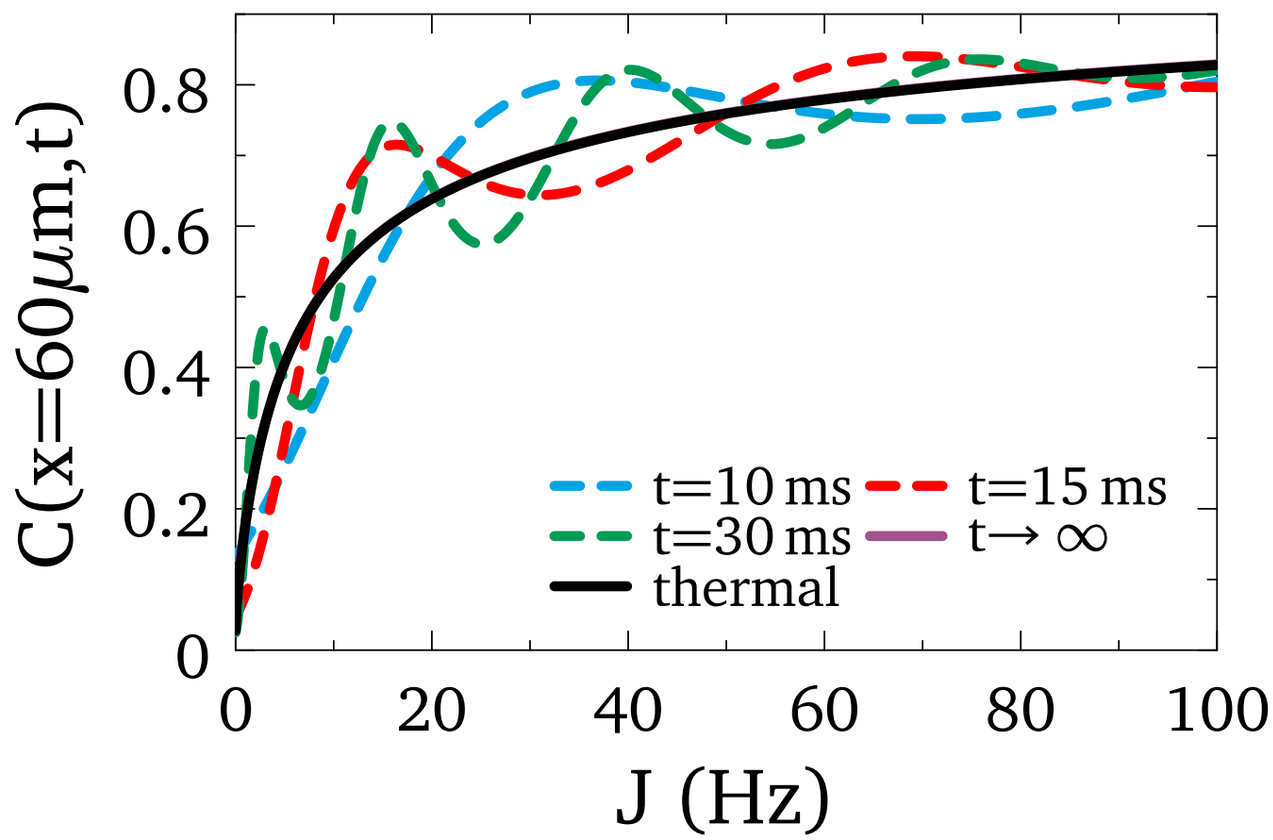
Correlation functions



Thermal at

$$T_{\text{eff}} = \frac{\pi \rho u}{4K}$$

Effective temperature



- Slow approach to steady value
- Asymptotic curve indistinguishable from thermic one
- T_{eff} not related to T_{in} and J

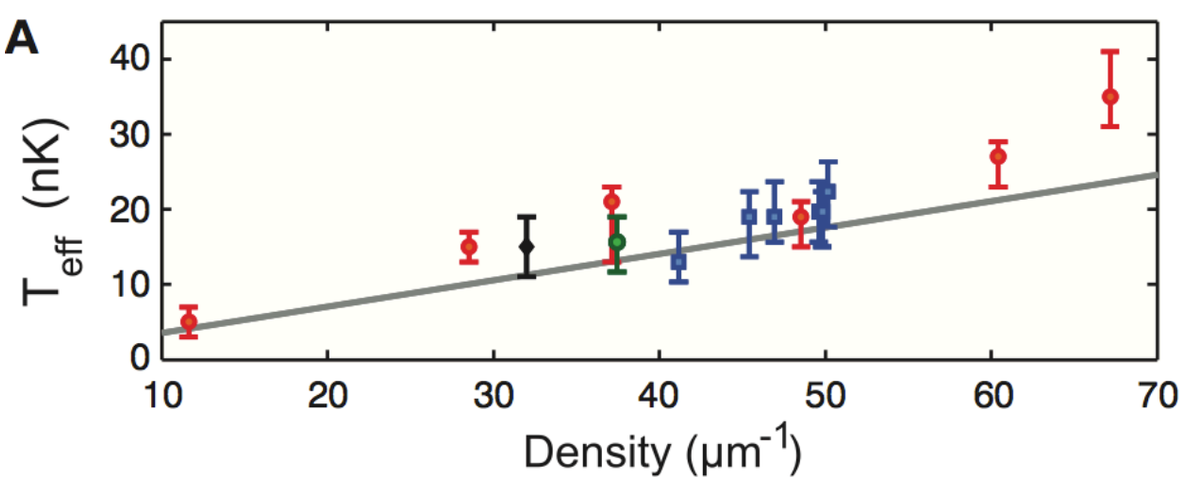
$$T_{\text{eff}} = \frac{\pi \rho u}{4K}$$

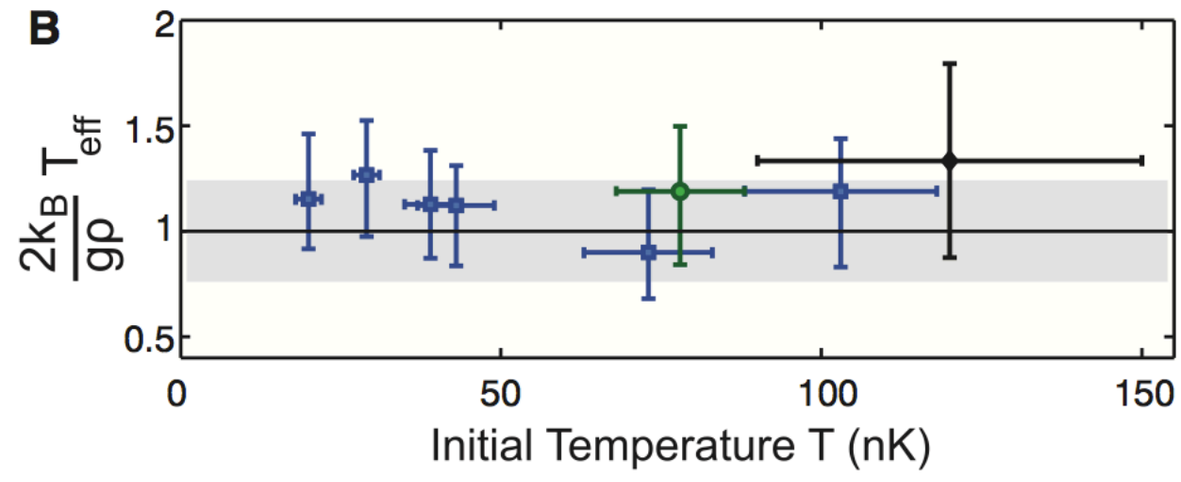
Effective temperature

$$T_{\rm eff} = \frac{\pi \rho u}{4K}$$ << T_{in}

Without tunneling

- T_{eff} not related to T_{in}
- T_{eff} linear in ρ





Gring et al., Science 337 (2012)

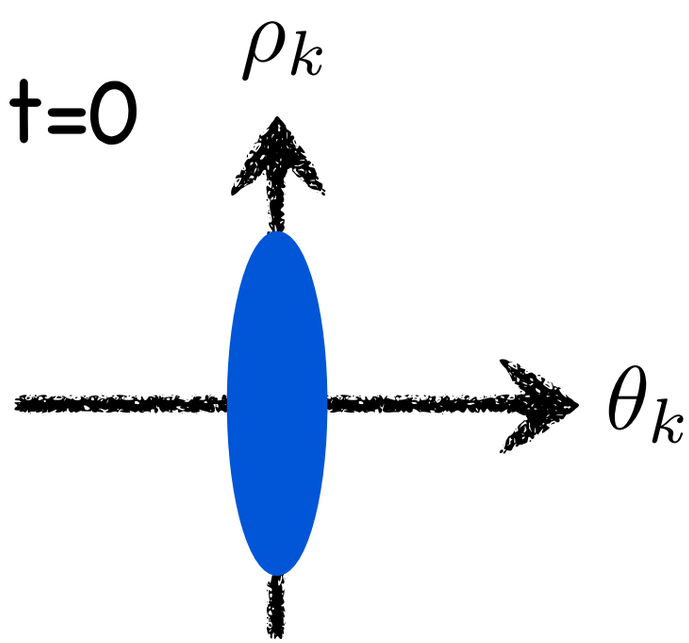
With tunneling

- T_{eff} independent of J?

Effective temperature

Energy modes

$$E_k(t) = E_k(0) = \frac{u\pi}{2K}\left[\langle\rho_k\rho_{-k}\rangle(0) + \left(\frac{K\omega_k}{\pi u}\right)^2 \langle\theta_k\theta_{-k}\rangle(0)\right]$$



Large

Small, but quantum mechanically non zero

Effective temperature

Energy modes

$$E_k(t) = E_k(0) = \frac{u\pi}{2K}\left[\langle\rho_k\rho_{-k}\rangle(0) + \left(\frac{K\omega_k}{\pi u}\right)^2 \langle\theta_k\theta_{-k}\rangle(0)\right]$$

"Classical approximations"

Equipartition

Zero

$$E_k = \frac{\pi\rho\, u}{4K} = T_{\text{eff}}$$

Effective temperature

``Classical approximation"

Equilibrium

$$\langle[\theta(x)-\theta(0)]^2\rangle_T = \frac{u}{K}\int \mathrm{d}p\; e^{-\alpha|p|}\frac{1}{\omega_p}(1-\cos px)\mathrm{cotanh}\left(\frac{\omega_p}{2T}\right) \xrightarrow[x\gg 1, T\gg 1]{} \frac{\pi T}{Km}$$

Non-equilibrium

$$\langle[\theta(x)-\theta(0)]^2\rangle \sim \frac{u\pi^2\rho}{4mK^2}$$

$$\omega_p = \sqrt{m^2+(up)^2} \quad m = \sqrt{\frac{uJ\rho}{K}}$$

$$E_k = \frac{\pi\rho\, u}{4K} = T_{\mathrm{eff}}$$

Two-time correlations

$$
\begin{aligned}
R(x,t;x',t') &= i\theta(t-t')\langle[\theta(x,t),\theta(x',t')]\rangle = \\
&= \theta(t-t')\frac{1}{2\pi}\int \mathrm{d}k\; e^{-ik(x-x')}\; e^{-\alpha^2k^2}\,\frac{\pi u}{\omega_k K}\,\sin(\omega_k(t-t'))
\end{aligned}
$$

$$
\begin{aligned}
C(x,t;x',t') &= \frac{1}{2}\langle\{\theta(x,t),\theta(x',t')\}\rangle = \\
&= \frac{1}{4\pi}\int \mathrm{d}k\; e^{-ik(x-x')}e^{-\alpha^2k^2}\,\Big((\cos\omega_k(t-t')-\cos\omega_k(t+t'))\,\left(\frac{u\pi}{\omega_k K}\right)^2\langle|n_k|^2\rangle(0) \\
&\quad +(\cos\omega_k(t-t')+\cos\omega_k(t+t'))\;\langle|\theta_k|^2\rangle(0)\Big)
\end{aligned}
$$

Two-time correlations

$$R(x,t;x',t') = i\theta(t-t')\langle[\theta(x,t),\theta(x',t')]\rangle =$$

$$= \theta(t-t')\frac{1}{2\pi}\int \mathrm{d}k\; e^{-ik(x-x')}\; e^{-\alpha^2k^2}\,\frac{\pi u}{\omega_k K}\,\sin(\omega_k(t-t'))$$

$$C(x,t;x',t') = \frac{1}{2}\langle\{\theta(x,t),\theta(x',t')\}\rangle =$$

$$= \frac{1}{4\pi}\int \mathrm{d}k\; e^{-ik(x-x')}e^{-\alpha^2k^2}\,\Big((\cos\omega_k(t-t') - \cancel{\cos\omega_k(t+t')})\,\left(\frac{u\pi}{\omega_k K}\right)^2 \langle|n_k|^2\rangle(0)$$

$$+(\cos\omega_k(t-t') + \cos\omega_k(t+t'))\;\cancel{\langle|\theta_k|^2\rangle(0)}\Big)$$

$$\frac{\rho}{2}$$

The classical FDT is recovered

$$R(t) = -\frac{1}{T_{\mathrm{eff}}}\theta(t)\;\frac{\mathrm{d}}{\mathrm{d}t}C(t)$$

$$T_{\mathrm{eff}} = \frac{\pi u\rho}{4K}$$

Conclusions

- Relaxation dynamics accessible in cold atoms
- One and two-time correlation functions
- Effect of tunneling on correlation functions
- Effective thermal behavior unaffected
- Classical FDT recovered in some limit

Perspectives

- What about coupling between symmetric and antisymmetric modes?
- And strong interactions/fermions?
- Go beyond semiclassical approximation
- Different splitting processes

Perspectives

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- And strong interactions/fermions?
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Thank you!