Jamming and hard spheres

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Spin glasses – Cargèse, August 28, 2014

D4PARTICLES

Outline

1 - Introduction: What is jamming?

2 - Early results

3 - Statistical mechanics approach

4 - Glassy phase diagram

5 - Microstructure of jammed packings

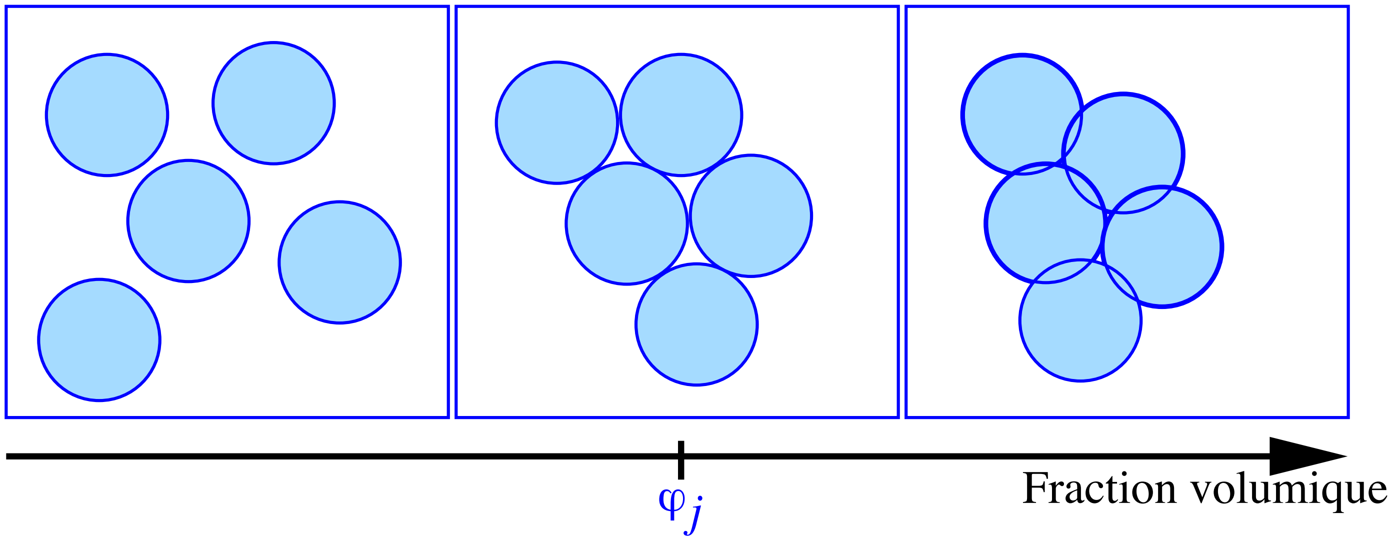
6 - Vibrational dynamics

7 - Rheology

I-Introduction: What is jamming?

A $T = 0$ geometric transition

• Athermal packing of soft repulsive spheres, e.g. $V(r < \sigma) = \epsilon(1 - r/\sigma)^2$.



Low φ: no overlap, fluid *Large φ: overlaps, solid*

• Clearly useful for: non-Brownian suspensions (below), hard grains (at), foams and emulsions of large droplets (above).

• Since $T = 0$, this is a purely geometric problem, possibly a nonequilibrium phase transition. A finite dimensional version of the ideal glass transition?

I . 1 – Spin glass perspective

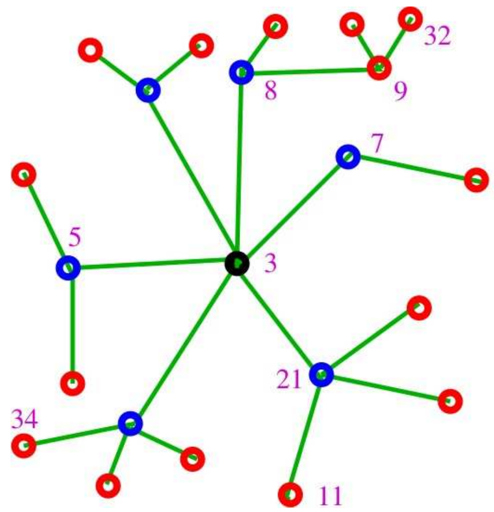
Constraint satisfaction problem

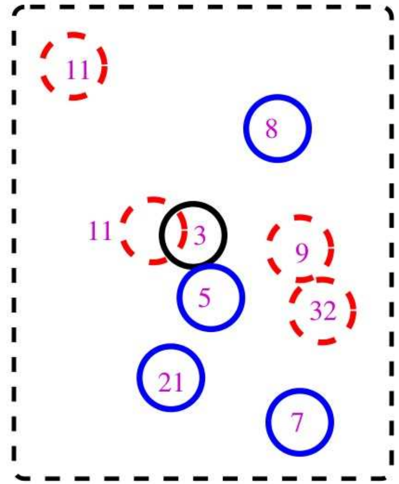
- Packing as a random constraint satisfaction problem $\rightarrow$ connection to spin glasses: Pack hard objects with no overlap. [Krzakala & Kurchan, PRE '07]

- A mean-field version of hard sphere fluid: each particle interact with $z \ll (N-1)$ neighbors.

- Interactions specified by quenched random graph.

- Aim: Understanding jamming within RFOT.





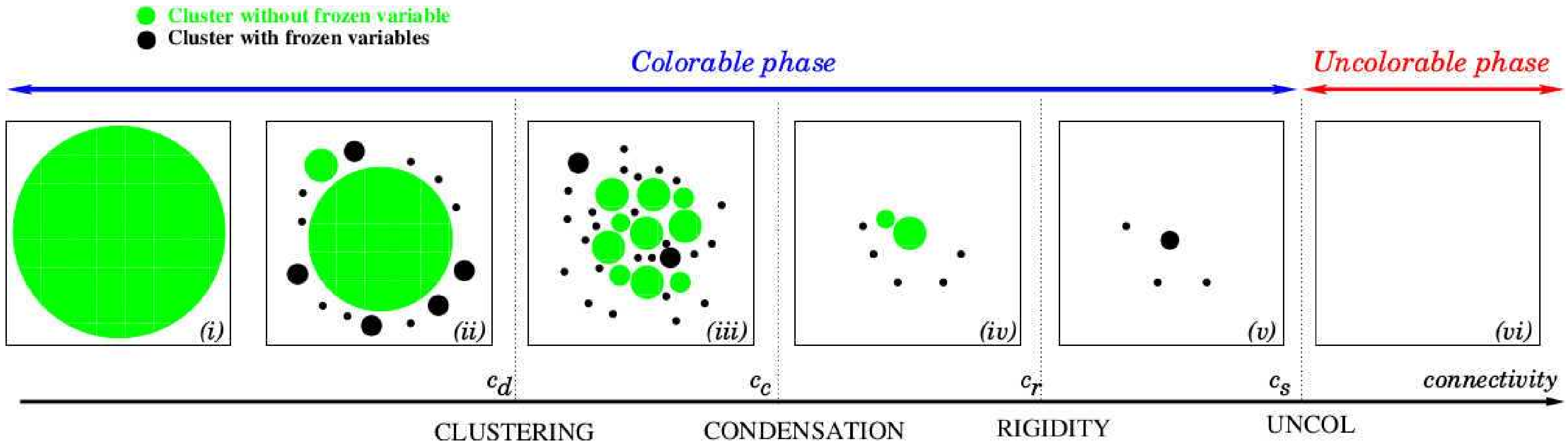
[Mari *et al.*, PRL '08]

- This model can then be solved as other random constraint satisfaction problems, e.g. cavity method. [Mézard *et al.*, JSTAT '11]

(*"This paper is meant to be read by specialists in the field, so we did not make much attempt to explain..."*)

Connection to Giulio's lecture

- Evolution of free energy landscape in the context of q-coloring problem on random graphs.



[Krzakala *et al.*, PNAS '07]

- "Clustering": Mode-coupling transition. Equilibrium relaxation time diverges in mean-field.

- "Condensation": Ideal glass (Kauzmann) transition in finite d.

- "Uncol": No solution found which satisfies all constraints. This is a jamming transition.

I . 2 – Broader perspective

Disordered solid states

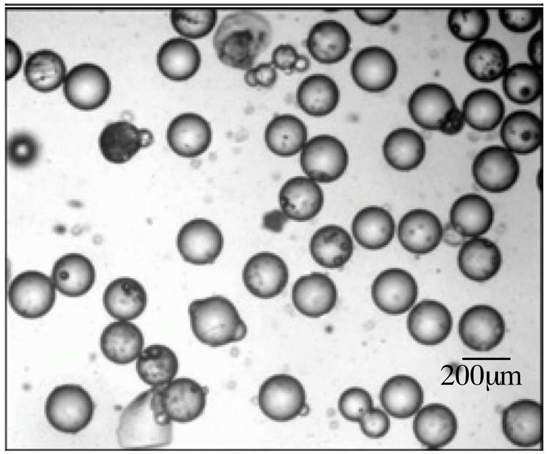

- Dense granular materials are disordered solids.

- Atomic glasses (window glasses, plastics) are solid materials frozen in an amorphous (non-crystalline, metastable) structure.

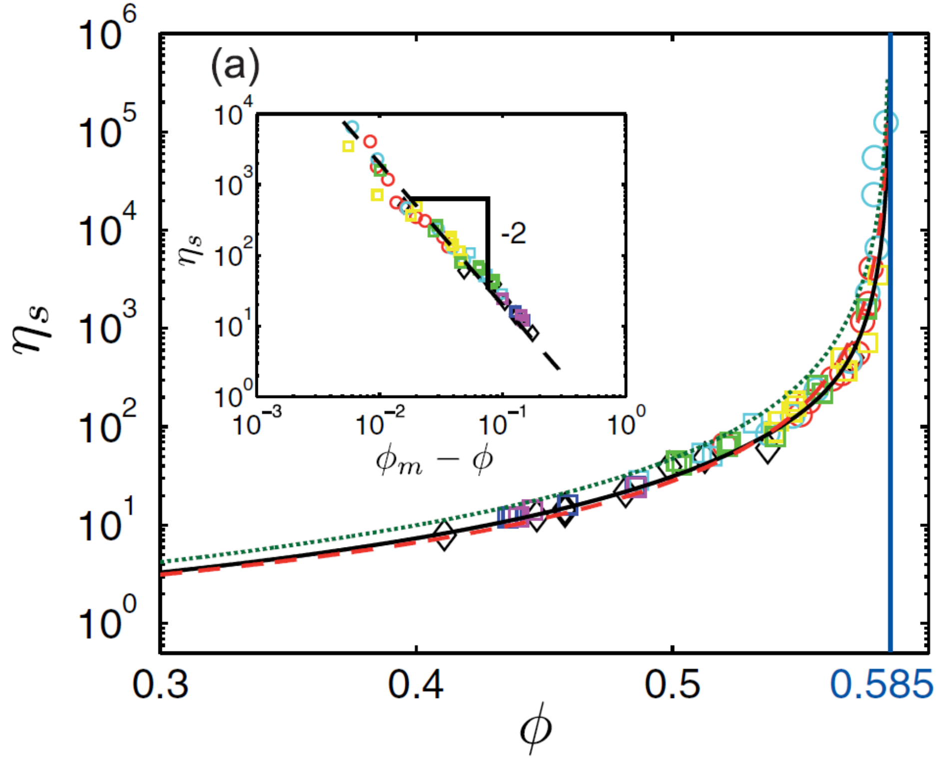
- Two possible pictures: Force chains and geometry of contact network versus complex energy landscape characteristic of disordered materials.

Jamming rheology

- Observed by compressing soft or hard macroscopic particles.
- Example: hard grain suspension. ‘Diverging’ athermal viscosity $\eta_0(\varphi)$.



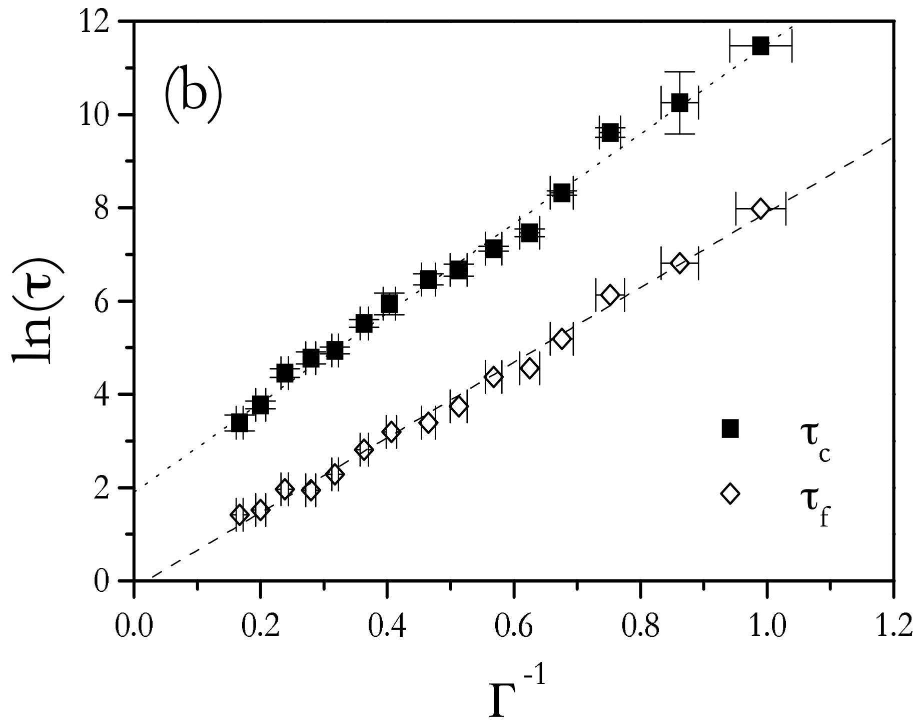
[Brown & Jaeger, PRL ’09]



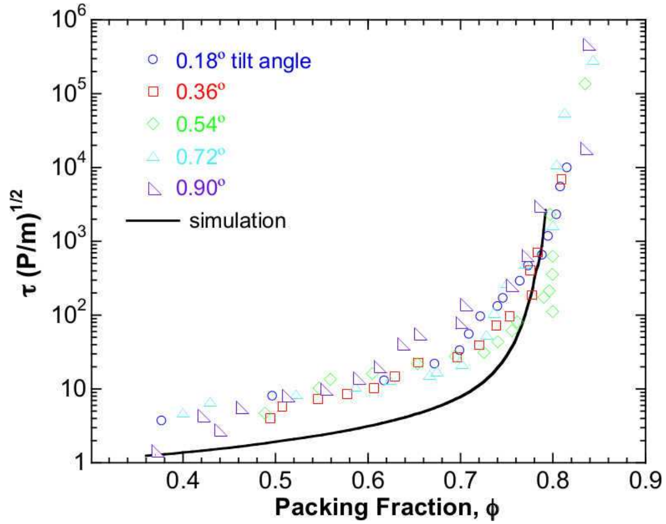
[Boyer & Pouliquen, PRL ’10]

- Simple rheology: no time scale competes with shear rate $\eta(\varphi,\dot\gamma) = \eta_0(\varphi) \sim |\varphi_J - \varphi|^{-\Delta}$.

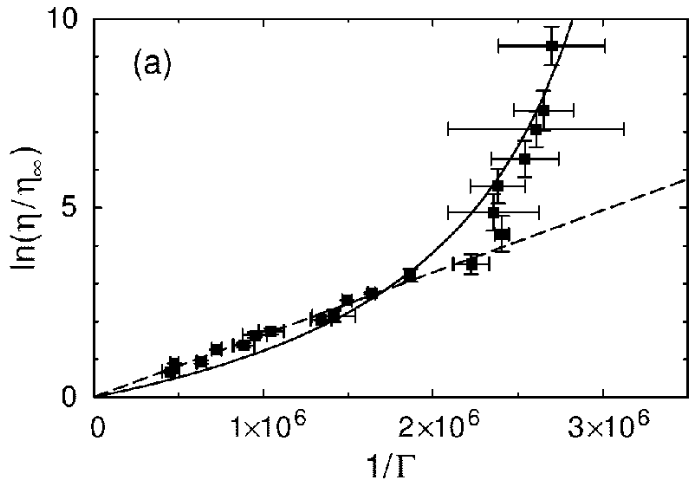
More ‘jamming’ transitions



Vibrated grains [Philippe & Bideau, EPL ’02]



Air fluidized granular bed
[Daniels *et al.*, PRL ’12]

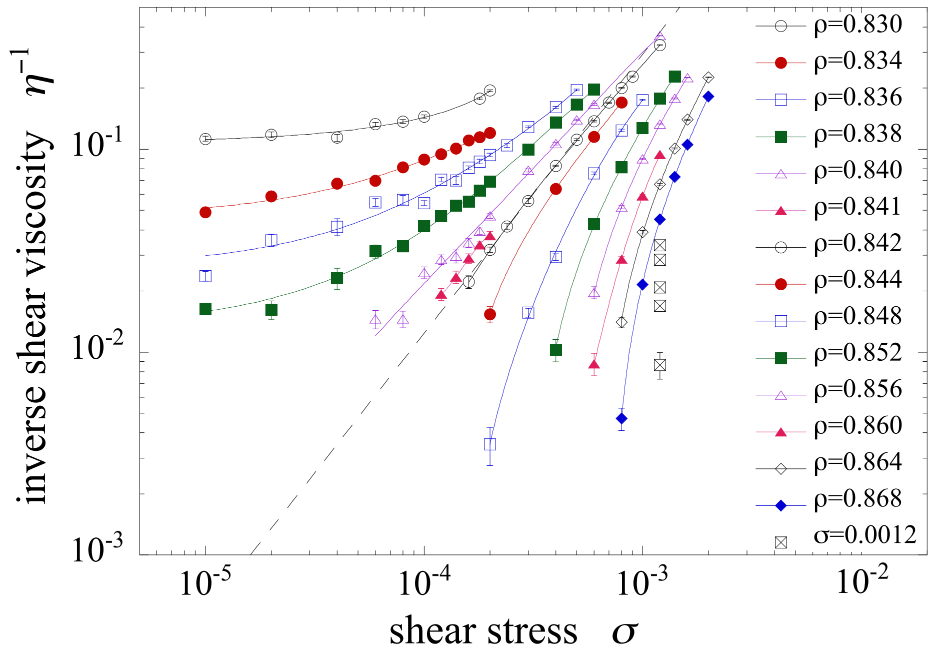


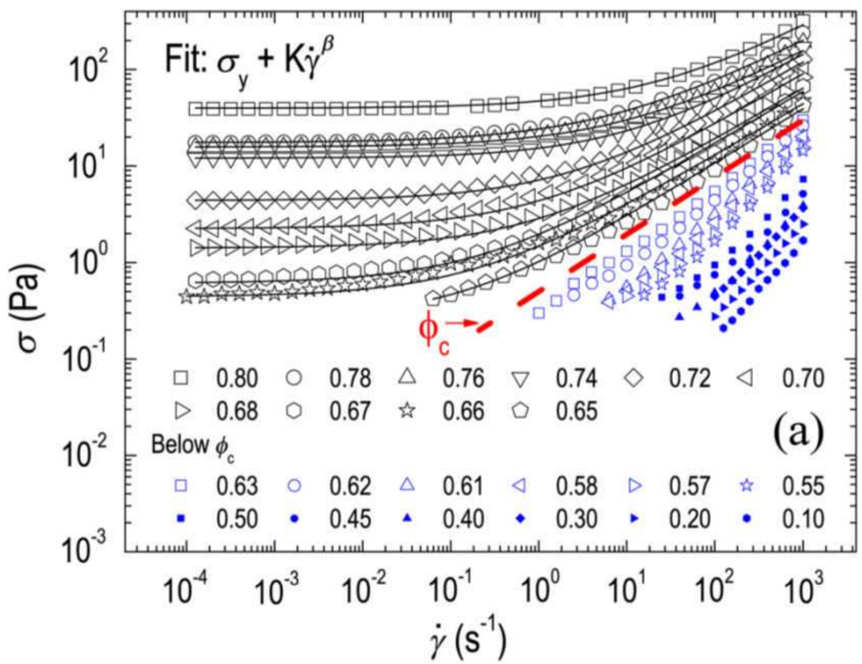
Sheared foam [Langer, Liu, EPL ’00]

- Dense assemblies of grains, (large) colloids and bubbles stop flowing.

Athermal rheology of soft particles

• Overdamped ($T = 0$) simulations of sheared harmonic spheres. Diverging viscosity, emergence of yield stress. $T = 0$ glass transition?





[Olsson & Teitel, PRL '07] [Paredes *et al.*, PRL '13]

• Scaling law: $\eta(\varphi, \dot{\gamma}) = |\varphi_J - \varphi|^{-\Delta} F(\dot{\gamma}|\varphi - \varphi_J|^\beta)$. Theoretical basis?

• Similar behaviour (and scaling laws?) observed in emulsion.

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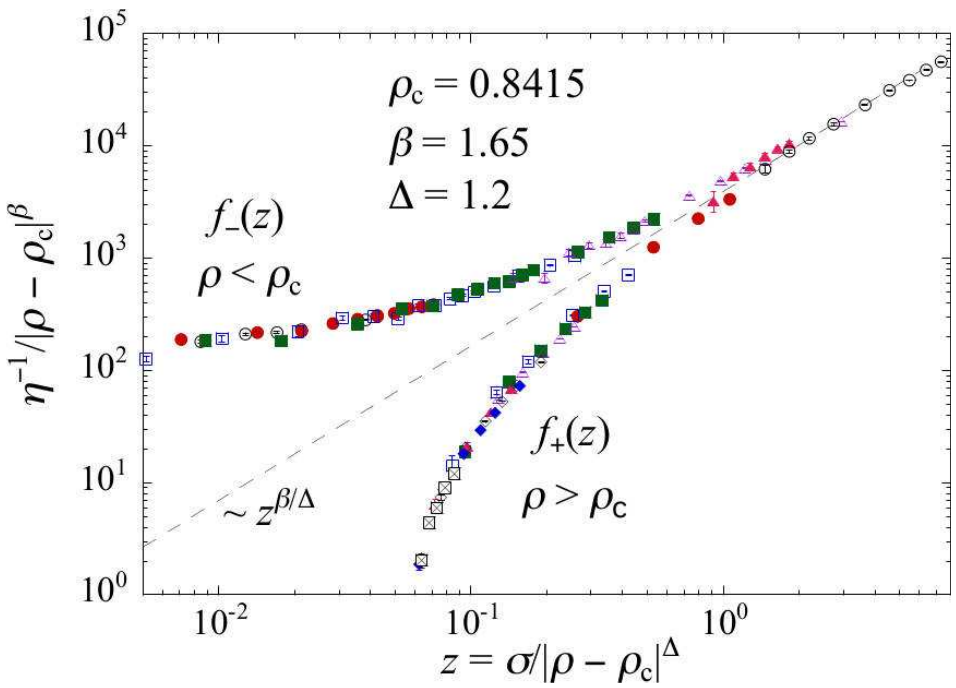
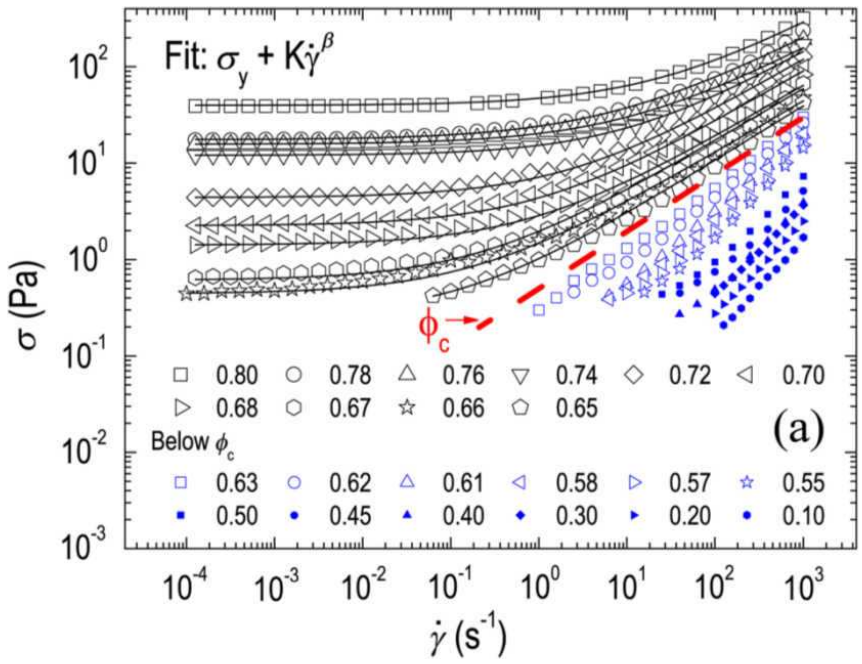


[Olsson & Teitel, PRL '07]



[Paredes *et al.*, PRL '13]

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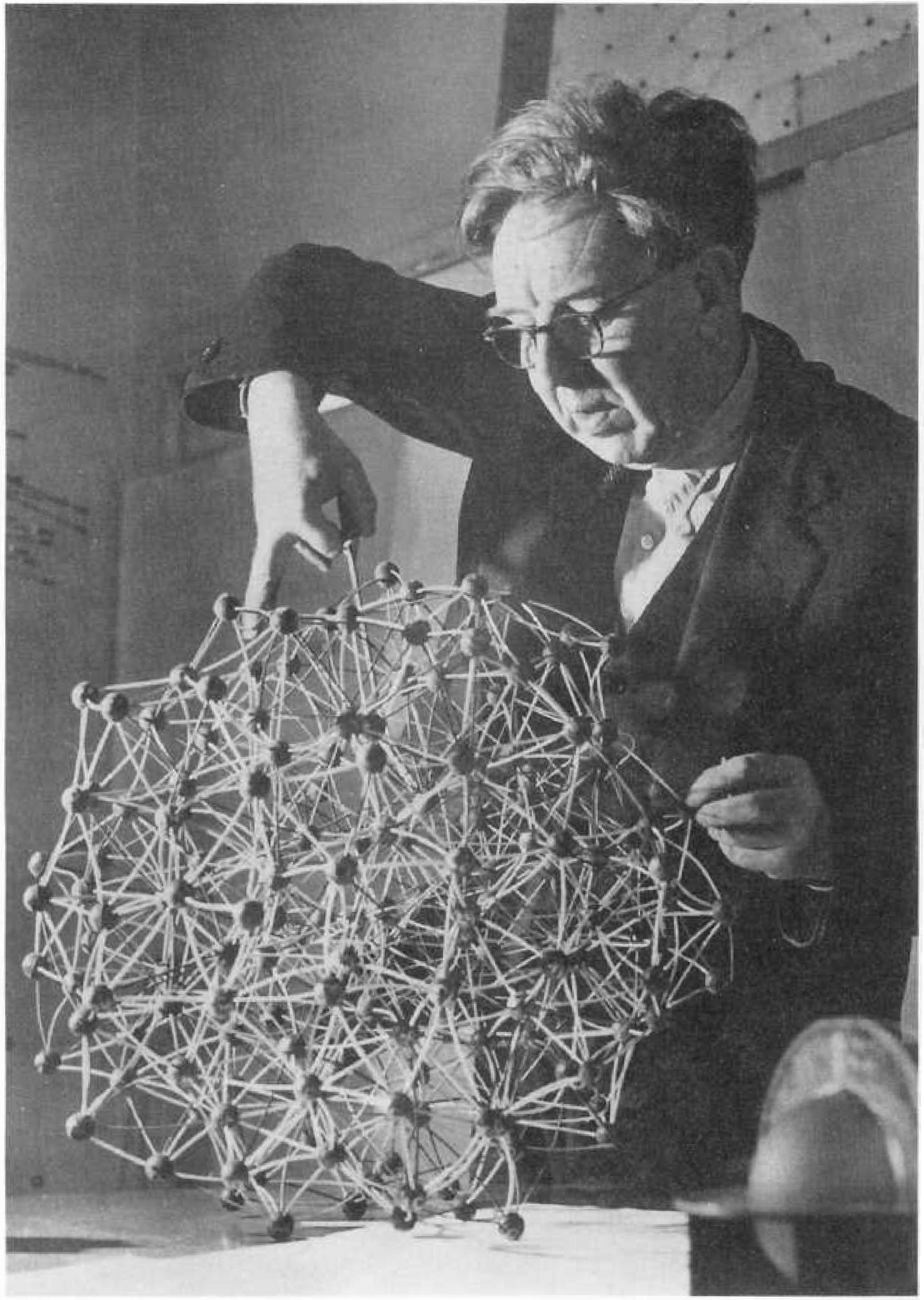


27. Sage constructs his first ball-and-spoke model of liquid structure, confident that he will be interrupted every few minutes so that the model would be disordered

Bernal's insight

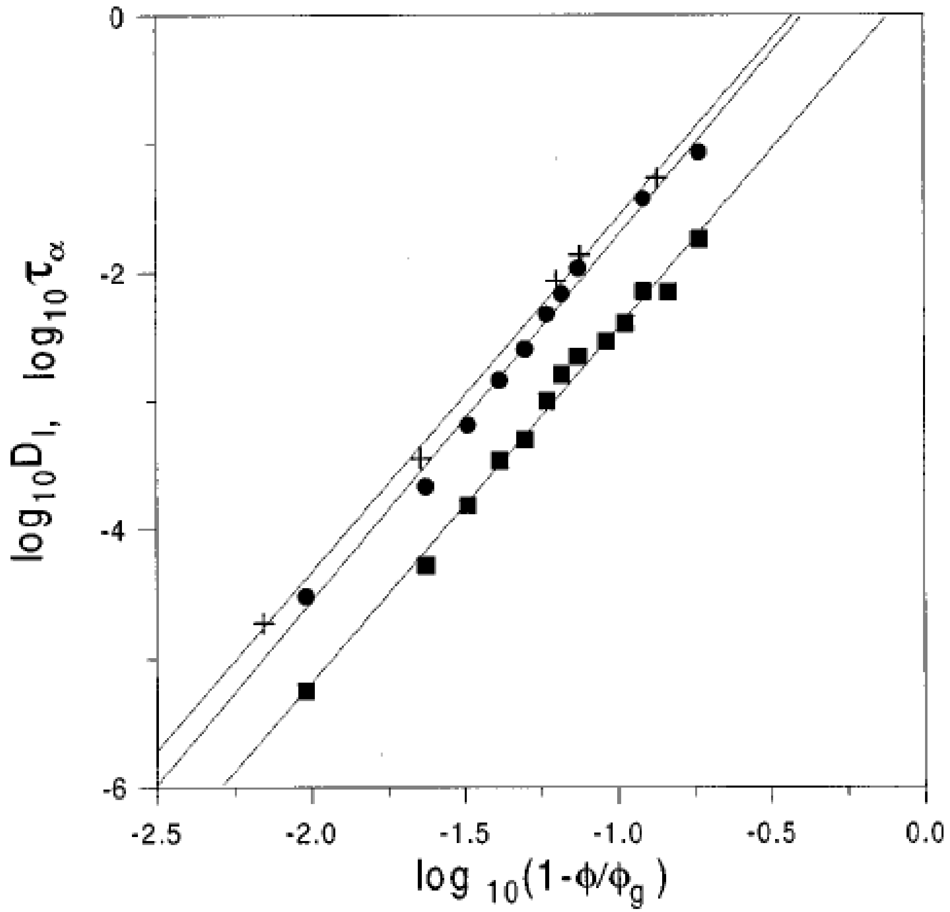
"*This theory treats liquids as homogeneous, coherent and irregular assemblages of molecules containing no crystalline regions or holes.*"

- Theory of liquids as a random packing problem.
- Experiments with grains, computer simulation.

[J. D. Bernal, "*A geometrical approach to the structure of liquids*", Nature (1959)]

Dynamics in colloidal hard spheres

- Glass 'transition': Dramatic increase in viscosity when φ decreases.



[van Megen *et al.*, PRE '98]

- Viscosity measurements difficult $\rightarrow$ light scattering.
- $\tau_T = \sigma^2/D_0 \sim 1$ ms.
- T value irrelevant, but $T > 0$ for thermal equilibrium $\neq$ jamming?
- Mode-Coupling Theory fit?
$\tau_\alpha \sim (\varphi_c - \varphi)^{-\gamma}$, $\varphi_c \approx 0.58$.
- 'Free volume' $\rightarrow 0$ at φ_J.
$\tau_\alpha \approx \exp(P/T) \sim \exp(A/|\varphi_J - \varphi|)$.
Jamming's back!

- No neat experimental answer from early work ('86 - '05).

Activated colloidal dynamics

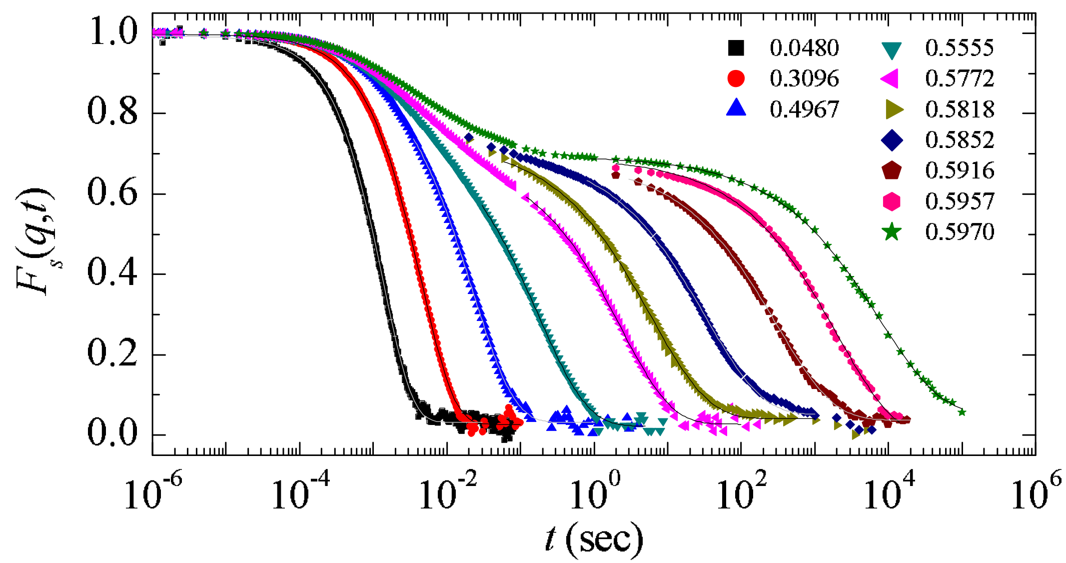
- Most recent set of light scattering data to date.

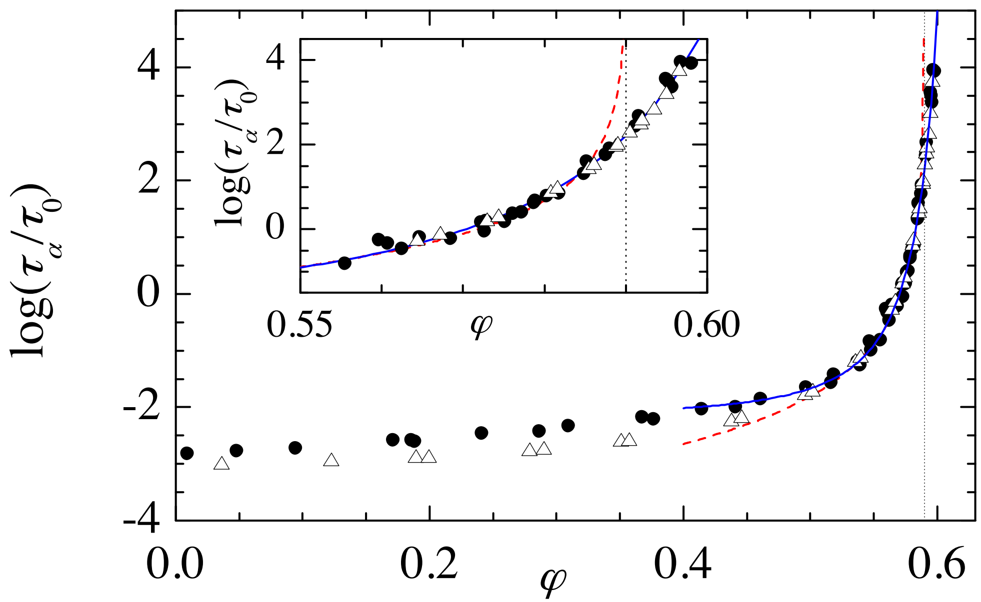
- Mode-coupling prediction fails, no algebraic divergence at φ_c.

- Best fit to ‘activated’ dynamics:
$\tau_\alpha \approx \exp(\frac{A}{(\varphi_0-\varphi)^\delta})$, $\delta \approx 2$.

- Simulations suggest φ_0 distinct from jamming point, but relie on extrapolations.

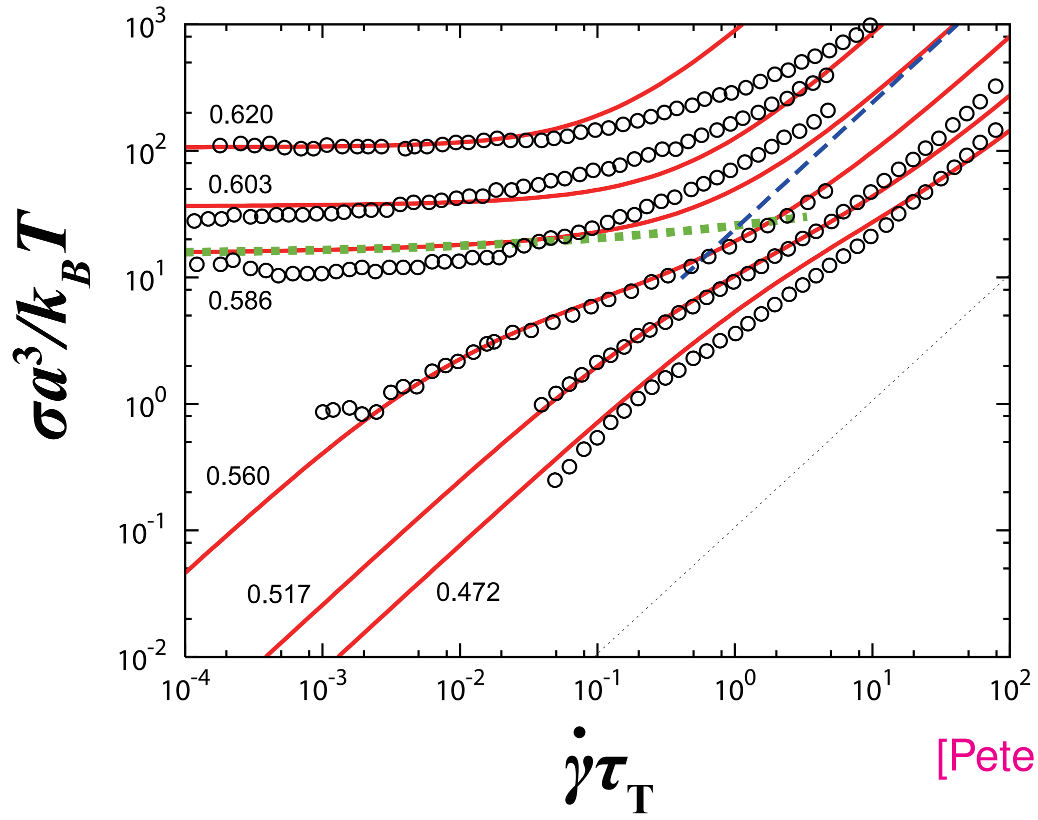
[Brambilla *et al.*, PRL ’09]





Colloidal rheology

- Non-linear rheological study: Transition from viscous fluid to yield stress amorphous solid, in the presence of thermal fluctuations, non-linear rheology: $\eta = \eta(\varphi, \dot{\gamma})$.

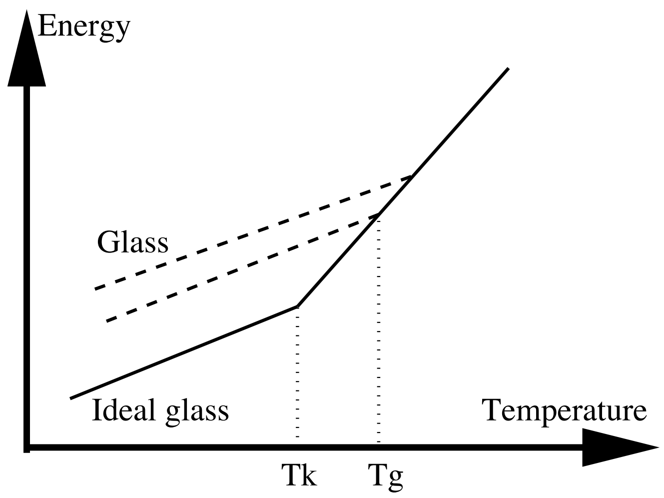


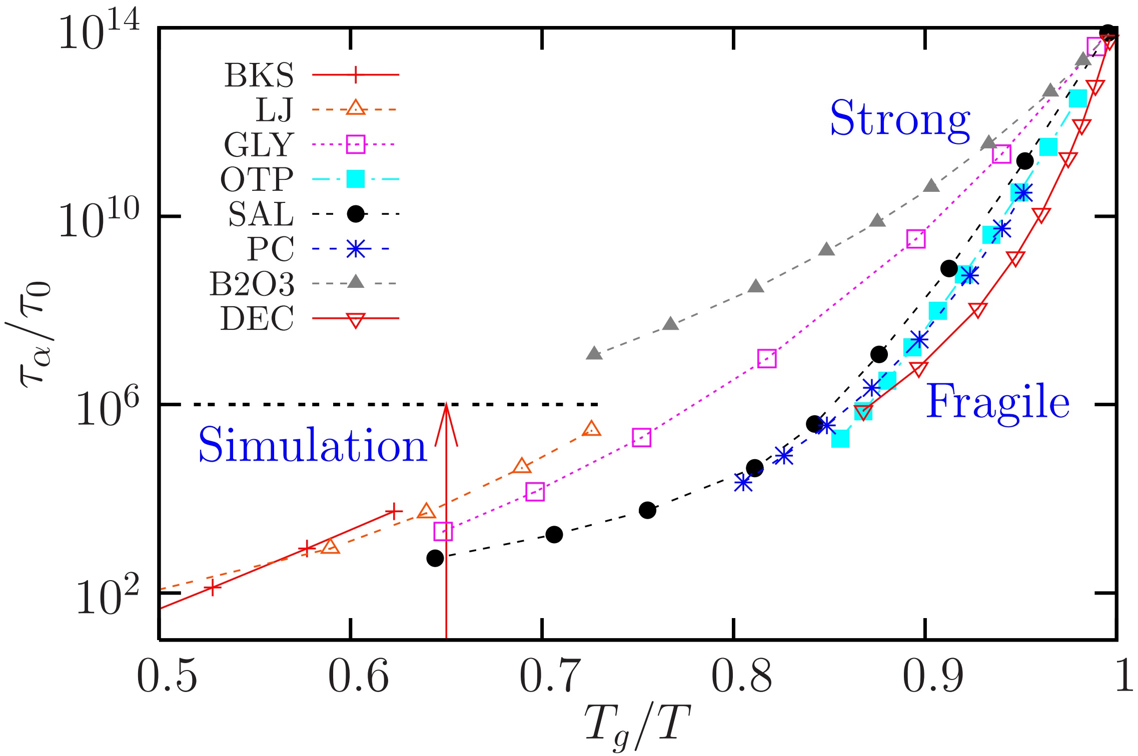
[Petekidis *et al.*, JPCM '04]

- Linear viscosity: $\eta_T(\varphi) = \eta(\dot{\gamma} \to 0, \varphi)$. Similar to $\eta_o(\varphi)$ for non-Brownian suspensions, or activated dynamics as found for τ_α??

The glass ‘transition’

- Many molecular materials become glasses at low temperature.





- Glass $\equiv$ liquid “too viscous” to flow. Glass formation is a gradual process with activated dynamics (not algebraic) in thermal equilibrium.

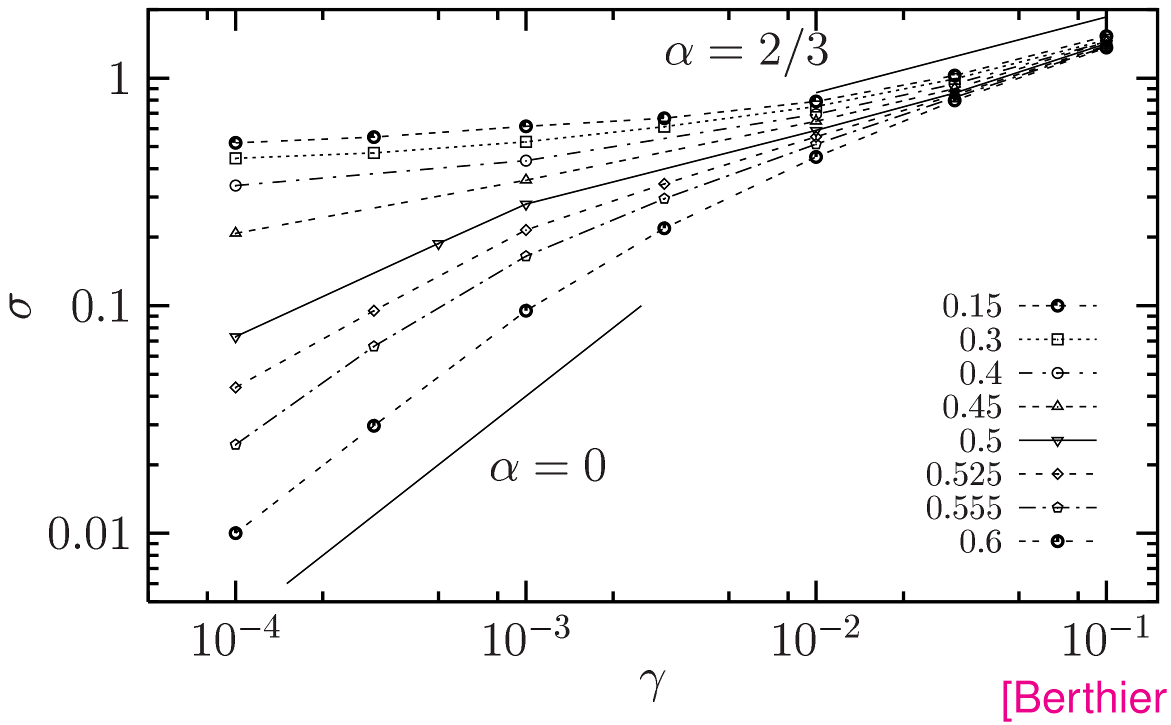
- ‘Ideal’ glass transition at equilibrium? Jamming point for molecules?

- Existence of many metastable states: glasses are many-body “complex” systems, due to disorder and geometric frustration.

Molecular glassy liquids rheology

• Flow curves at finite shear rate $\dot{\gamma}$ in simple shear flow: $\sigma = \sigma(\dot{\gamma}) = \eta(\dot{\gamma})\dot{\gamma}$, in binary LJ mixture.

• Transition from viscous fluid to solid material (finite yield stress), with non-linear behaviour (shear-thinning).

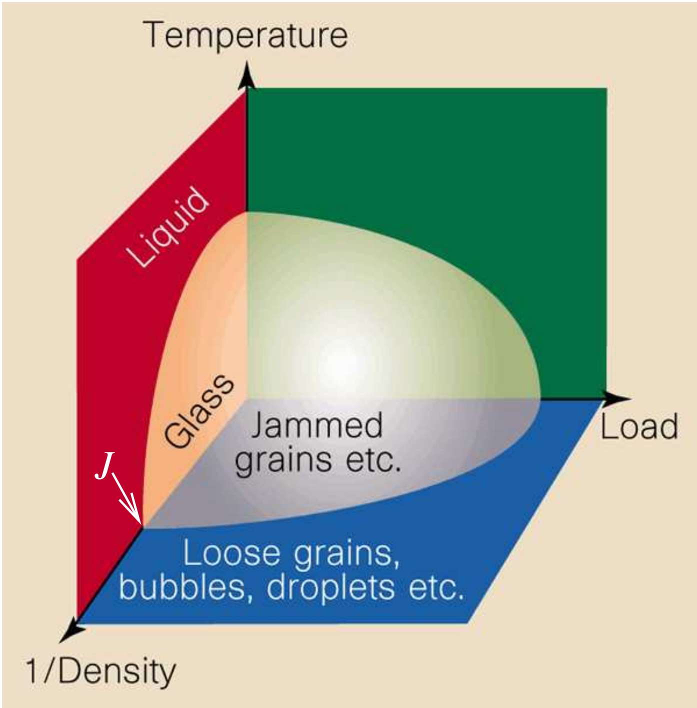


[Berthier & Barrat JCP '01]

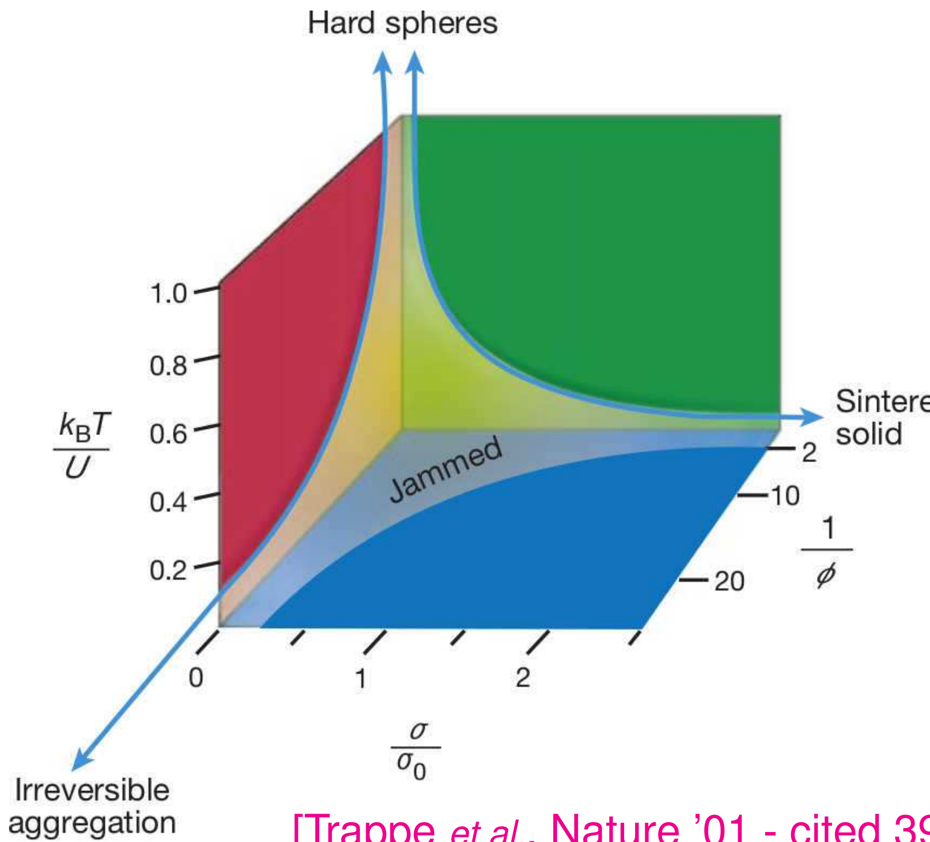
• Rheological behaviour (again) very similar to colloidal hard spheres and non-Brownian soft particles. Viscosity shows activated dynamics.

Jamming phase diagram

• Suggested by similarity of rheological behaviour observed in non-Brownian & Brownian suspensions, and molecular liquids.



[Liu & Nagel, Nature '98 - cited 846]



[Trappe *et al.*, Nature '01 - cited 398]

• Has given rise to a whole field of 'jamming' studies, many experimental measurements in connection to jamming phase diagram.

II-Early results

Harmonic spheres

- “Bubble model” introduced by Durian in ’95 to study wet foams: $V(r < \sigma) = \epsilon(1 - r/\sigma)^2$. Can be used to explore the complete (T, φ, σ) “jamming phase diagram” at once.

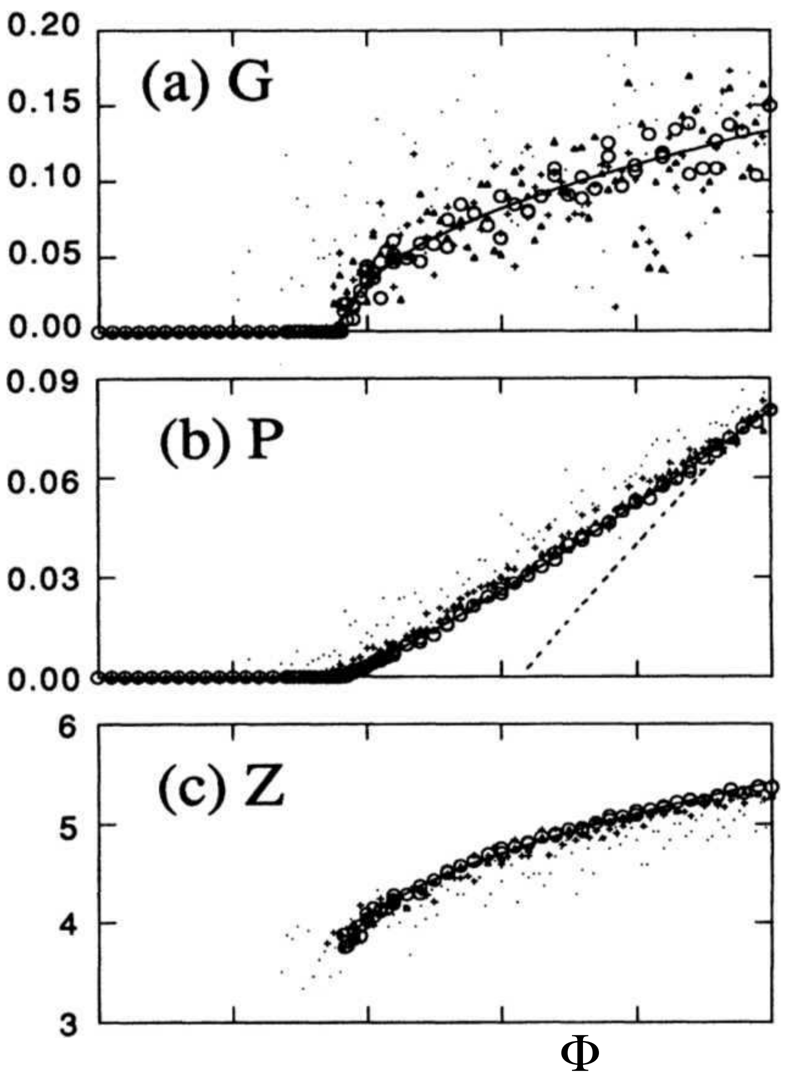
- Two ways of going athermal, $\epsilon/T \to \infty$.

(i) $T = const.$ and $\epsilon \to \infty$: Brownian hard spheres (e.g. colloids).

(ii) $\epsilon = const.$, $T \to 0$: athermal soft suspensions (e.g. foams), equivalent to hard spheres below jamming.

- For thermal systems, T/ϵ quantifies the particle softness: large T/ϵ = soft particles. Useful for emulsions, microgels, simple liquids.

Durian’s bubble model: ’95



[Durian, PRL ’95]

- Nice early work:
  [Bolton & Weaire, PRL 65, 3449 (1990)]

- Durian: Rheological study at $T = 0$ using computer simulations.

- Transition from fluid to solid behaviour at φ_J with scaling properties.

- Scaling laws for emergence of shear modulus G, packing pressure P and number of contacts per particle z at critical density φ_J.

Point J: '02

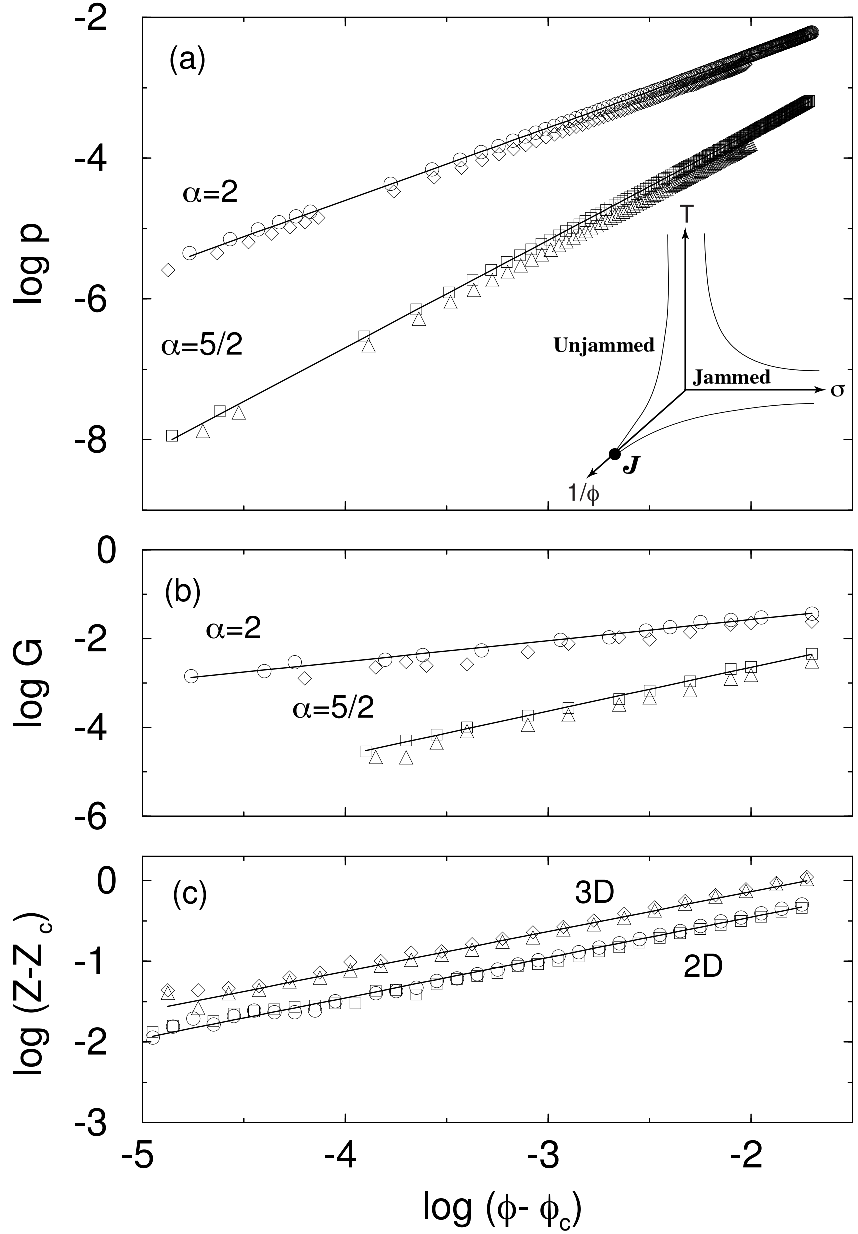
- J-point from random packing properties of soft repulsive particles at $T = 0$. [O'Hern *et al.* PRL '02, PRE '03]

- Scaling laws and structure of packings near jamming [vanHecke JPCM'10]

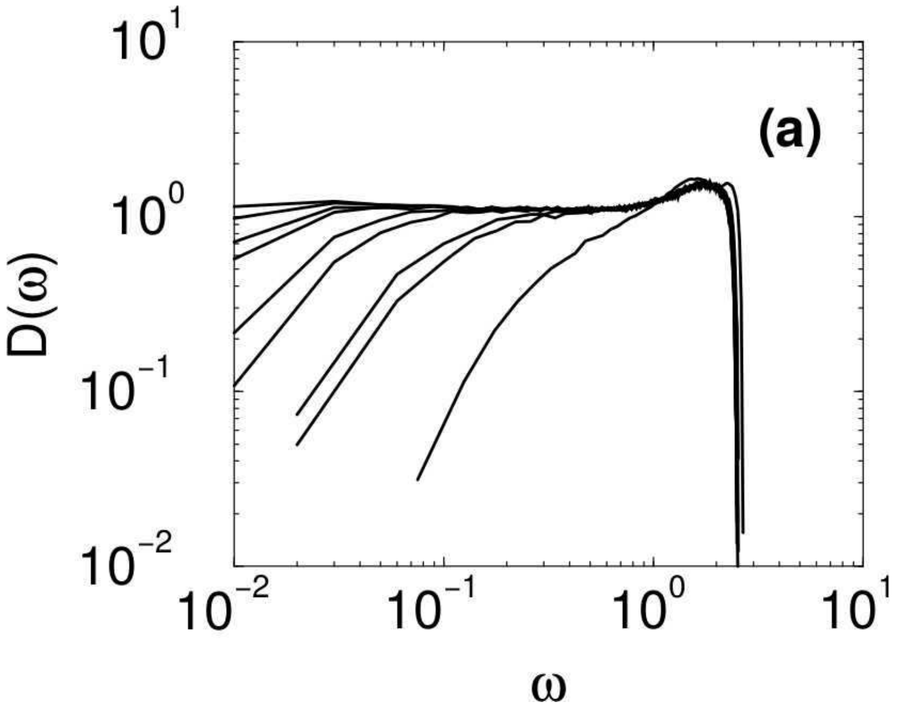
Energy: $E = 0$ for $\varphi < \varphi_J$; $E \sim (\varphi - \varphi_J)^\alpha$ for $\varphi > \varphi_J$.

Contact number: $z = 0 \to z = z_c + a(\varphi - \varphi_J)^{1/2}$ with $z_c = 2d$: isostaticity.

- Major numerical and experimental effort over the last decade; considered as a new nonequilibrium phase transition with relevant physical consequences.



Density of states

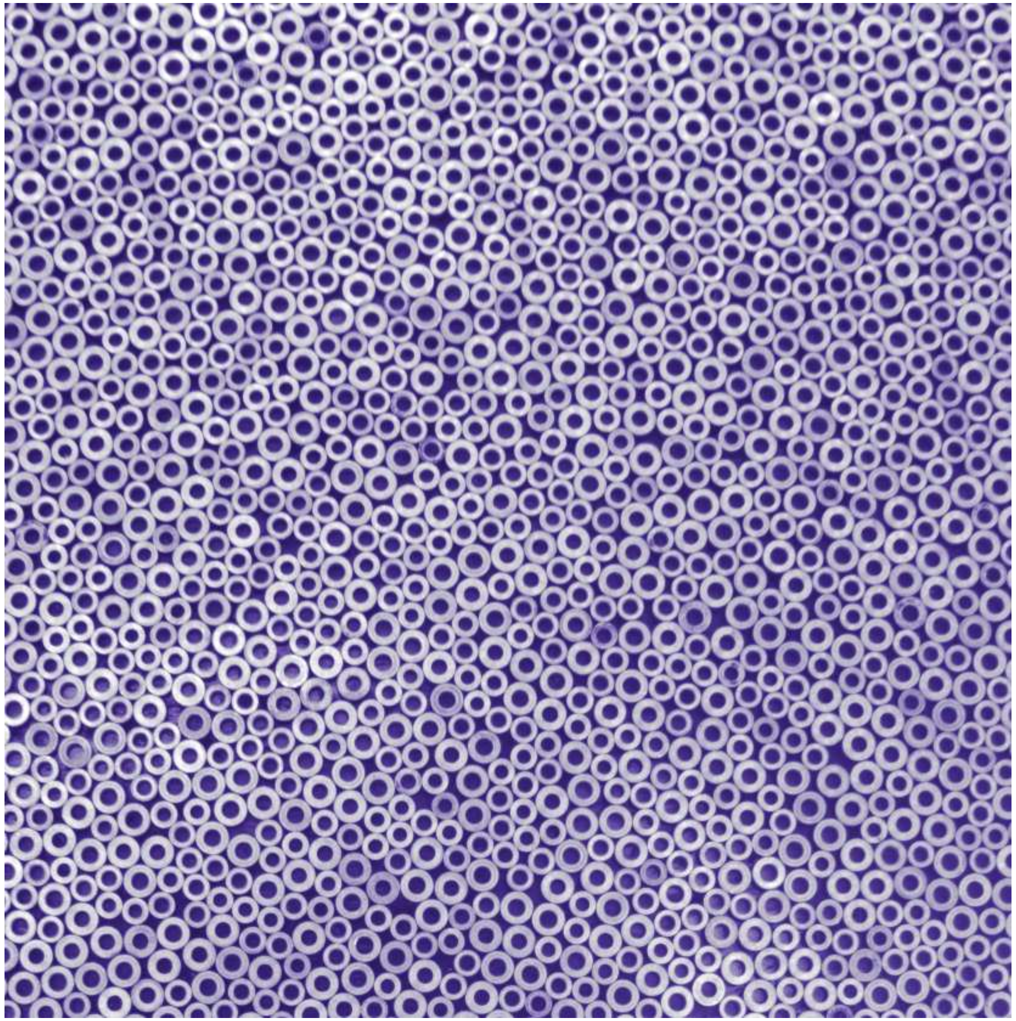


- Density of state above jamming transition reveals the presence of 'soft modes', with $\omega^\star \sim (\varphi - \varphi_J)^{1/2}$.
- Deep link with isostaticity: $z \to z_c = 2d$.
- $z = z_c$: each particle has just enough contacts to maintain mechanical stability (i.e. $N_c \equiv z_c N/2 = Nd$).

- Isostatic packings have flat density of states; compressed packings are flat down to $\omega^\star \sim (z - z_c) \sim (\varphi - \varphi_J)^{1/2}$. [Wyart *et al.*, EPL '05]

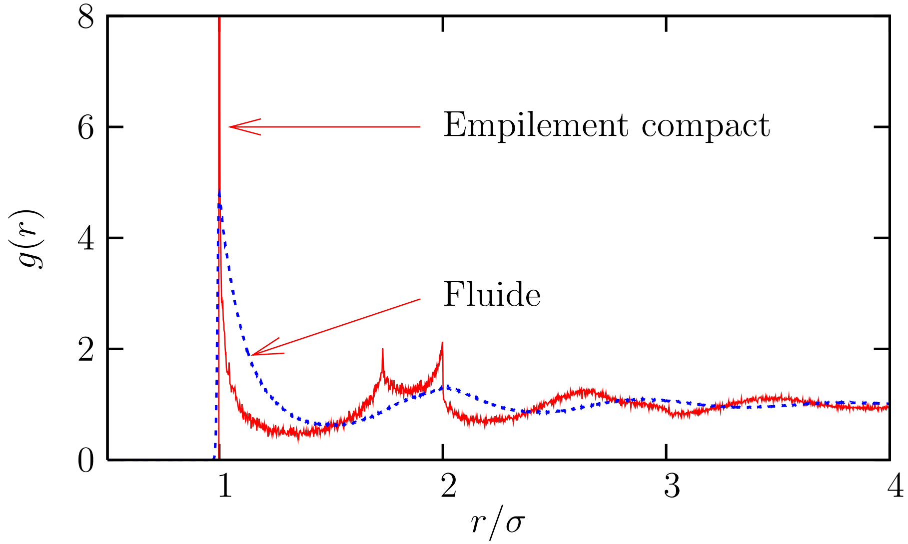
- Marginally stable solids: abundance of soft modes. Counting arguments of soft modes using isostaticity yields the above scaling relations, observed in simulations. Challenge for theory: predicting isostaticity.

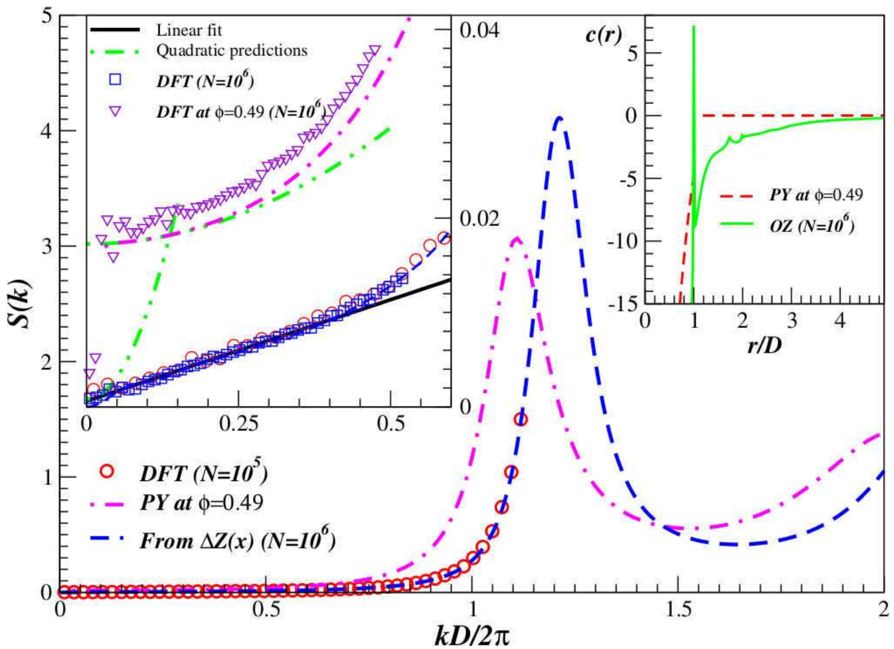
Structure of packings



[Courtesy O. Dauchot]

Structure of packings





- Pair correlation function: $g(r) = (\rho N)^{-1} \sum_{ij} \delta(r - |r_i - r_j|)$ and structure factor $S(q)$ quantify two-point density correlation functions.

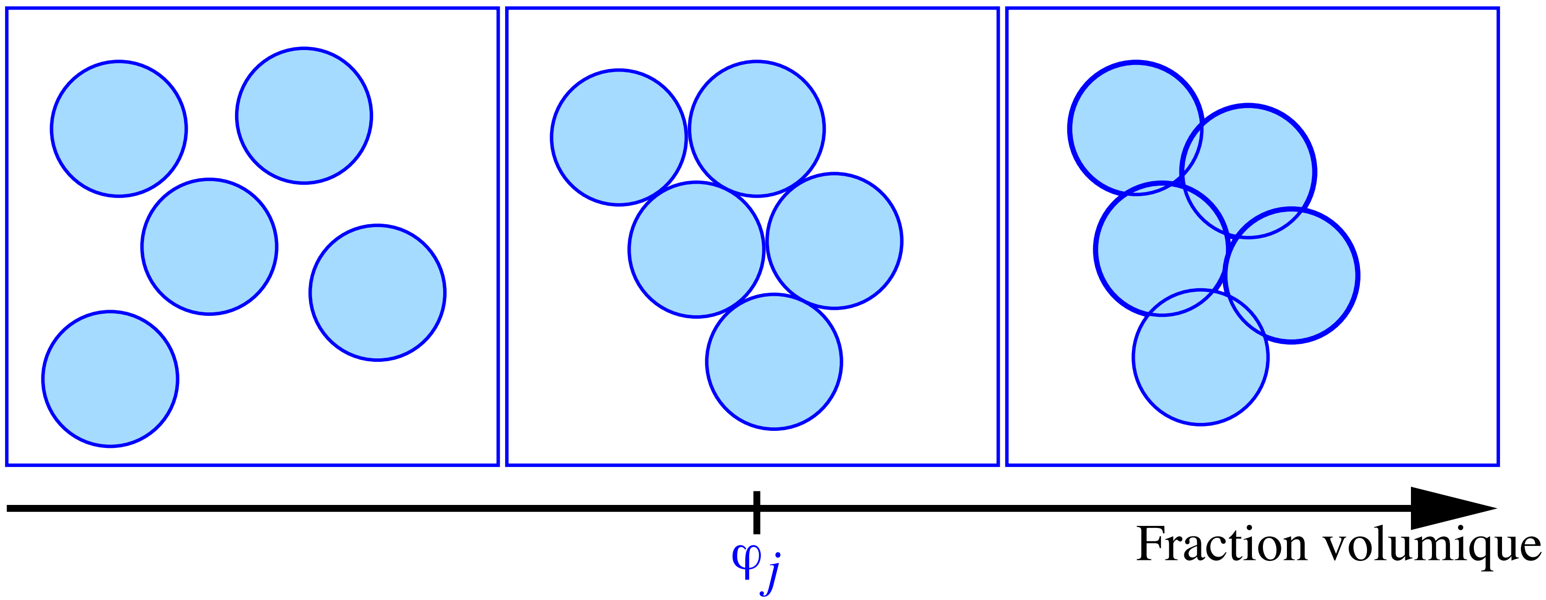
- Multiple singularities revealed by $g(r)$ analysis. The "structure" is not boring!

- Physics near contact $r \sim \sigma$ is crucial and reveals 3 different power laws–described later.

- Suppressed density fluctuations at large scale: $S(q) \sim q$ at low q. 'Hyperuniform' state with $V\langle \delta\varphi^2 \rangle_V \to 0$. [Donev *et al.*, PRL '05]

III-Statistical mechanics approach

Geometric problem



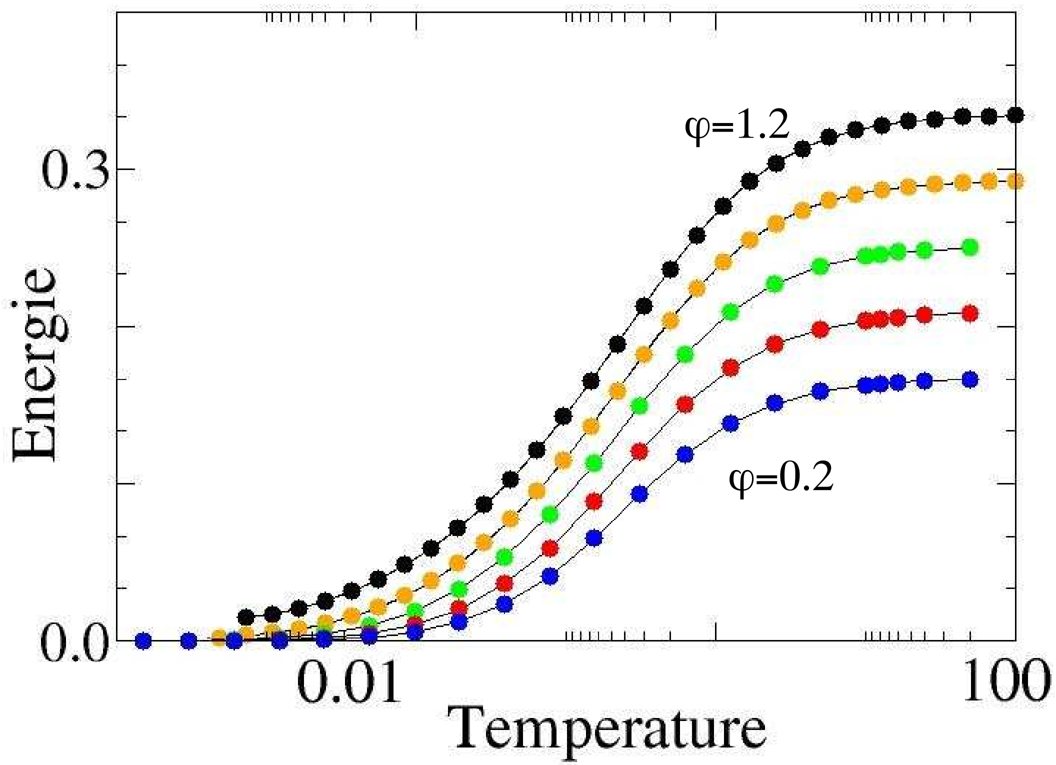
- Equilibrium statistical mechanics approach to jamming transition: Introduce a Hamiltonian penalizing overlaps (e.g. harmonic spheres) and a temperature T.

- Take the $T \to 0$ limit of the constructed (T, φ) phase diagram.

Start with the fluid

• Consider the fluid, $V(r) = (1 - r)^2$, at equilibrium at $(T > 0, \varphi)$.

• Liquid state theory: solves the structure, $g(r)$, thus the thermodynamics using integral equations. We use (for instance) HNC.



[Jacquin & Berthier, Soft Matter '10]

• $e(T, \varphi) \sim T^{3/2}$.

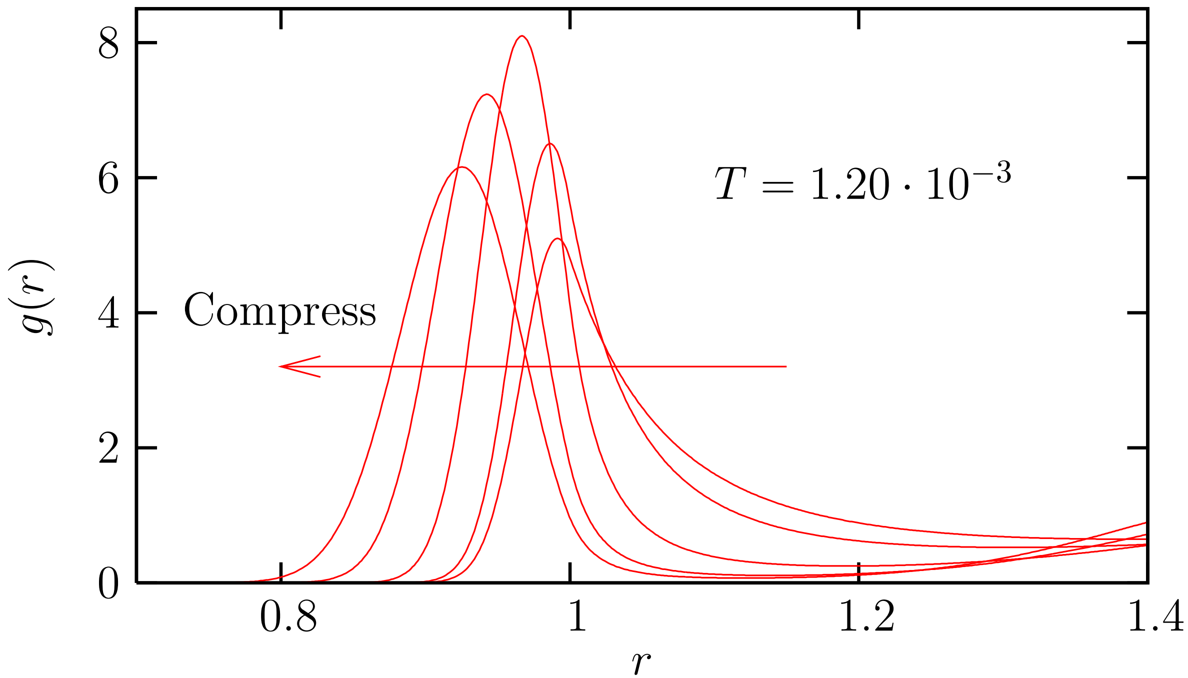
• Pressure is finite at $T = 0$ and continuous function of φ.

• No jamming (or glass) transition.

• Liquid state theory is essentially blind to relevant phase transitions. Useful?

Nonmonotonic structure

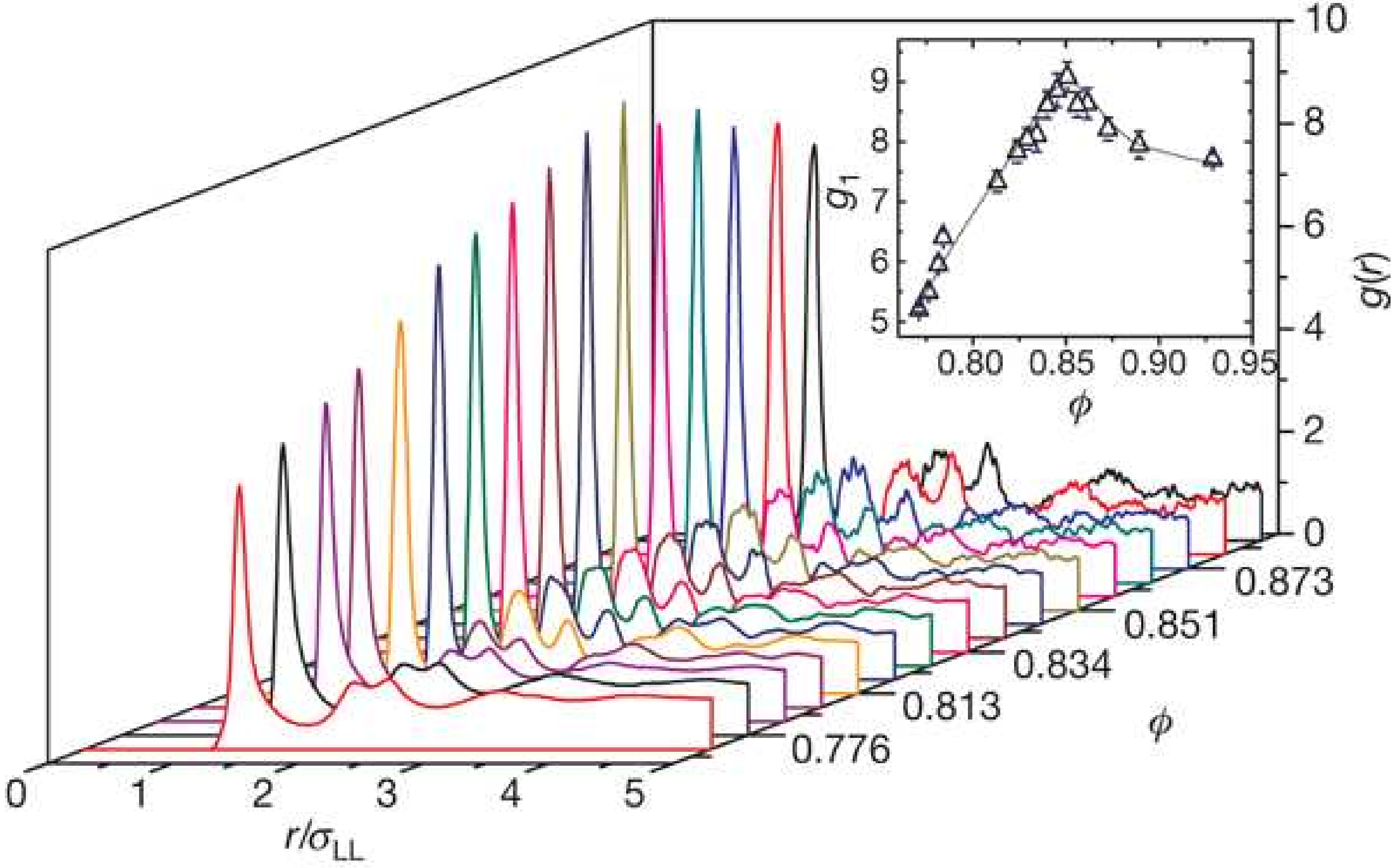
- Anomalous structural evolution at constant $T = 1.2 \cdot 10^{-3}$. The system orders, $\varphi = 0.55 \to 0.78$, then disorders $\varphi = 0.78 \to 0.96$.



- $F = E - TS$: Avoiding overlaps to reduce energy becomes difficult (entropically disfavoured) at large φ. Solution: increase overlap using particle softness (lose energy) to make space and gain entropy.

- Smooth version of jamming ('thermal vestige') arising at equilibrium.

Structure of soft colloids



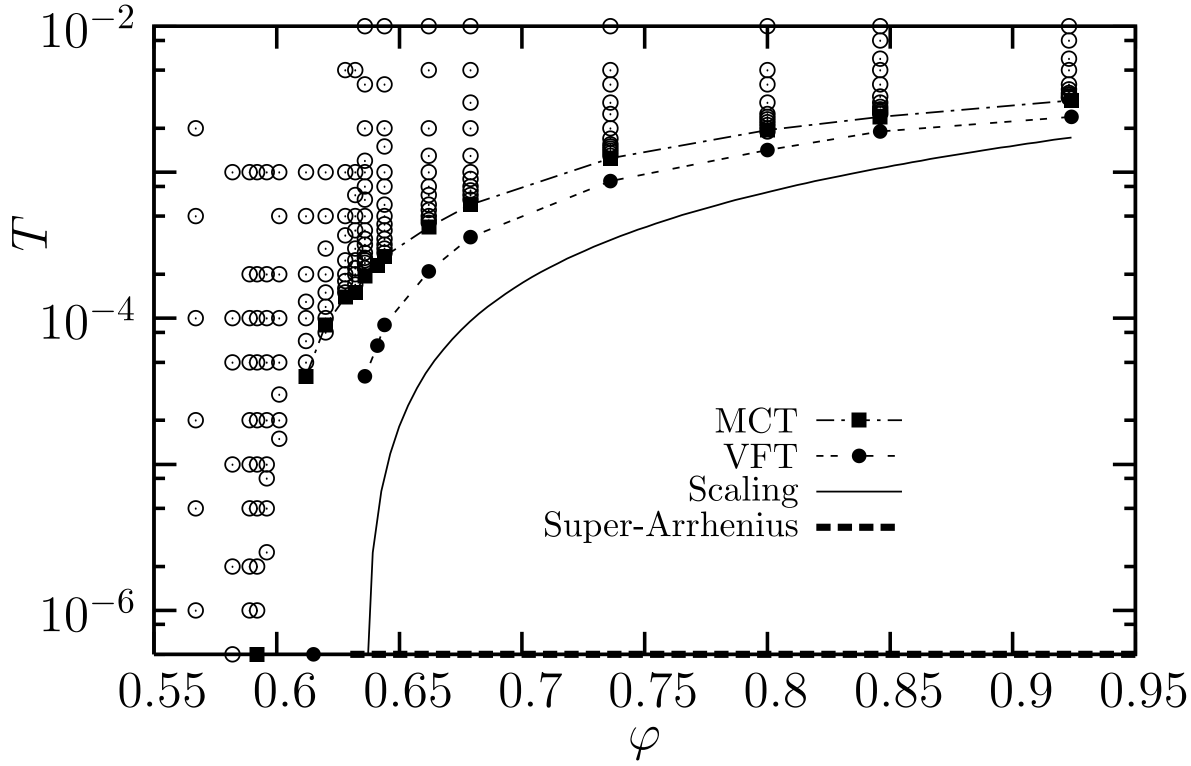
[Zhang *et al.*, Nature '09]

- The structural anomaly seen in jammed colloids is naturally explained by equilibrium concepts—in fact seen in many ultrasoft materials.

Why liquid state theory fails

- Numerical phase diagram of soft harmonic spheres

[Berthier & Witten, EPL, PRE '09]



- Glassy 'phase' is found. Glasses jam, fluids do not.

- Statmech needs to first handle the complexity of the glass phase before addressing the jamming transition.

IV-Glassy phase diagram

Statistical mechanics of glasses

- Assuming exponential number of metastable states exists:

$$f(T) = -\frac{T}{V} \log \int df' \exp\left[-\frac{Nf'}{T} + N s_{\rm conf}(f',T)\right].$$

- Replicas are then introduced to 'recognize' multiple amorphous metastable states.

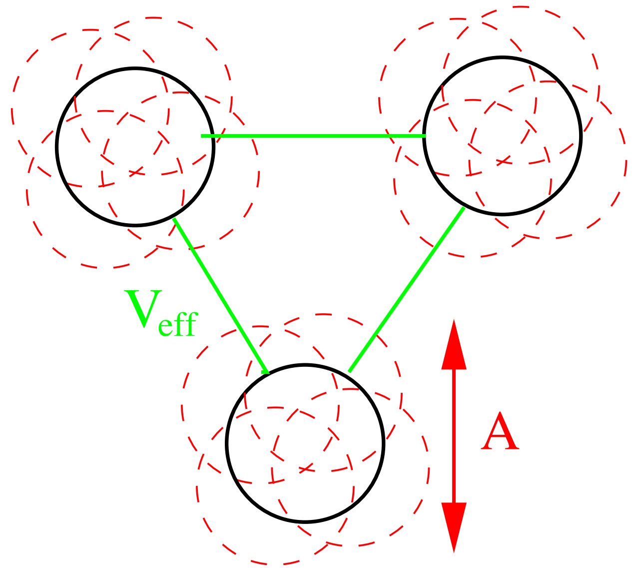
- Physical idea: $-T s_{\rm conf}$ represents the free energy cost of 'forcing' two replicas within the same state. [cf all lectures on Tuesday]

- In practice, take m replica(s) and minimize the replicated free energy. [Monasson, PRL '95]

$$f(m,T) = -\frac{T}{V} \log \int df' \exp\left[-\frac{Nf'm}{T} + N s_{\rm conf}(f',T)\right].$$

- In the 'ideal' glass: $s_{\rm conf}(T \le T_K) = 0$ and $f_{\rm glass} = f(m^\star(T),T)/m^\star(T)$, with $m^\star(T) < 1$ [Mézard & Parisi, PRL '99]. One-step replica symmetry breaking.

Replicated liquid state theory



- Liquid state theory of replicated liquid.

- Mézard-Parisi's 'small cage' (harmonic) expansion breaks down near jamming.

- Zamponi-Parisi 'small (but hard) cage' approximation is correct for hard spheres.
[Zamponi & Parisi, RMP '10].

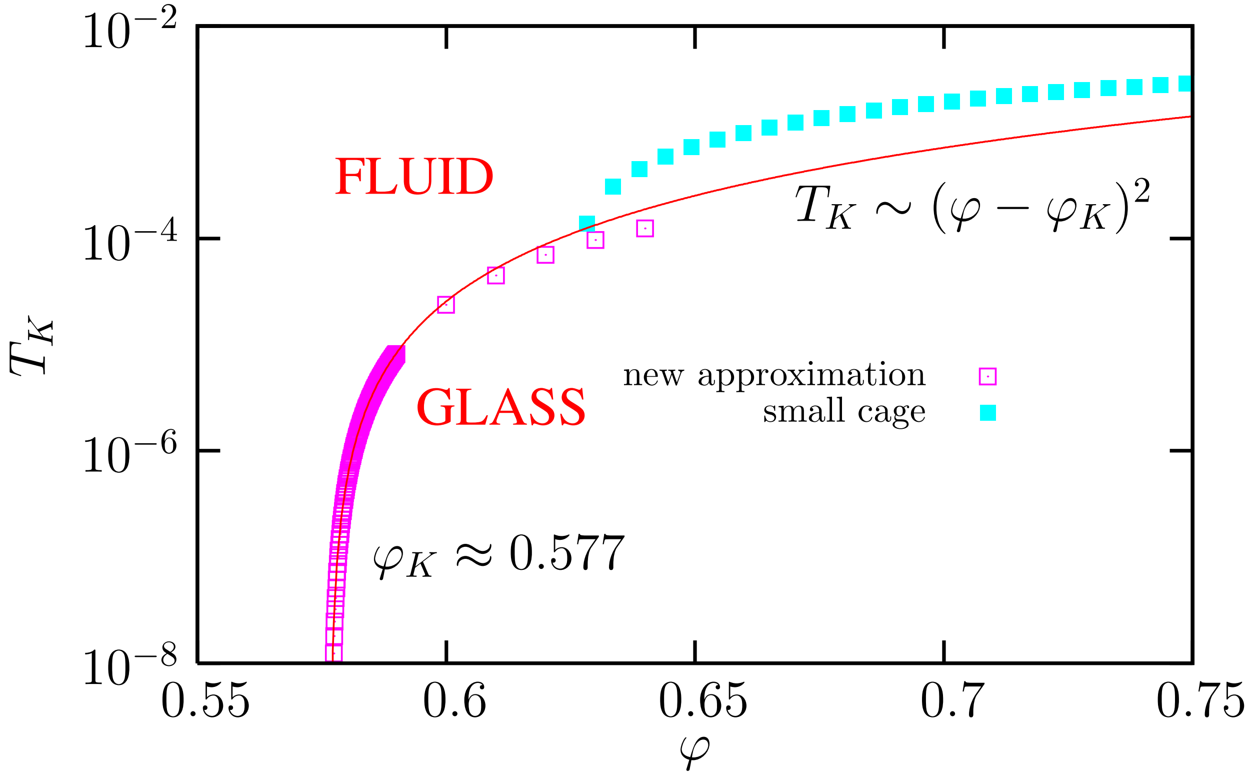
- An effective potential valid for both hard spheres ($T \to 0$ small φ) and soft glasses ($T \to 0$ large φ) [Jacquin *et al.*, PRL '11].

$$f(m, A, \varphi, T) =$$
$$f_{\text{harm}}(m, A) + f_{\text{liquid}}(\varphi, T/m) - \frac{\rho}{2} \int drg(r)[e^{-\beta(V_{\text{eff}}(r) - mV(r))} - 1]$$

- Low-T approximation to treat analytically the glass & jamming transitions of harmonic spheres.

The 'ideal' glass transition

- High T fluid: $m = 1$, $s_{\mathrm{conf}}(T) > 0$ and simple liquid theory is enough.
- $s_{\mathrm{conf}}(T)$ vanishes at $T_K(\varphi) > 0$ for $\varphi > \varphi_K \equiv$ hard sphere glass transition.



- Low-T scaling: $T_K \sim (\varphi - \varphi_K)^2$ (robust scaling) with $\varphi_K \approx 0.577$ (value depends on specific approx). Correct phase diagram.

The ‘ideal’ jamming transition

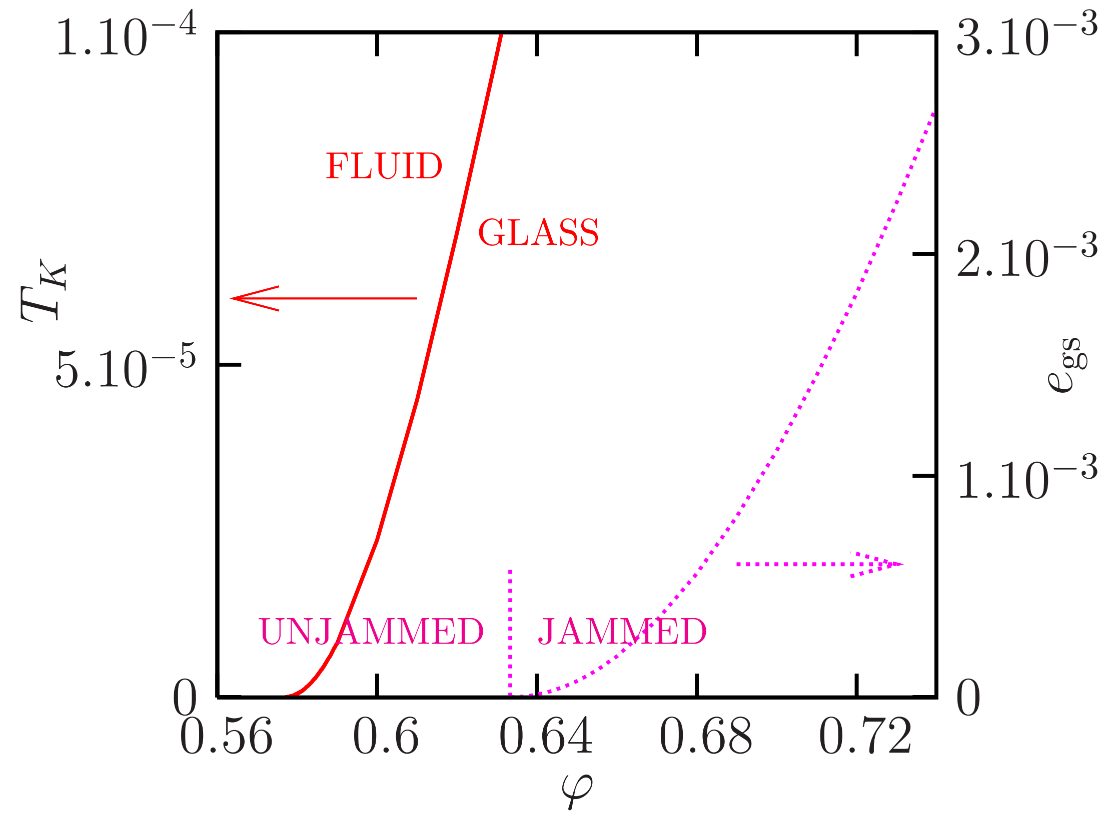
- Glass thermodynamics: energy, pressure, specific heat, fragility...

- Jamming at $T = 0 \Leftrightarrow$ Change in ground state glass properties.

- $\varphi_{\rm GCP} = 0.633353...$ such that:
  $E_{\rm GS} = 0$ below,
  $E_{\rm GS} \simeq a(\varphi - \varphi_{GCP})^2$ above.

- Glass Close Packing: densest $T = 0$ glass with no overlap.
  [Zamponi & Parisi, RMP '10]

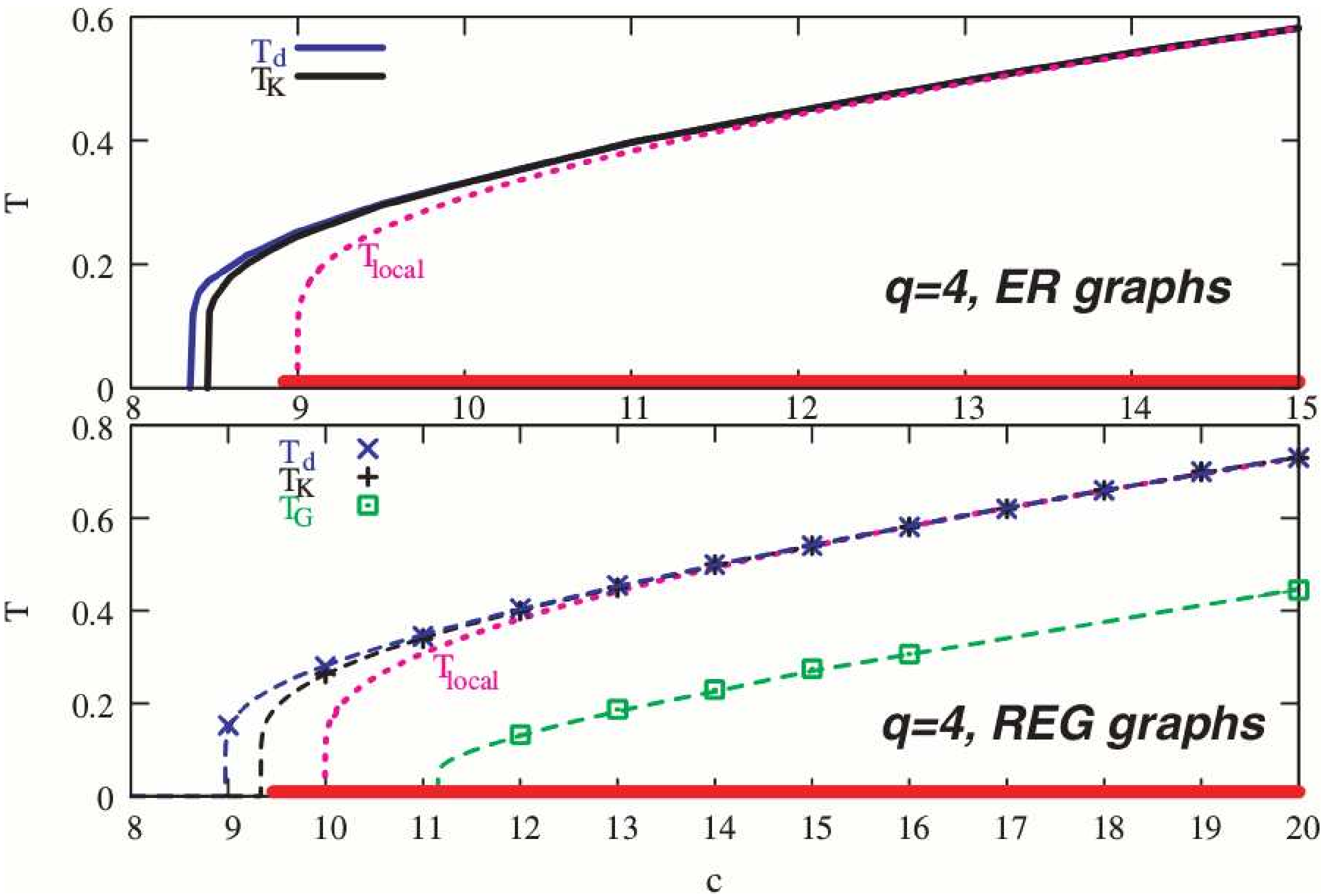
- $P_{\rm GS} \sim (\varphi - \varphi_{GCP})$.



- Existence, location(s), and scaling laws of jamming from ‘first principles’.

Constraint satisfaction problem

- Similar phase diagram for Potts glass model on random graphs, obtained using the cavity method (q-coloring problem as $T \to 0$).

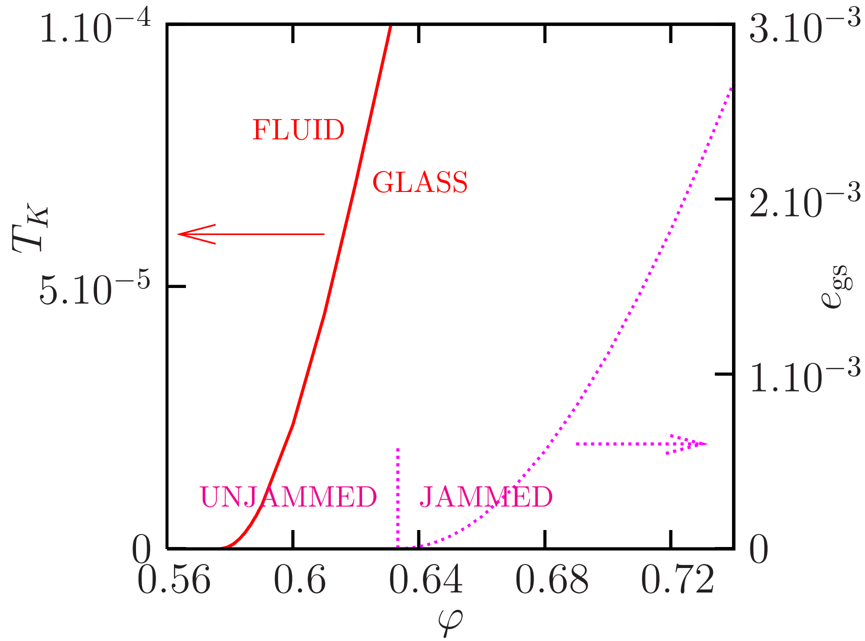


[Krzakala & Zdeborova, EPL '08]

Locating the jamming transition

- Glass Close Packing buried deep inside glass phase.

- Locating it means solving a very hard computational problem.



- A standard algorithm will fall out of equilibrium before reaching glass transition and be trapped in one of many metastable state.

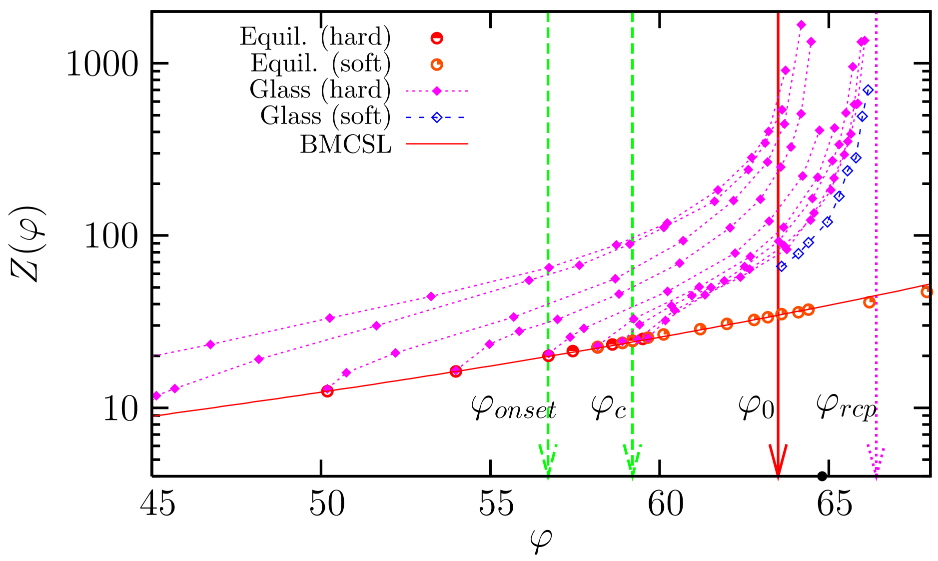
- Chosen state is followed down to $T = 0$, jams at $\varphi < \varphi_{GCP}$.

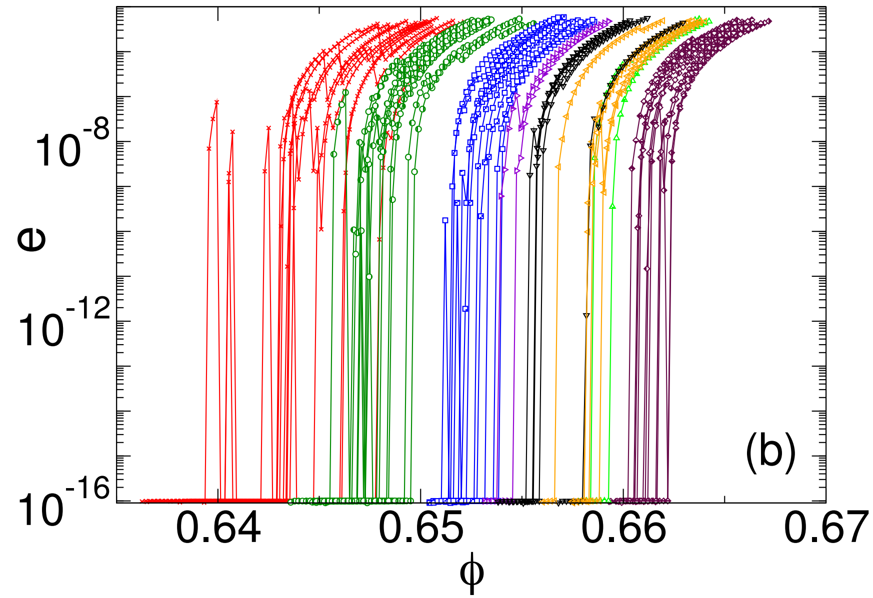
- Measured location of jamming point is therefore strongly algorithm-dependent. Point J is one of the 'worst'!

Numerical evidence: ‘J-line’

- Rapid compressions of hard sphere fluid configurations, starting from various packing fractions reveals continuous range of jamming densities: J-point $\to$ J-line.

- The normalized pressure, $Z = P/(\rho k_B T)$, diverges when $\varphi \to \varphi_J$.





[Chaudhuri *et al.*, PRL ’11]

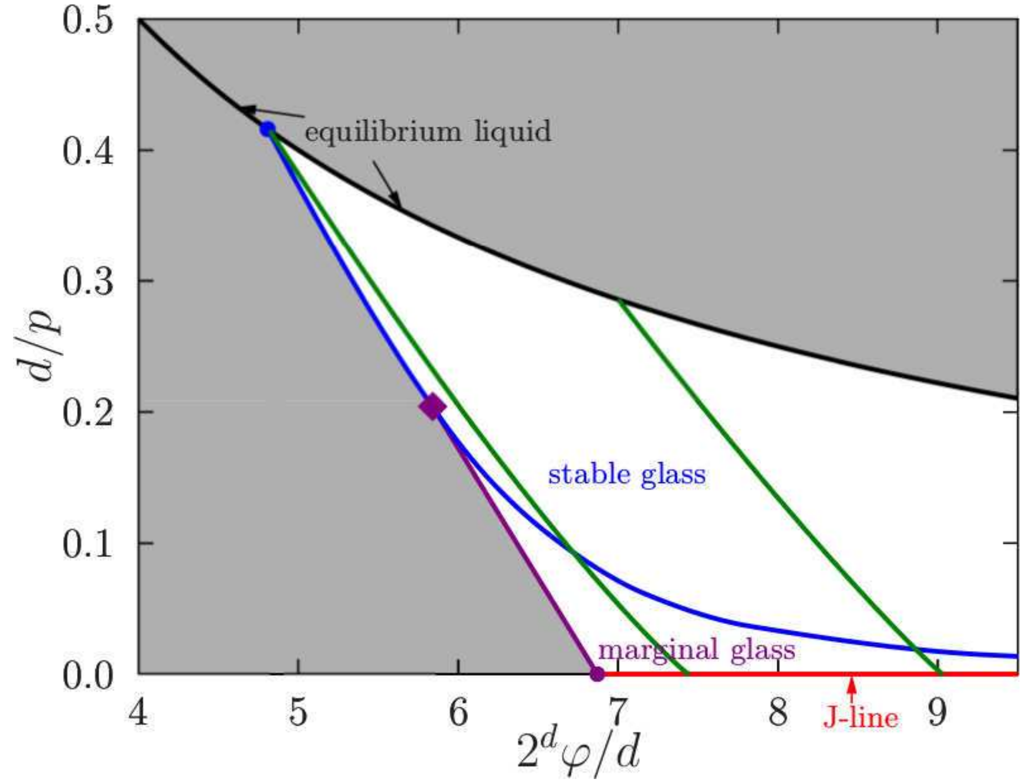
- Location of jamming transition is not uniquely defined but its physical & scaling properties seem ‘universal’, i.e. shared by all packings.

Gardner transition

- Problems of 1-RSB solution revealed by comparison to numerical work.

- Suggests that 1-RSB might be inconsistent... full RSB needed below the Gardner transition. Hard sphere limit has been treated in $d = \infty$.

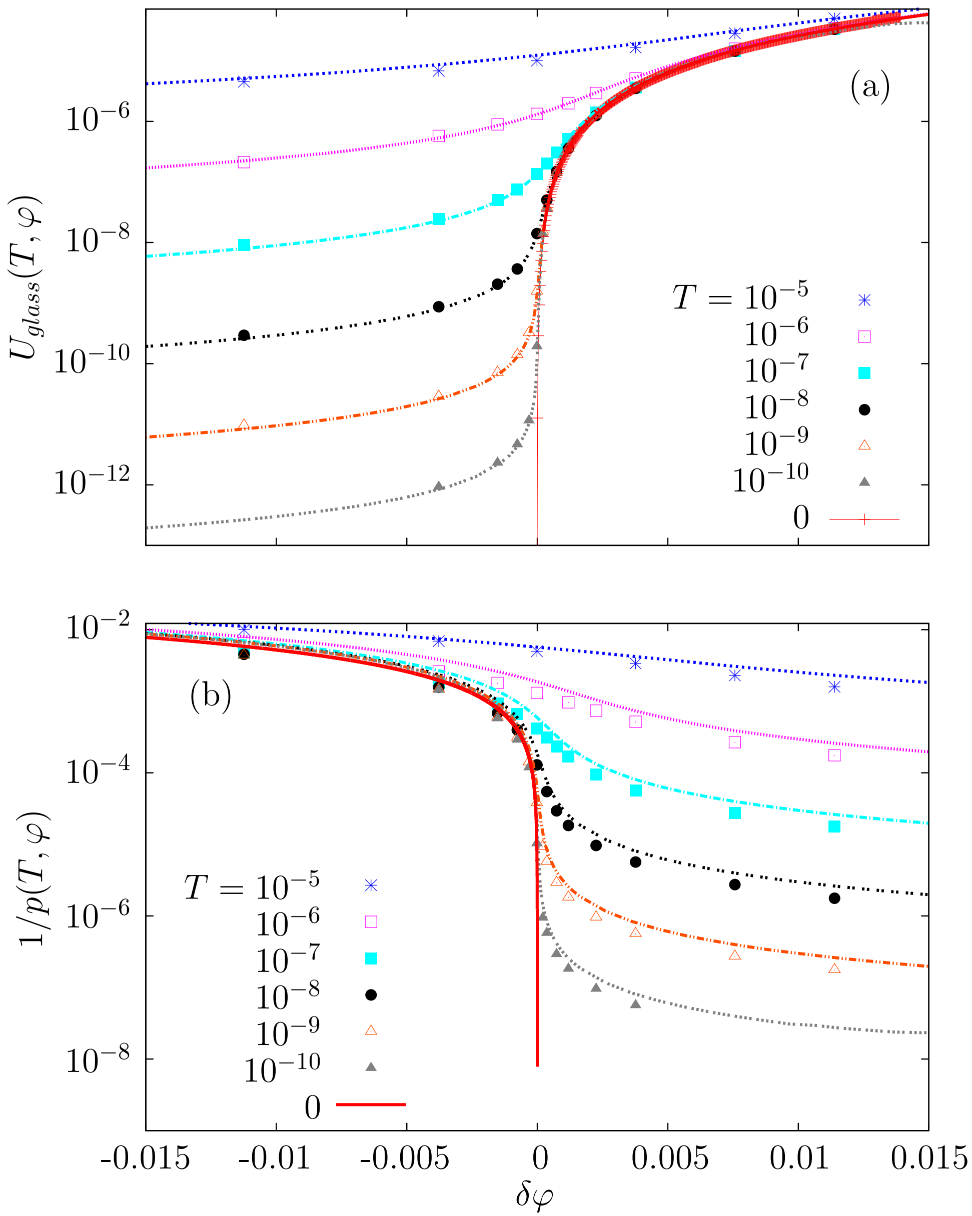
[cf. P. Urbani, this afternoon]



[Kurchan, Zamponi, Parisi, Urbani, Charbonneau... ’13-’14]

V-Microstructure of jammed packings

Thermodynamic properties



- Emergence of $T = 0$ discontinuity as $T \to 0$ in harmonic spheres.

- Scaling laws for several properties are predicted within 1-RSB solution.

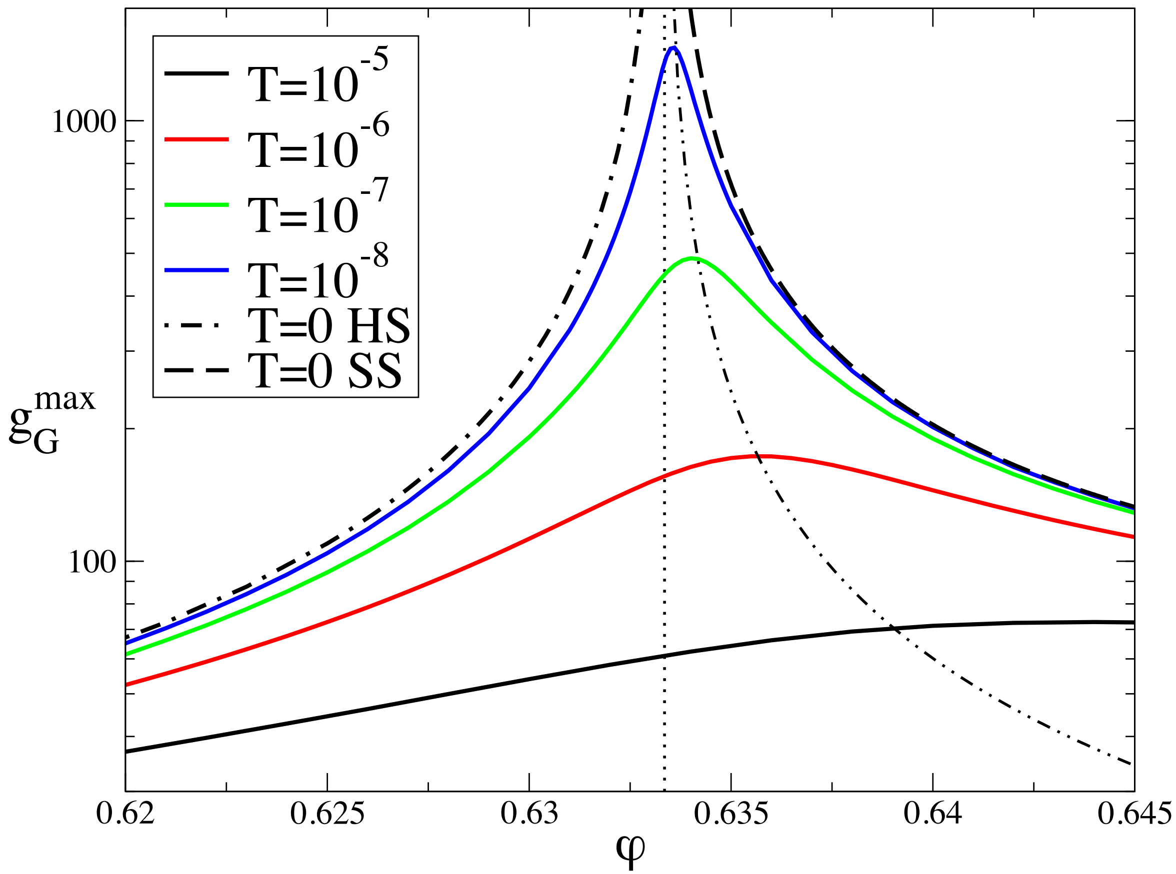
- $U(T,\varphi) = T\mathcal{F}_U(\frac{|\varphi-\varphi_J|}{\sqrt{T}})$,
  $P = \sqrt{T}\mathcal{F}_P(\frac{|\varphi-\varphi_J|}{\sqrt{T}})$.

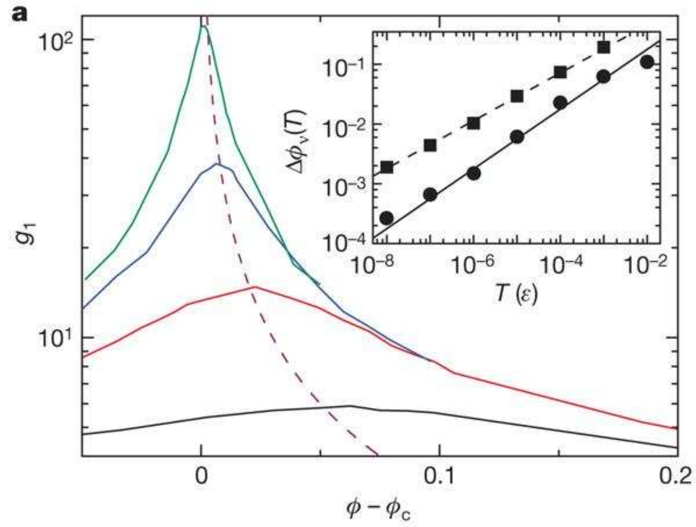
- Natural matching between jammed and unjammed regions.

[Berthier, Jacquin, Zamponi, PRE '11]

Nonmonotonic glass structure

- Evolution of maximum of $g(r)$ at low T has been measured numerically.



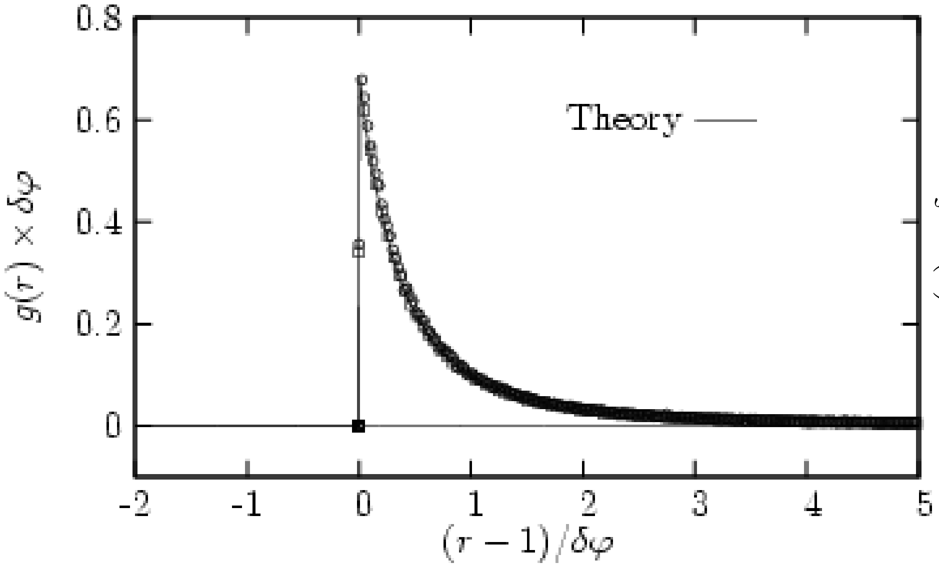


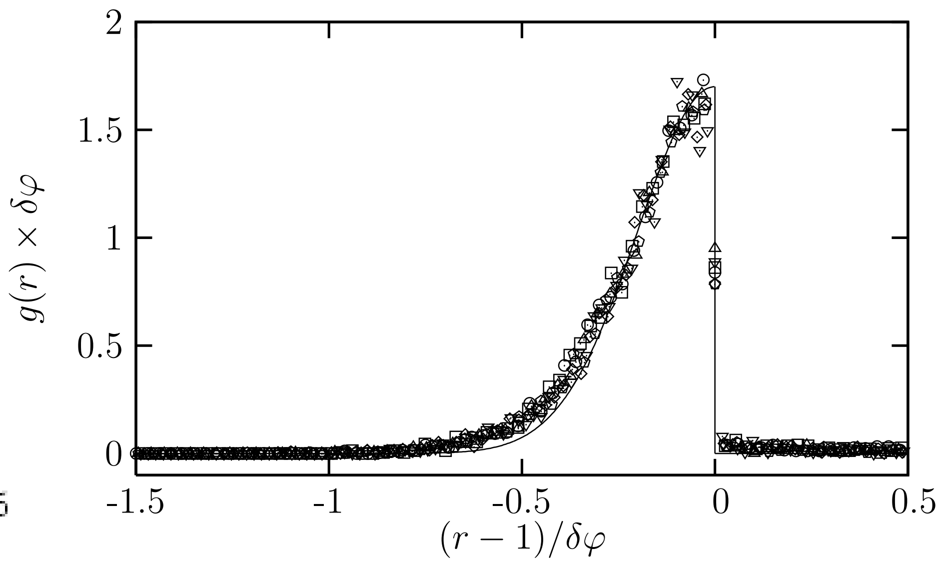
[Zhang *et al.* Nature '09]

- $g(r)$ near contact is quite well described–other distances are harder to treat...

$T = 0$ pair correlation

- Predictions for $g(r)$ near contact at all (T, φ).
- $g(r) \approx z_c \delta(r-1)$ at $(T=0, \varphi = \varphi_{\rm GCP})$ with $z_c = 6$: isostaticity is predicted.
- Asymmetric scaling: $g(r) \approx |\delta\varphi|^{-1} \mathcal{F}_{\pm} \left[\dfrac{r-1}{|\delta\varphi|} \right]$ at $(T=0, \varphi = \varphi_{\rm GCP} \pm \delta\varphi)$.

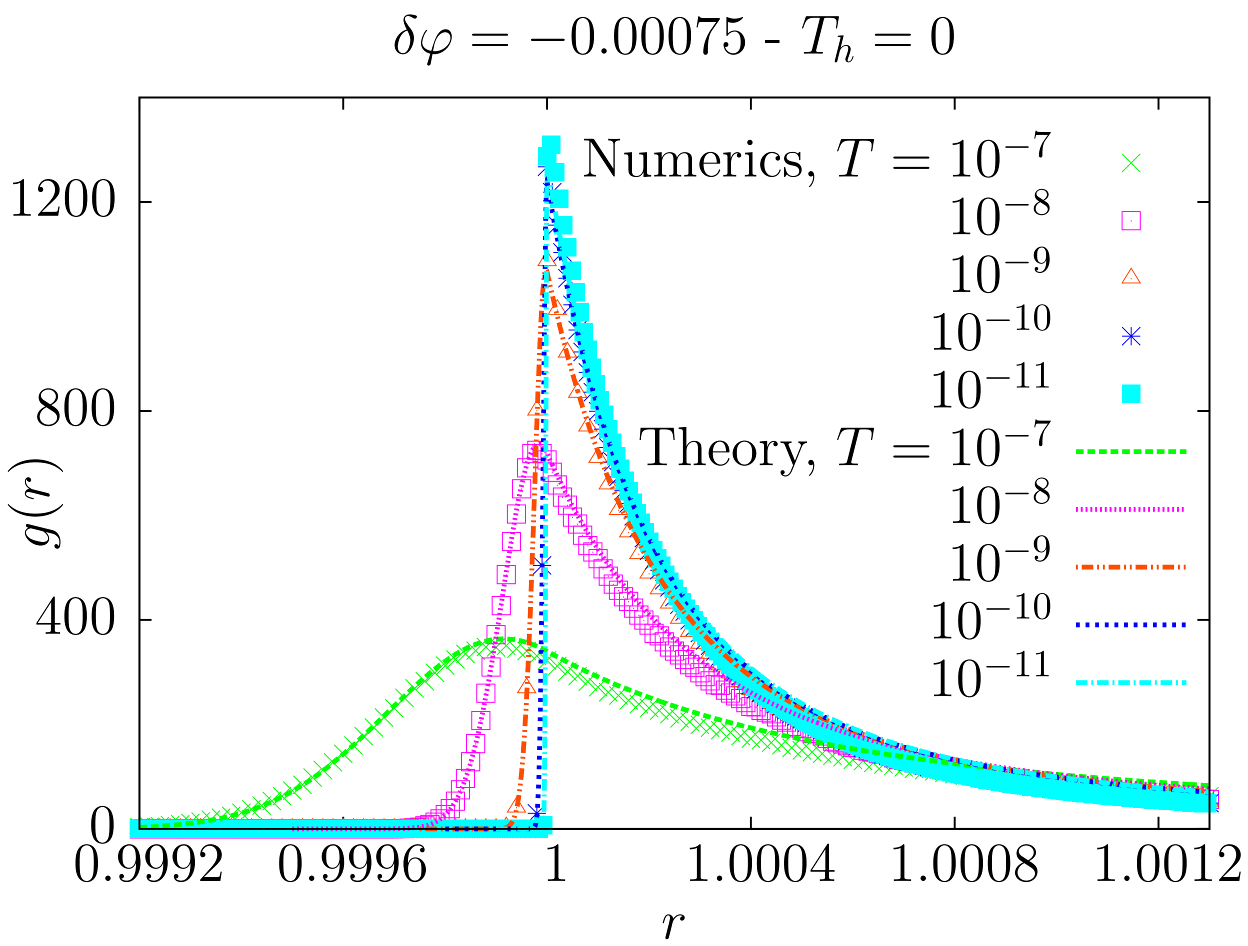




- $T = 0$: Critical version of fluid ‘anomaly’. Occurs for the same physical, ‘equilibrium’, reason (energy/entropy competition).

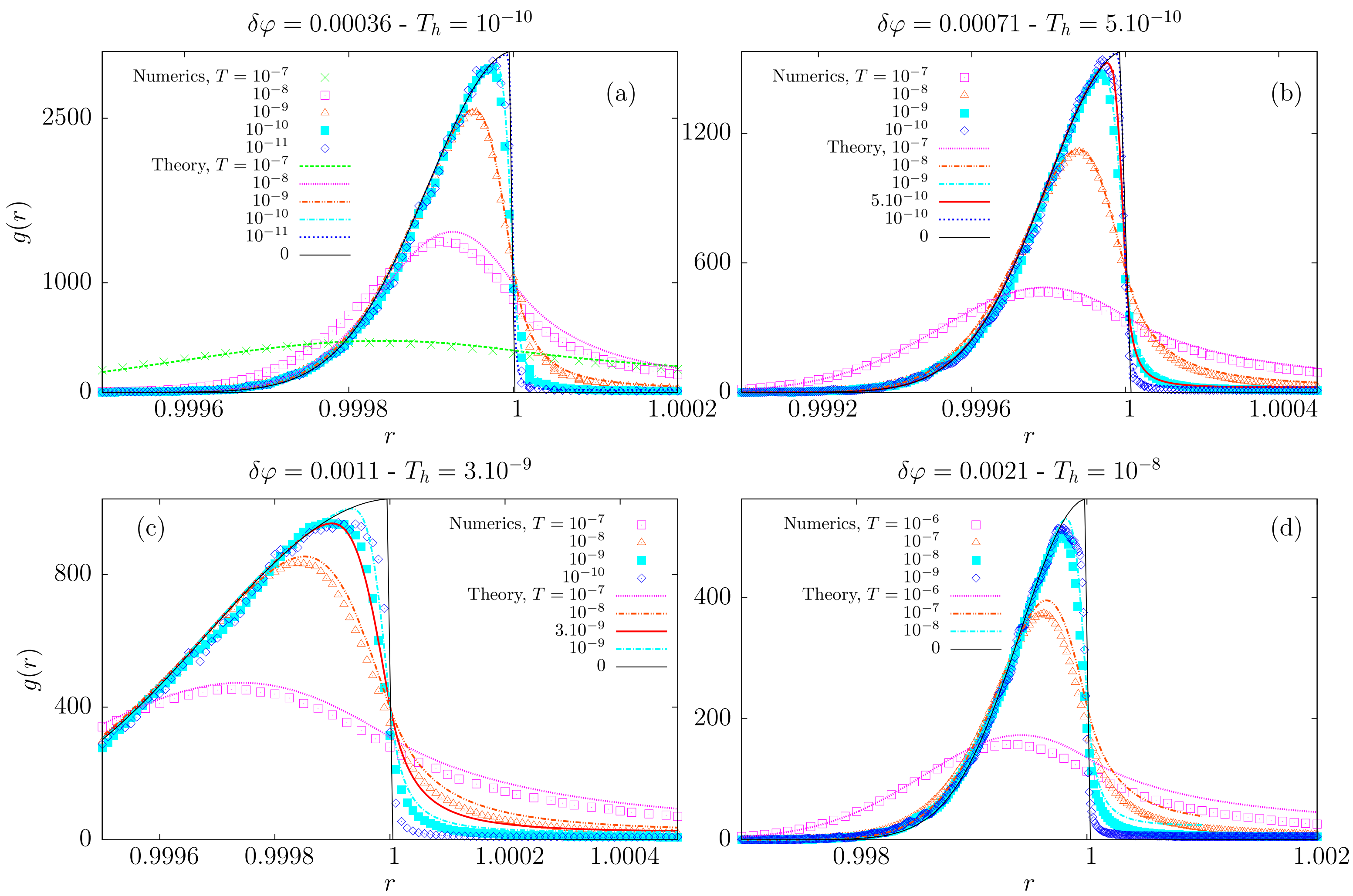
Low temperature structure

- 1-RSB theory seems correct on hard sphere side, $\varphi < \varphi_J$.



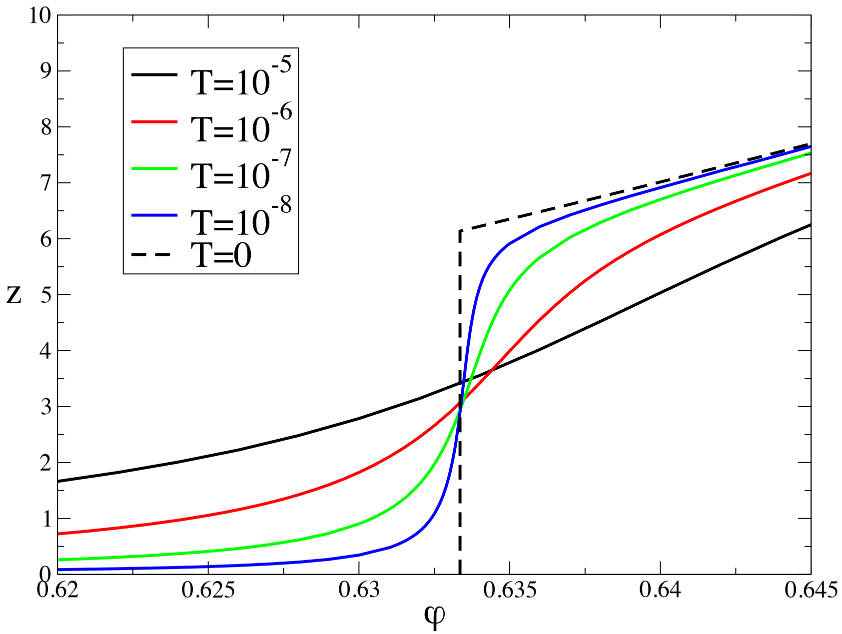
Low temperature structure

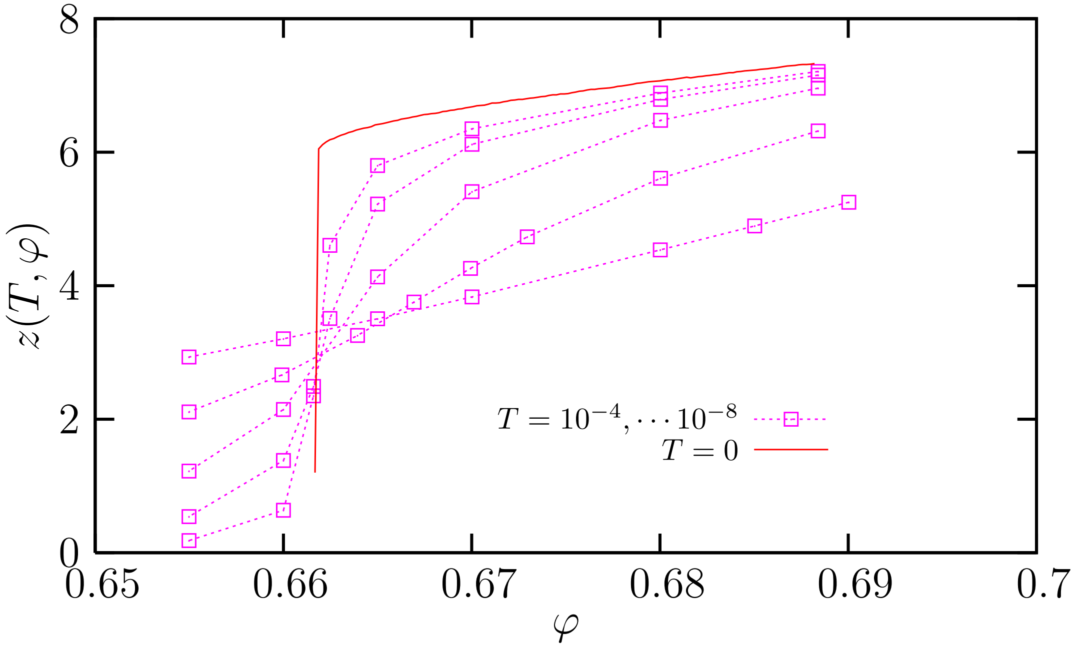
- OK above jamming if $T < T_h(\varphi) \sim (\varphi - \varphi_J)^2$: Limit of 1-RSB validity?



1-RSB biggest failure

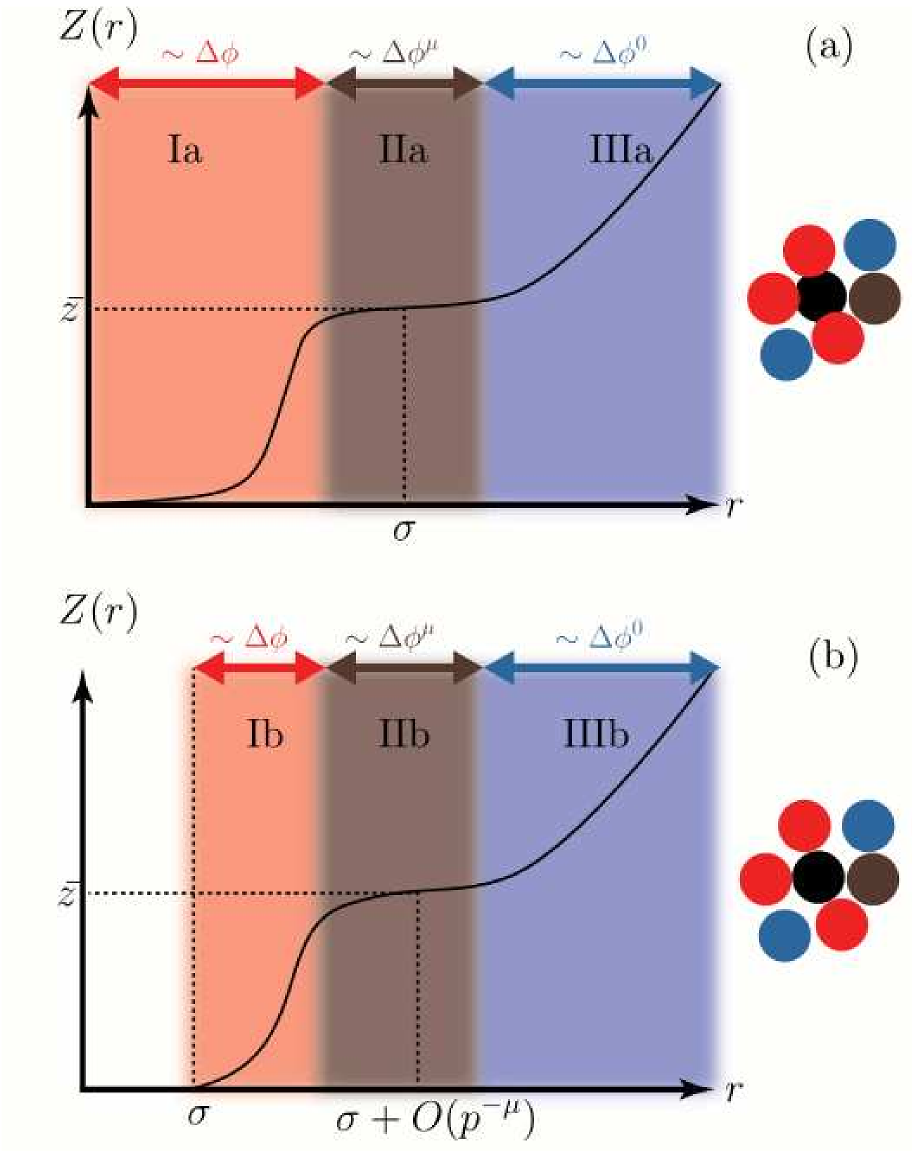
- $z(\varphi, T) = \rho \int_0^1 dr 4\pi r^2 g(r)$ is the number of contacts.
- $z(\varphi > \varphi_{GCP}, T = 0) \approx z_c + c(\varphi - \varphi_{GCP})^\alpha$ with incorrect exponent $\alpha = 1$.
- Large number of 'near contacts' below $\varphi_{\rm GCP}$ at $T = 0$ is missed by 1-RSB effective potential approach.





- This failure is fully cured by the full RSB solution.

Final (?) results–this afternoon



[Charbonneau *et al.*, PRL '12]

- $z(r) = \rho \int_0^r 4\pi r^2 g(r) dr$

- Power law approach to isostatic plateau, corresponds to weak inter-particle forces, $P(f \ll \langle f \rangle) \sim f^{\theta}$.

- Abundance of near contact, $g(r) \sim (r-1)^{-\gamma}$.

- Non-trivial full-RSB $d = \infty$ predictions for $\theta = 0.42311$ and $\gamma = 0.41269$ [cf P. Urbani]

- Non-trivial scaling relations from stability requirements [cf M. Wyart]

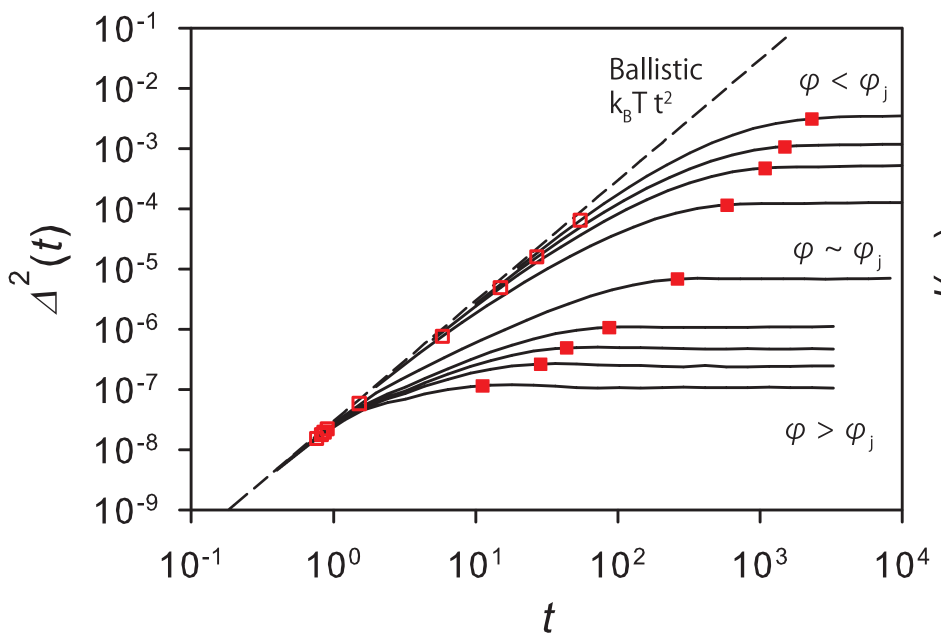
- Precise numerical measurements in many d [cf P. Charbonneau]

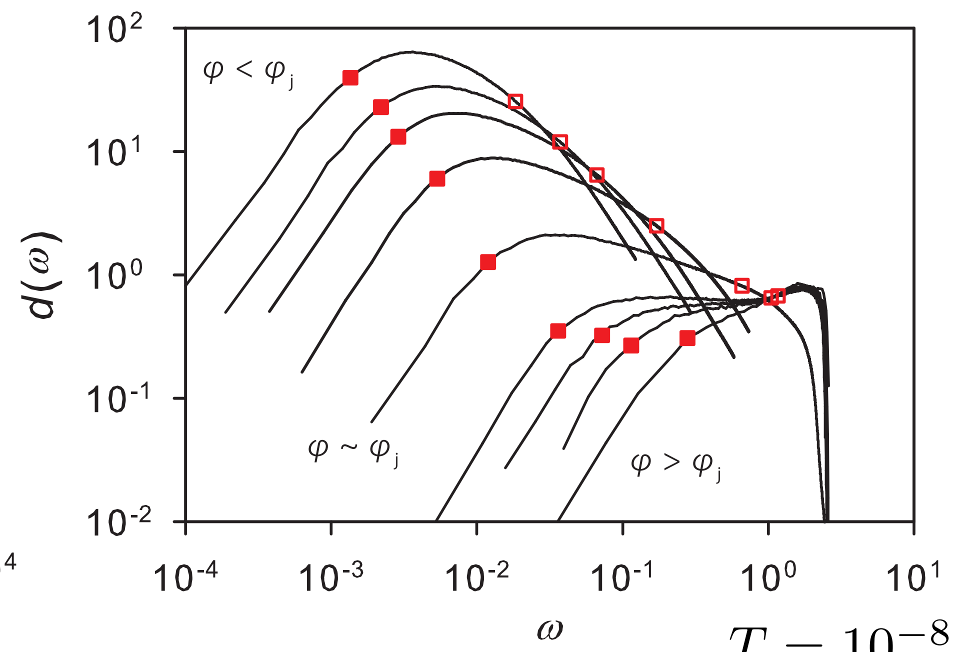
VI-Vibrational Dynamics

Vibrational dynamics in glass

- First step towards dynamics: mean-squared displacements at finite T.

[Ikeda, Berthier, Biroli, JCP'13]



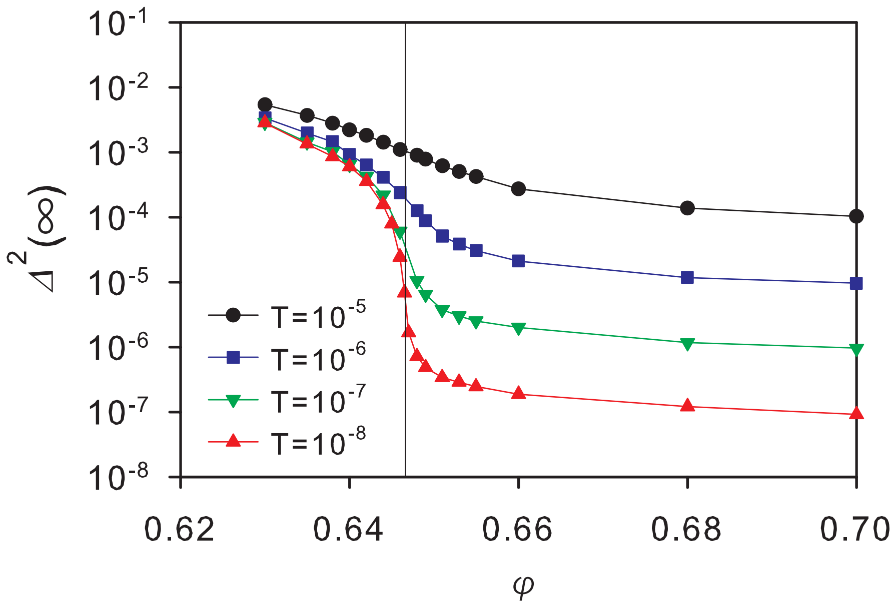


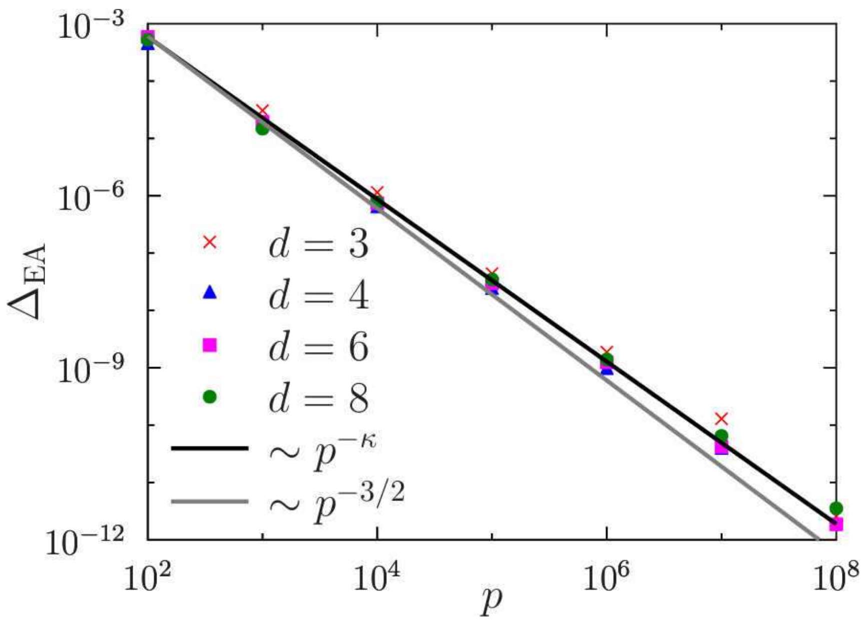
$T = 10^{-8}$

- Ballistic time: $\tau_0 \sim |\varphi - \varphi_J|/\sqrt{T}$ for $\varphi < \varphi_J$, $\tau_0 \sim const.$ for $\varphi > \varphi_J$.

- "Anomalous" timescale to reach plateau: $t^\star \sim \dfrac{2\pi}{\omega^\star}$, where $\omega^\star$ is the frequency of soft modes if harmonic approximation works.

Anomalous vibrational dynamics





- Amplitude of vibration much larger than naive guess $\ell_0^2 = T\tau_0^2$.

- Non-trivial power law scaling: $\Delta^2(\infty) \sim (\varphi_J - \varphi)^\kappa$ with $\kappa \sim 1.5$.
[Brito & Wyart, JCP '09]

- $\kappa = 1.41574$ directly predicted from full RSB $d = \infty$ hard sphere.
[Charbonneau *et al.*, JSTAT '14]

- Density of states and MSD related to $P(f)$ and θ via scaling arguments.
[DeGiuli *et al., arxiv:1402.3834*]

Dynamic criticality near jamming

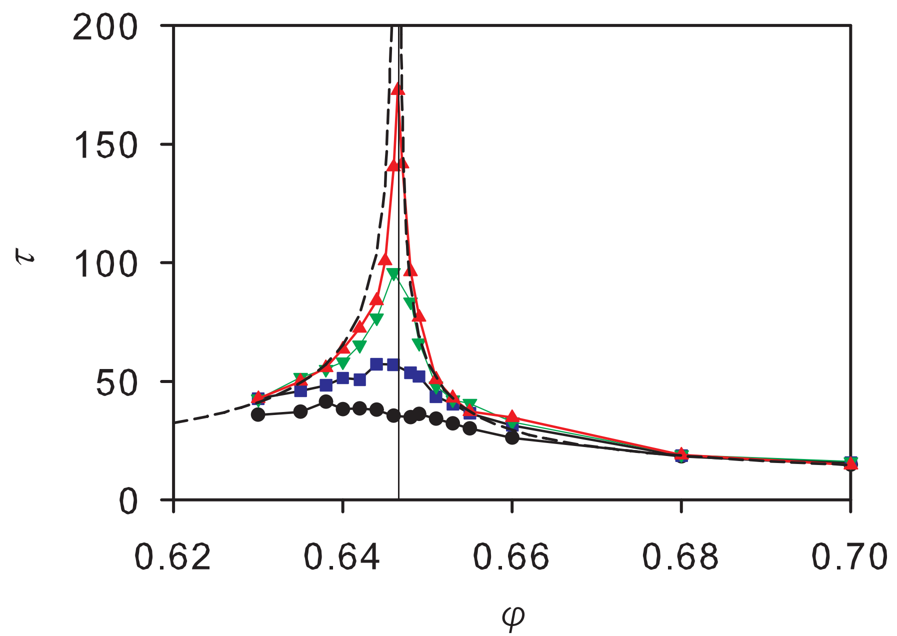
- Diverging (rescaled) timescale: $\tau = \dfrac{t^\star}{\tau_0} \sim |\varphi - \varphi_J|^{-1/2}$.

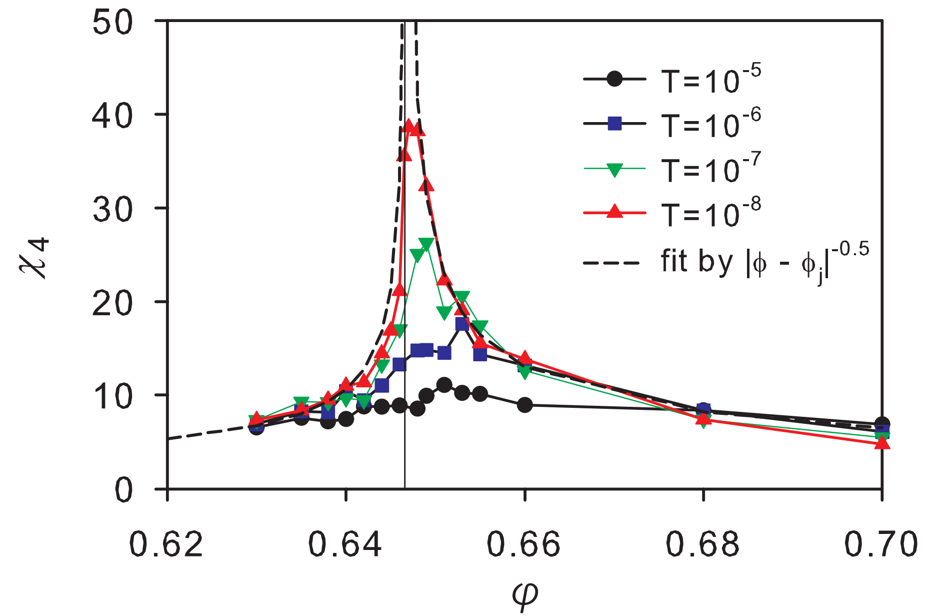
- Diverging dynamic susceptibility: $\chi_4 \sim |\varphi - \varphi_J|^{-1/2}$.

- Diverging dynamic correlation length: $\xi_4 \sim |\varphi - \varphi_J|^{-1/4}$.

- Time and length scales associated to anomalous vibrational motion diverge near jamming.

- Criticality observed if T is very low (particles not too soft), and very near φ_J.

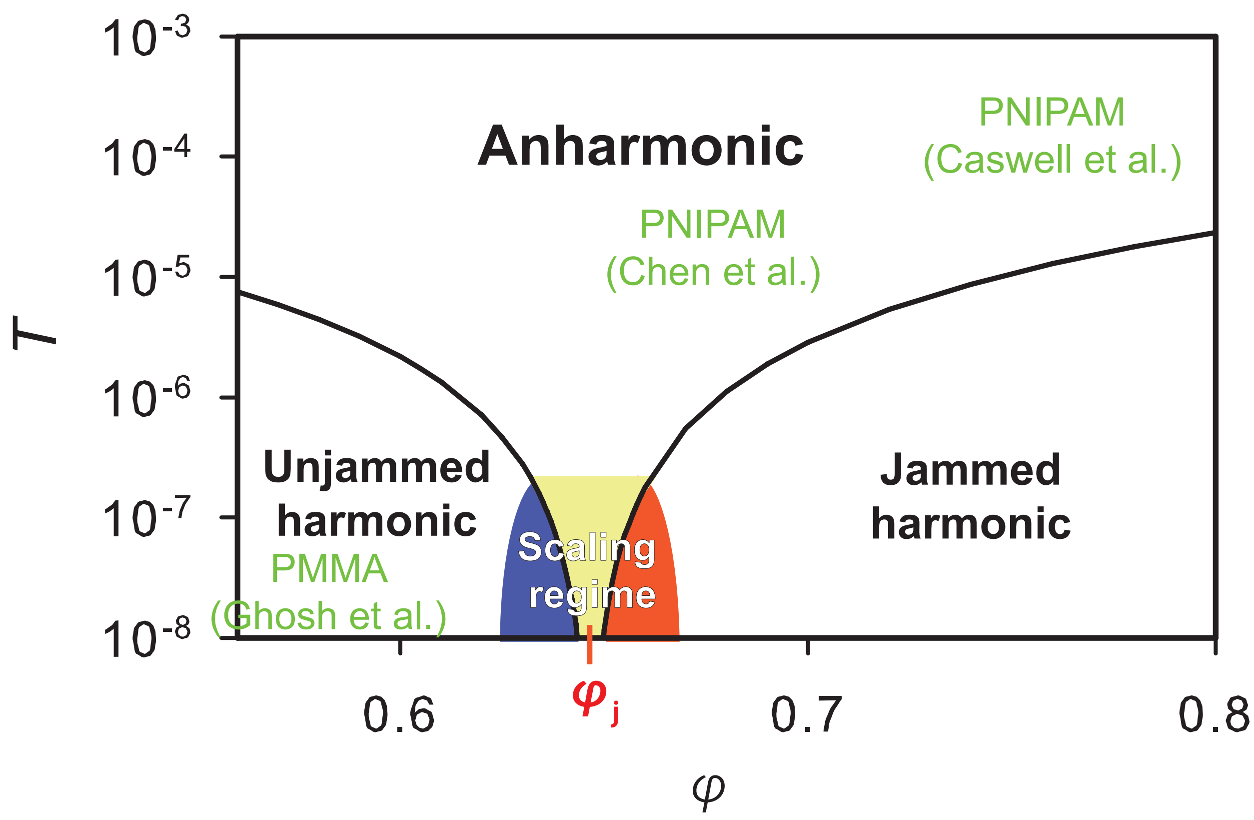




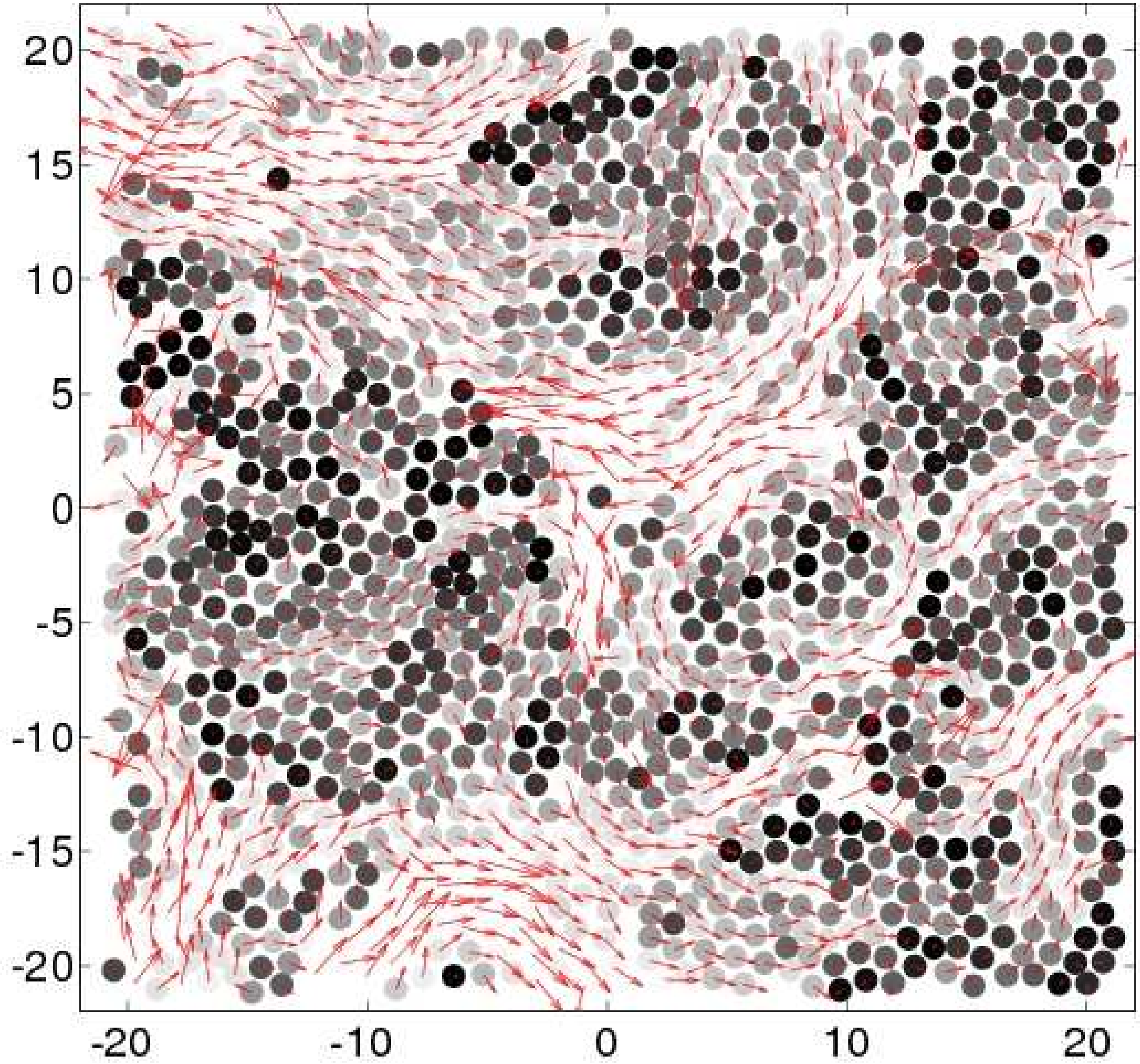
Connection to experiments?

- Many experiments have measured “soft modes” (hard spheres, microgels, grains...).
- What is the extension of the critical regime?
- When does the harmonic approximation work?

[cf O. Dauchot’s lecture]



Critical motion in grains



[O. Dauchot]

VII-Rheology

Revisit jamming phase diagram

- Aim: Study the full (σ, T, φ) phase diagram of harmonic spheres.

- Langevin dynamics with shear and thermostat in $d = 3$:

$$\xi\left(\frac{d\mathbf{r}_i}{dt} - \dot{\gamma} y_i \mathbf{e}_x\right) = -\sum_j \frac{dV(|\mathbf{r}_i - \mathbf{r}_j|)}{d\mathbf{r}_i} + \eta_i,$$

with $\langle \eta_i(t)\eta_j(t')\rangle = 2k_B T \xi \mathbf{1} \delta(t - t')$.

- Two important microscopic timescales:

(i) dissipation: $\tau_0 = \xi a^2/\epsilon = 1$, our time unit.
(ii) thermal time: $\tau_T = \xi a^2/(k_B T) \to \infty$ when $T \to 0$. $Pe \equiv \dot{\gamma}\tau_T$ (Peclet).

- Two stress scales: $\sigma_T = k_B T/a^3$ (entropy / thermal),

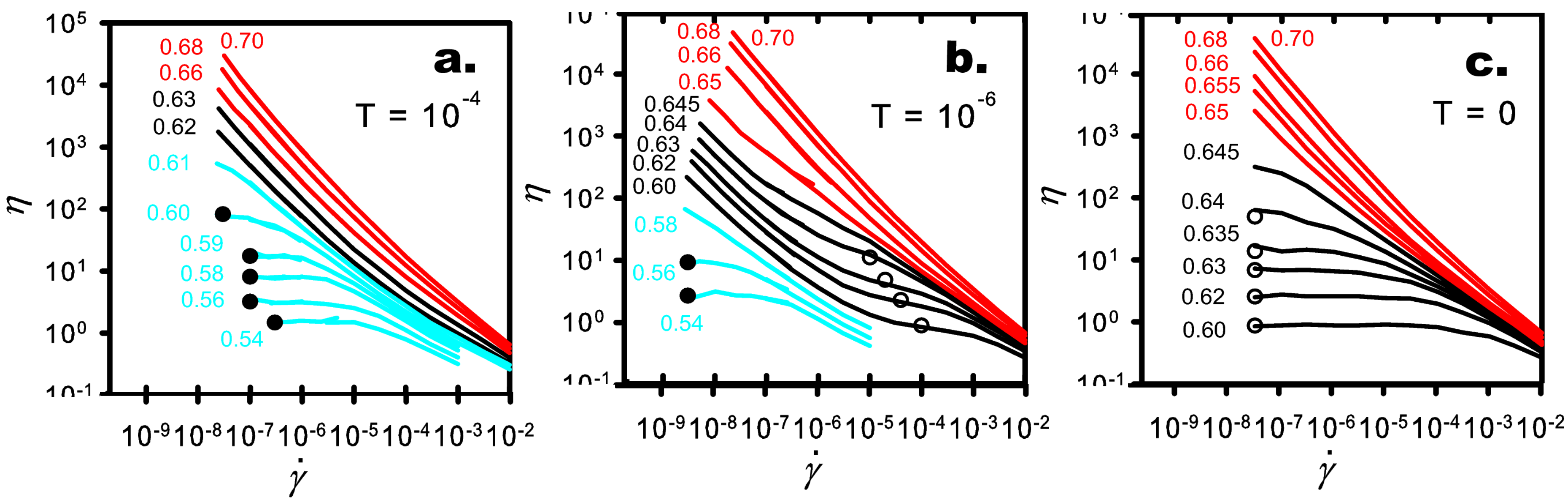
$\sigma_0 = \epsilon/a^3$ (energy / athermal).

- Study both finite and zero temperatures, both thermal ($Pe < 1$) and athermal ($Pe > 1$) rheologies at once.

[Ikeda *et al.*, PRL '12]

Flow curves

- Steady state rheology at $T = 10^{-4}$, $T = 10^{-6}$, and $T = 0$.

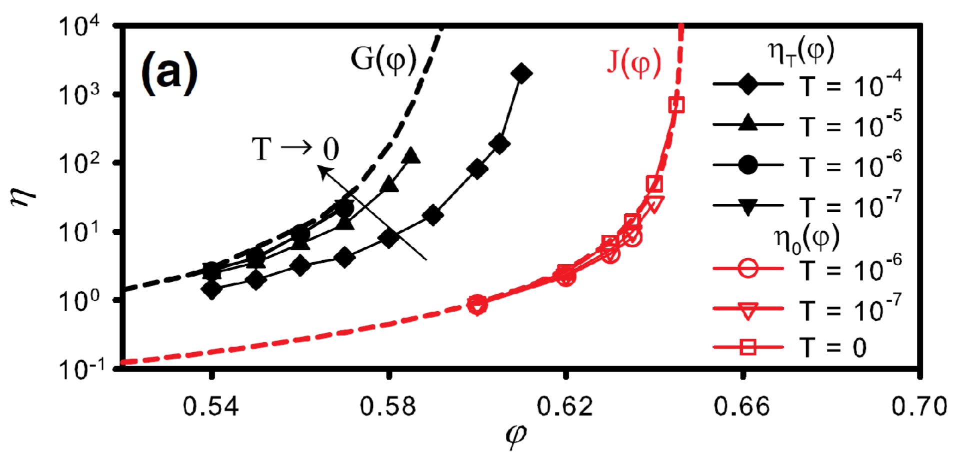


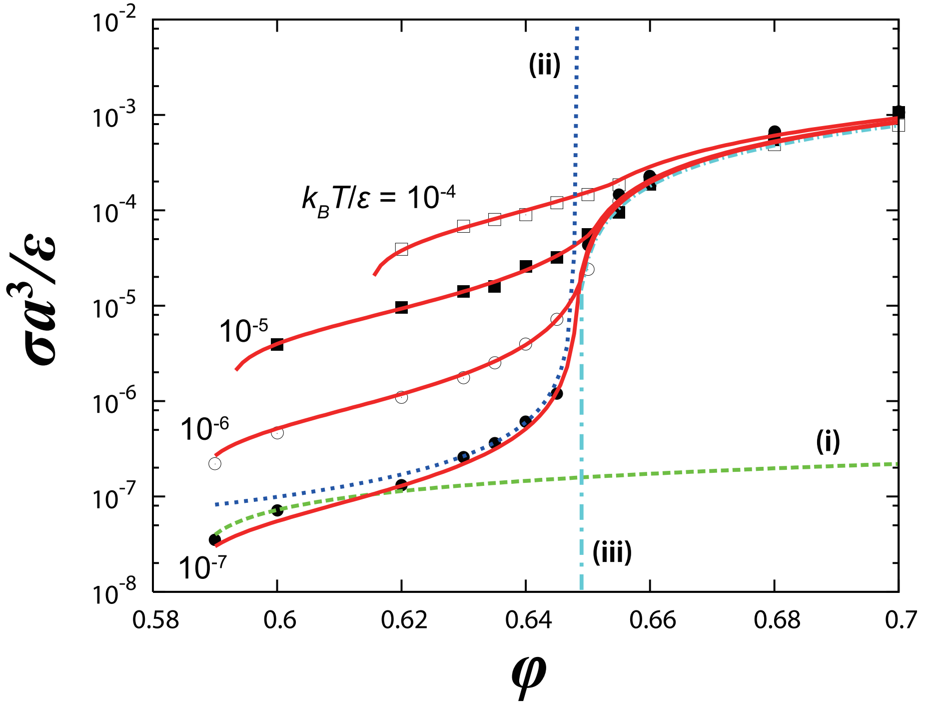
- From glass to jamming rheology, with interesting crossover.

- Two types of Newtonian regimes, depending on the Péclet number and particle softness.

- Striking similarity between glass & jamming flow curves.

Viscosity & yield stress

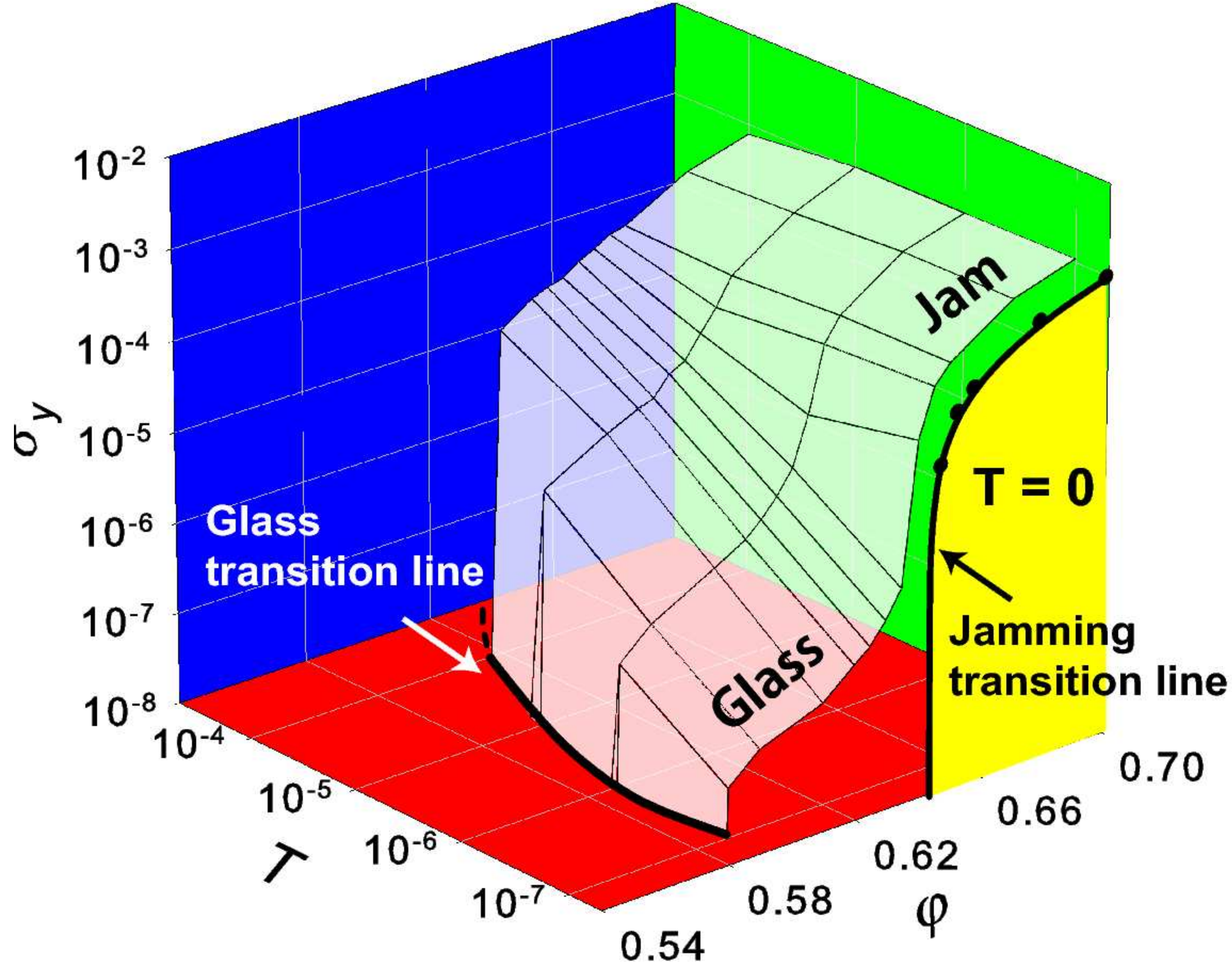




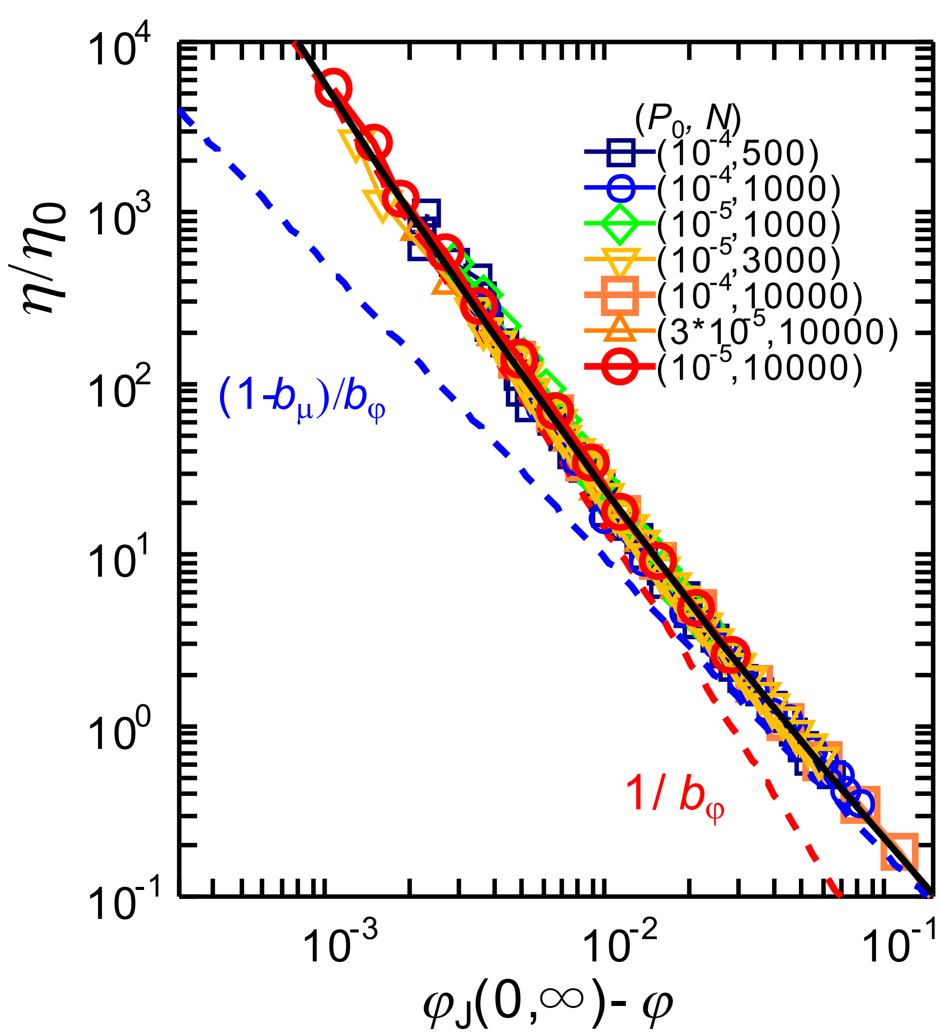
- $\eta_T(T > 0)$ does not converge to $\eta_0(T = 0)$ when $T \to 0$. These are distinct divergences at distinct densities with distinct physics.

- Solidity (yield stress) emerges at the glass transition at any $T > 0$, and jamming transition seen in density dependence of σ_Y.

‘Jamming phase diagram’



Non-Brownian hard spheres



[T. Kawasaki]

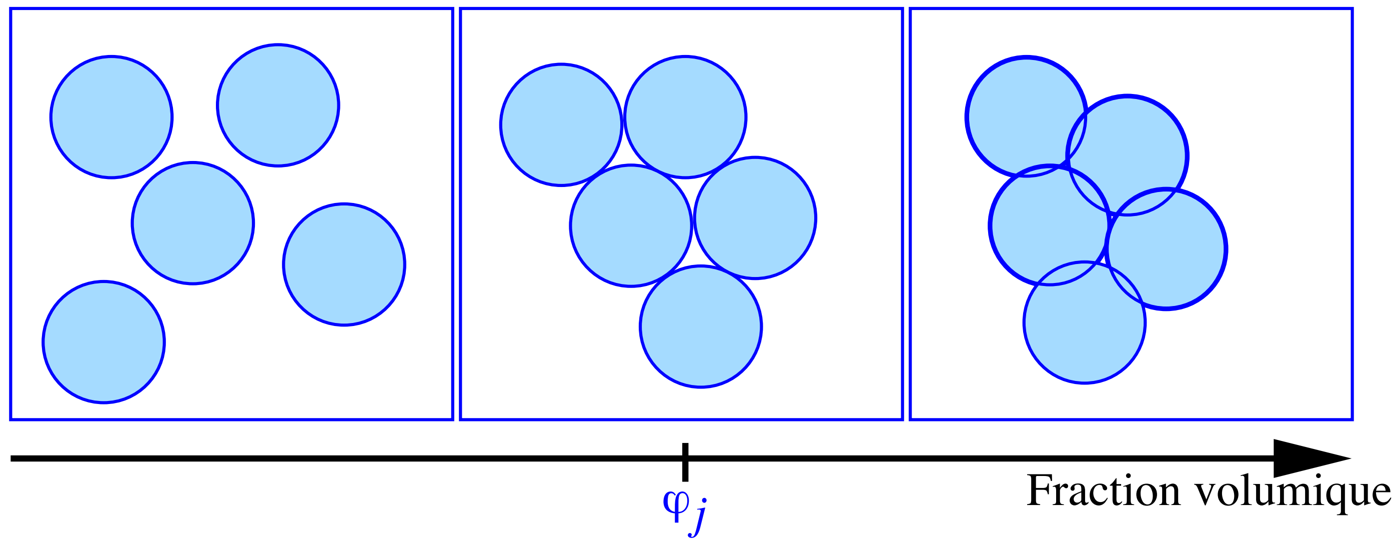
- Nature of viscosity divergence at $T = 0$?

- Simulations suggest genuine power law divergence (i.e. no crossover to some activated dynamics).

- Analogy with (nearly isostatic) elastic network relates viscosity to structure. (cf Wyart this afternoon.) [Lerner *et al.* '11]

- Structure of sheared suspensions seems difficult to analyse from first principles.

Conclusion



- Nature of jamming transition has been greatly clarified over recent years, salient features identified, consequences and generalisations explored.

- Remarkable success of replica approach... in $d = \infty$.

- Remarkably compatible with geometrical approach using marginal stability and isostaticity.