Ising-like effective theory for the glass transition

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Spin glasses: an old tool for new problems
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with G. Biroli, C. Cammarota, and G. Tarjus

Introduction & motivation

Effective theory of the glass transition in terms the overlap $p(x)$ with an equilibrium configuration

•It allows to focus on the relevant field and on the "physical" order parameter and to obtain a more intuitive description of the glass transition

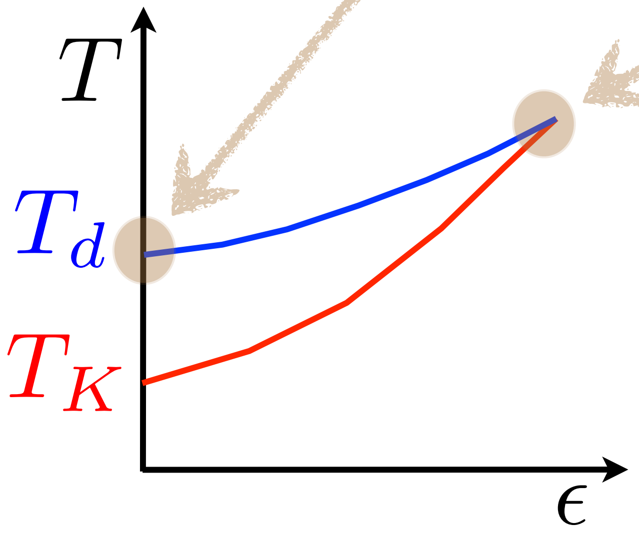
•It leads naturally to a scalar field theory in presence of quenched disorder, which is easier to handle than the original replica field theory. It can be studied using standard tools of statistical physics (nonperturbative RG, numerical simulations, ...) cfr Gille's lecture

•It allows to go beyond mean-field theory and RFOT, and to study the nature and the critical properties of the critical points. It naturally allows to identify the possible mechanisms that could destroy the glass transition in finite dimensions

Known results: Self-induced disorder and RFIM

- Analysis of perturbation theory of the Replica Field Theory

Critical overlap fluctuations close to the dynamical transition in the β-regime are in the same universality class of the spinodal point of the RFIM Franz, Parisi, Ricci-Tersenghi, Rizzo



The terminal critical point in the T-ϵ phase diagram is in the same universality class of the RFIM
Franz & Parisi; Biroli, Cammarota, Tarjus, MT

The same result holds for the continuous glass transition found at the terminal point of the random pinning phase diagram
Cammarota & Biroli; Nandi & Biroli

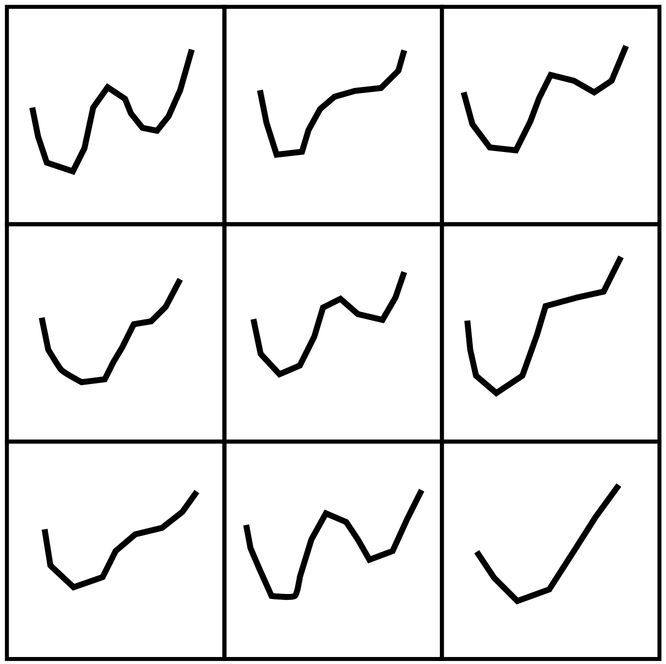
- The distribution of the overlap fluctuations has been computed in numerical simulations of glass forming systems and have been interpreted in terms of an effective RFIM Stevenson & al

Intuituve arguments

cfr Giulio's lecture and Silvio's talk

The equilibrium reference configuration acts as a random field

Local fluctuations of the Franz-Parisi potential due to the density fluctuations of the reference configuration



↓

Local fluctuations of the configurational entropy and of the surface tension

↓

Random field random bond Ising model

overlap p
configurational entropy s_c
height of the barrier

→

magnetization m
magnetic field h
ferromagnetic coupling J

What do we want/need to compute?

Choose an equilibrium reference configuration $\mathcal{C}_0$ at random (according to $e^{-\beta\mathcal{H}(\mathcal{C}_0)}/Z$)

Compute the probability that a copy of the system has an overlap profile $p(x)$ with the reference configuration:

$$e^{-\mathcal{S}[p(x)|\mathcal{C}_0]} \propto \sum_{\mathcal{C}} e^{-\beta\mathcal{H}(\mathcal{C})}\delta[p(x) - Q_x(\mathcal{C},\mathcal{C}_0)]$$

The cumulants of $\mathcal{S}[p(x)|\mathcal{C}_0]$ can be computed through an expansion in free replica sums: Tarjus & Tissier (cfr Gille's lecture)

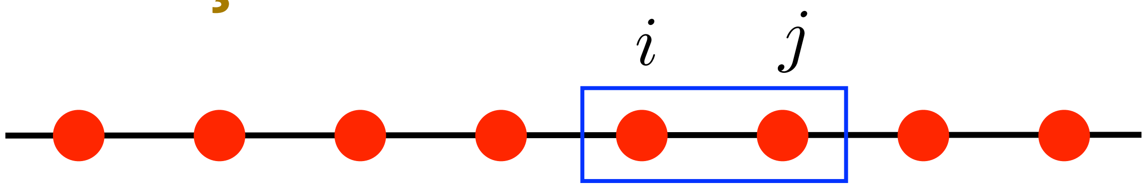
$$e^{-\mathcal{S}_{rep}[\{p_a(x)\}]} = \overline{e^{-\sum_{a=1}^{n}\mathcal{S}[p(x)|\mathcal{C}_0]}}^{\mathcal{C}_0}$$

$$\mathcal{S}_{rep}[\{p_a(x)\}] = \sum_{a=1}^{n}\mathcal{S}_1[p_a(x)] - \frac{1}{2}\sum_{a,b=1}^{n}\mathcal{S}_2[p_a(x),p_b(x)] + \cdots$$

$$\mathcal{S}_1[p(x)] = \overline{\mathcal{S}[p(x)|\mathcal{C}_0]}\,; \qquad \mathcal{S}_2[p_1(x),p_2(x)] = \overline{\mathcal{S}[p_1(x)|\mathcal{C}_0]\mathcal{S}[p_2(x)|\mathcal{C}_0]}^{c}$$

The Kaç version of the Random Energy Model

Franz, Parisi, Ricci-Tersenghi



2^M configurations on each site: $\mathcal{C}_i = \{1, \ldots, 2^M\}$

$$\mathcal{H} = \sum_{\langle ij \rangle} E_{\langle ij \rangle}(\mathcal{C}_i, \mathcal{C}_j)$$

Random energy on each link $\langle ij \rangle$

$E_{\langle ij \rangle} = E(\mathcal{C}_i, \mathcal{C}_j)$ iid Gaussian

$$\overline{E(\mathcal{C}_i, \mathcal{C}_j)} = 0$$

$$\overline{E(\mathcal{C}_i, \mathcal{C}_j) E(\mathcal{C}'_i, \mathcal{C}'_j)} = M \delta_{\mathcal{C}_i, \mathcal{C}'_i} \delta_{\mathcal{C}_j, \mathcal{C}'_j}$$

Compute the replicated action $\alpha = 0, \ldots, n \longrightarrow n+1$ replicas

$$e^{-\mathcal{S}_{rep}[\{p_a(i)\}]} \propto \overline{\sum_{\{\mathcal{C}_i^\alpha\}} \exp\left(-\beta \sum_{\langle ij \rangle, \alpha} E_{\langle ij \rangle}(\mathcal{C}_i^\alpha, \mathcal{C}_j^\alpha) \right) \prod_{a,i} \delta_{p_a(i), q(\mathcal{C}_i^0, \mathcal{C}_i^a)}}$$

$$= \sum_{\{\mathcal{C}_i^\alpha\}} \exp\left(\frac{\beta^2 M}{2} \sum_{\langle ij \rangle} \sum_{\alpha, \beta} \delta_{\mathcal{C}_i^\alpha, \mathcal{C}_i^\beta} \delta_{\mathcal{C}_j^\alpha, \mathcal{C}_j^\beta} \right) \prod_{a,i} \delta_{p_a(i), q(\mathcal{C}_i^0, \mathcal{C}_i^a)}$$

The overlap matrix

$$\delta_{\mathcal{C}^\alpha,\mathcal{C}^\beta} = \begin{pmatrix} 1 & p_1 & p_2 & \cdots & p_n \\ p_1 & 1 & q_{12} & \cdots & q_{1n} \\ p_2 & q_{21} & 1 & \cdots & q_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_n & q_{n1} & q_{n2} & \cdots & 1 \end{pmatrix}$$

Overlap p_a with the reference configuration

Overlap q_{ab} among the free replicas

$\mathcal{C}_i^a = \mathcal{C}_i^0$ and $\mathcal{C}_i^b = \mathcal{C}_i^0 \longrightarrow \mathcal{C}_i^a = \mathcal{C}_i^b$

$\mathcal{C}_i^a = \mathcal{C}_i^0$ and $\mathcal{C}_i^b \neq \mathcal{C}_i^0 \longrightarrow \mathcal{C}_i^a \neq \mathcal{C}_i^b$

$\mathcal{C}_i^a \neq \mathcal{C}_i^0$ and $\mathcal{C}_i^b \neq \mathcal{C}_i^0 \longrightarrow$??????

example ($n = 5$):

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & ? & 0 & ? \\ 0 & 0 & ? & 1 & 0 & ? \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & ? & ? & 0 & 1 \end{pmatrix}$$

In the simplest approximation consists in assuming that $\mathcal{C}_i^a \neq \mathcal{C}_i^b$ in this case (justified in the Kaç limit for large M)

$$q_{ab}(i) = p_a(i) p_b(i)$$

The annealed approximation

$$\mathcal{S}_{rep}^{MF} = \sum_a \left(-\frac{M\beta^2}{2} \sum_{\langle ij \rangle} p_a(i) p_a(j) + M \log 2 \sum_i p_a(i) \right)$$

$$- M\beta^2 \sum_{ab} \sum_{\langle ij \rangle} p_a(i) p_b(i) p_a(j) p_b(j)$$

Going back to a spin model $\longrightarrow$ $p(i) = (1 + \sigma_i)/2$

Random field random bond Ising model (with correlated disorder)

$$\mathcal{H}^{MF} = -\sum_{\langle ij \rangle} (J + J_{ij}) \sigma_i \sigma_j + \sum_i (H_{ext} - h_i) \sigma_i$$

$H_{ext} = Md(\beta_K^2 - \beta^2)/4 \propto M s_c$

$J = M\beta^2/8$

$\Delta_J^2 = M\beta^2$ $\Delta_h^2 = M\beta^2 d/8$

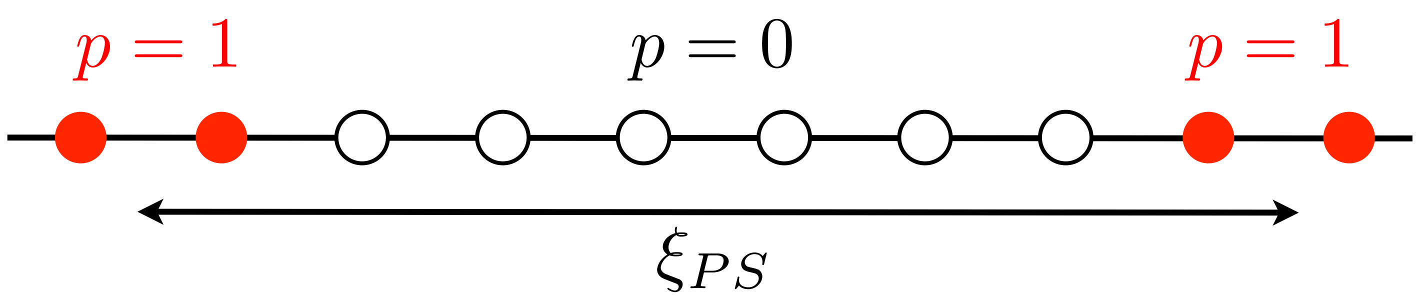
$\overline{h_i h_j} = \overline{h_i J_{ij}} = M\beta^2/16$

$\beta_K = \sqrt{2 \ln 2/d}$

Mean-field ($M \to \infty$) critical temperature

Beyond the annealed computation

Consider $p_a(i) = p(i) \quad \forall a$ (first cumulant of the effective action)



Regions with $p = 1$ closer than the point-to-set length → Locally decrease s_c → Induce a spontaneous RSB among the n replicas forced to have $p = 0$

A variational Ground-State approximation

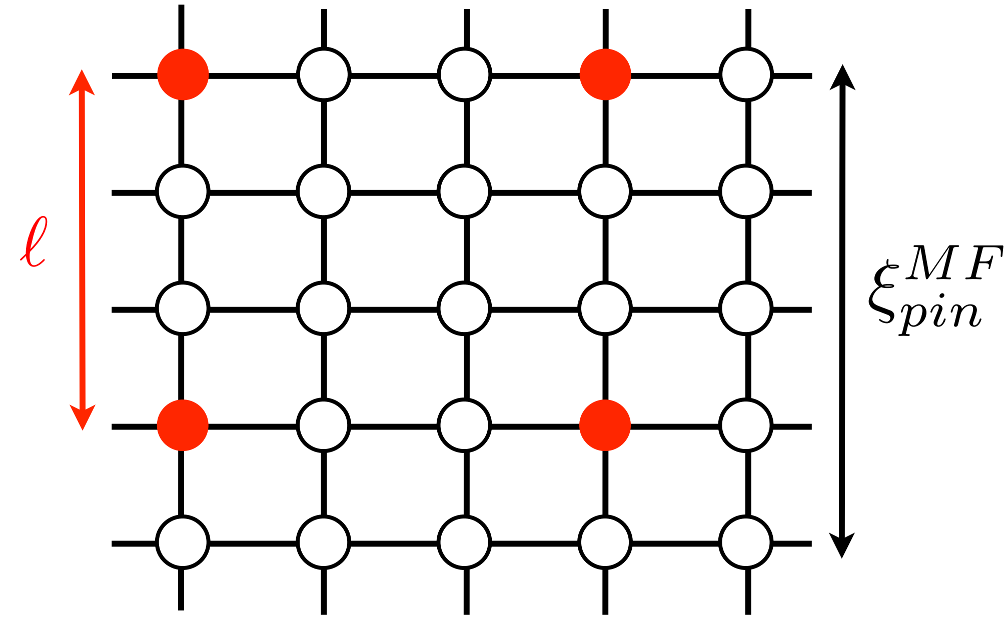
$$p(i) = 0 \rightarrow q_{ab}(i) = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & \boxed{1} & & 0 & \\ \vdots & & \boxed{1} & & \\ \vdots & 0 & & \boxed{1} & \\ 0 & & & & \end{pmatrix} \updownarrow m$$

combinatorial factor

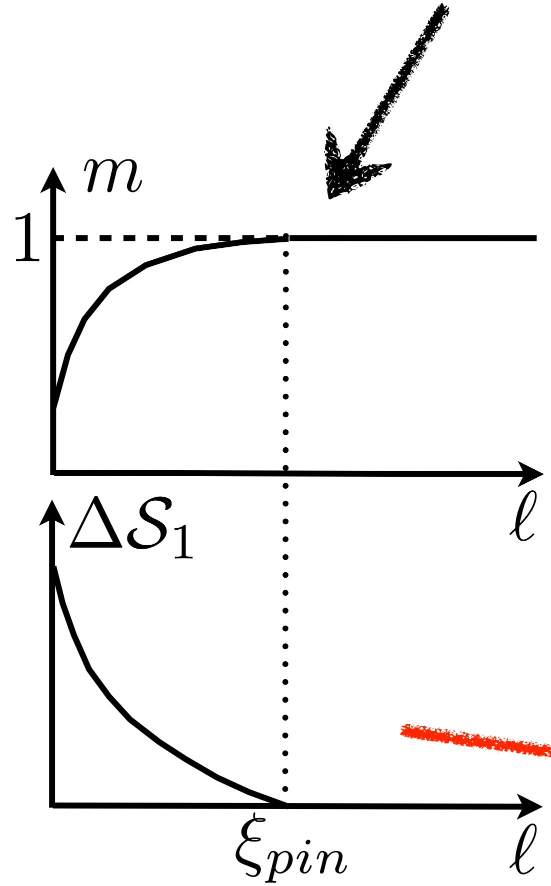
$$\binom{2^M - 1}{n/m}$$

The periodic pinning

Set $p(x) = 1$ on the vertices of a d-dimensional hyper-cube of size ℓ and $p(x) = 0$ elsewhere



Use the variational ansatz and optimize over m



$$\mathcal{S}_1[p(i)] = \mathcal{S}_1^{MF}[p(i)] + \Delta\mathcal{S}_1[p(i)]$$

$$\xi_{pin}^{MF} = \left(\frac{\beta_K^2 - \beta^2}{\beta_K^2}\right)^{\frac{1}{d}}$$

Additional effective interaction among the $p(i)$

Effective long-range antiferromagnetic interaction

Approximate ansatz for the effective interaction → possibly long-range pair interaction $K(r)$ + external field ΔH

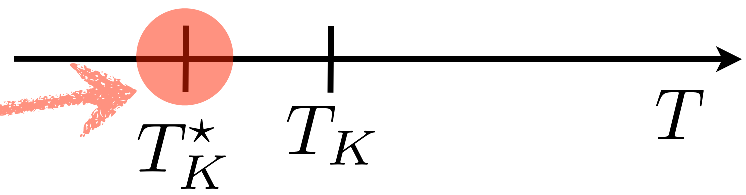
$$\mathcal{H} = -\sum_{\langle ij \rangle} (J + J_{ij})\sigma_i\sigma_j + \sum_{i,j} K(|i-j|)\,\sigma_i\sigma_j + \sum_i (H_{ext} + \Delta H + h_i)\sigma_i$$

$$K(r) \simeq \frac{Mc}{r^{2d}}\,\theta(\xi_{pin}^{MF} - r)$$

ΔH lowers the transition temperature with respect to mean-field (by renormalizing the configurational entropy)

$$\beta_K \longrightarrow H_{ext}(\beta_K^\star) + \Delta H(\beta_K^\star) = 0 \qquad \beta_K^\star > \beta_K$$

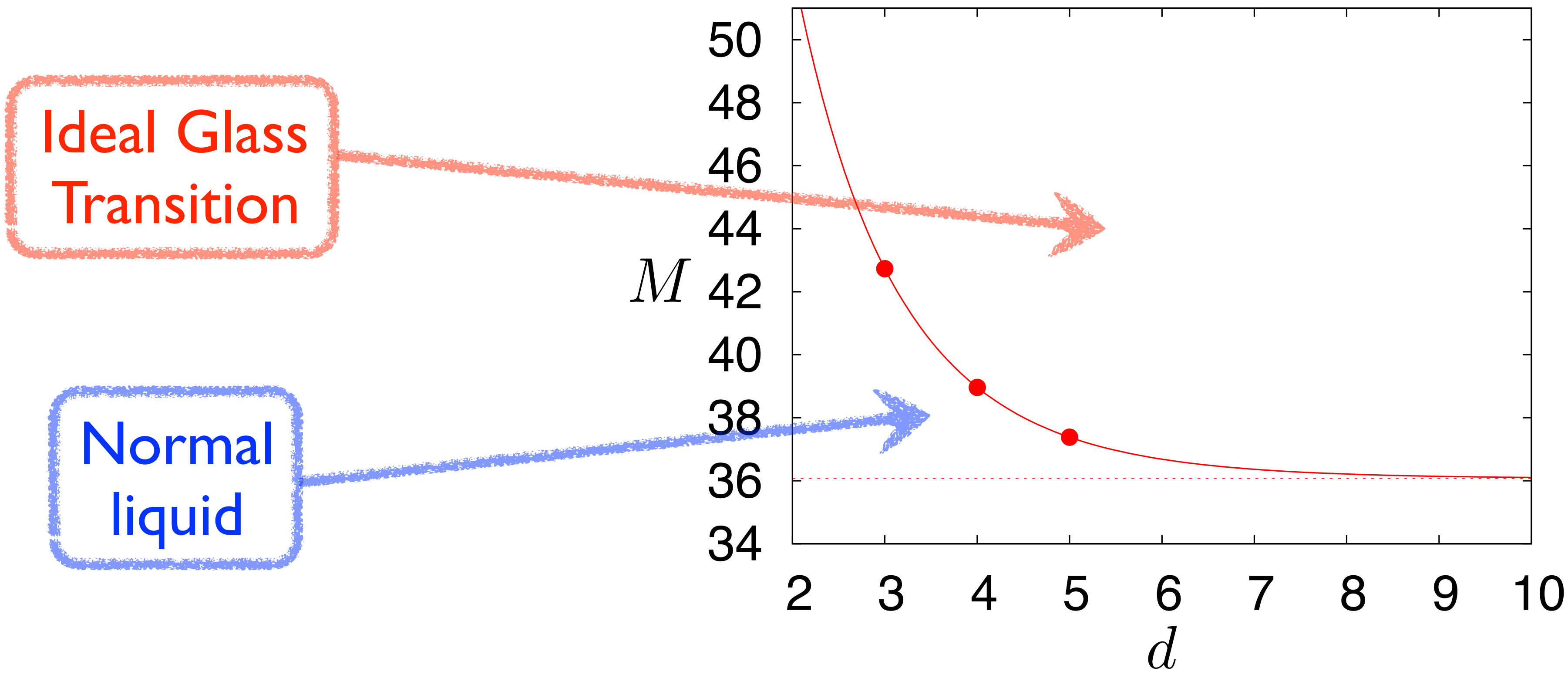
Study the properties of the critical point using the effective Hamiltonian



Estimation of the disorder at the critical point

Numerical simulations in $d = 3$ $\longrightarrow$ $\frac{\sqrt{\overline{\Delta_h^2}}}{Jd} \simeq 0.4$

- Neglect the random bonds (ok for large M)
- Neglect correlations between n.n. random fields and random bonds
- Use the result of $d = 3$ in higher dimensions
- $(Jd)_{eff} = Jd - \frac{1}{2} \int r^{d-1} K(r) \mathrm{d}r$



Conclusions & Perspectives

•First steps towards the derivation of an Ising-like effective theory for the glass transition close to T_K
•Random field random bond Ising model (with correlated disorder) + long-range antiferromagnetic interaction
•Go beyond RFOT. Use the effective Hamiltonian to study the properties of the original model (length scales, critical points, ...)
•Identify mechanisms which may destroy the Ideal glass transition

•Check the robustness of the results with respect to other geometry of the overlap profile
•Extend the computation to the second cumulant of the effective action (possibly long-range correlated disorder?)
•Extend the calculation to other models (similar results are found for a generic effective Ginzburg-Landau replicated action)
•Improve the variational ground state approximation (low temperature expansion?)