On glassy dynamics as a melting process…

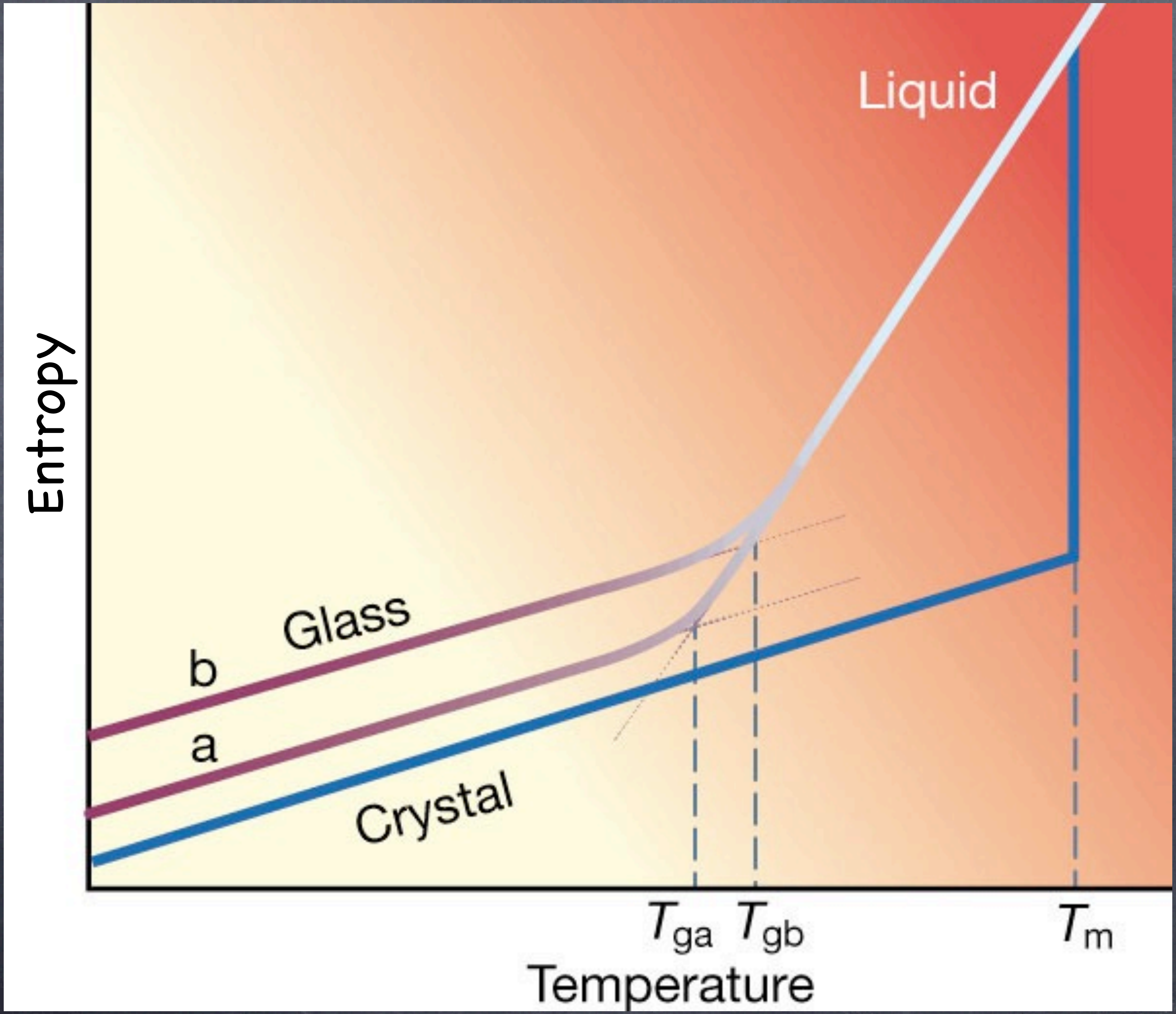
Florent Krzakala & Lenka Zdeborová

- Introduction to (some) glassy phenomenology
- The bulk melting problem
- Glassy and melting dynamics are (sometimes!) in the same university class

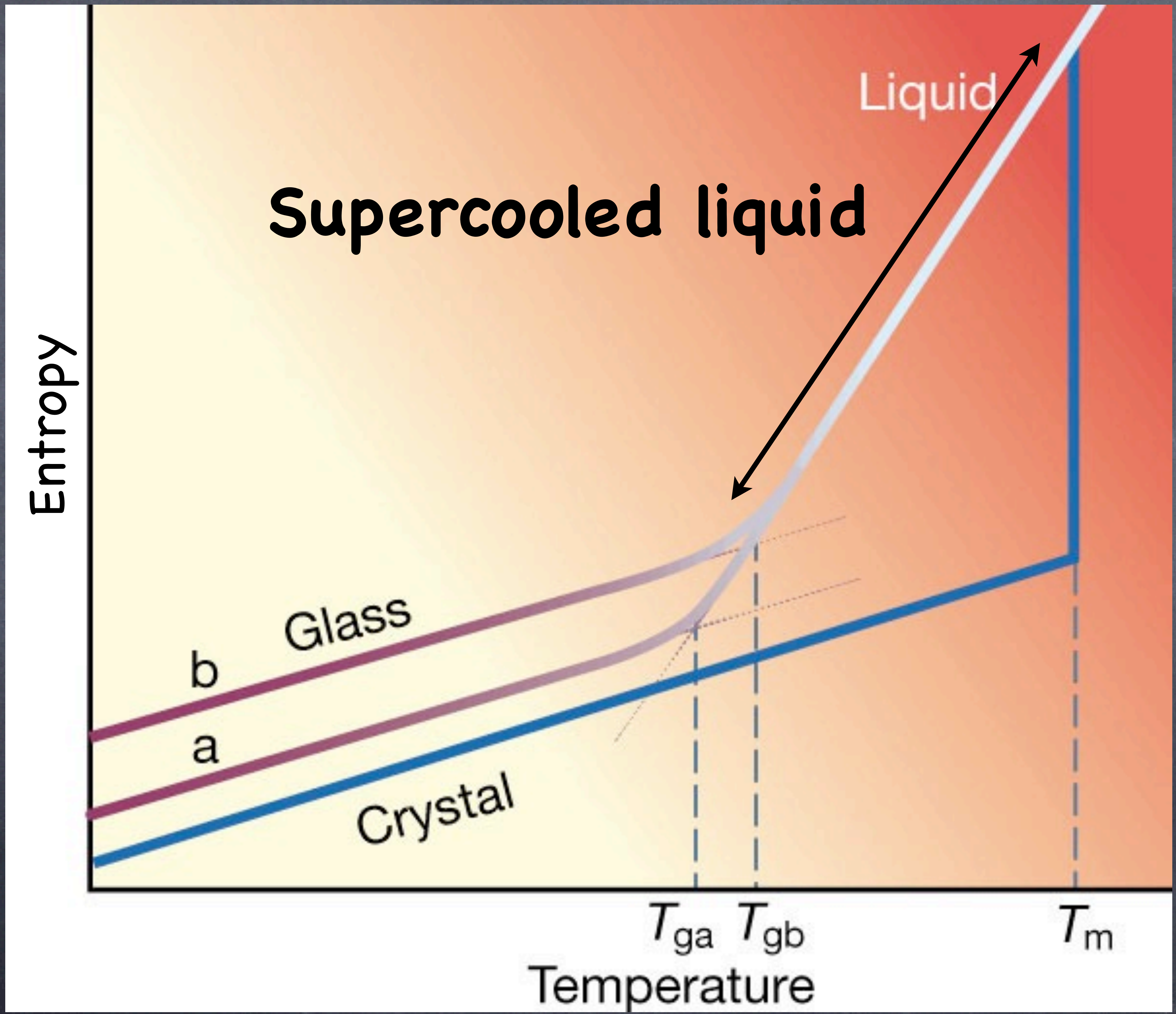
What is a glass ?

"The deepest and most interesting unsolved problem in solid state theory is probably the nature of glass and the glass transition". P.W. Anderson, Science '95

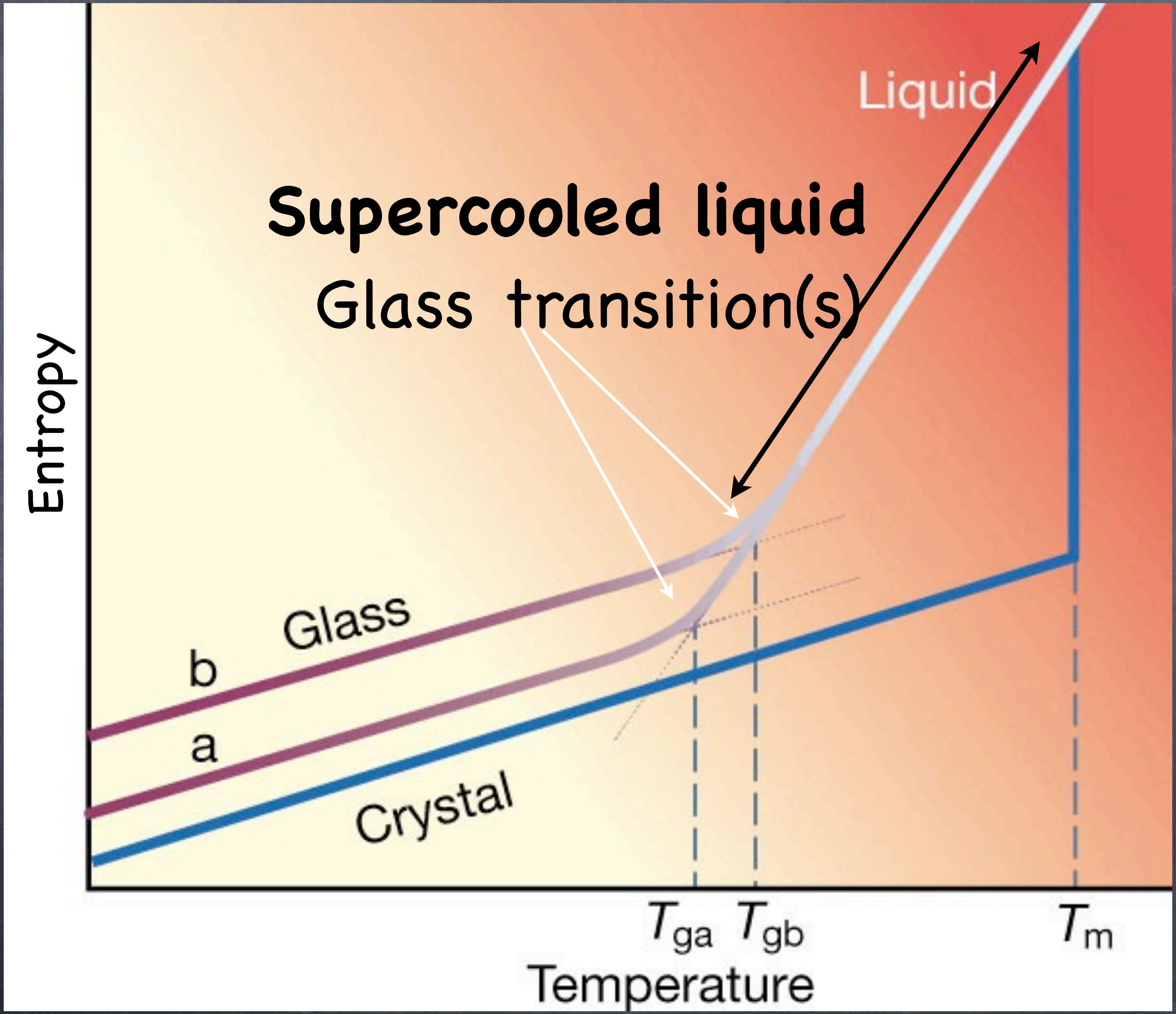
Supercooled liquids



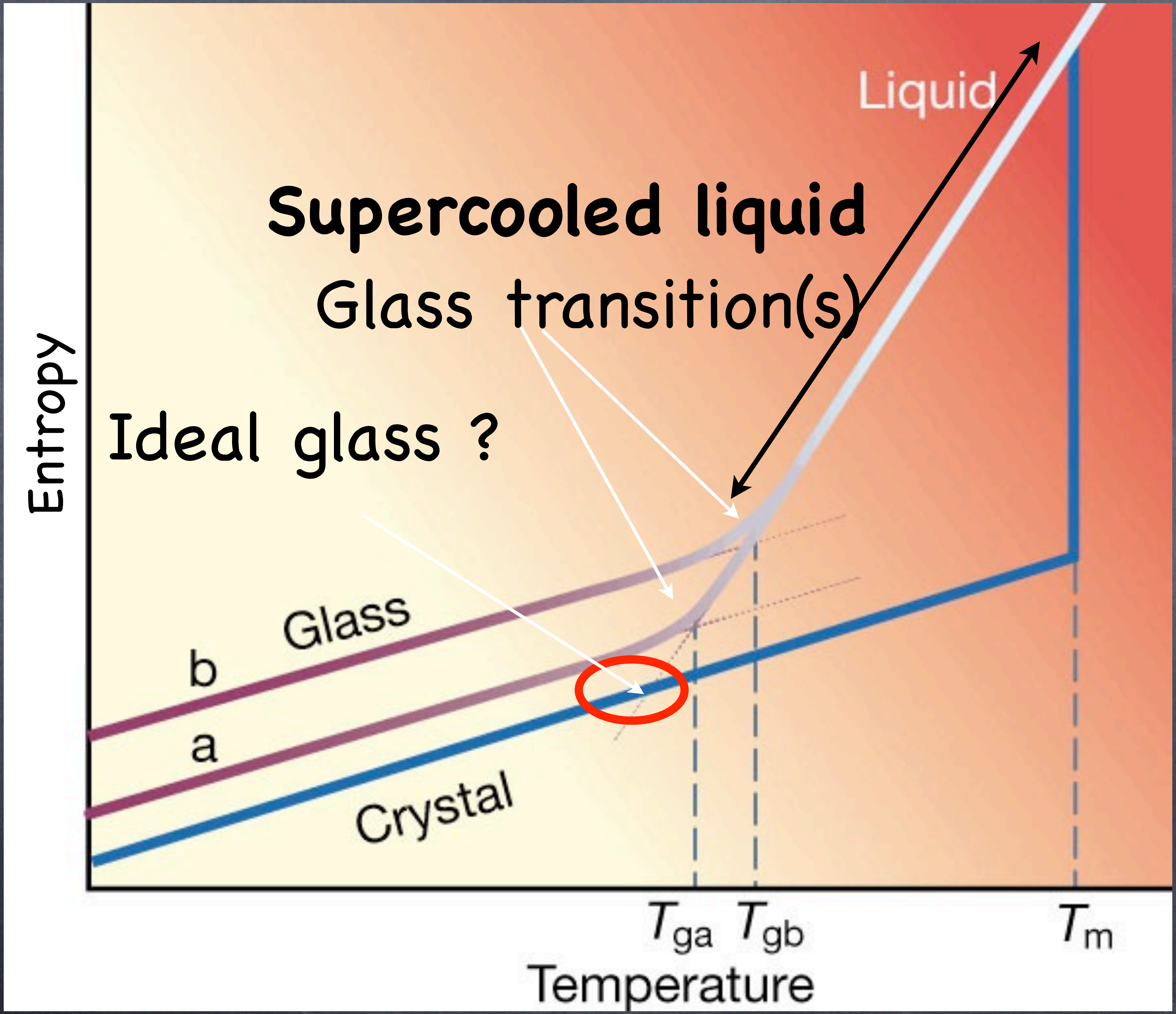
Supercooled liquids



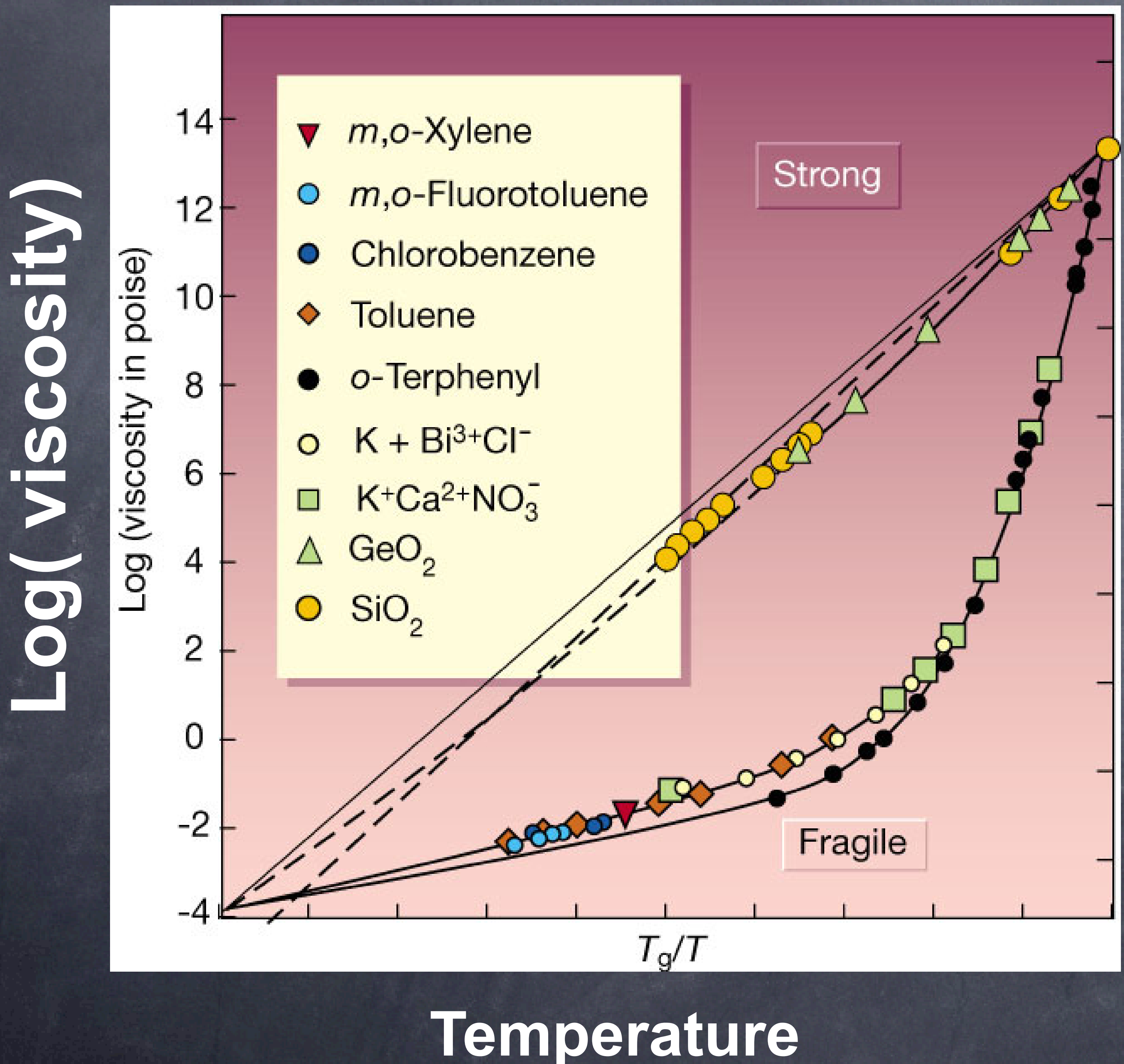
Supercooled liquids



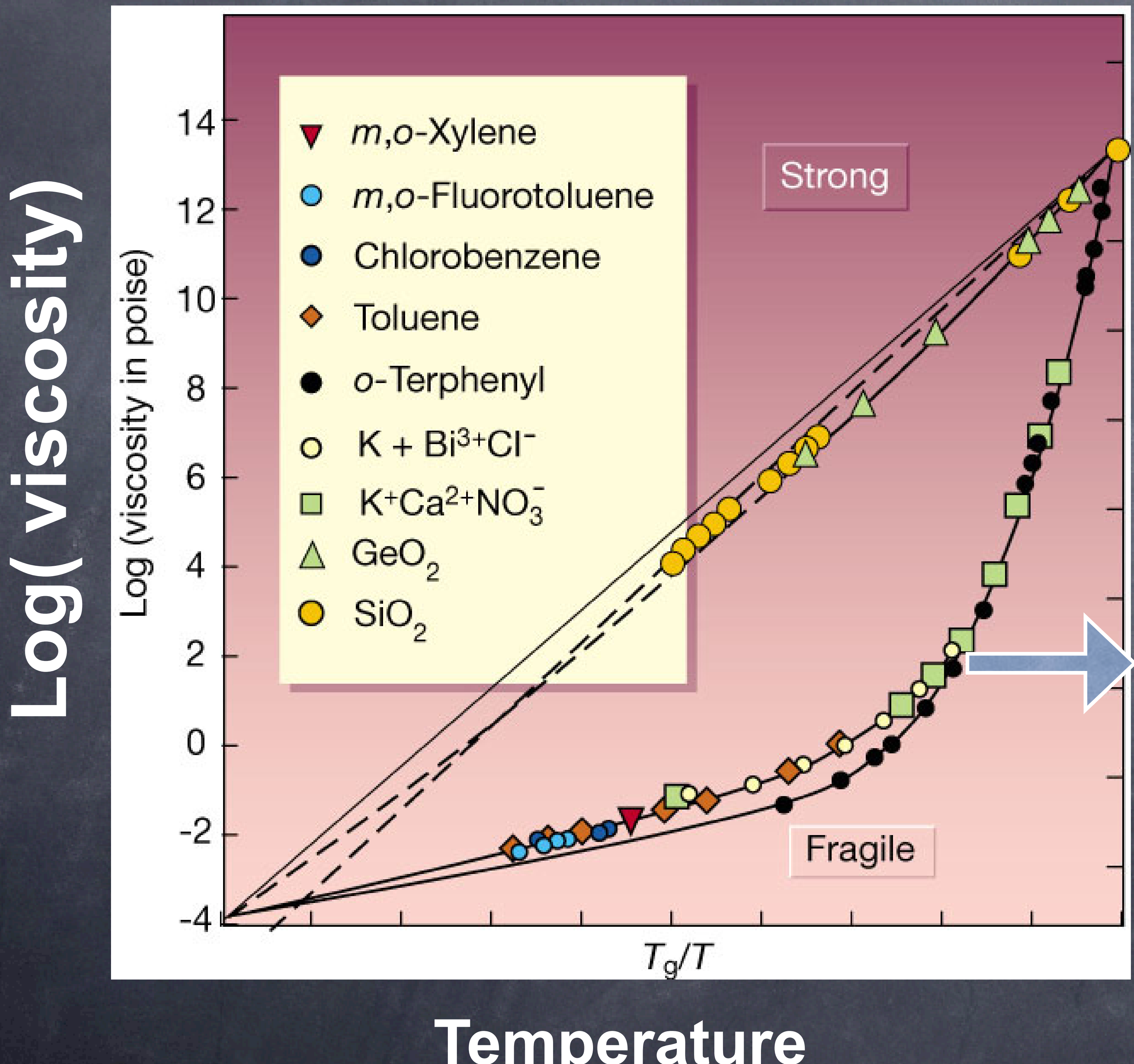
Supercooled liquids



Supercooled liquids



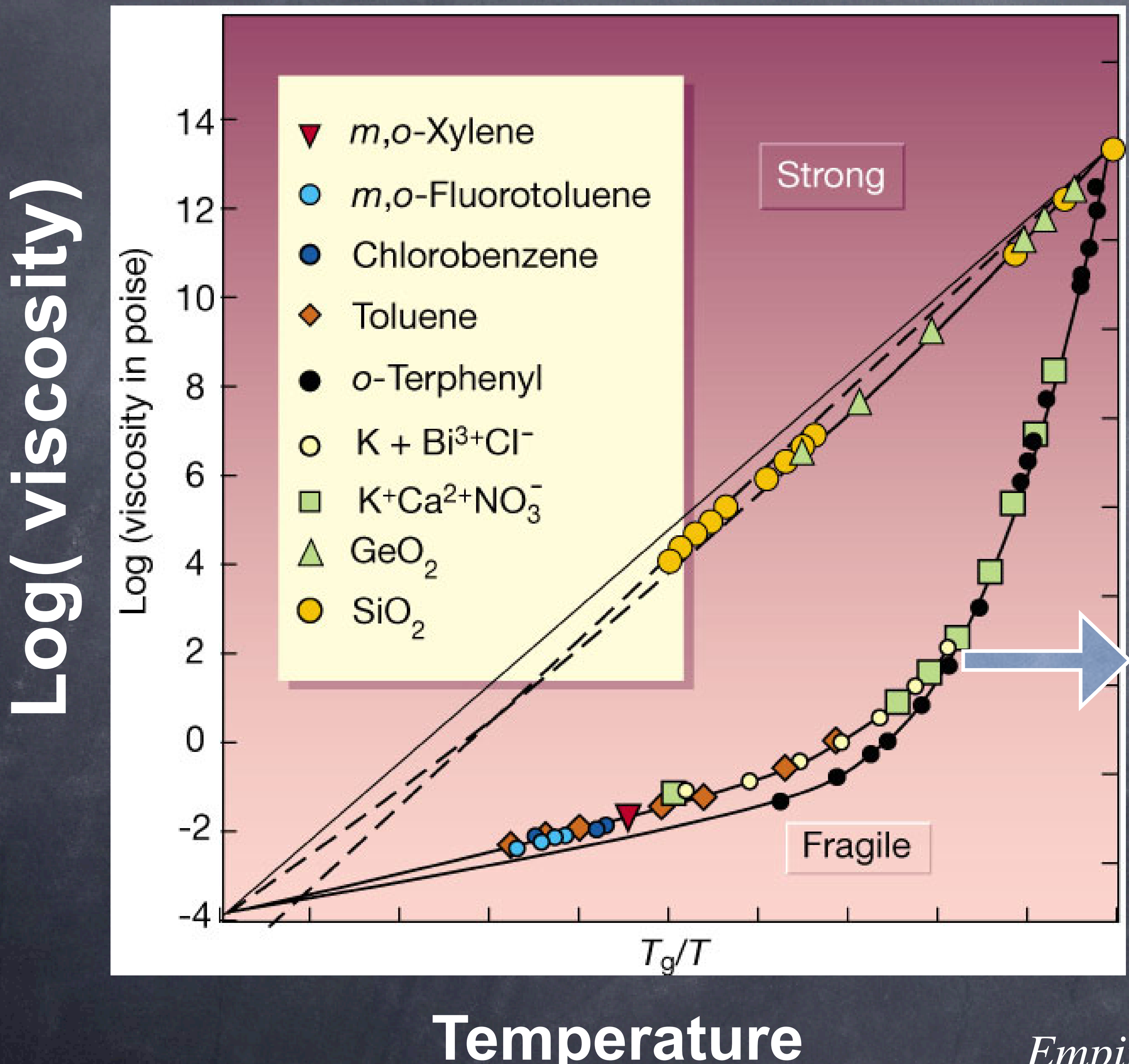
Supercooled liquids



Vogel-Fulscher

$$\eta \approx e^{\frac{A}{(T-T_K)}}$$

Supercooled liquids



Vogel-Fulscher

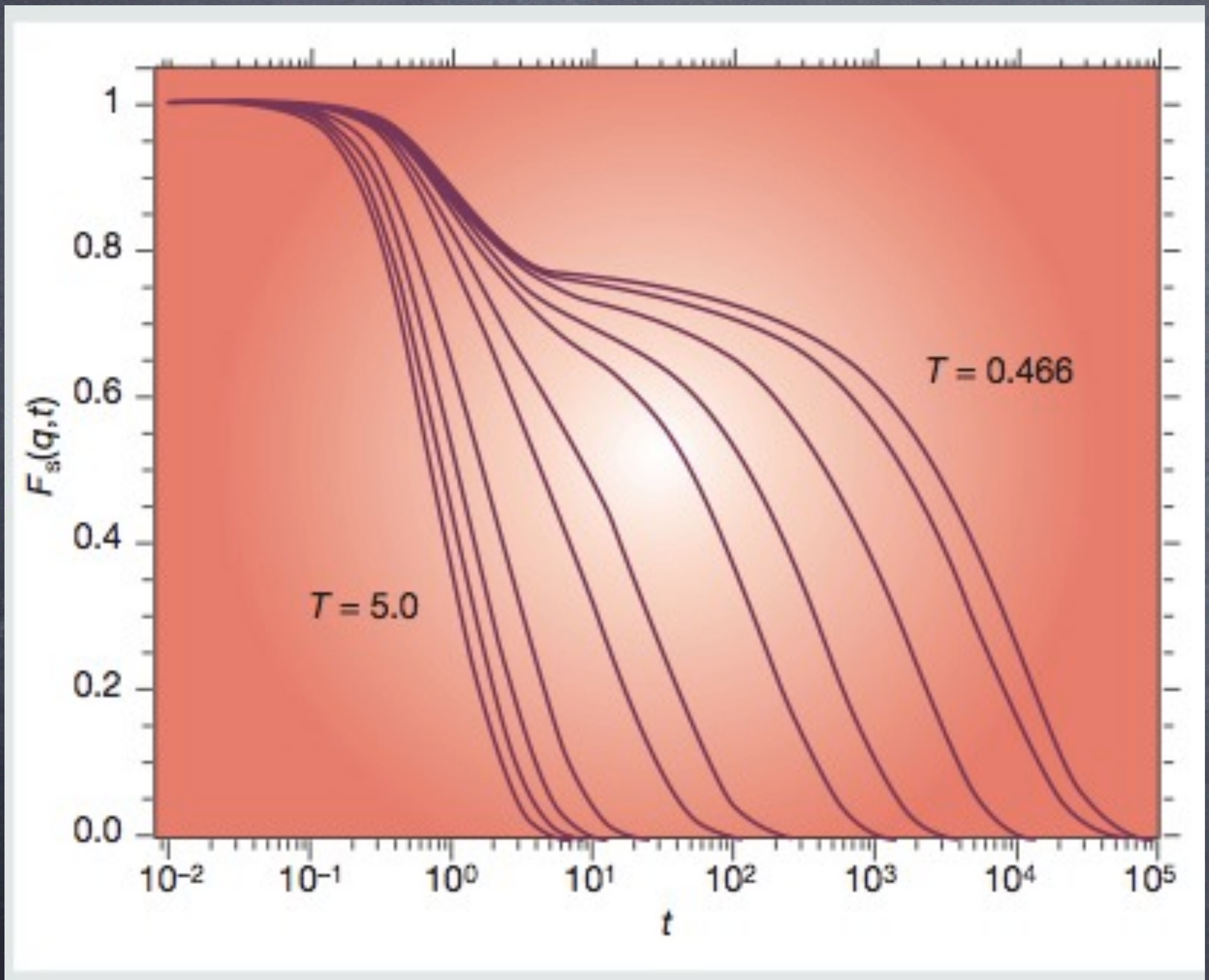
$$\eta \approx e^{\frac{A}{(T-T_K)}}$$

$$\eta \approx e^{\frac{A}{T\Delta S}}$$

Empirical ***Adam-Gibbs relation***

between viscosity and excess entropy !

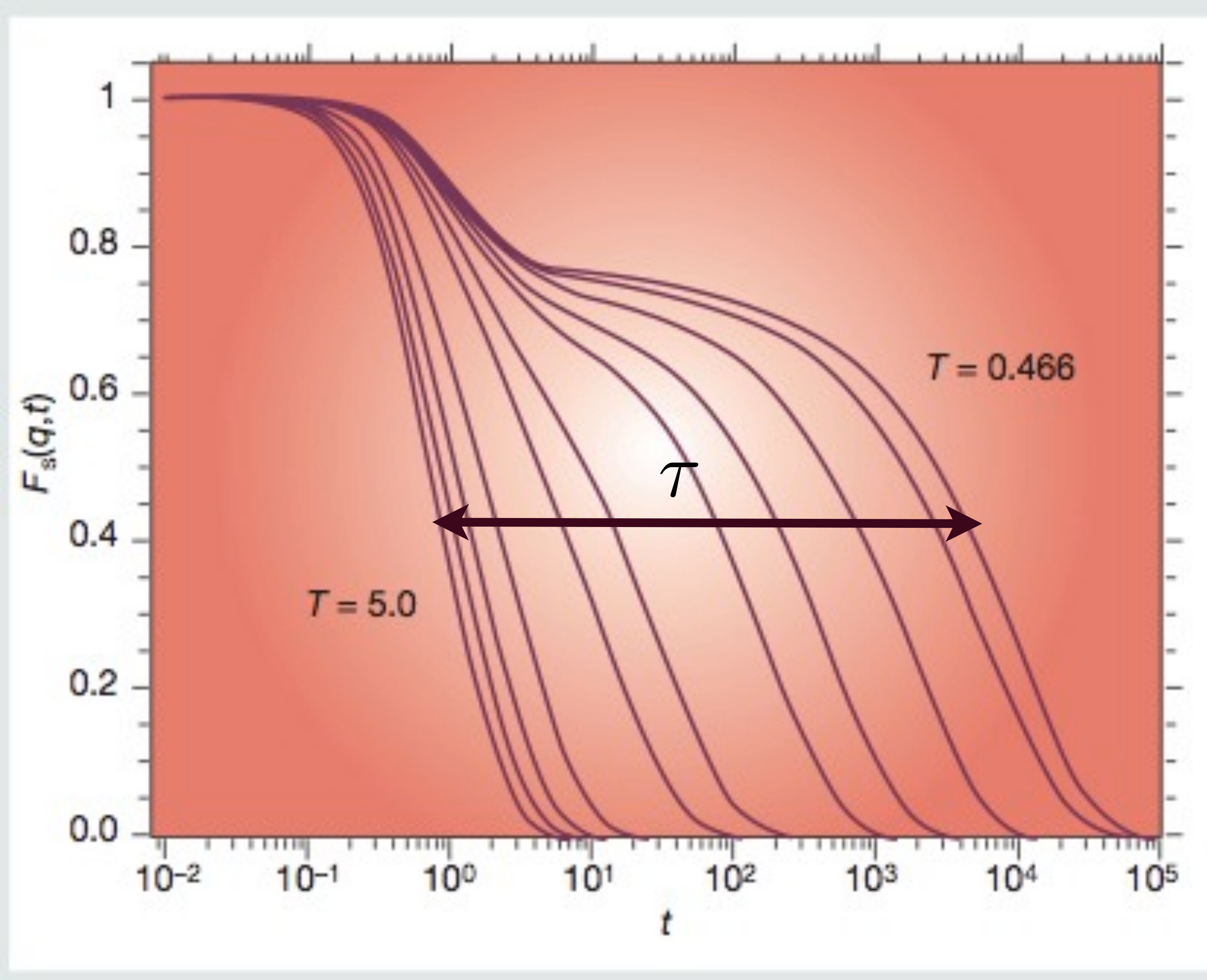
“Two-steps”
relaxation in time correlation function



Time
Correlation
Function

"Two-steps"

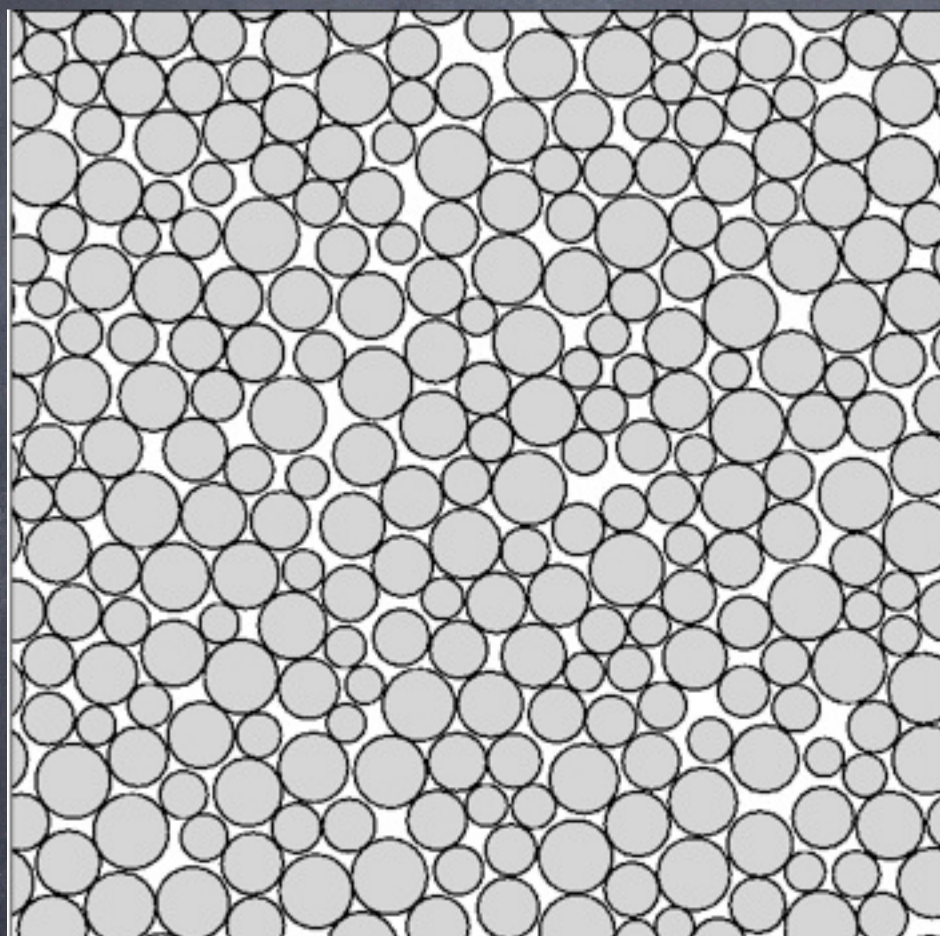
relaxation in time correlation function



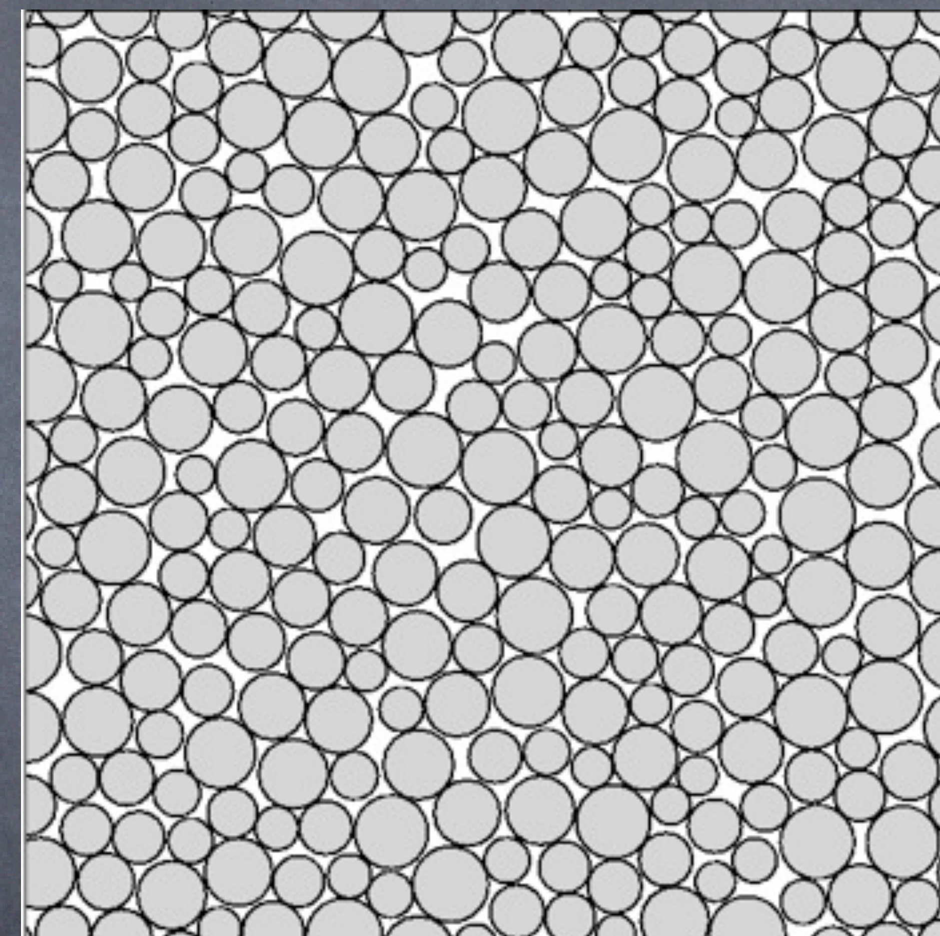
Time Correlation Function

$$\tau \approx \tau_0 e^{\frac{A}{T-T_K}} \propto e^{\frac{B}{T\Delta s}}$$

No apparent sign of order of correlation lengths...



"Liquid"



"Glass"

The hard spheres problem: Pictures from Werner Krauth

… but growing of subtle correlation lengths!

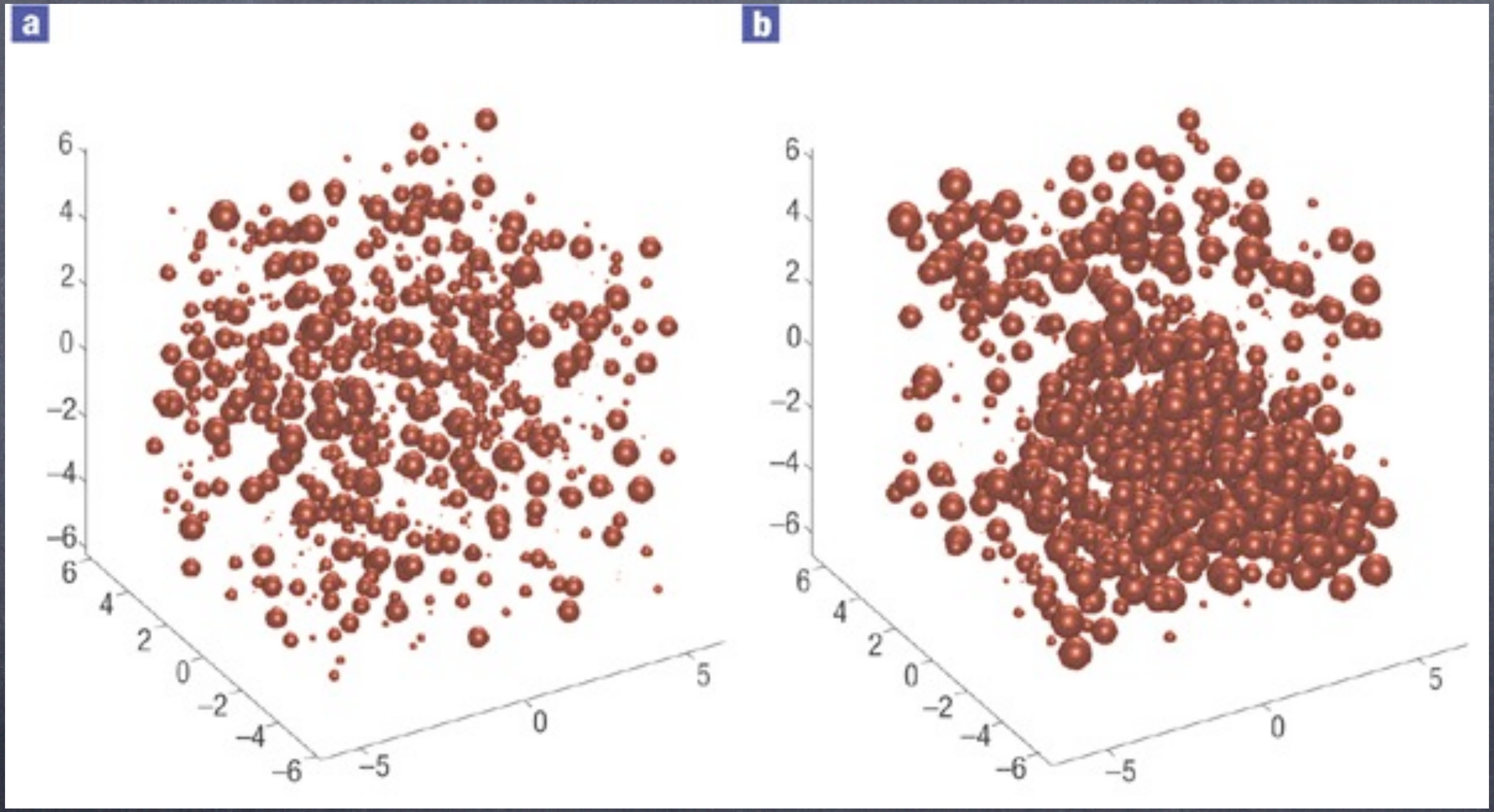
A <u>**dynamic**</u> correlation length: heterogeneous dynamics

Evolution between time t and t + τ

… but growing of subtle correlation lengths!

A **dynamic** correlation length: heterogeneous dynamics

Evolution between time t and t + τ



Berthier et al. 2004

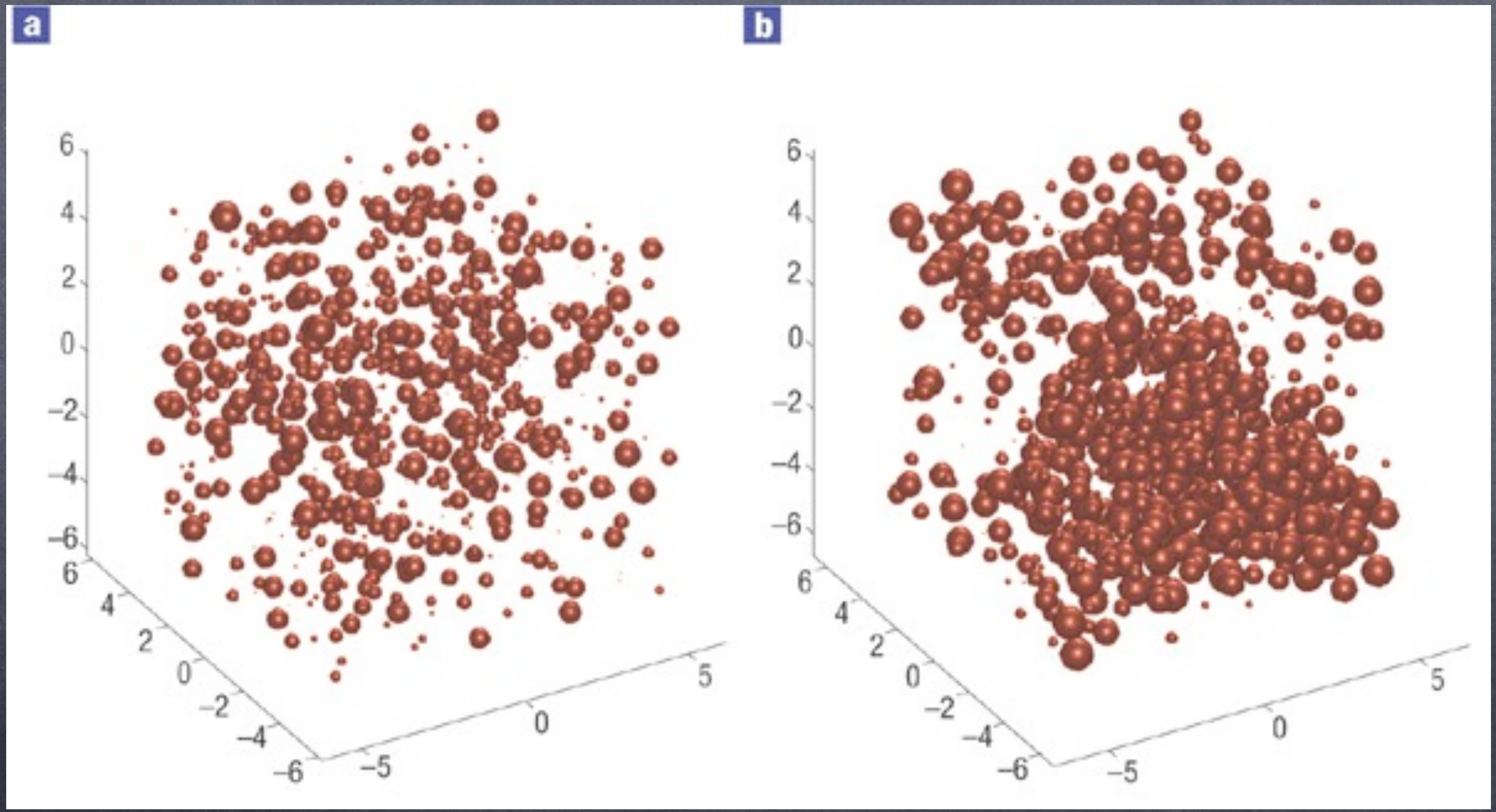
... but growing of subtle correlation lengths!

A **dynamic** correlation length: heterogeneous dynamics

Evolution between time t and t + τ

Large T

Lower T



Berthier et al. 2004

... but growing of subtle correlation lengths!

A **dynamic** correlation length: heterogeneous dynamics

1) Consider the following 4-points correlation

$$C_4(t_1, t_2, r_1, r_2) = \langle \rho(t_1, r_1)\rho(t_1, r_2)\rho(t_2, r_1)\rho(t_2, r_2)\rangle$$

… but growing of subtle correlation lengths!

A **dynamic** correlation length: heterogeneous dynamics

1) Consider the following 4-points correlation

$$C_4(t_1, t_2, r_1, r_2) = \langle \rho(t_1, r_1)\rho(t_1, r_2)\rho(t_2, r_1)\rho(t_2, r_2)\rangle$$

2) Define the following susceptibility (or correlated volume)

$$\chi_4(t_1, t_2) = \frac{1}{V}\int dr_1 dr_2 C_4(t_1, t_2, r_1, r_2)$$

… but growing of subtle correlation lengths!

A **dynamic** correlation length: heterogeneous dynamics

1) Consider the following 4-points correlation

$$C_4(t_1, t_2, r_1, r_2) = \langle \rho(t_1, r_1)\rho(t_1, r_2)\rho(t_2, r_1)\rho(t_2, r_2)\rangle$$

2) Define the following susceptibility (or correlated volume)

$$\chi_4(t_1, t_2) = \frac{1}{V}\int dr_1 dr_2 C_4(t_1, t_2, r_1, r_2)$$

3) At equilibrium, time translational invariance impose that

$$\chi_4(t_1, t_2) = \chi_4(\Delta t = t_2 - t_1)$$

… but growing of subtle correlation lengths!

A **dynamic** correlation length

1) Consider the … n

$$C_4(t_1, t_2, r_1, r_2)$$

2) Define the following s…

$$\chi_4(t_1, t_2) = $$

3) At equilibrium, time tran…

$$\chi_4(t_1, t_2) = $$

Direct Experimental Evidence of a Growing Length Scale Accompanying the Glass Transition

L. Berthier,[1*] G. Biroli,[2] J.-P. Bouchaud,[3,4] L. Cipelletti,[1]
D. El Masri,[1] D. L'Hôte,[4] F. Ladieu,[4] M. Pierno[1]

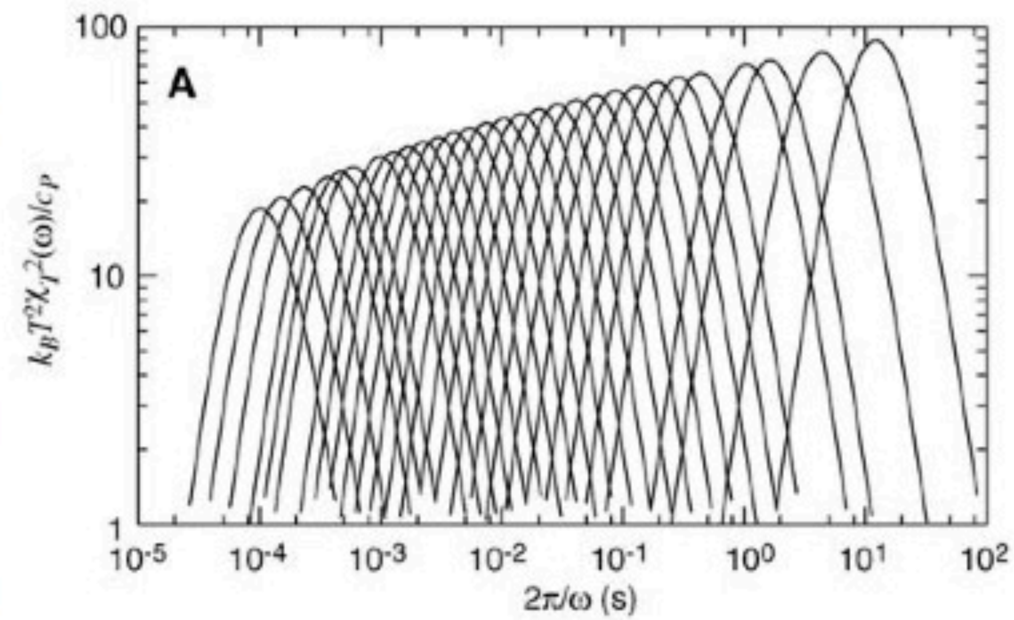
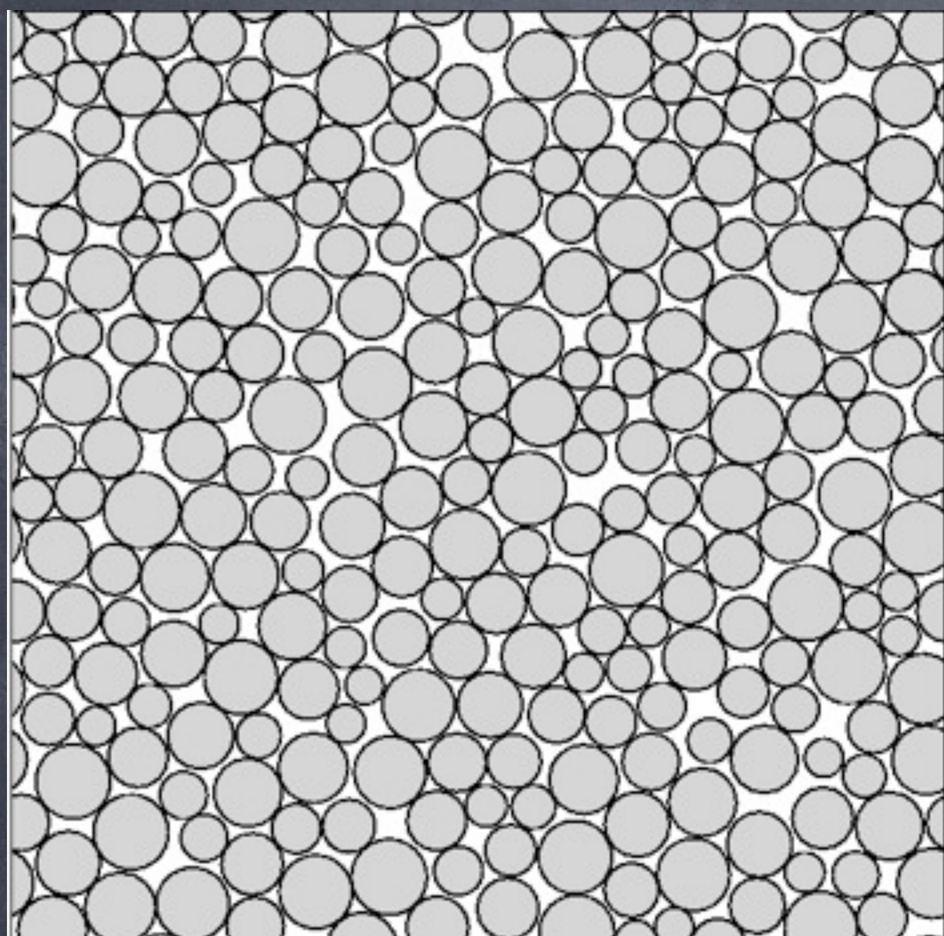


Fig. 1. Dynamic susceptibilities in "χ_4 units," right side of relations 5 and 6 for three glass formers. (**A**) $\chi_T(\omega)$ was obtained for 99.6% pure supercooled glycerol in a desiccated Argon environment to prevent water absorption by using standard capacitive dielectric measurements for 192 K $\leq T \leq$ 232 K ($T_g \approx$ 185 K). (**B**) $\chi_\varphi(t)$ was obtained in colloidal hard spheres by dynamic light scattering. The static prefactor, $\rho k_B T \kappa_T$, was evaluated from the Carnahan-Starling equation of state (*20*). From left to right, φ = 0.18, 0.34, 0.42, 0.46, 0.49, and 0.50. (**C**) $\chi_T(t)$ was obtained in a binary Lennard-Jones (LJ) mixture by numerical simulation. From left to right, T = 2.0, 1.0, 0.74, 0.6, 0.5, and 0.465 [in reduced LJ units (*24*, *25*)]. Relative errors at the peak are at most about 10% for (A) and (C) and 30% for (B). For all of the systems, dynamic susceptibilities display a peak at the average relaxation time whose height increases when the dynamics slows down, which is direct evidence of enhanced dynamic fluctuations and a growing dynamic length scale.

$\chi_4(\Delta t = \tau)$ increase strongly when approaching the transition (and is expected to diverge!)

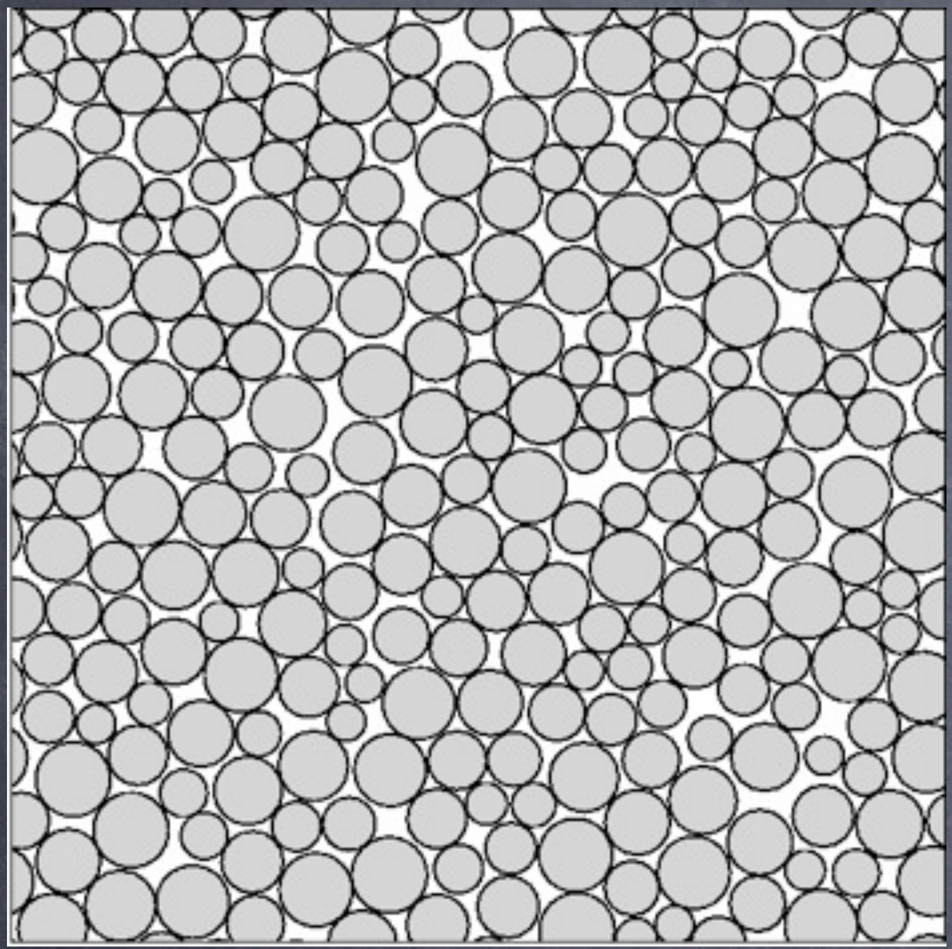
… but growing of subtle correlation lengths!

An <u>**equilibrium**</u> correlation length



… but growing of subtle correlation lengths!

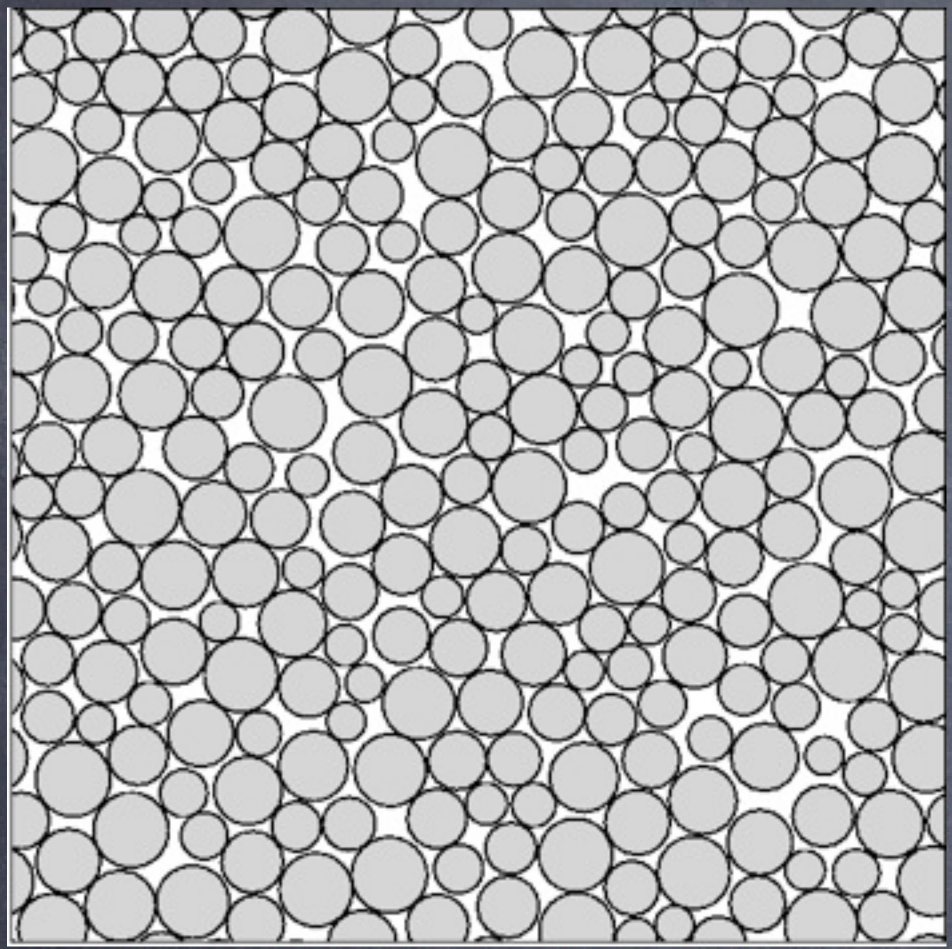
An **equilibrium** correlation length



1) Consider an equilibrium configuration

… but growing of subtle correlation lengths!

An **equilibrium** correlation length

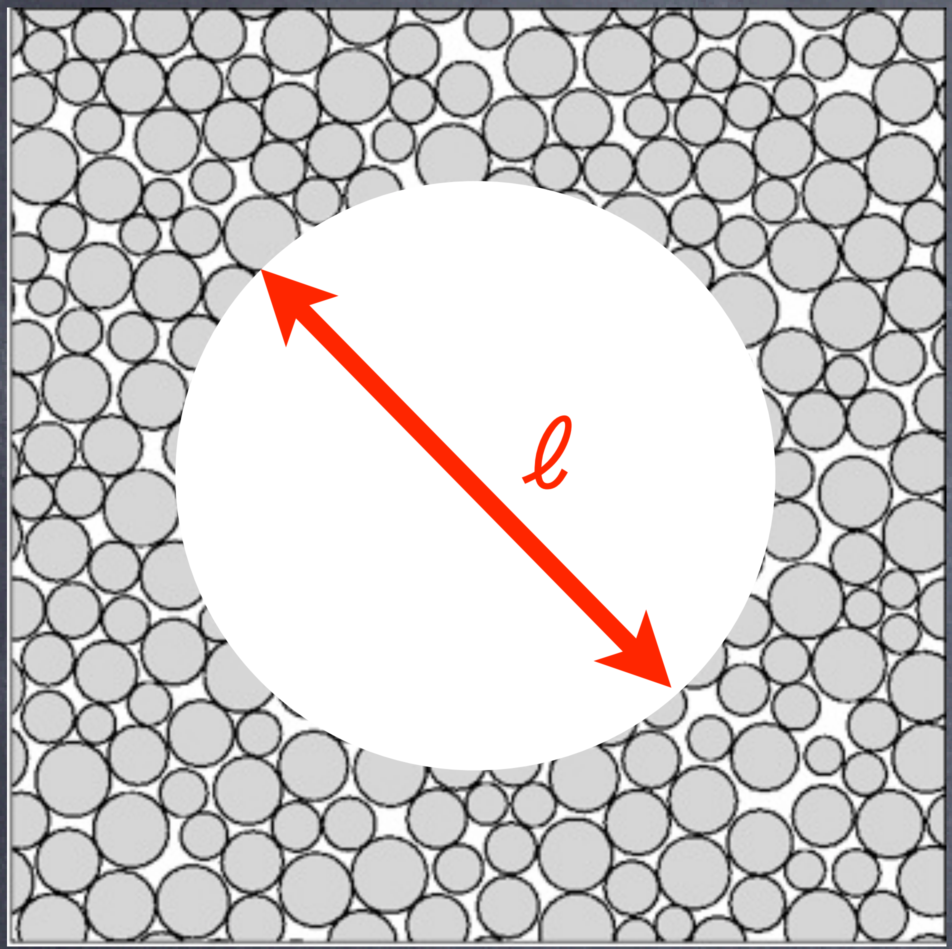


1) Consider an equilibrium configuration

2) Freeze the system and make a hole (cavity) of size ℓ inside the system

… but growing of subtle correlation lengths!

An **equilibrium** correlation length

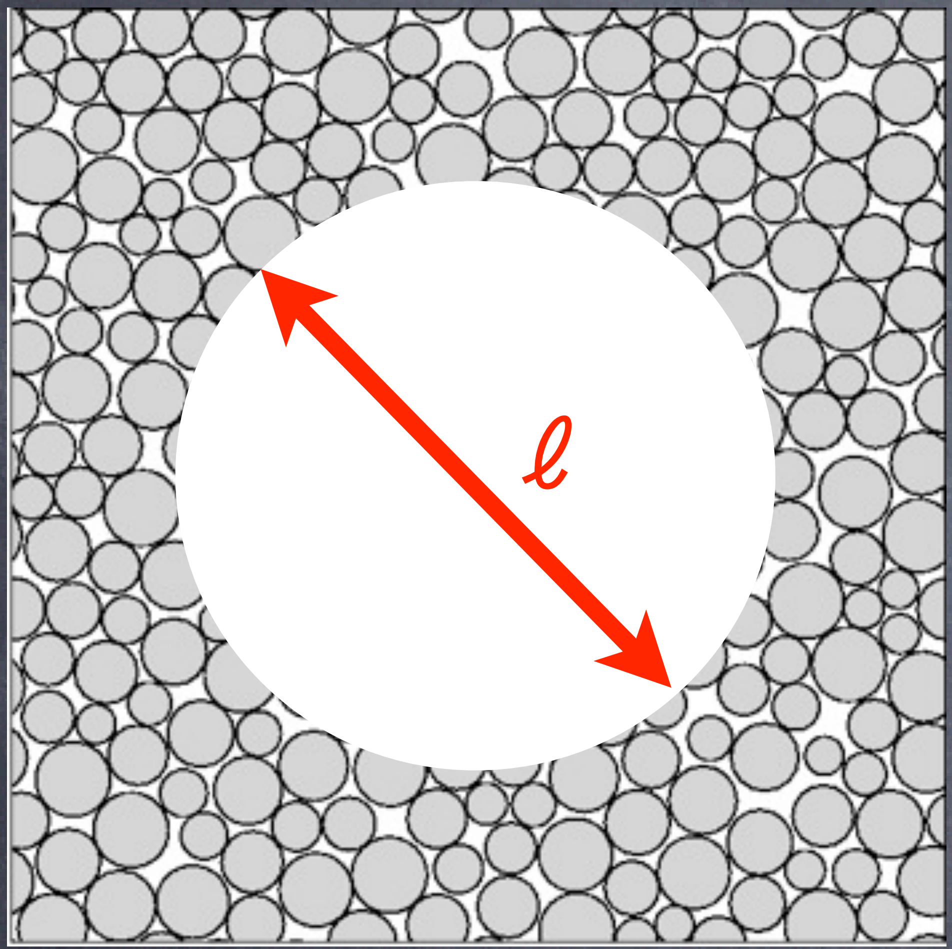


1) Consider an equilibrium configuration

2) Freeze the system and make a hole (cavity) of size ℓ inside the system

… but growing of subtle correlation lengths!

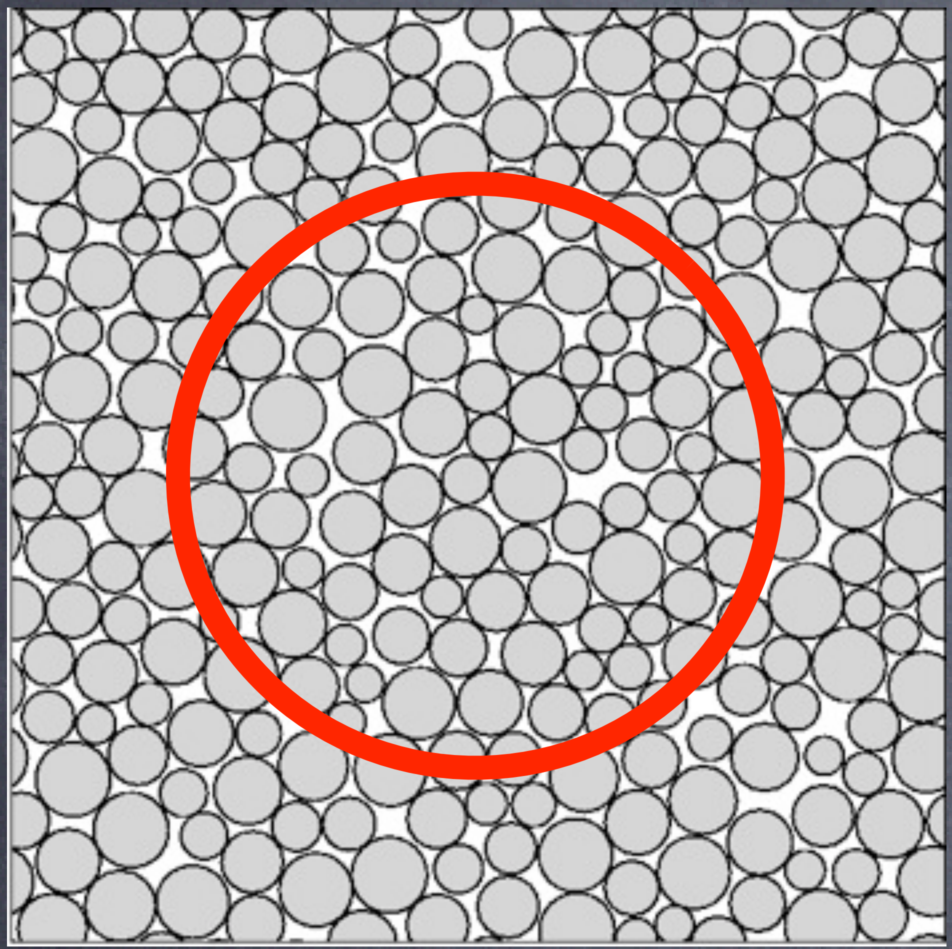
An **equilibrium** correlation length



1) Consider an equilibrium configuration
2) Freeze the system and make a hole (cavity) of size ℓ inside the system
3) Un-freeze the system inside the cavity: does it stay close to the original configuration ?

… but growing of subtle correlation lengths!

An **equilibrium** correlation length



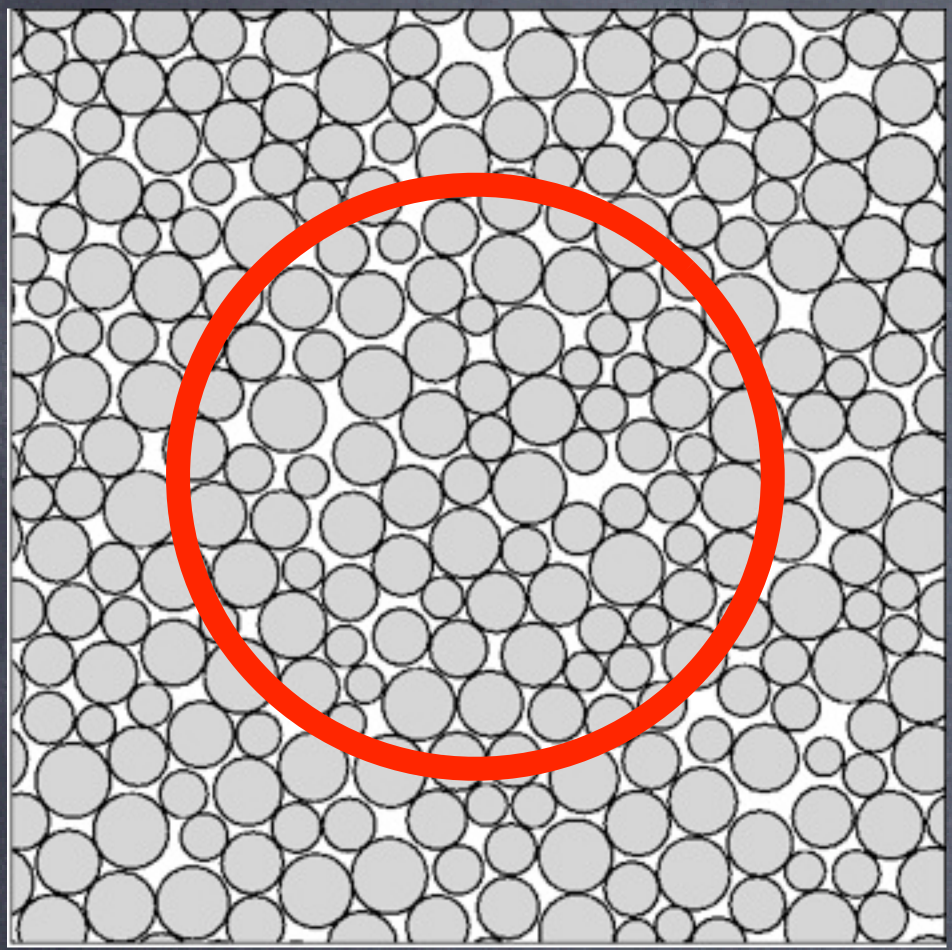
1) Consider an equilibrium configuration

2) Freeze the system and make a hole (cavity) of size ℓ inside the system

3) Un-freeze the system inside the cavity: does it stay close to the original configuration ?

… but growing of subtle correlation lengths!

An **equilibrium** correlation length



1) Consider an equilibrium configuration

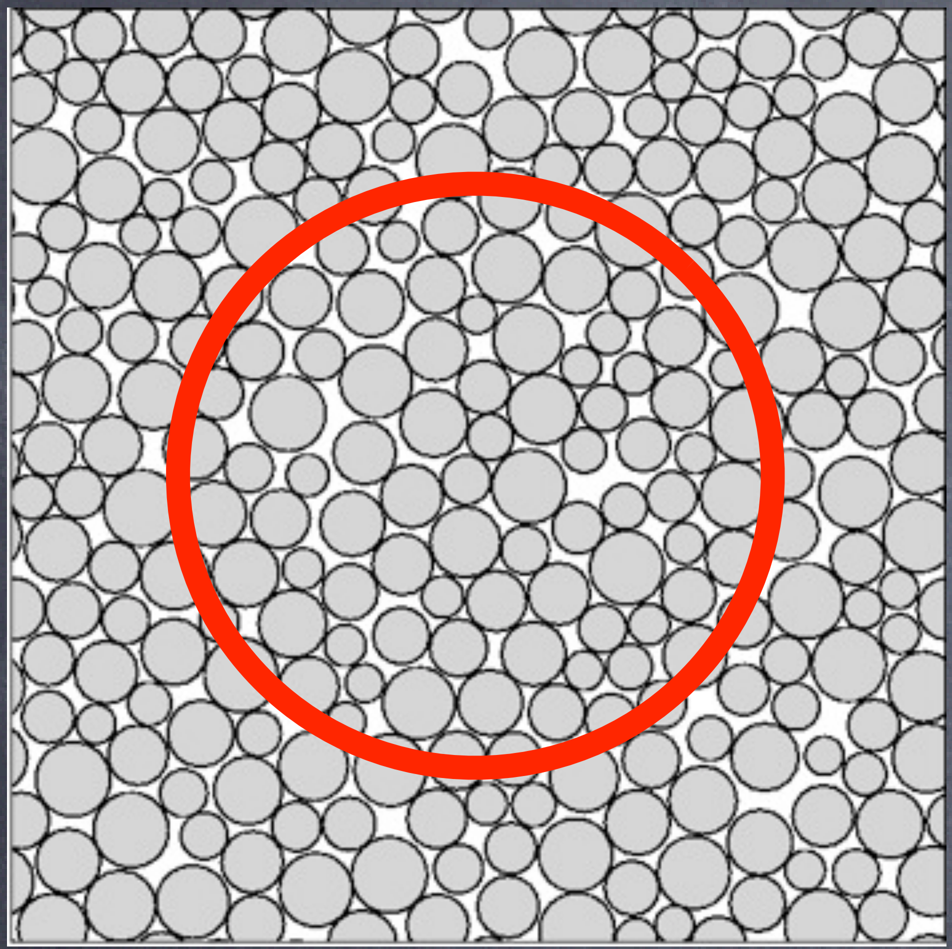
2) Freeze the system and make a hole (cavity) of size ℓ inside the system

3) Un-freeze the system inside the cavity: does it stay close to the original configuration ?

4) The length beyond which the system in the cavity decorrelates is ℓ_c

… but growing of subtle correlation lengths!

An **equilibrium** correlation length



1) Consider an equilibrium configuration

2) Freeze the system and make a hole (cavity) of size ℓ inside the system

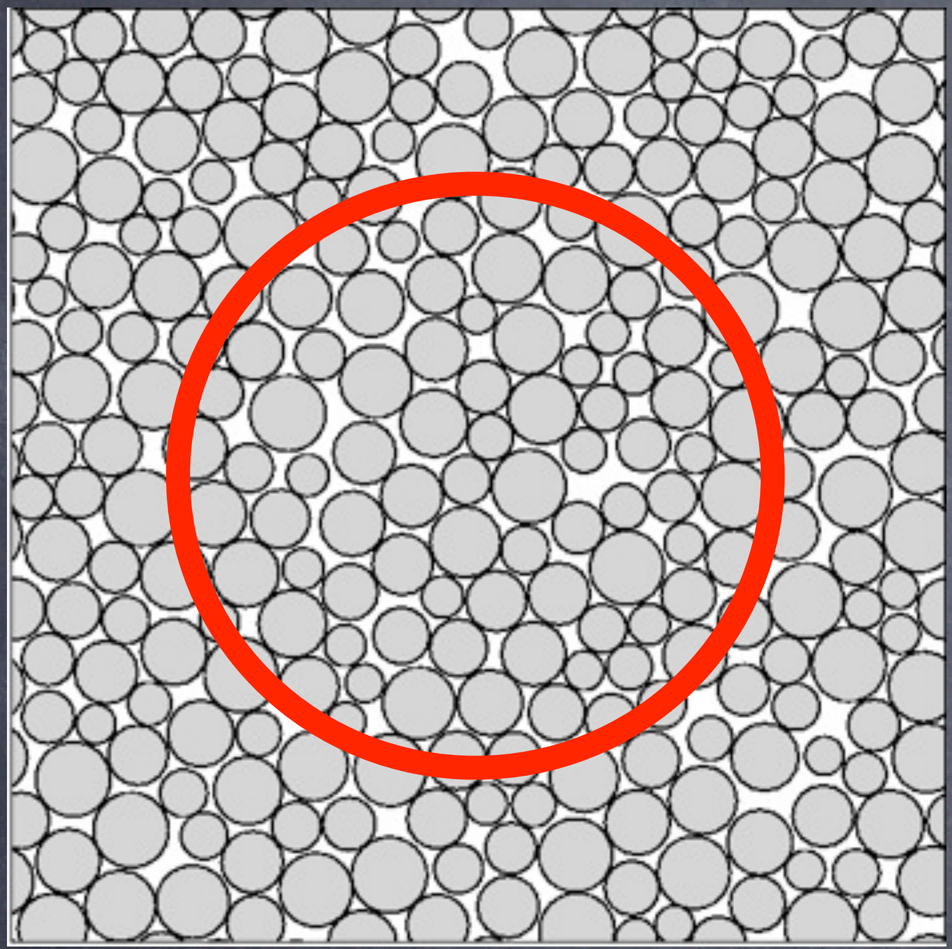
3) Un-freeze the system inside the cavity: does it stay close to the original configuration ?

4) The length beyond which the system in the cavity decorrelates is ℓ_c

Point-to-Set correlations !

... but growing of subtle correlation lengths!

An **equilibrium** correlation length



1) Consider an equilibrium configuration

2) Freeze the system and make a hole (cavity) of size ℓ inside the system

3) Un-freeze the system inside the cavity: does it stay close to the original configuration ?

4) The length beyond which the system in the cavity decorrelates is ℓ_c

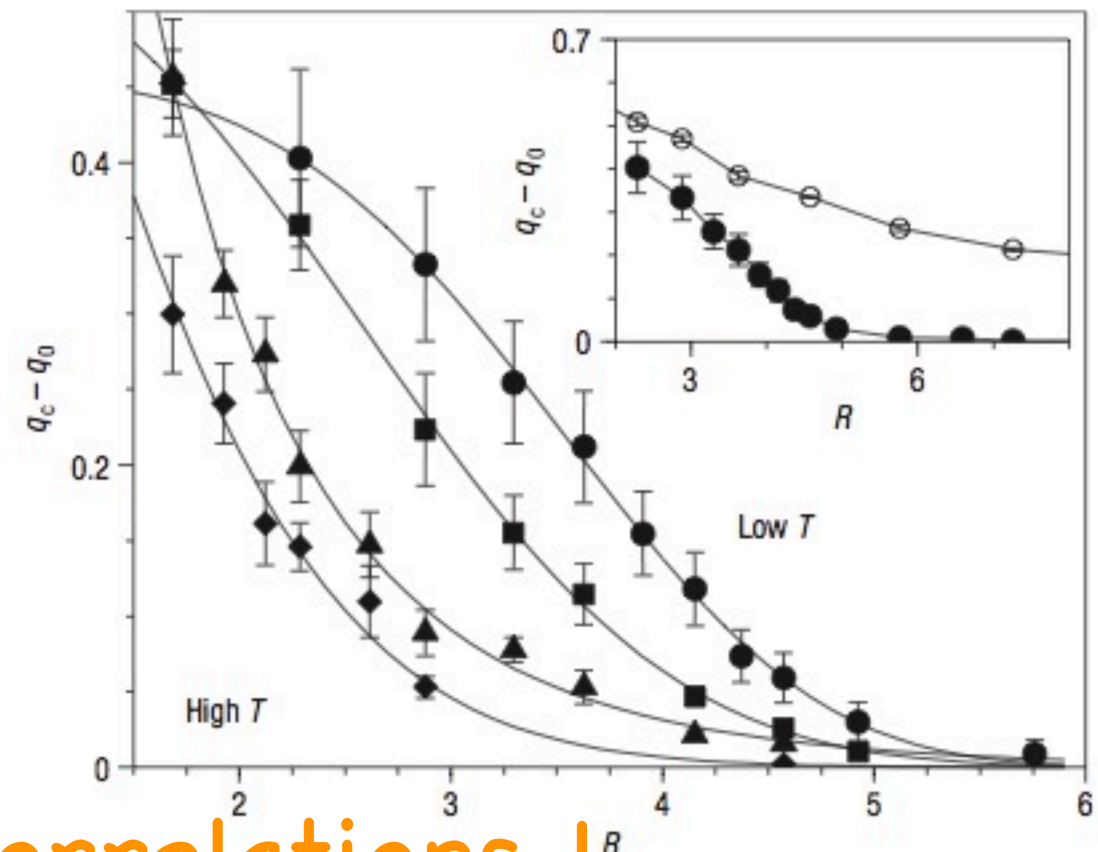
Point-to-Set correlations !

ℓ_c increase strongly when approaching the transition (and is expected to diverge if there is a genuine transition)

…

Thermodynamic signature of growing amorphous order in glass-forming liquids

G. BIROLI[1], J.-P. BOUCHAUD[2], A. CAVAGNA[3], T. S. GRIGERA[4,5]* AND P. VERROCCHIO[6]
[1]CEA, DSM, Institut de Physique Théorique, IPhT, CNRS, MPPU, URA2306, Saclay, F-911
[2]Science & Finance, Capital Fund Management, 6 Bd Haussmann, 75009 Paris, France
[3]Centre for Statistical Mechanics and Complexity (SMC), CNR-INFM, Via dei Taurini 19,
[4]Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas (INIFTA –CCT La Plat
Universidad Nacional de La Plata, c.c. 16, suc. 4, 1900 La Plata, Argentina
[5]Consejo Nacional de Investigaciones Científicas y Técnicas, c.c. 16, suc. 4, 1900 La Pl
[6]Dipartimento di Fisica, Università di Trento, via Sommarive 14, 38050 Povo, Trento, Ita
*e-mail: tgrigera@inifta.unlp.edu.ar



4) The leng
in the

Point-to-Set correlations !

ℓ_c increase strong
the transition (and is
if there is a ge

Figure 1 Change of the overlap with mobile cavity size. Overlap at the centre of the mobile cavity versus radius R of the cavity, for temperatures $T = 0.482$ (diamonds), 0.350 (triangles), 0.246 (squares) and 0.203 (circles). Lines are fits to equation (1). Inset: Comparison of $q_c(R) - q_0$ at $T = 0.203$ (filled circles) with the overlap $Q(R) - q_0$ integrated over the whole sphere (open circles, data ref. 23). The local observable $q_c(R)$ shows a much sharper behaviour. Error bars were obtained from a jack-knife estimate from sample-to-sample fluctuations.

Phenomenology of glass former liquids

- Super exponentially relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- Dynamical heterogeneities
- “Divergence” of a length scale (Point-To-Set correlations)

Phenomenology of glass former liquids

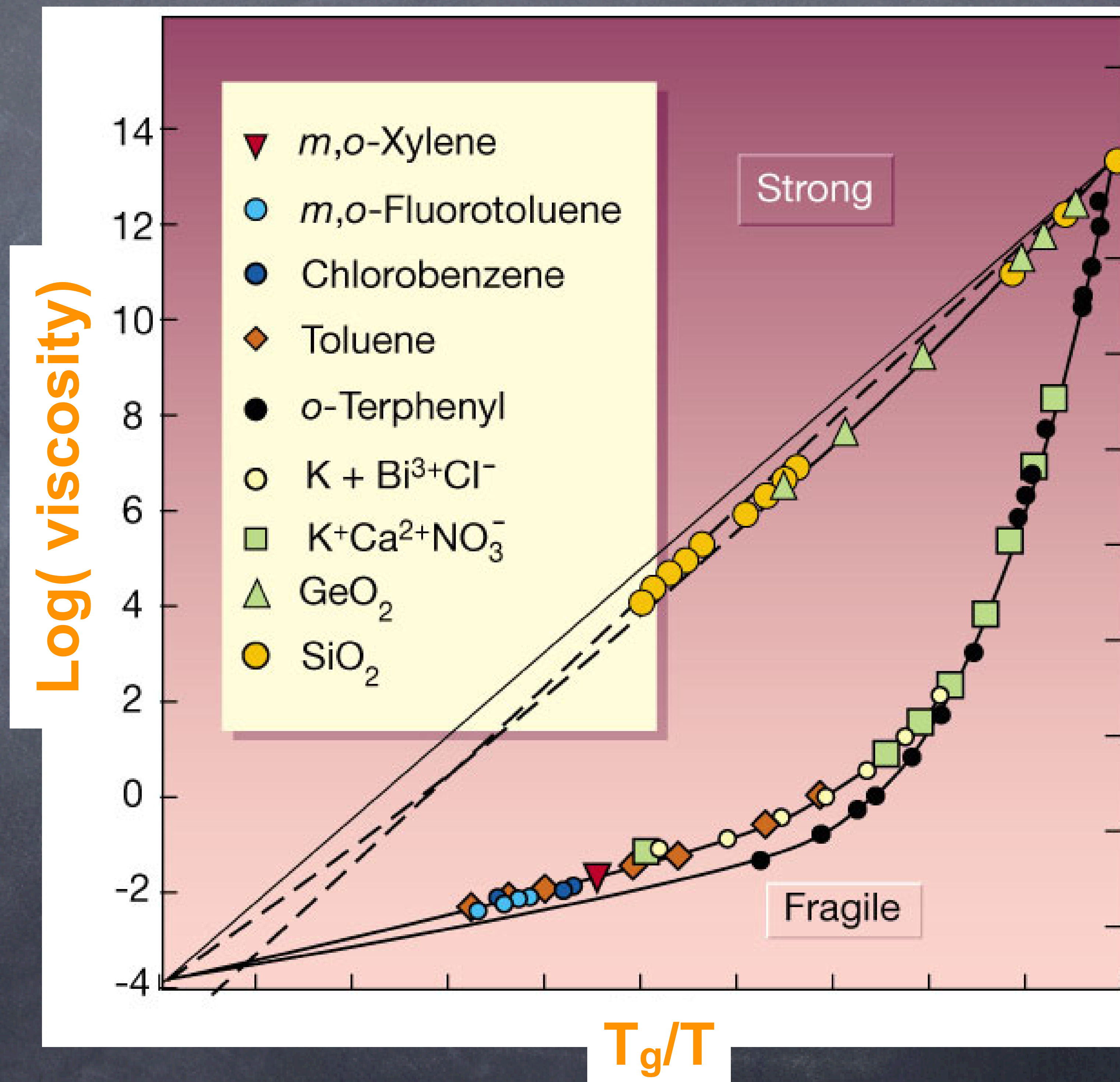
- **Super exponential relaxation**
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- Dynamical heterogeneities
- "Divergence" of a length scale (Point-To-Set correlations)

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- Dynamical heterogeneities
- "Divergence" of a length scale (Point-To-Set correlations)

Good fit:
Vogel-Fulcher "law"

$$\tau \approx \tau_0 e^{\frac{A}{T-T_K}}$$



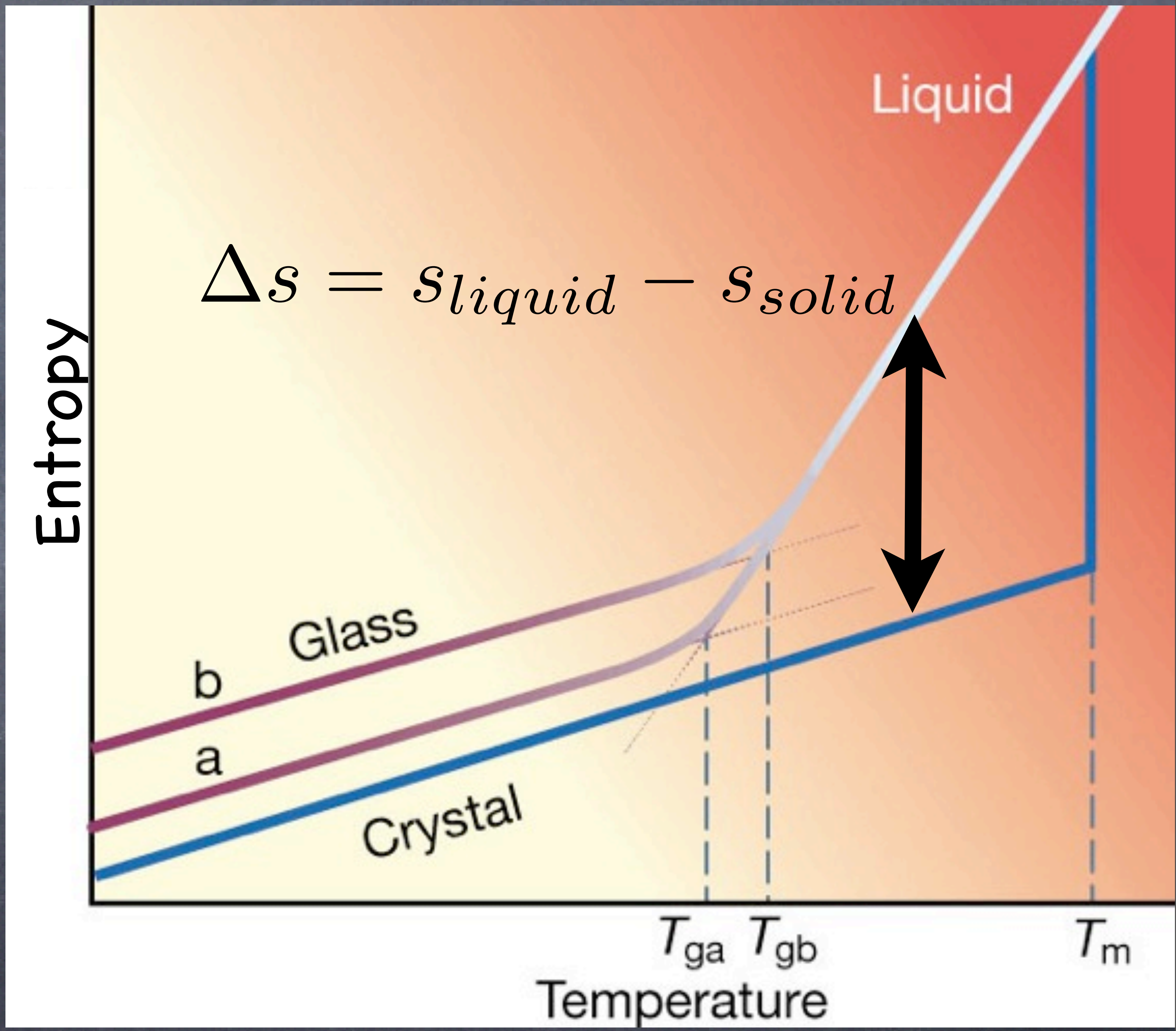
Phenomenology of glass former liquids

- Super exponential relaxation
- **Kauzmann paradox & Adam-Gibbs relation**
- Two steps correlation function
- Dynamical heterogeneities
- "Divergence" of a length scale (Point-To-Set correlations)

Phenomenology of glass former liquids

- Super exponential relaxation
- **Kauzmann paradox & Adam-Gibbs relation**
- Two steps correlation function
- Dynamical heterogeneities
- "Divergence" of a length scale (Point-To-Set correlations)

$$\tau \approx \tau_0 e^{\frac{A}{T-T_K}} \propto e^{\frac{B}{T\Delta s}}$$



Δs is called the "Configurational entropy" or "Complexity"

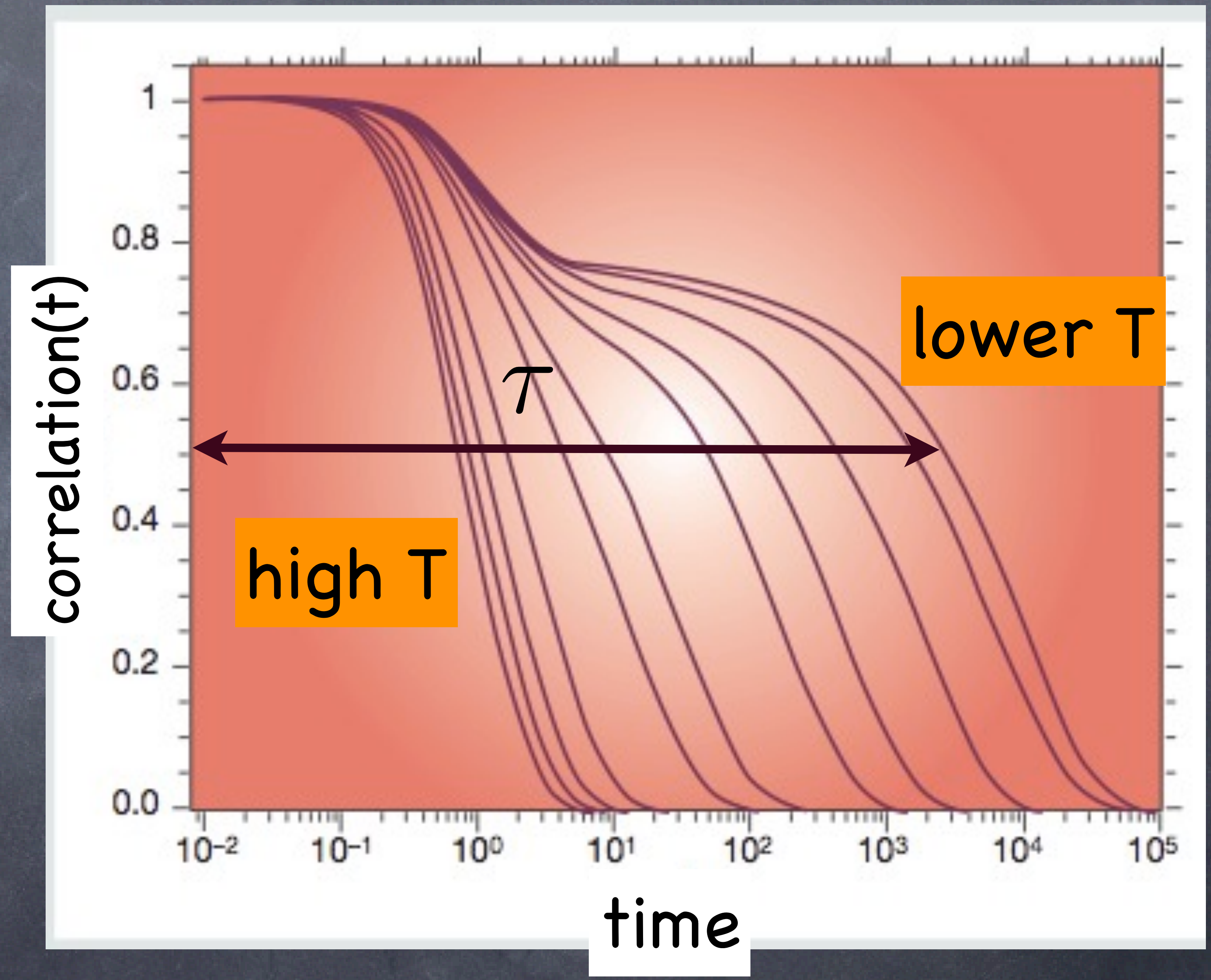
Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- **Two steps correlation function**
- Dynamical heterogeneities
- “Divergence” of a length scale (Point-To-Set correlations)

$$\tau \approx \tau_0 e^{\frac{A}{T-T_K}} \propto e^{\frac{B}{T\Delta s}}$$

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- **Two steps correlation function**
- Dynamical heterogeneities
- "Divergence" of a length scale (Point-To-Set correlations)



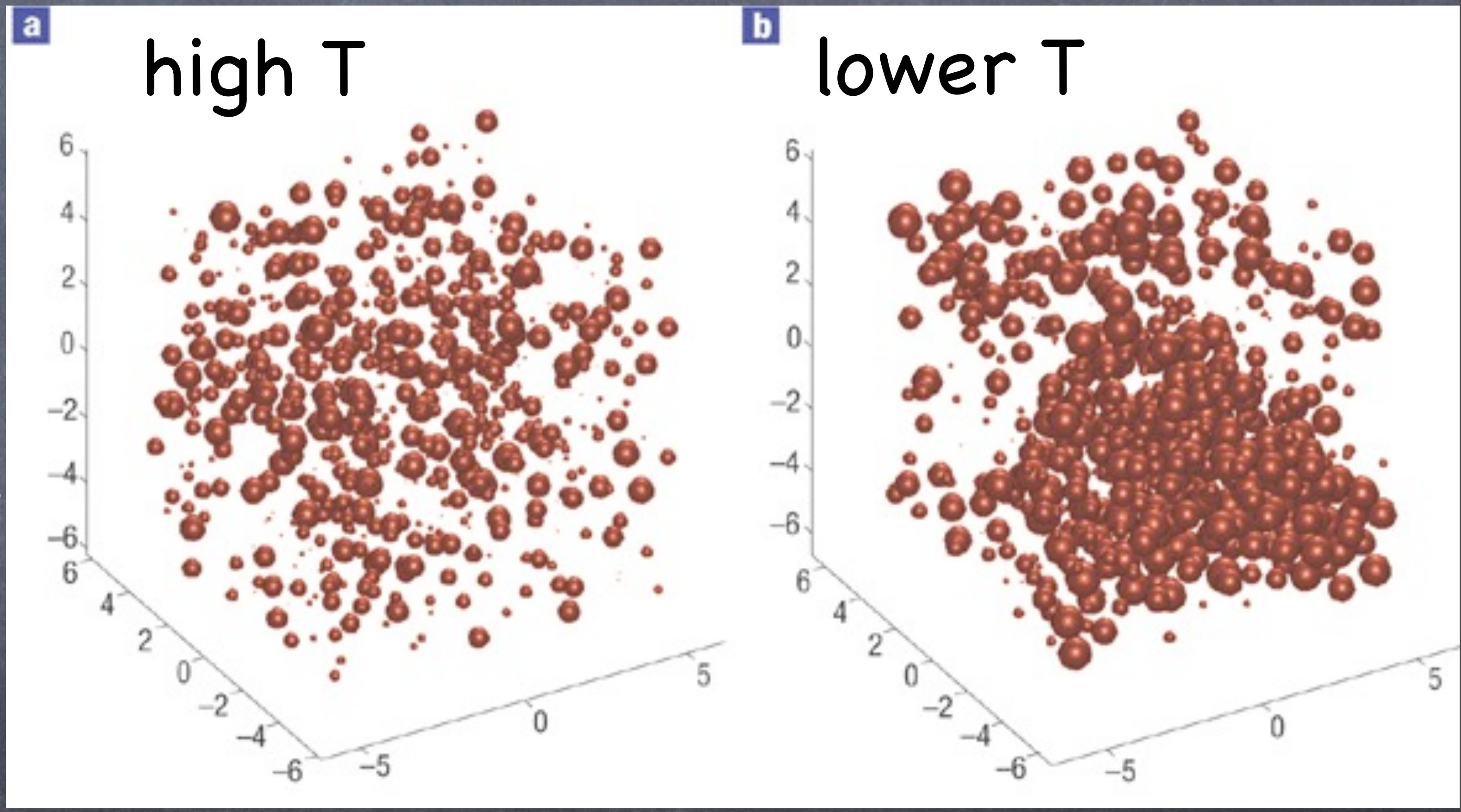
$$\tau \approx \tau_0 e^{\frac{A}{T-T_K}} \propto e^{\frac{B}{T\Delta s}}$$

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- **Dynamical heterogeneities**
- "Divergence" of a length scale (Point-To-Set correlations)

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- **Dynamical heterogeneities**
- "Divergence" of a length scale (Point-To-Set correlations)



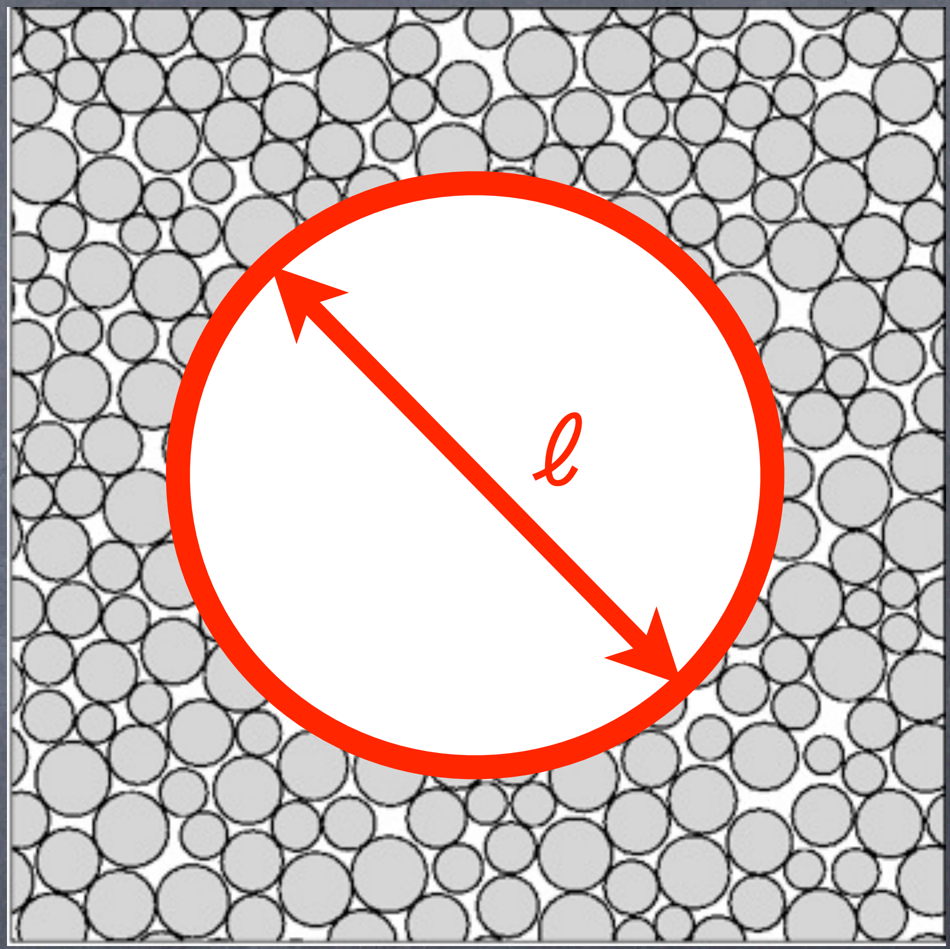
$$\chi_4(t_1,t_2) = \frac{1}{V}\int dr_1 dr_2 \langle \rho(t_1,r_1)\rho(t_1,r_2)\rho(t_2,r_1)\rho(t_2,r_2)\rangle_c$$

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- Dynamical heterogeneities
- **"Divergence" of an equilibrium length scale (Point-To-Set correlations)**

Phenomenology of glass former liquids

- Super exponential relaxation
- Kauzmann paradox & Adam-Gibbs relation
- Two steps correlation function
- Dynamical heterogeneities
- **"Divergence" of an equilibrium length scale (Point-To-Set correlations)**



Phenomenology of glass former liquids

- Super exponential relaxation ?
- Kauzmann paradox & Adam-Gibbs relation ?
- Two steps correlation function
- Dynamical heterogeneities
- "Divergence ?" of a length scale (Point-To-Set correlations)

Still many debates on how to describe this transition

↓

Random First Order Phenomenology ?

Thirumalai, Kirkpatrick, Wolynes (87-89)

↓

(replica theory) First principles computations in glasses

Mézard-Parisi (99')

A very debated question....

HOME PAGE | TODAY'S PAPER | VIDEO | MOST POPULAR | TIMES TOPICS

Try Electronic Edition | Log In | Register Now

The New York Times

Monday, March 23, 2009 Last Update: 11:31 AM ET

Search

Try Our EXTRA Home Page

Special Offer! Try Times Reader | Personalize Your Weather

JOBS
REAL ESTATE
AUTOS
ALL CLASSIFIEDS

WORLD
U.S.
POLITICS
N.Y./REGION
BUSINESS
TECHNOLOGY
SPORTS
SCIENCE
HEALTH
OPINION
ARTS
Books
Movies
Music
Television
Theater
STYLE
Dining & Wine
Fashion & Style
Home & Garden
Weddings/ Celebrations
TRAVEL

Blogs
Cartoons / Humor
Classifieds

HOME PAGE | MY TIMES | TODAY'S PAPER | VIDEO | MOST POPULAR | TIMES TOPICS

Log In | Register Now

The New York Times

Science

Search All NYTimes.com

Go

WORLD | U.S. | N.Y. / REGION | BUSINESS | TECHNOLOGY | SCIENCE | HEALTH | SPORTS | OPINION | ARTS | STYLE | TRAVEL | JOBS | REAL ESTATE | AUTOS

ENVIRONMENT | SPACE & COSMOS

The Nature of Glass Remains Anything but Clear

Mark Interrante

ENIGMA Molten glass being worked into an ornament. Understanding glass could lead to better products and offer headway in other scientific problems.

By KENNETH CHANG
Published: July 29, 2008

SIGN IN TO E-MAIL OR SAVE THIS

PRINT

Correction Appended

More Articles in Science »

Today's Headlines Daily E-Mail

Sign up for a roundup of the day's top stories, sent every morning.

Sign Up

See Sample | Privacy Policy

MOST POPULAR

E-MAILED | BLOGGED | SEARCHED

1. Having a Bat Mitzvah in Their 90s Because It's a Hoot
2. Frank Rich: Has a 'Katrina Moment' Arrived?

- Krugman: Policy Despair
  Comments
- Cohen: From Tehran to Tel Aviv
- Editorial: Exxon Valdez
- Op-Ed: Economy on a Cliff
- The Opinionator: Geithner's Plan

MARKETS » At 11:33 AM ET

S.&P. 500	Dow	Nasdaq
798.78	7,559.93	1,510.42
+30.24	+281.55	+53.15
+3.93%	+3.87%	+3.65%

GET QUOTES My Portfolios »

Stock, ETFs, Funds Go

"I think we have a very good constructive theory of that these days," Dr. Wolynes said. "Many people tell me this is very contentious. I disagree violently with them."

The New York Times

11:31 AM ET

The Nature of Glass Remains Anything but Clear

Mark Interrante

ENIGMA Molten glass being worked into an ornament. Understanding glass could lead to better products and offer headway in other scientific problems.

By KENNETH CHANG
Published: July 29, 2008

Correction Appended

SIGN IN TO E-MAIL OR SAVE THIS

PRINT

More Articles in Science »

Today's Headlines Daily E-Mail

Sign up for a roundup of the day's top stories, sent every morning.

Sign Up

See Sample | Privacy Policy

MOST POPULAR

E-MAILED | BLOGGED | SEARCHED

1. Having a Bat Mitzvah in Their 90s Because It's a Hoot
2. Frank Rich: Has a 'Katrina Moment' Arrived?

- Krugman: Policy Despair
  Comments
- Cohen: From Tehran to Tel Aviv
- Editorial: Exxon Valdez
- Op-Ed: Economy on a Cliff
- The Opinionator: Geithner's Plan

MARKETS » At 11:33 AM ET

S.&P. 500	Dow	Nasdaq
798.78	7,559.93	1,510.42
+30.24	+281.55	+53.15
+3.93%	+3.87%	+3.65%

GET QUOTES My Portfolios »

Stock, ETFs, Funds Go

“I think we have a very good constructive theory of that these days,” Dr. Wolynes said. “Many people tell me this is very contentious. I disagree violently with them.”

Dr. Wolynes and his collaborators so insisted they were right that “you had the impression they were trying to sell you an old car,” said Jean-Philippe Bouchaud of the Atomic Energy Commission in France.

"I think we have a very good constructive theory of that these days," Dr. Wolynes said. "Many people tell me this is very contentious. I disagree violently with them."

Dr. Wolynes and his collaborators so insisted they were right that "you had the impression they were trying to sell you an old car," said Jean-Philippe Bouchaud of the Atomic Energy Commission in France.

David A. Weitz, a physics professor at Harvard, joked, "There are more theories of the glass transition than there are theorists who propose them."

In this talk: Two Statements

In this talk: Two Statements

ONE: All this complex "glassy" phenomenology can be observed in the **bulk melting problem.**

In this talk: Two Statements

ONE: All this complex "glassy" phenomenology can be observed in the **bulk melting problem.**

TWO: Melting dynamics and equilibrium dynamics are exactly **equivalent** in some models.

(in particular the Random First Order Theory is mappable to a melting problem of some sort...)

ONE

Melting dynamics of superheated solids

Dynamics from fully ordered initial conditions

Dynamics from fully ordered initial conditions

1) Consider a problem with a first-order transition at T_c

Dynamics from fully ordered initial conditions

1) Consider a problem with a first-order transition at Tc

→

- Liquid-Solid
- Potts models
- Spin models with 3-body interactions

Dynamics from fully ordered initial conditions

1) Consider a problem with
a first-order transition at Tc

2) Initialized your system
in the fully ordered configuration
(i.e. the ground state configuration)

- Liquid-Solid
- Potts models
- Spin models with
3-body interactions

Dynamics from fully ordered initial conditions

1) Consider a problem with
a first-order transition at Tc

- Liquid-Solid
- Potts models
- Spin models with 3-body interactions

2) Initialized your system
in the fully ordered configuration
(i.e. the ground state configuration)

- Crystal state
- All spins equal

Dynamics from fully ordered initial conditions

1) Consider a problem with
a first-order transition at Tc

- Liquid-Solid
- Potts models
- Spin models with 3-body interactions

2) Initialized your system
in the fully ordered configuration
(i.e. the ground state configuration)

- Crystal state
- All spins equal

Set the temperature to T>Tc and observe the melting dynamics
of the ordered phase into the less ordered one

Dynamics from fully ordered initial conditions

1) Consider a problem with
a first-order transition at Tc

- Liquid-Solid
- Potts models
- Spin models with 3-body interactions

2) Initialized your system
in the fully ordered configuration
(i.e. the ground state configuration)

- Crystal state
- All spins equal

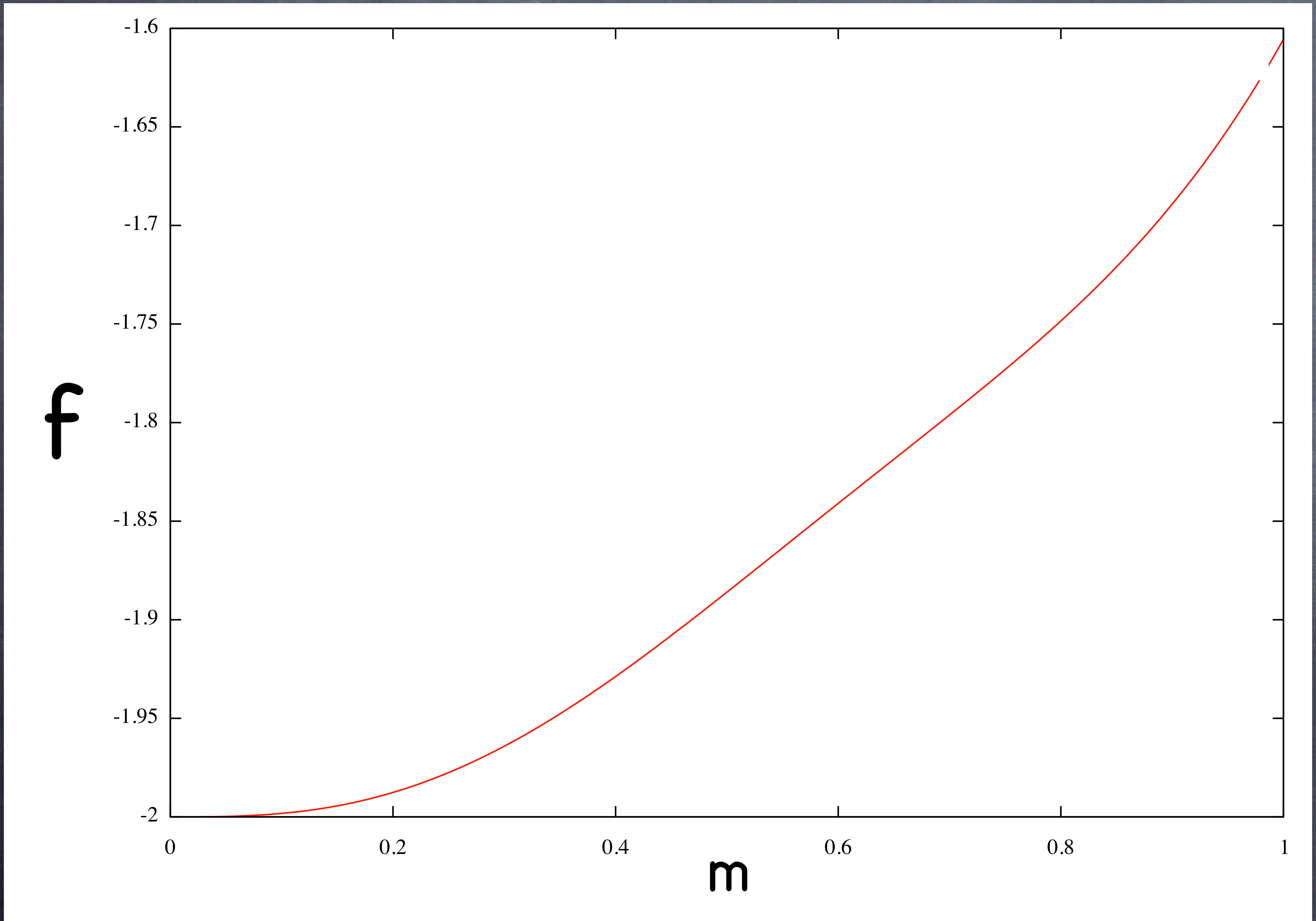
Set the temperature to T>Tc and observe the melting dynamics
of the ordered phase into the less ordered one

Periodic Boundary
conditions ⇒ No boundaries!

No "surface melting"

First order transitions

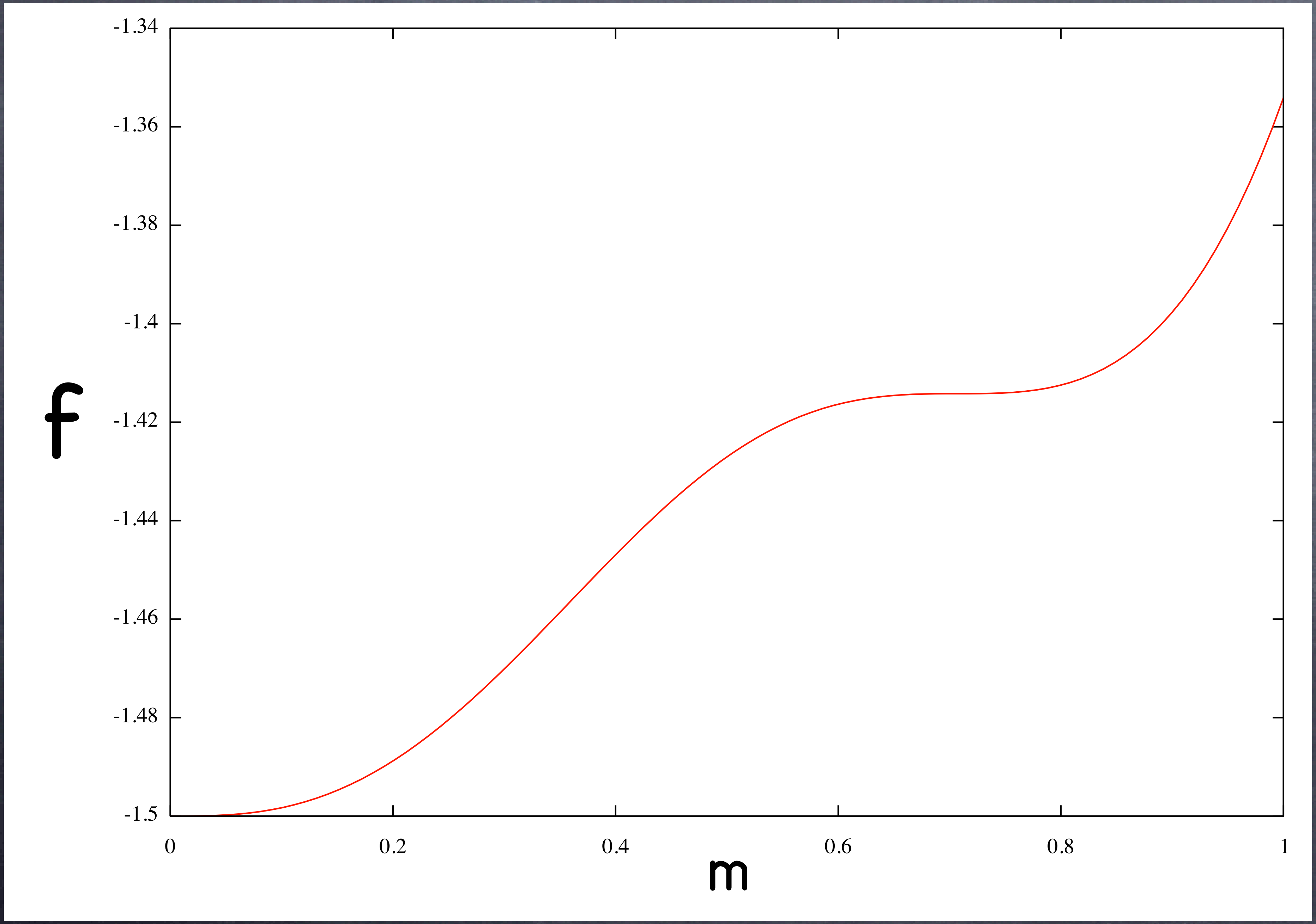
free energy landscape



very high T

First order transitions

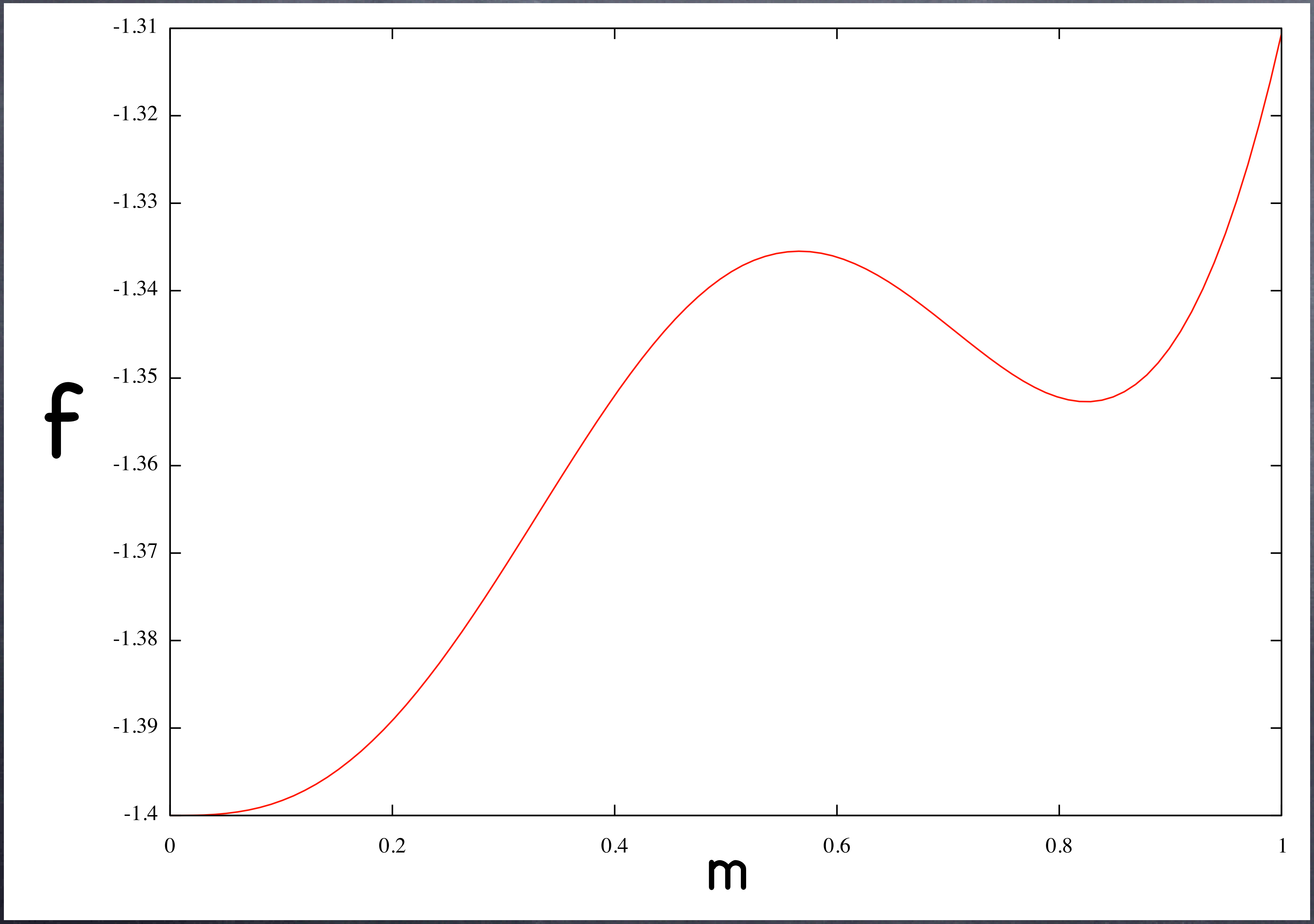
free energy landscape



high T

First order transitions

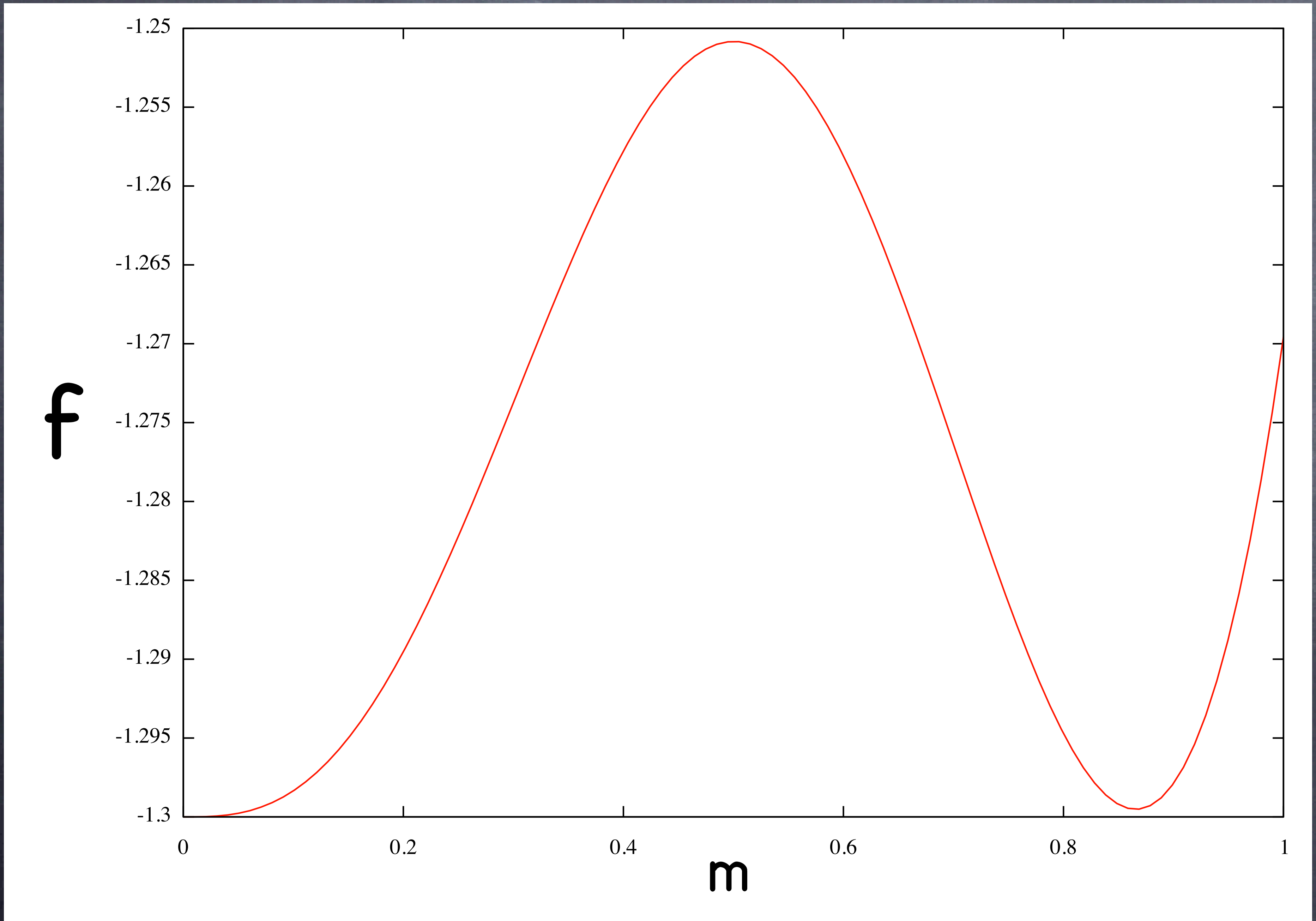
free energy landscape



lower T

First order transitions

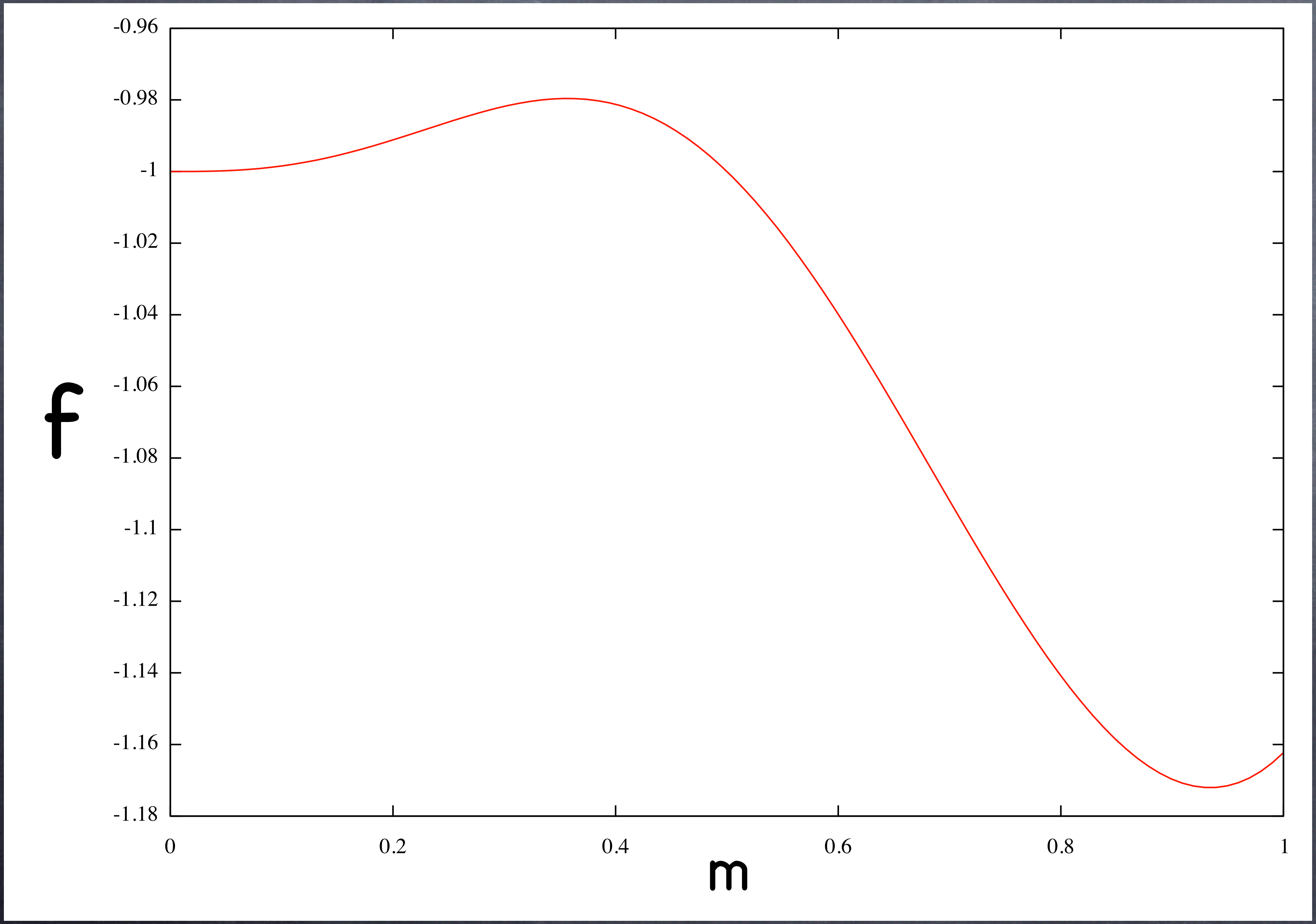
free energy landscape



Transition Tc

First order transitions

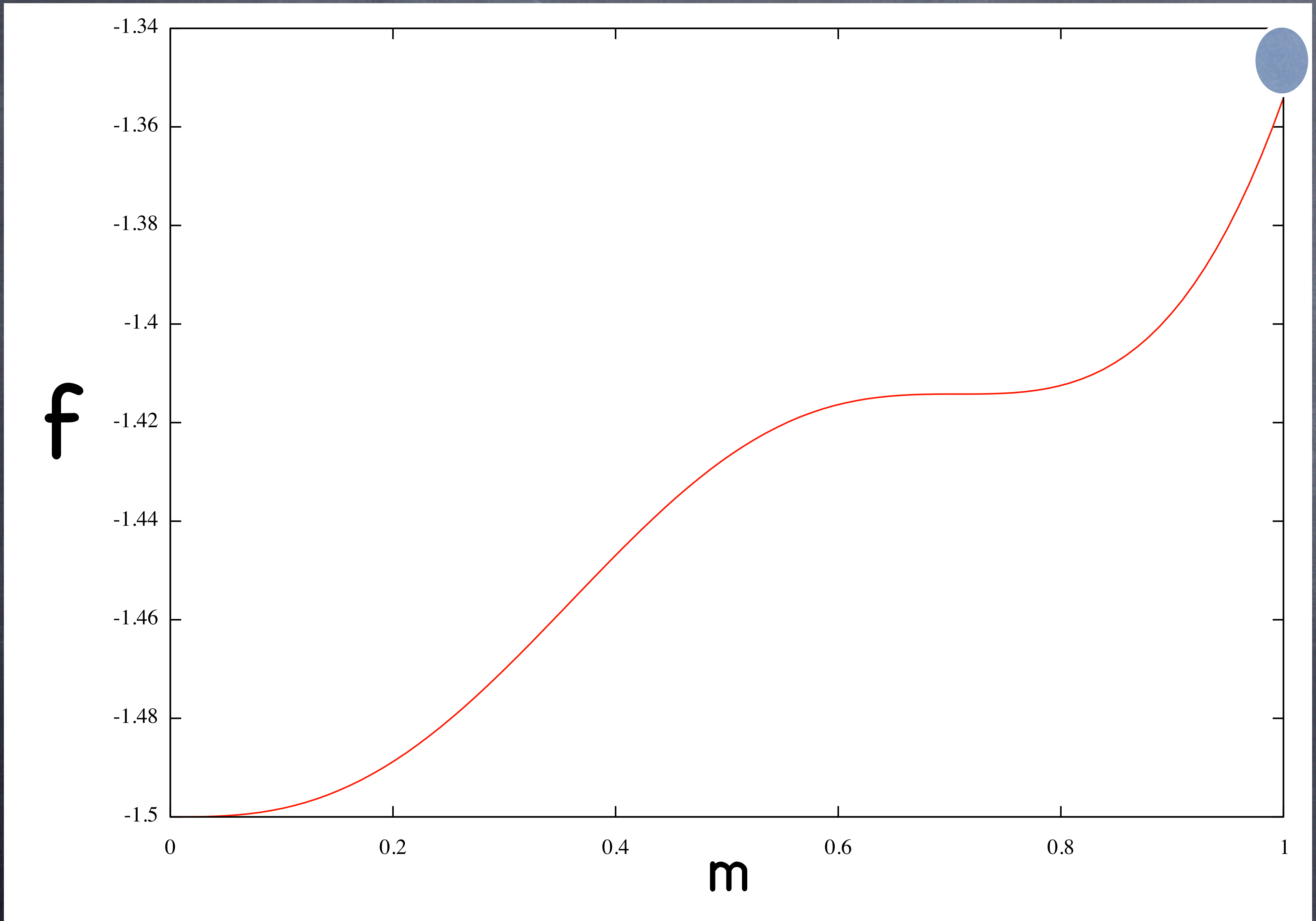
free energy landscape



Low T

Melting dynamics

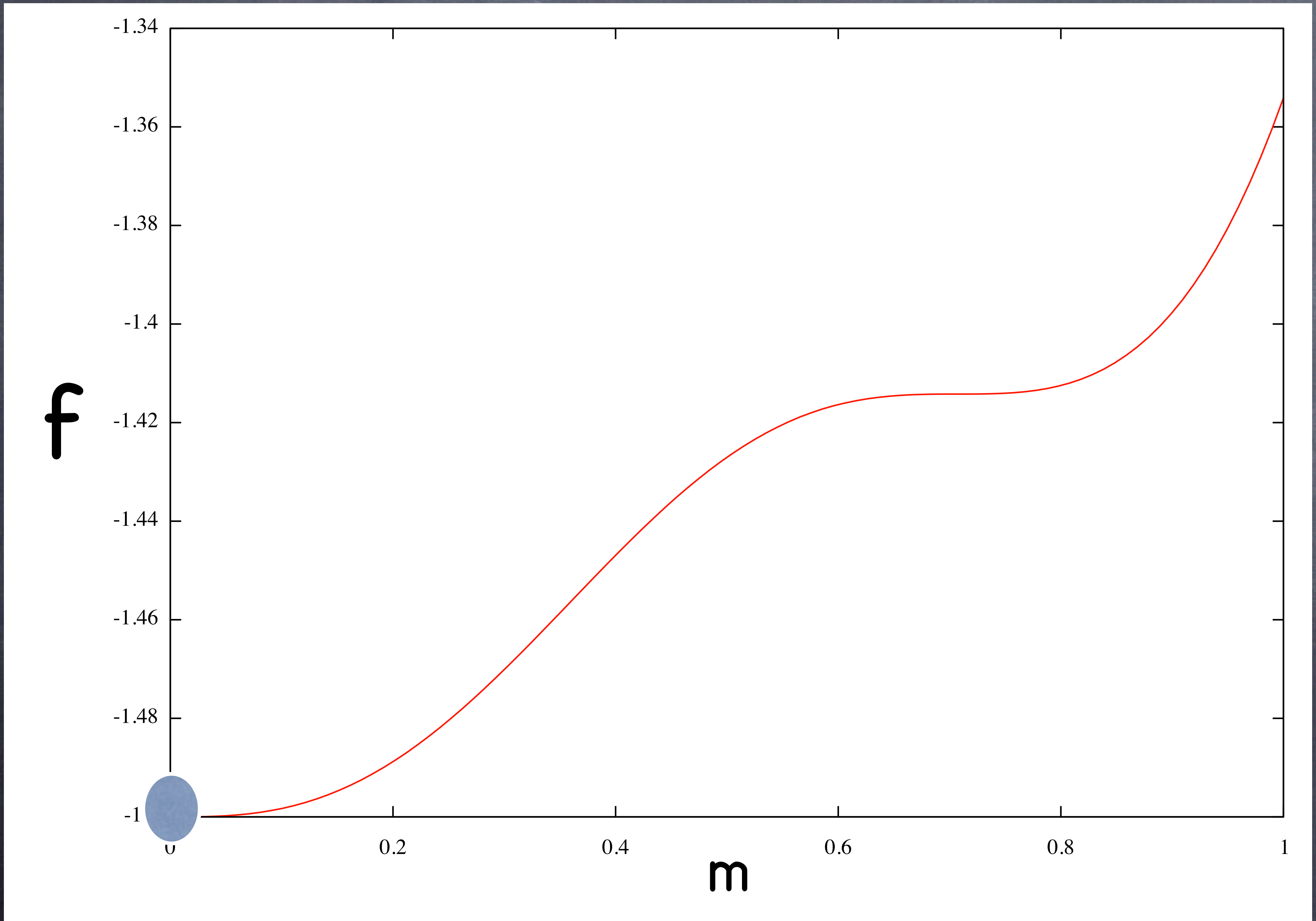
free energy landscape



Large enough Temperature

Melting dynamics

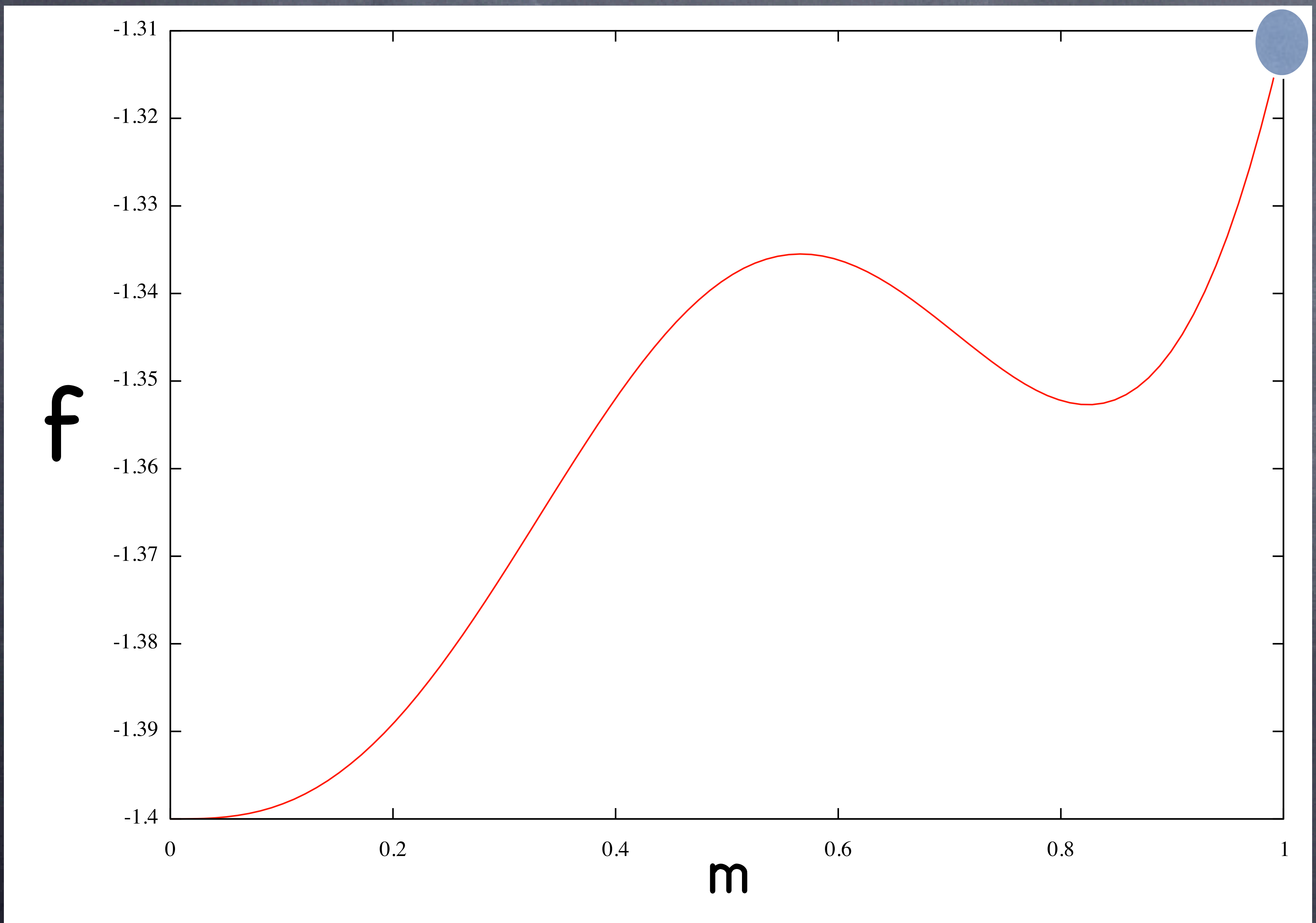
free energy landscape



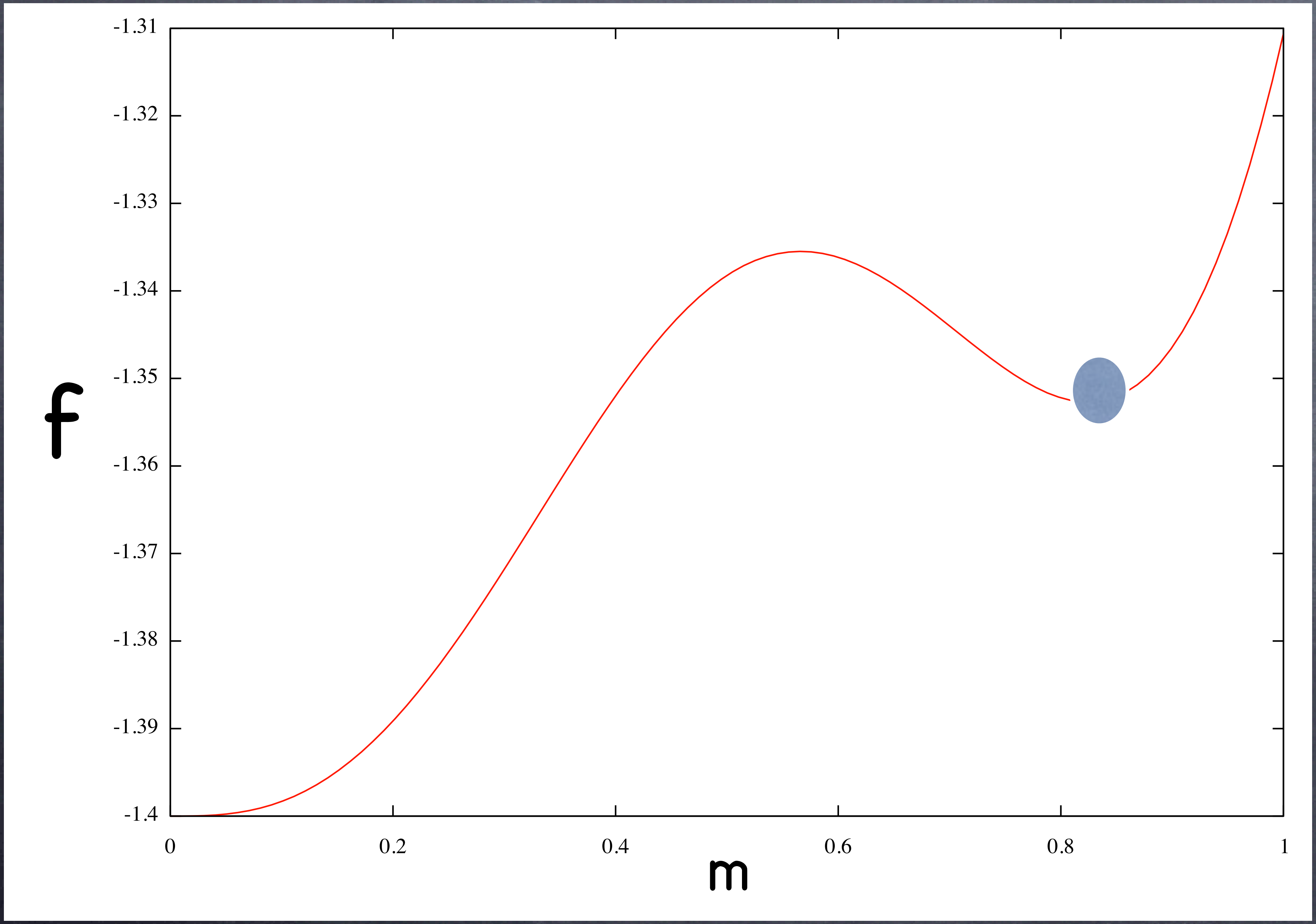
Large enough Temperature

Sunday, January 30, 2011

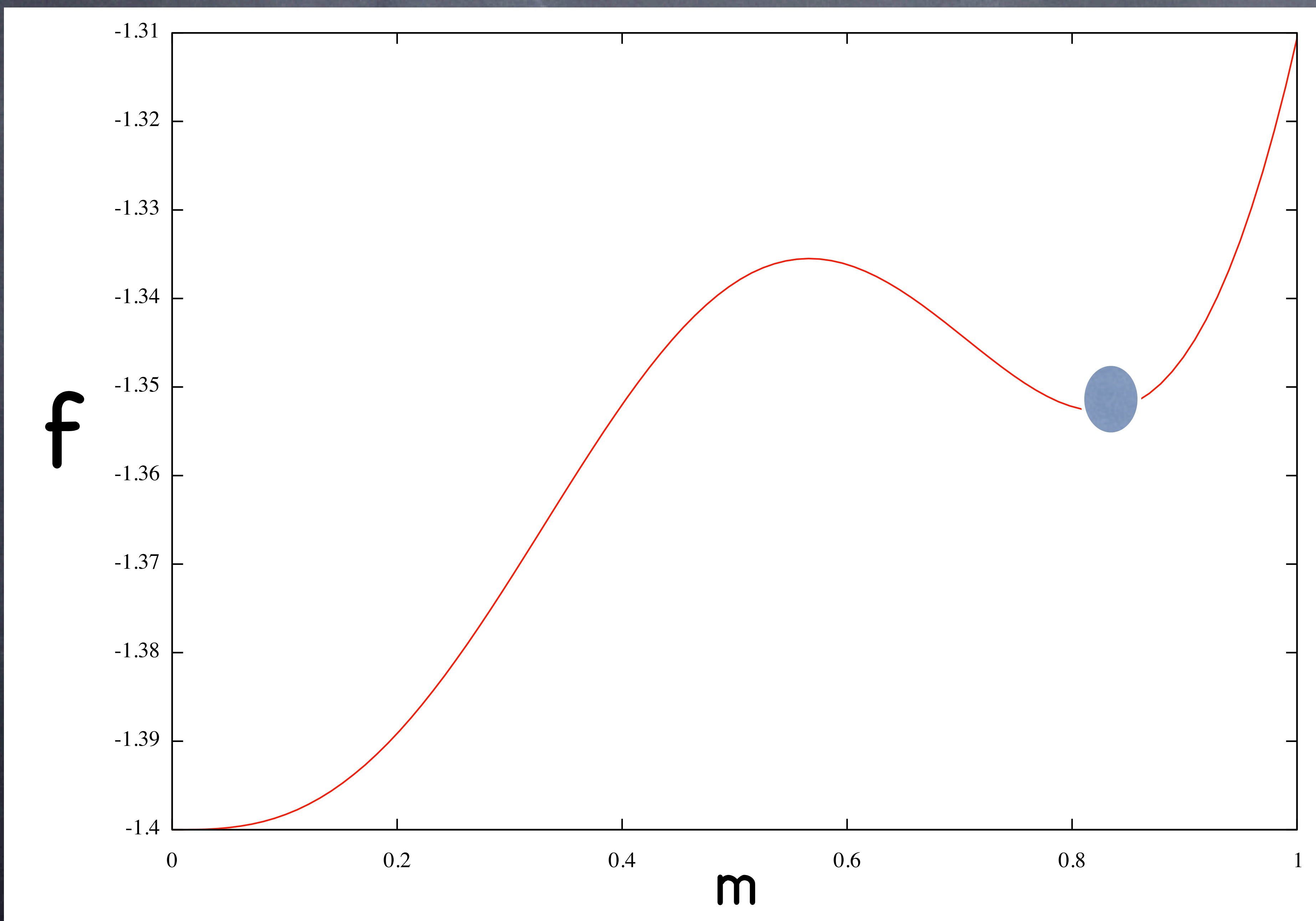
Melting dynamics



Melting dynamics

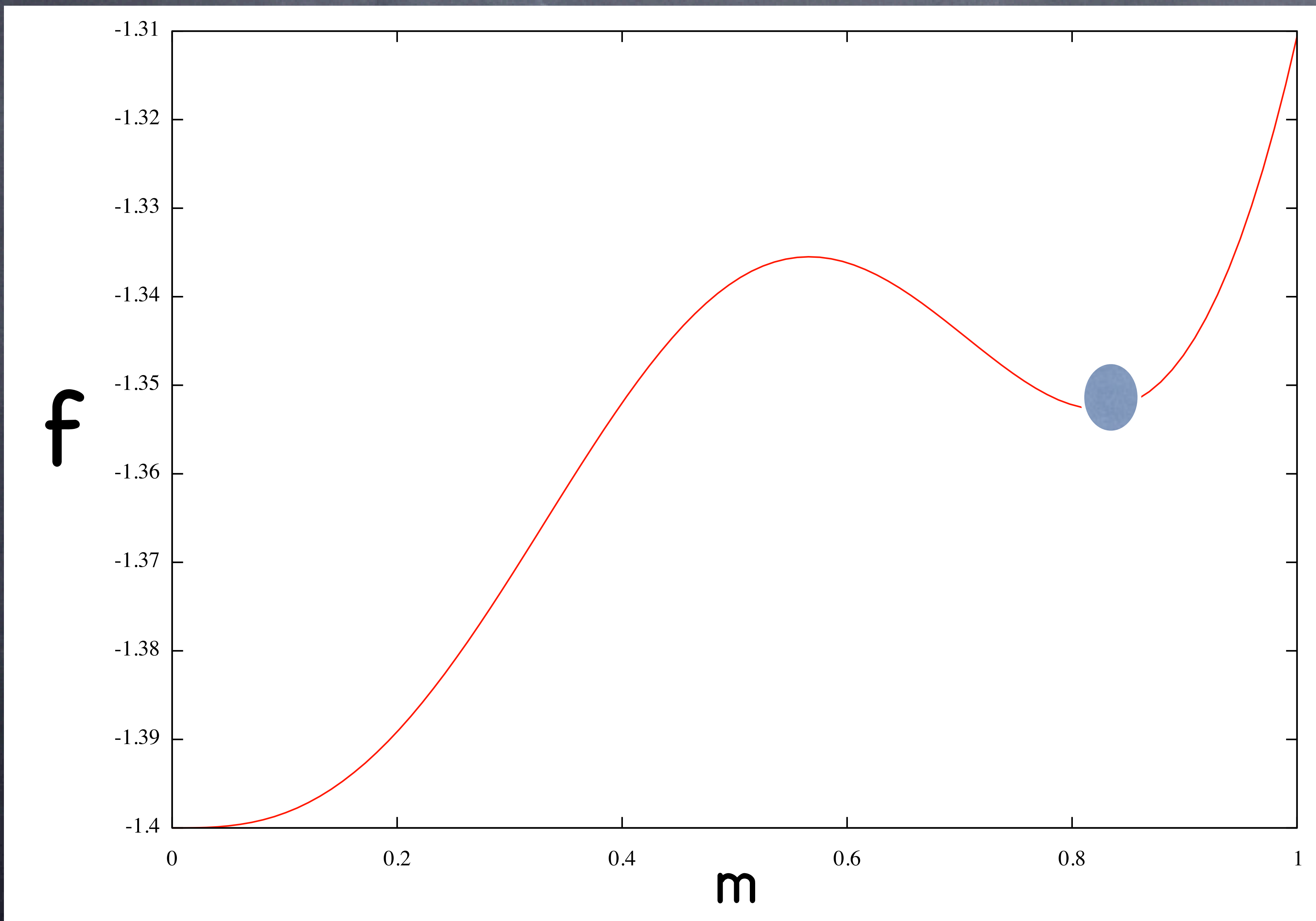


Melting dynamics



In mean-field:
Melting dynamics is trapped by the high free-energy state

Melting dynamics

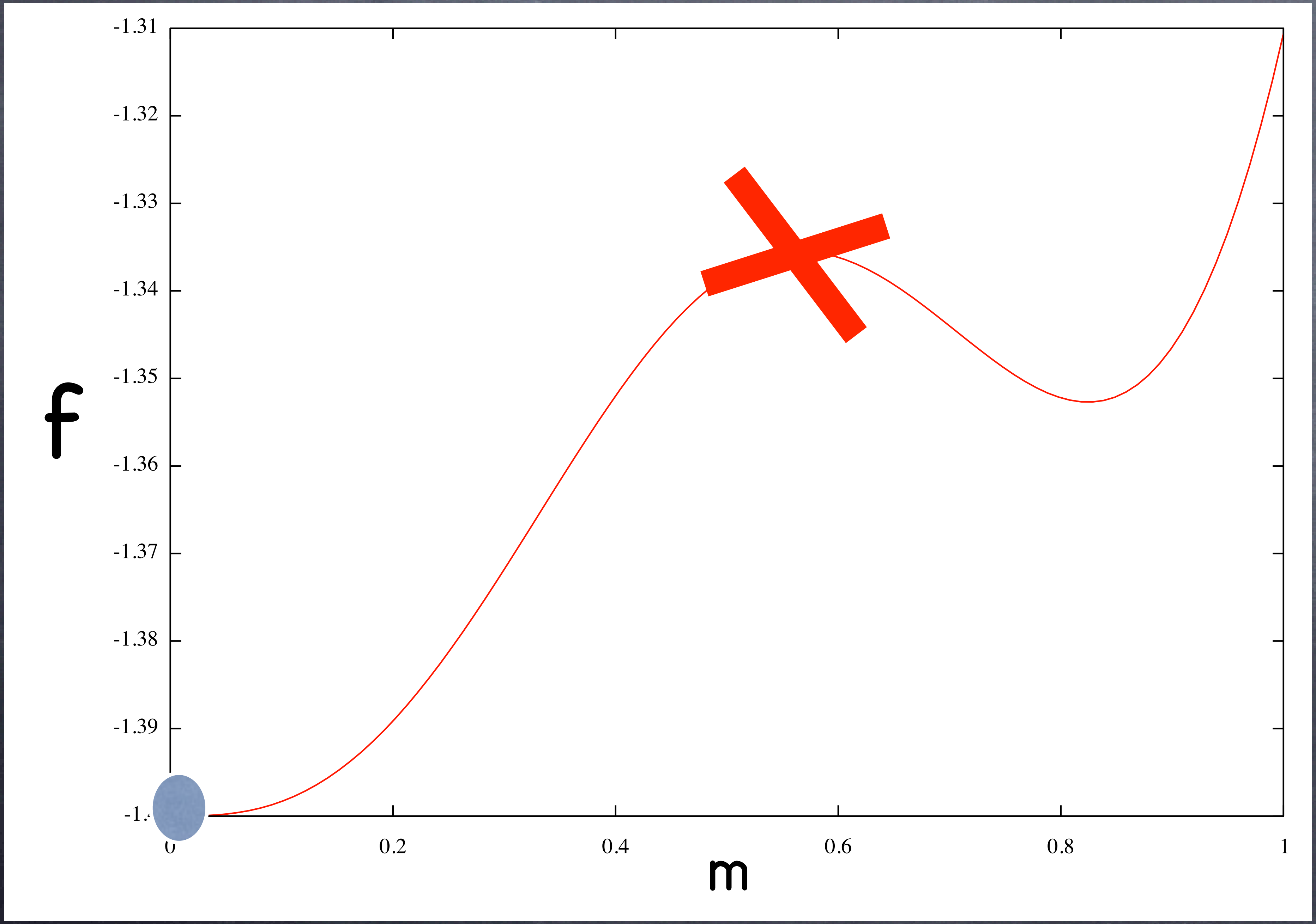


In mean-field:
Melting dynamics is trapped by the high free-energy state

In finite dimension:
Metastability & Activation process

Melting dynamics

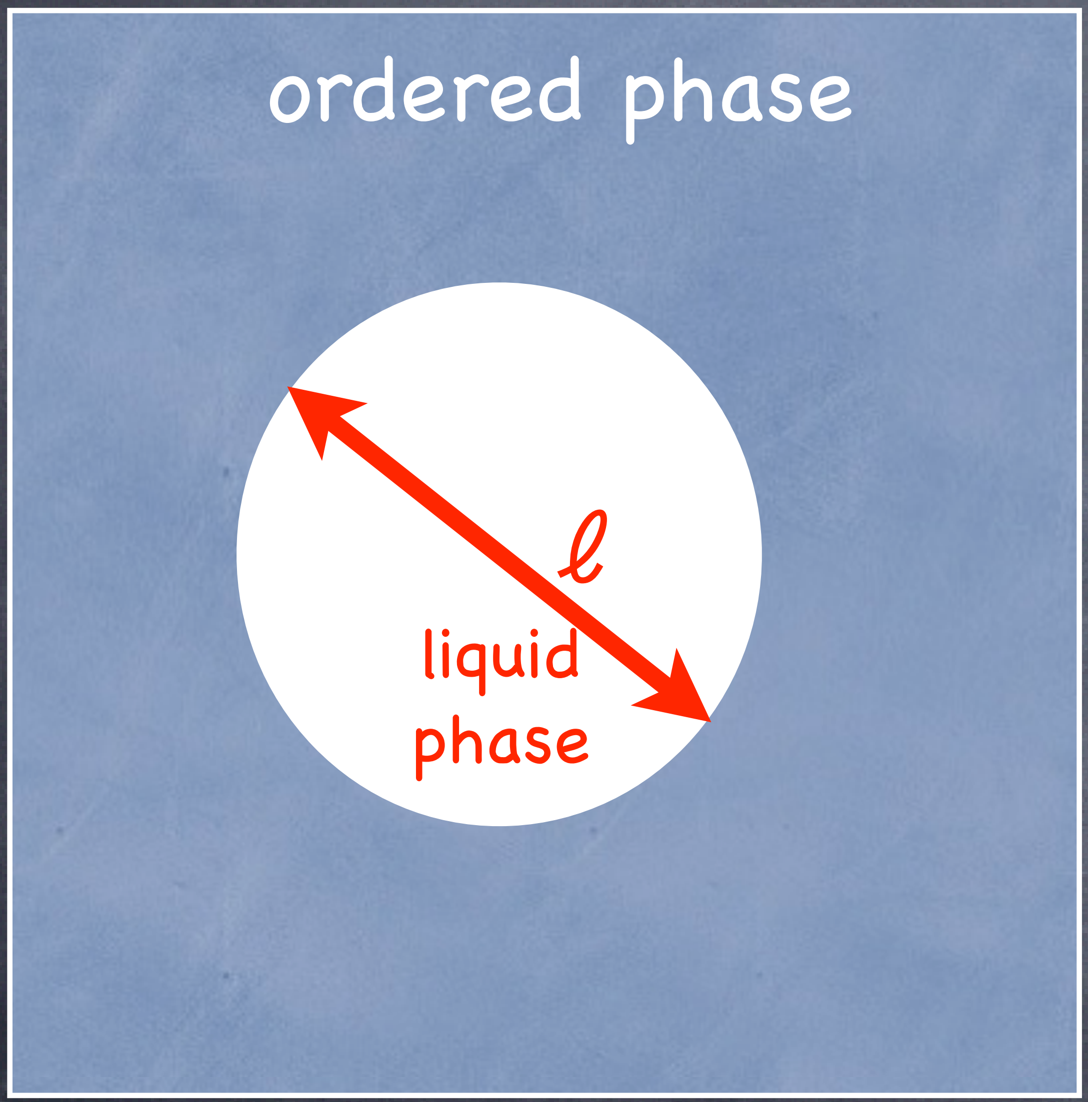
No extensive barrier in finite d!



In mean-field:
Melting dynamics is trapped by the high free-energy state

In finite dimension:
Metastability &
Activation process

Nucleation argument



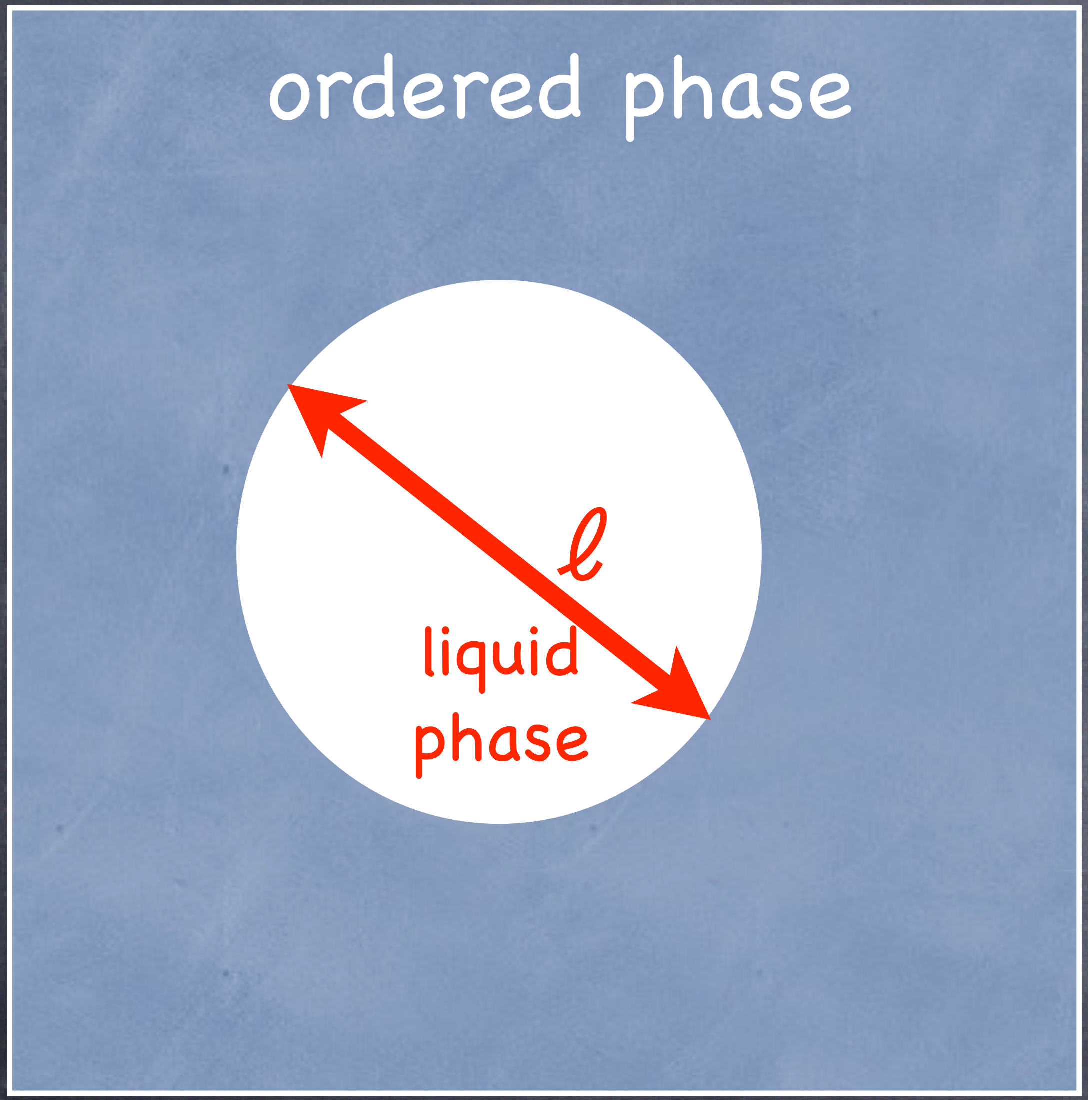
Cost:

$$F_s = \gamma \ell^{d-1}$$

Gain:

$$F_v = \Delta f \ell^d$$

Nucleation argument



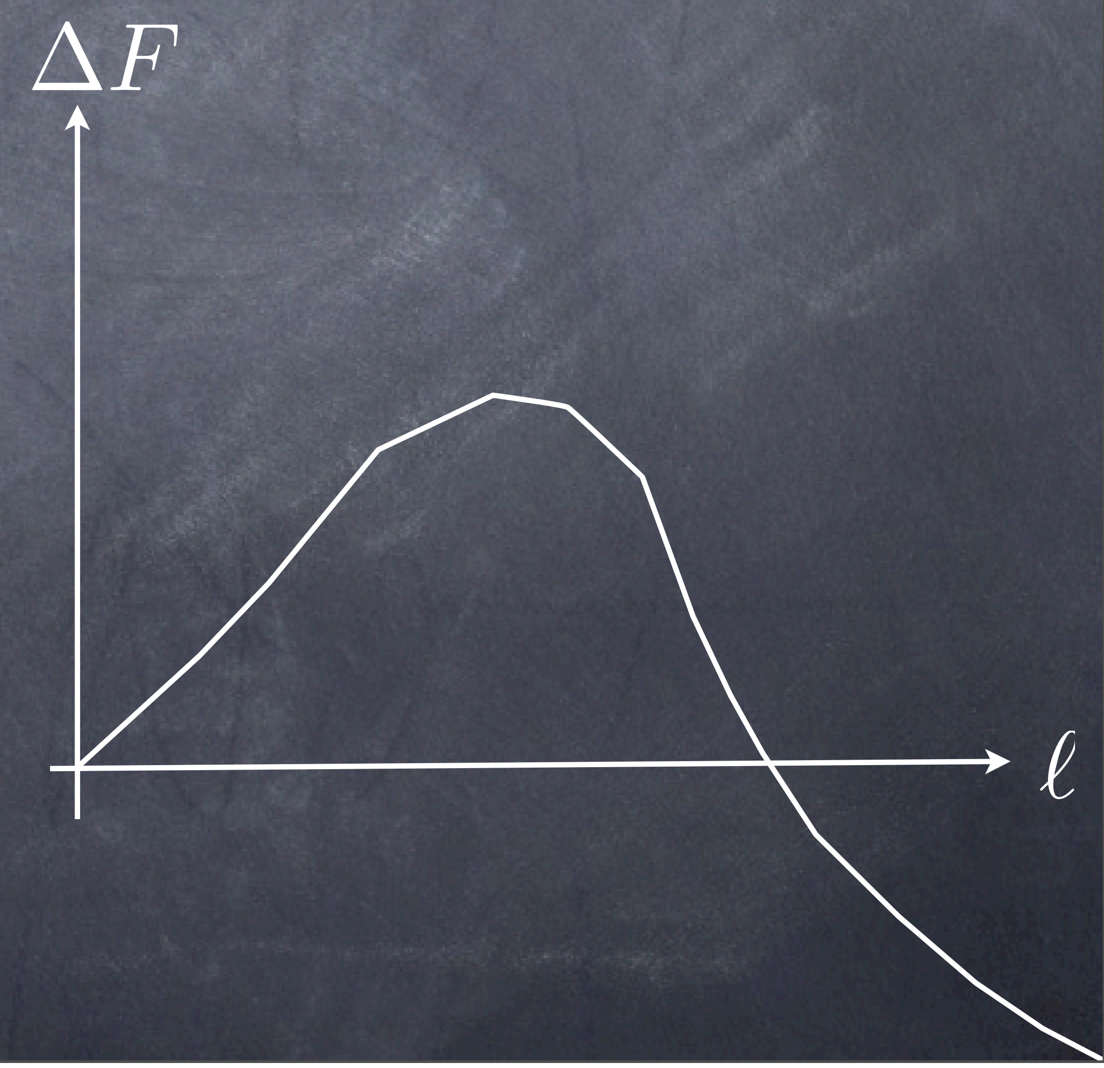
Cost:

$$F_s = \gamma \ell^{d-1}$$

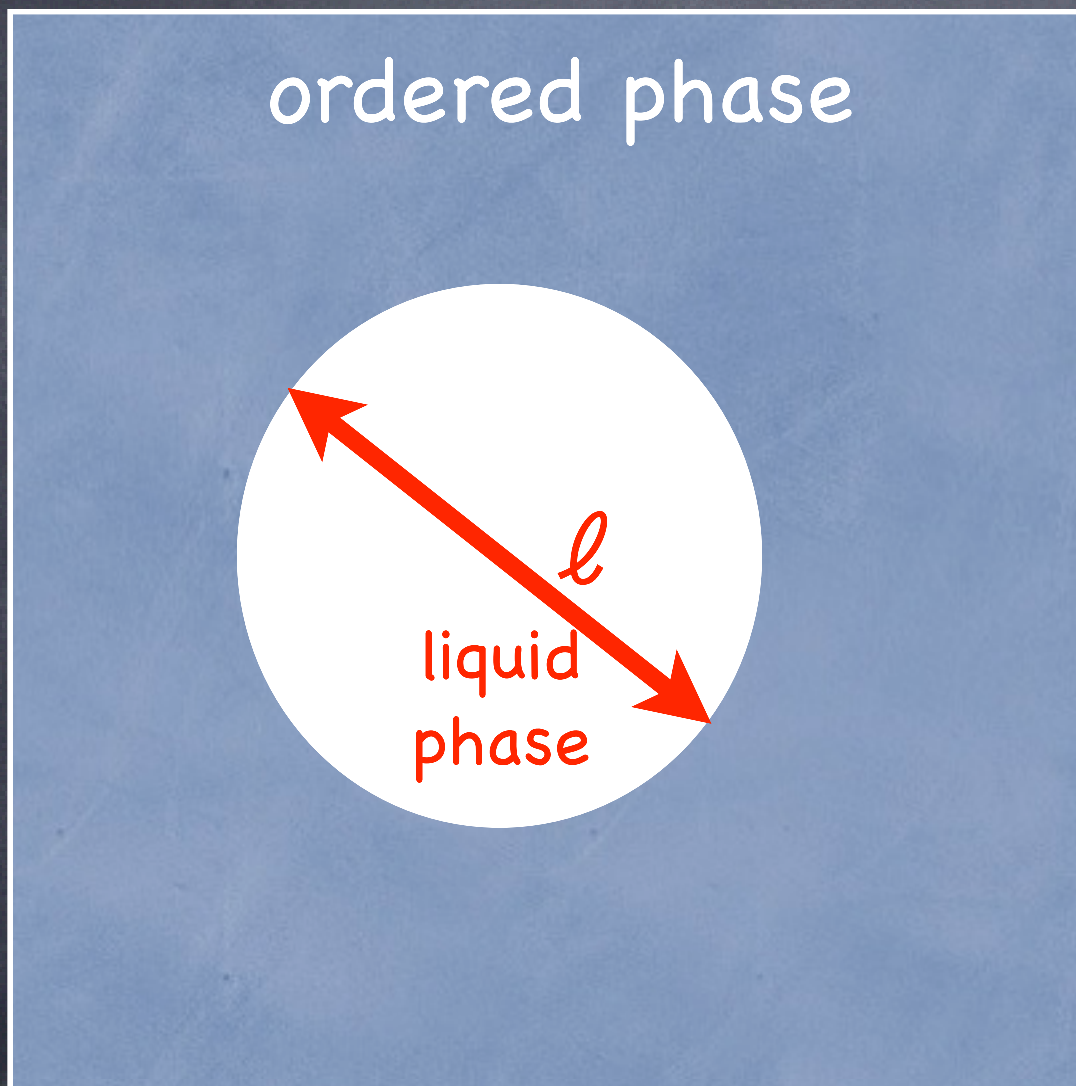
Gain:

$$F_v = \Delta f \ell^d$$

Total Free energy cost of the droplet



Nucleation argument



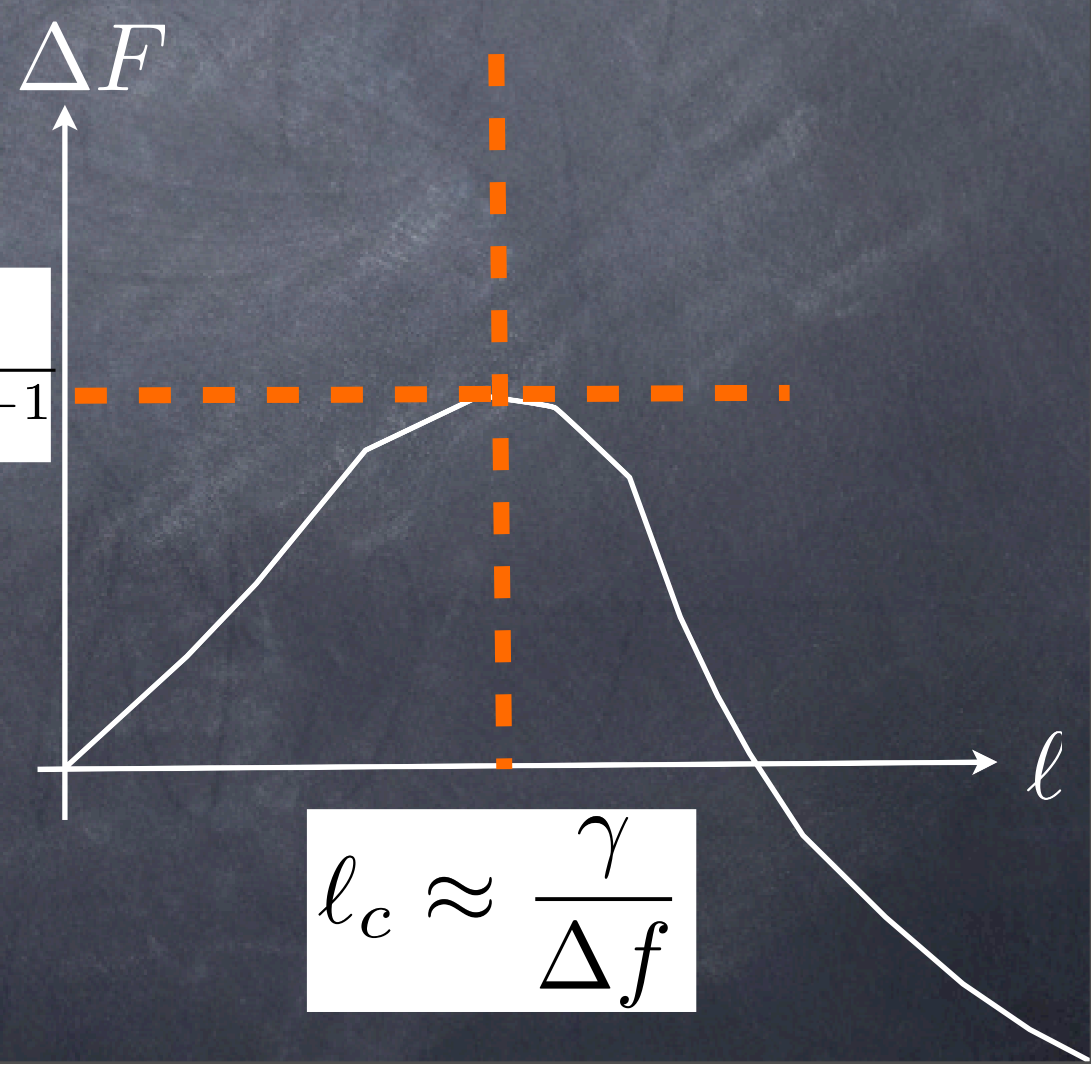
Cost:

$$F_s = \gamma \ell^{d-1}$$

Gain:

$$F_v = \Delta f \ell^d$$

Total Free energy cost of the droplet



$$\Delta F_{max} \approx \frac{\gamma^d}{\Delta f^{d-1}}$$

$$\ell_c \approx \frac{\gamma}{\Delta f}$$

The nucleation argument

Free energy barrier

$$\Delta F_{max} \approx \frac{\gamma^d}{\Delta f^{d-1}}$$

Arrhenius factor

$$\tau \propto e^{\beta A / \Delta f^{d-1}}$$

The nucleation argument

Free energy barrier

$$\Delta F_{max} \approx \frac{\gamma^d}{\Delta f^{d-1}}$$

Arrhenius factor

$$\tau \propto e^{\beta A/\Delta f^{d-1}}$$

$$\Downarrow$$

$$\tau \propto e^{\beta A/(T-Tc)^{d-1}}$$

The nucleation argument

Free energy barrier

$$\Delta F_{max} \approx \frac{\gamma^d}{\Delta f^{d-1}}$$

Arrhenius factor

$$\tau \propto e^{\beta A/\Delta f^{d-1}}$$

$$\tau \propto e^{\beta A/(T-Tc)^{d-1}}$$

Potts model D=2

$$\mathcal{H} = -\sum_{<ij>} \delta s_i, s_j \qquad s = 1, \ldots, q$$

1st order for q>4 $\qquad \beta_c = \ln(1+\sqrt{q})$

q=10, Tc=0.70123...

The nucleation argument

Free energy barrier

$$\Delta F_{max} \approx \frac{\gamma^d}{\Delta f^{d-1}}$$

Arrhenius factor

$$\tau \propto e^{\beta A/\Delta f^{d-1}}$$

⇓

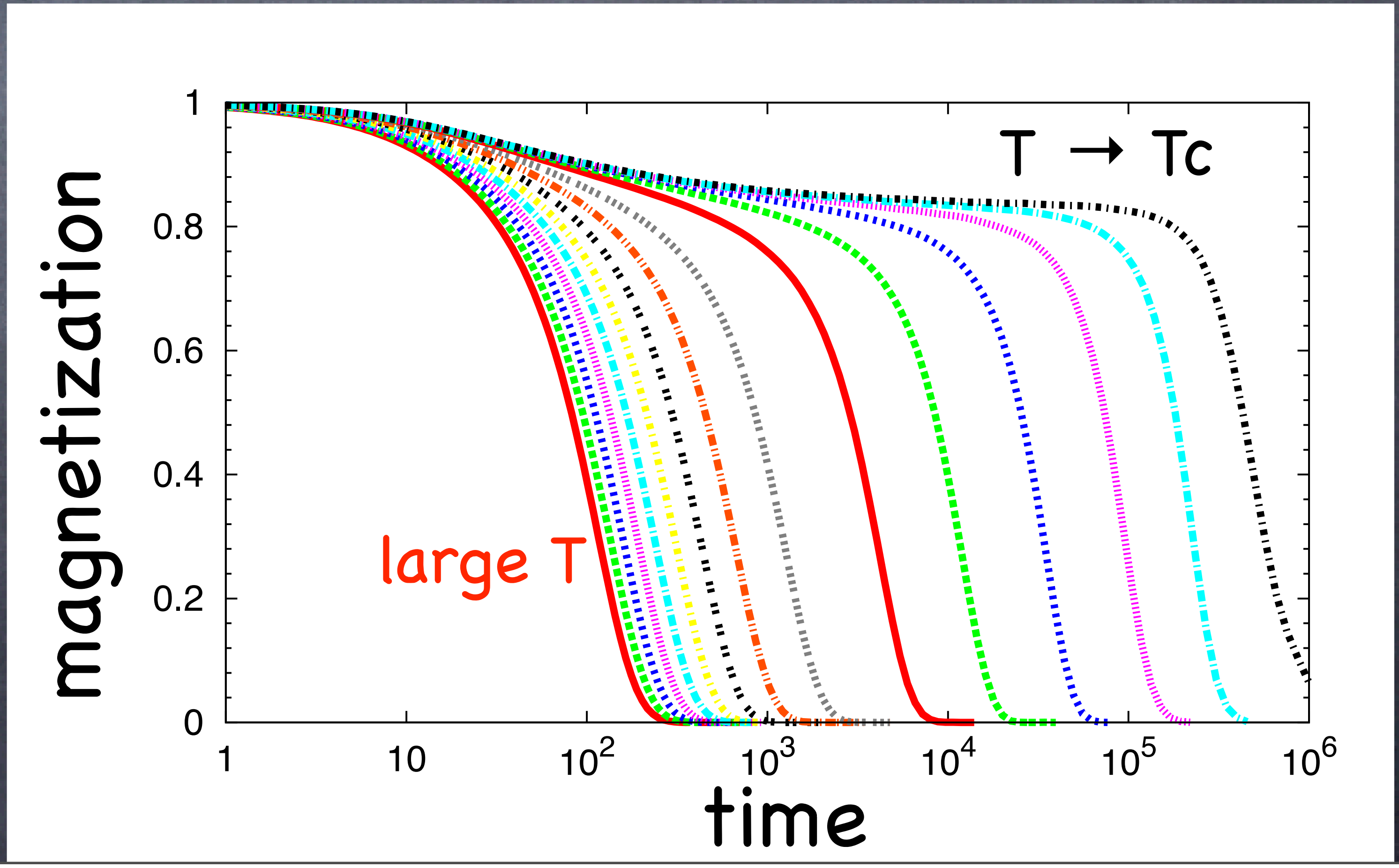
$$\tau \propto e^{\beta A/(T-Tc)^{d-1}}$$

Potts model D=2

$$\mathcal{H} = -\sum_{<ij>} \delta s_i, s_j \qquad s = 1, \ldots, q$$

1st order for q>4 $\qquad \beta_c = \ln(1+\sqrt{q})$

q=10, Tc=0.70123...



The nucleation argument

Potts model D=2

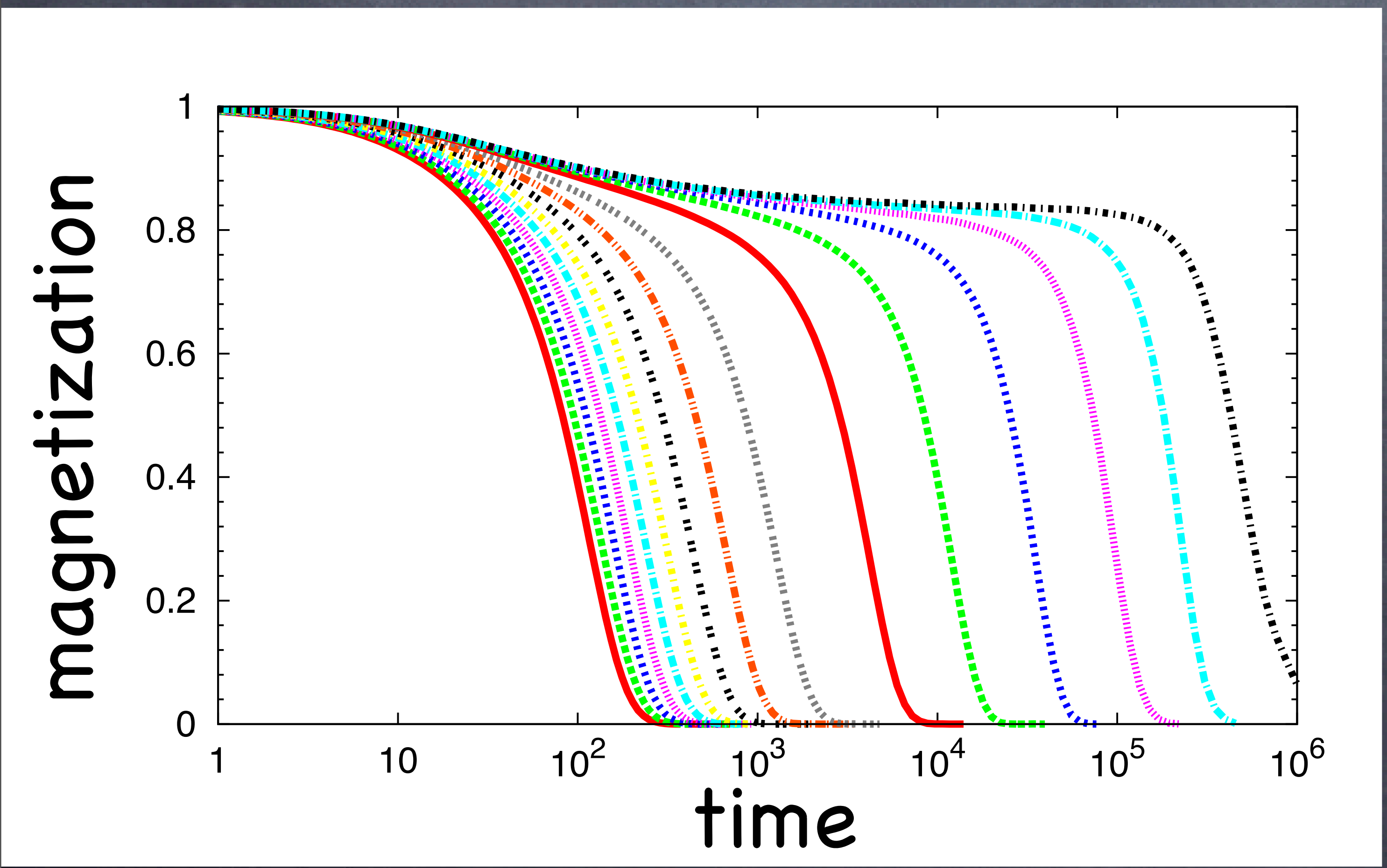
Arrhenius factor

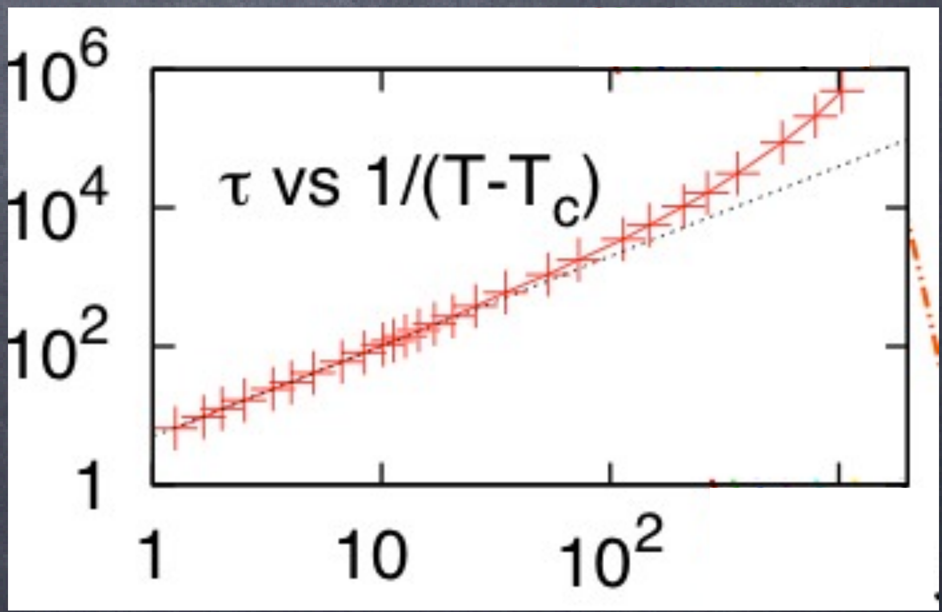
$$\tau \propto e^{\beta A/(T-Tc)^{d-1}}$$

$$\mathcal{H} = -\sum_{<ij>} \delta s_i, s_j \qquad s = 1, \ldots, q$$

1st order for q>4 $\qquad \beta_c = \ln(1+\sqrt{q})$

q=10, Tc=0.70123...





The nucleation argument

Potts model D=2

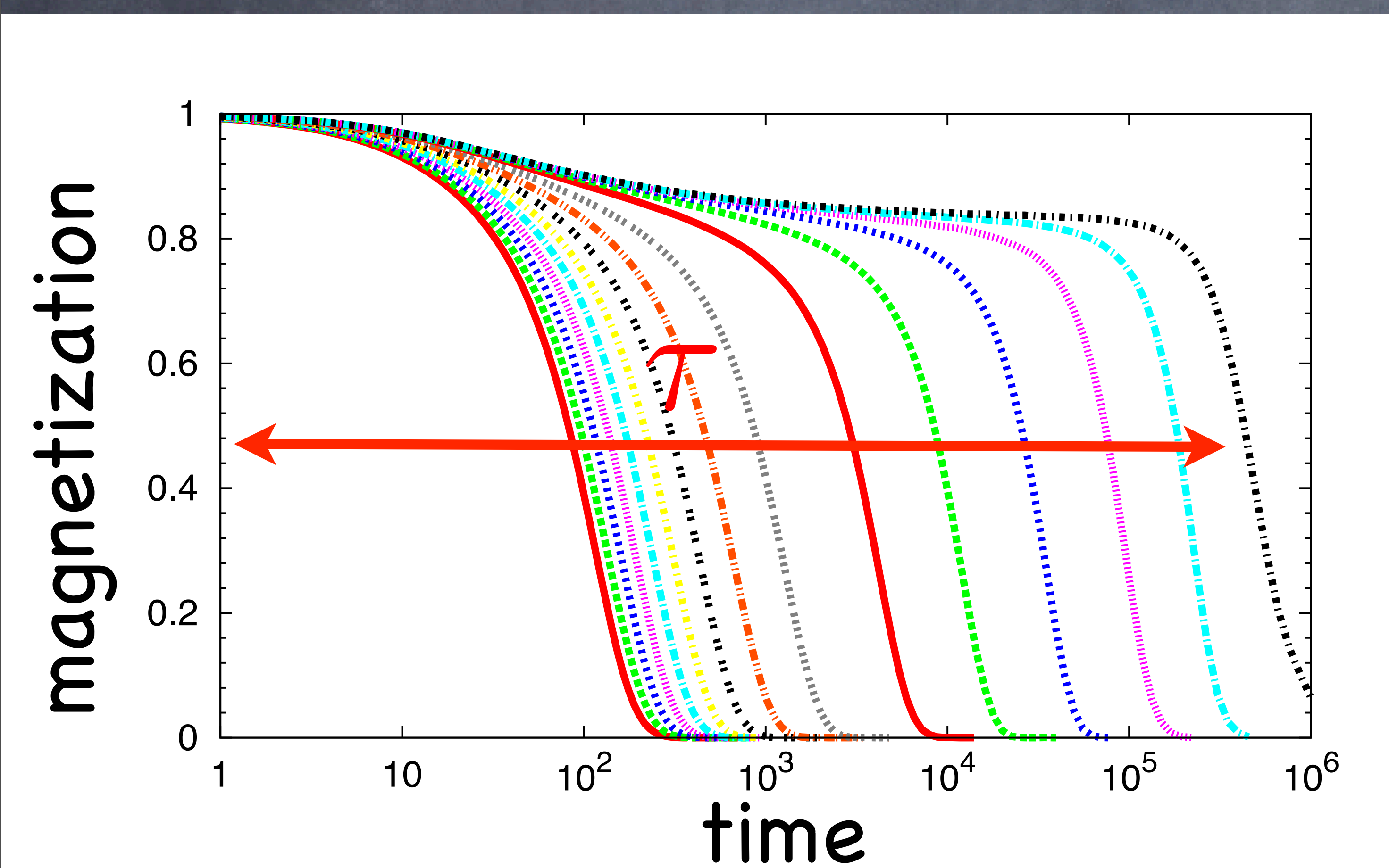
Arrhenius factor

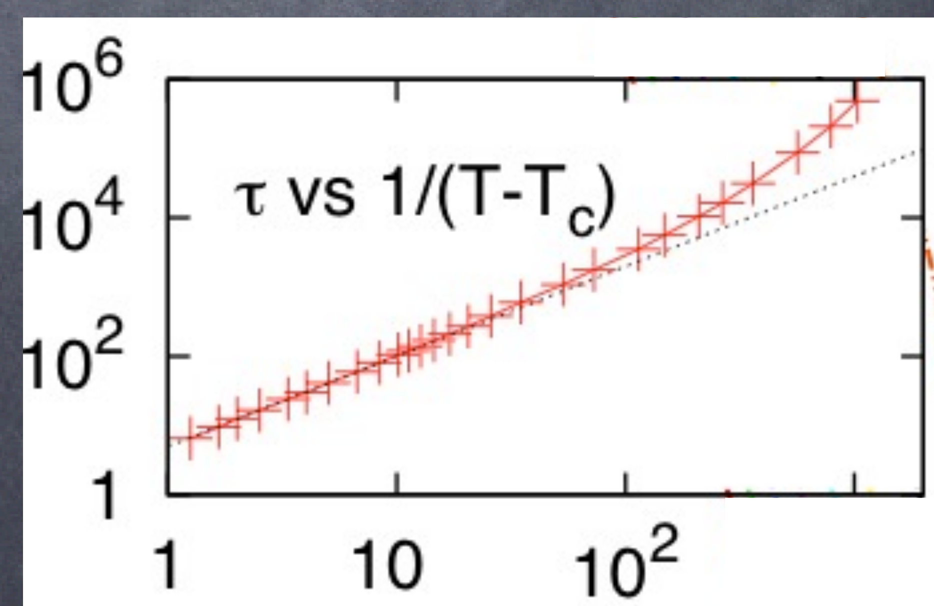
$$\tau \propto e^{\beta A/(T-Tc)^{d-1}}$$

$$\mathcal{H} = -\sum_{<ij>} \delta s_i, s_j \qquad s = 1, \ldots, q$$

1st order for q>4 $\qquad \beta_c = \ln(1+\sqrt{q})$

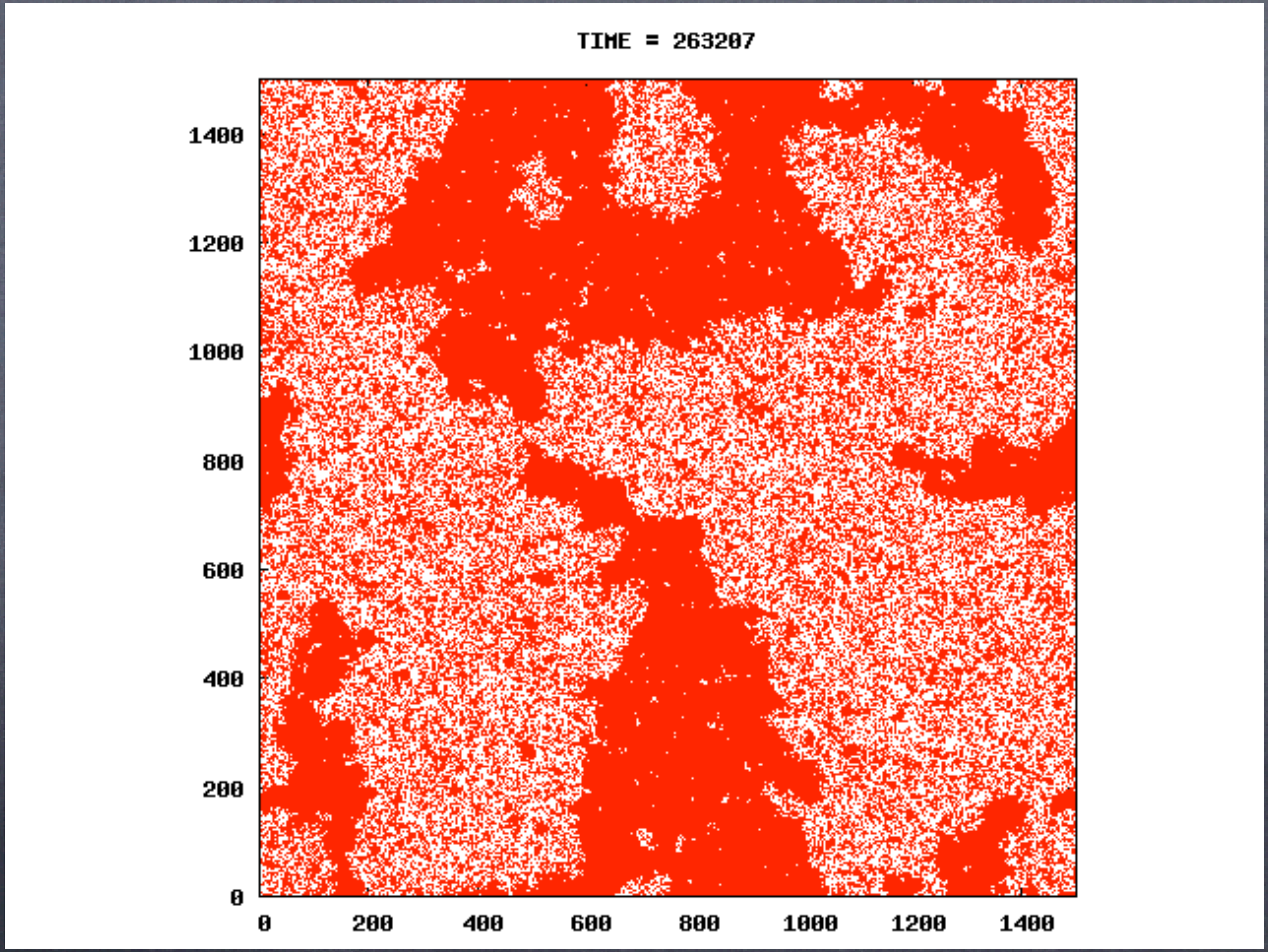
q=10, Tc=0.70123...



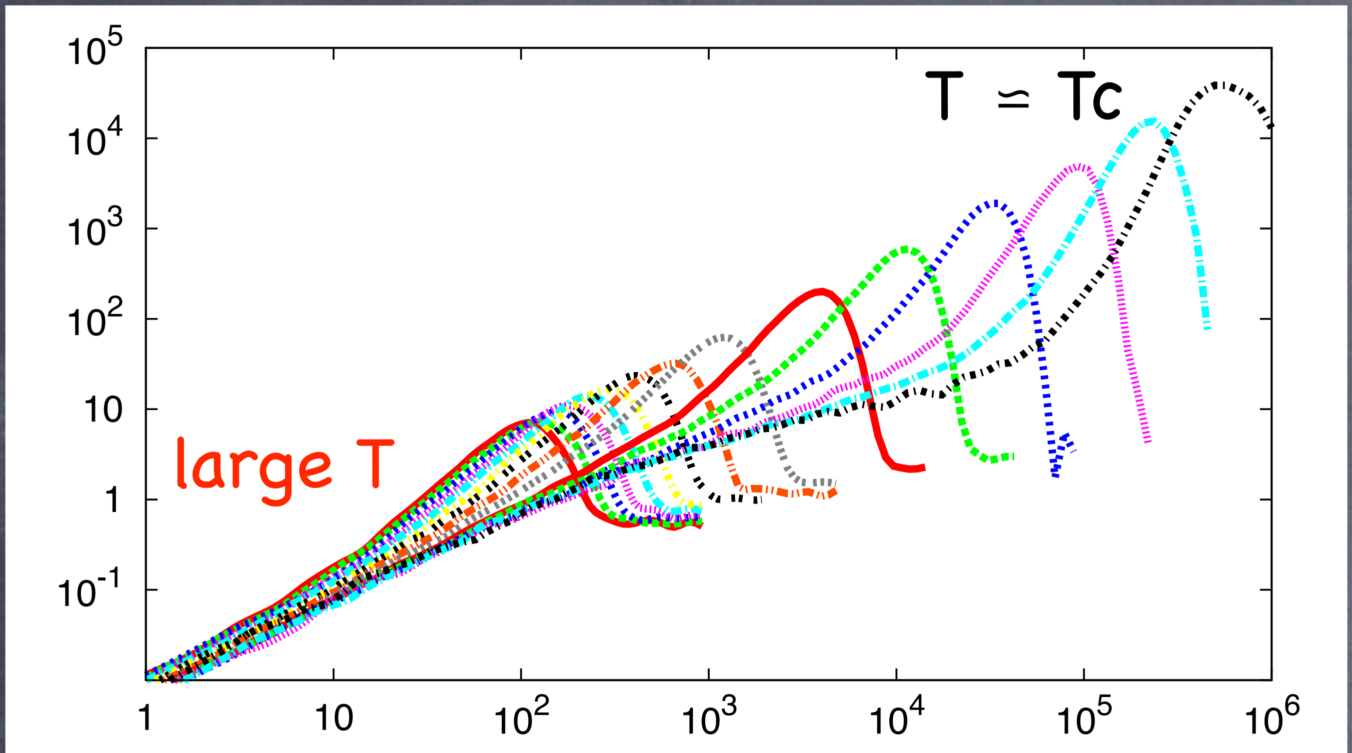


Melting in the 2d Potts model: nucleation and growth

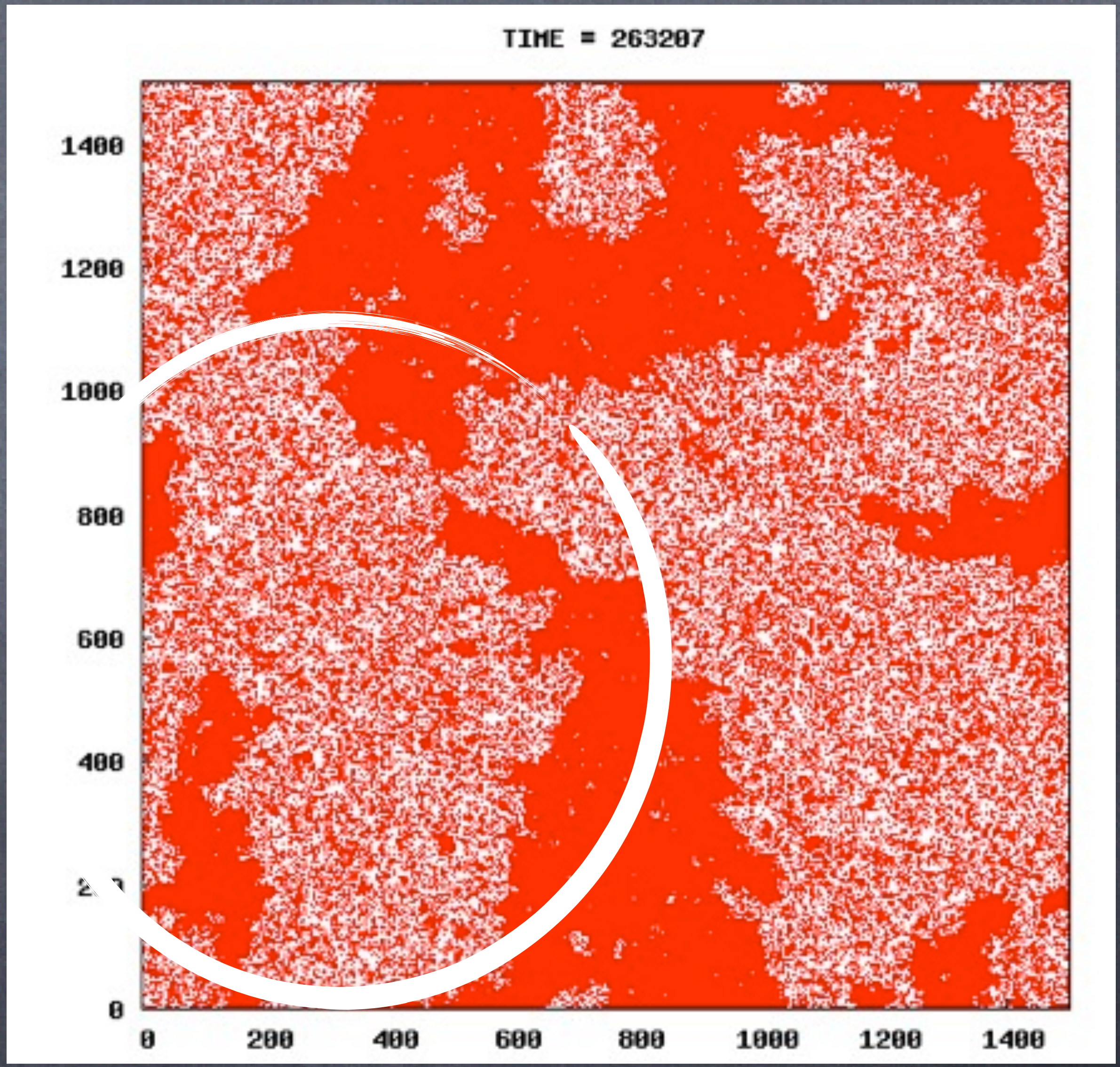
Melting in the 2d Potts model: nucleation and growth



Melting in the 2d Potts model: nucleation and growth



Maximum heterogeneities



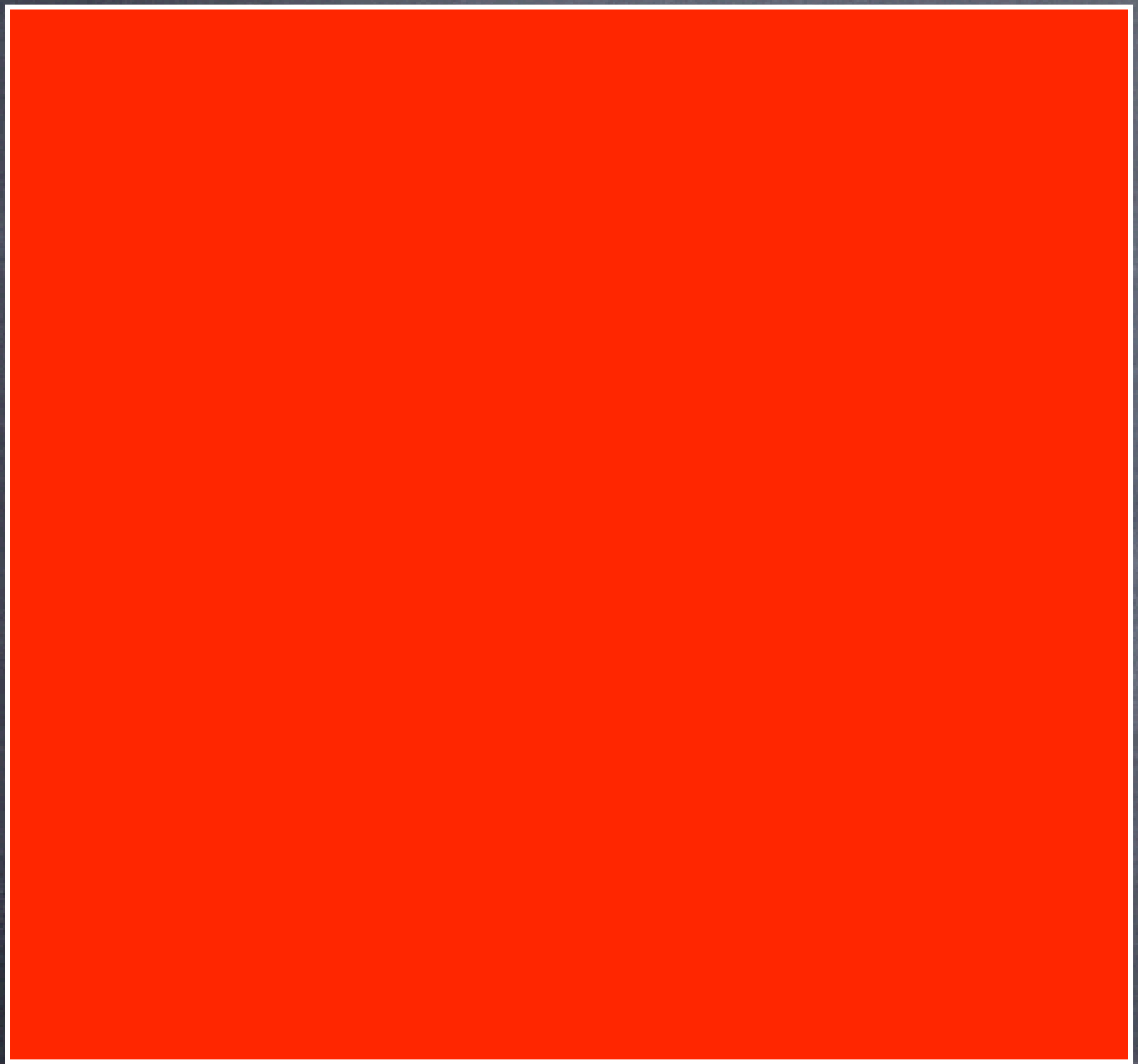
Growing of dynamical heterogeneities

$$\chi_4(t_1,t_2) = \frac{1}{V}\int dr_1 dr_2 C_4(t_1,t_2,r_1,r_2)$$

$$\chi_4(t) = \chi_F(t) = N(< m(t)^2 > - < m(t) >^2)$$

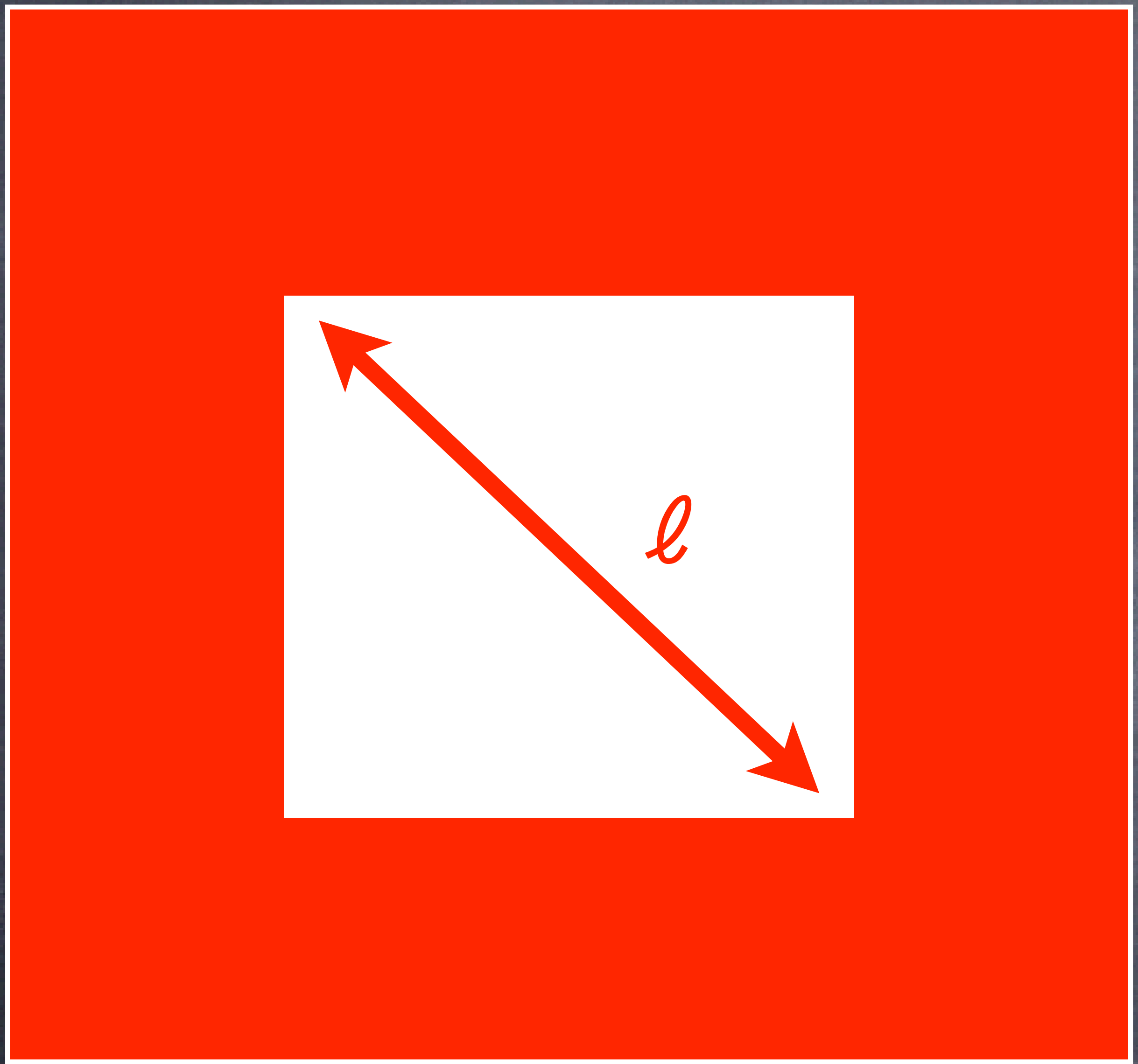
Divergence of an equilibrium Correlation length ?

Divergence of an equilibrium Correlation length ?



1) Consider the initial configuration

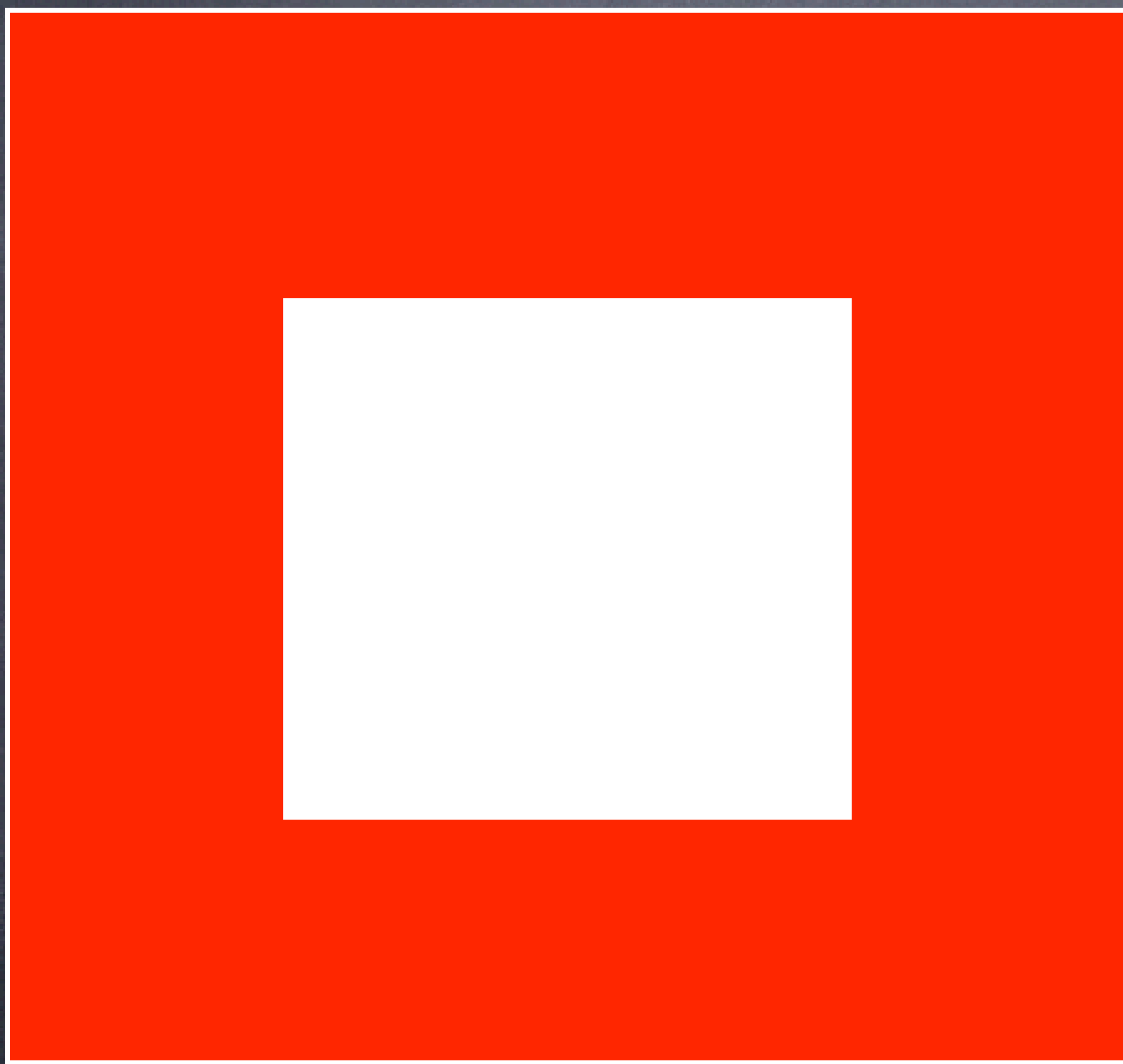
Divergence of an equilibrium Correlation length ?



1) Consider the initial configuration

2) Freeze the system and make a hole of size ℓ

Divergence of an equilibrium Correlation length ?



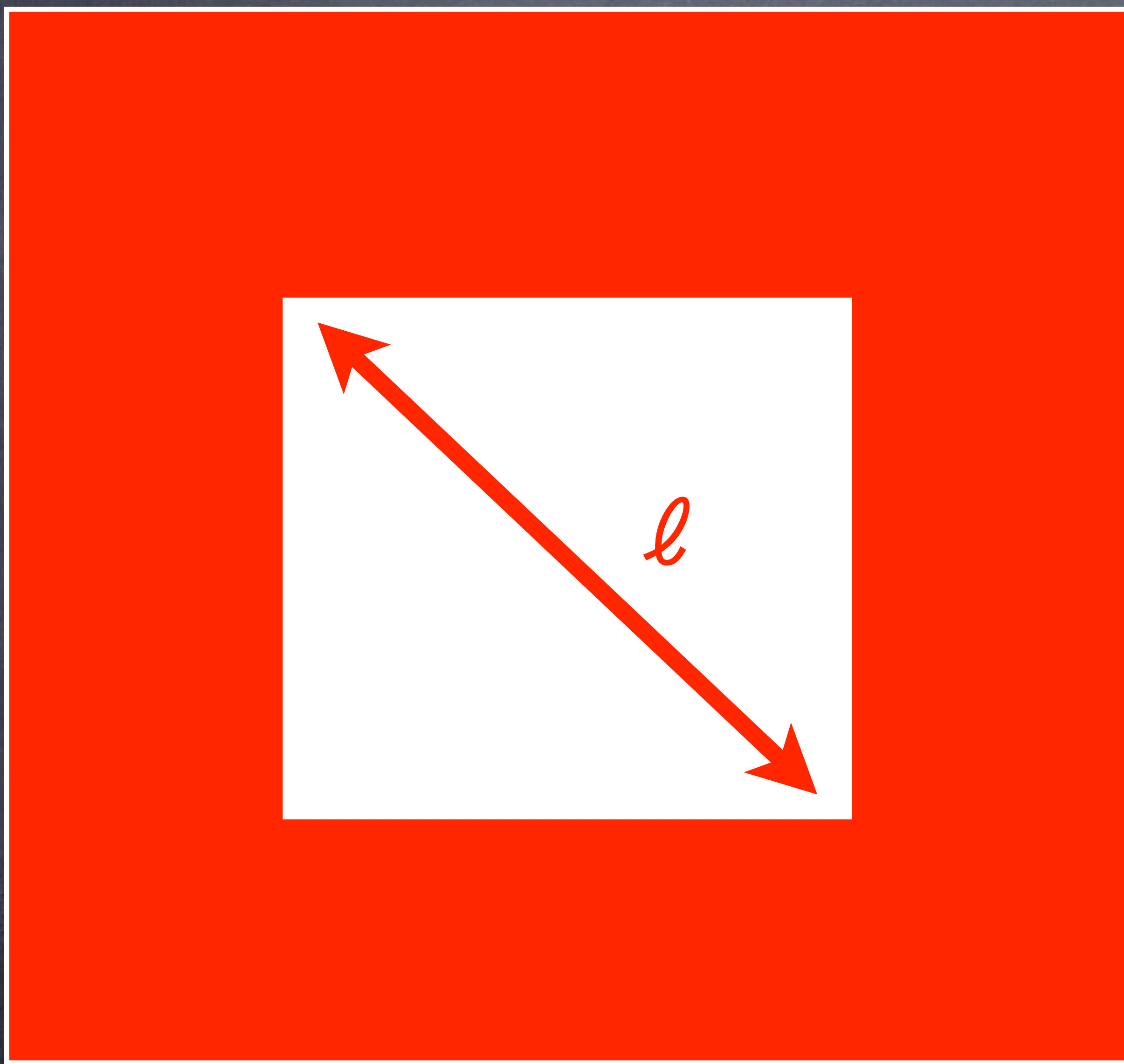
1) Consider the initial configuration

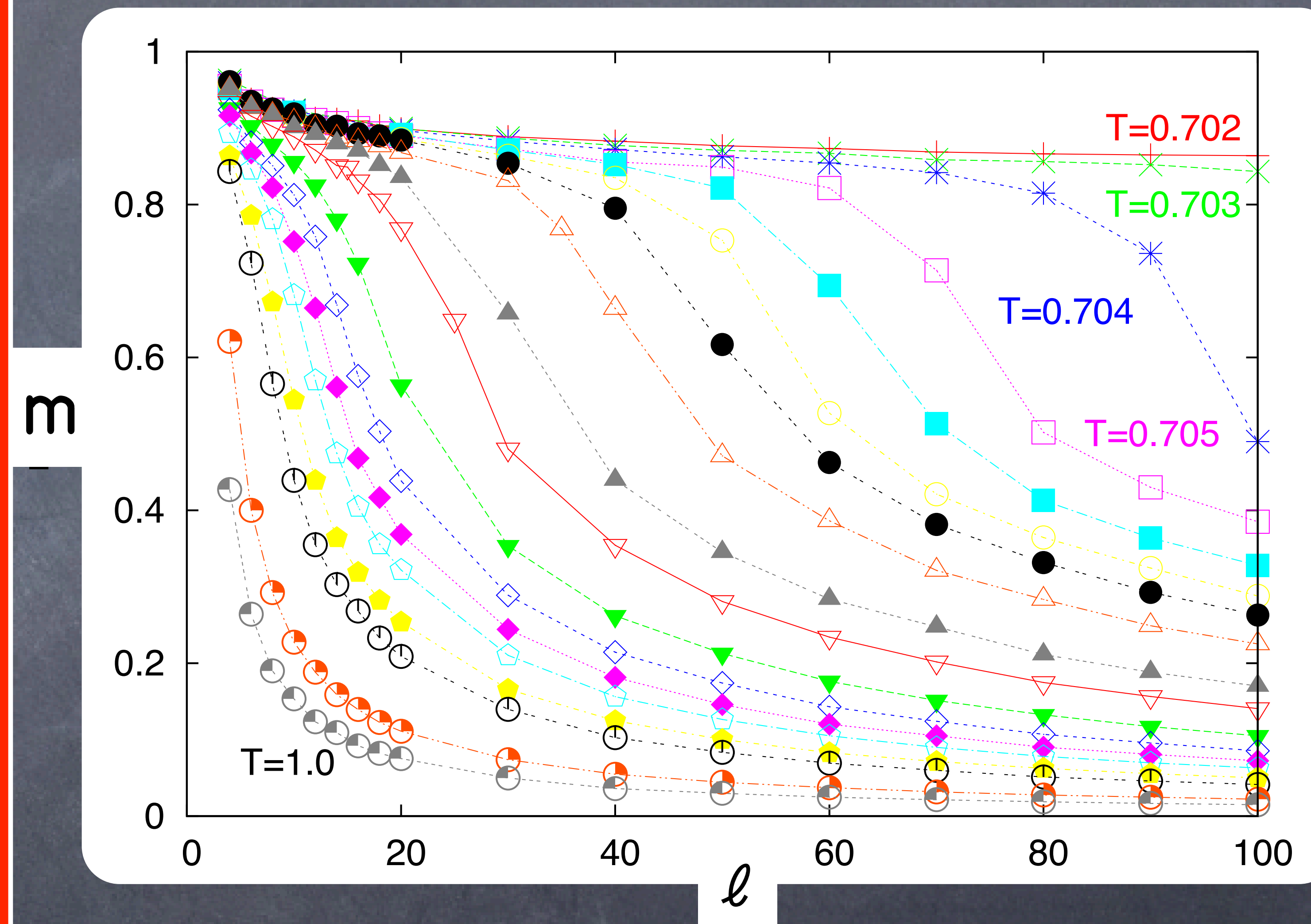
2) Freeze the system and make a hole of size ℓ

3) Un-freeze the system inside the cavity

Divergence of an equilibrium Correlation length !

Tc=0.70123





Growing and divergence of a (large) equilibrium correlation length....

Melting phenomenology...

- Plateau in the correlation function
- From Power-law (mean-field) to Vogel-Fulcher (finite dimension)
- Relation between static and dynamic $\tau \propto e^{\beta A/\Delta f^{d-1}}$
- Heterogeneous dynamics
- Divergence of a "static" length scale

Melting phenomenology...

- Plateau in the correlation function
- From Power-law (mean-field) to Vogel-Fulcher (finite dimension)
- Relation between static and dynamic $\tau \propto e^{\beta A/\Delta f^{d-1}}$
- Heterogeneous dynamics
- Divergence of a "static" length scale

... just like glass phenomenology!

Differences between melting dynamics and the equilibrium dynamics of glass formers

Melting		**glass forming liquids**
Out-of equilibrium process	→	Equilibrium dynamics
Happens only once	→	Equilibrium Stationary process
Free energy difference	→	Entropy difference
$\tau \propto e^{\beta A/\Delta F^{d-1}}$		$\tau \propto e^{\beta A/\Delta S}$
Latent heat in first order transition	→	No Latent heat at the glass transition

TWO

Glassy dynamics can be sometime mapped exactly to a melting problem...

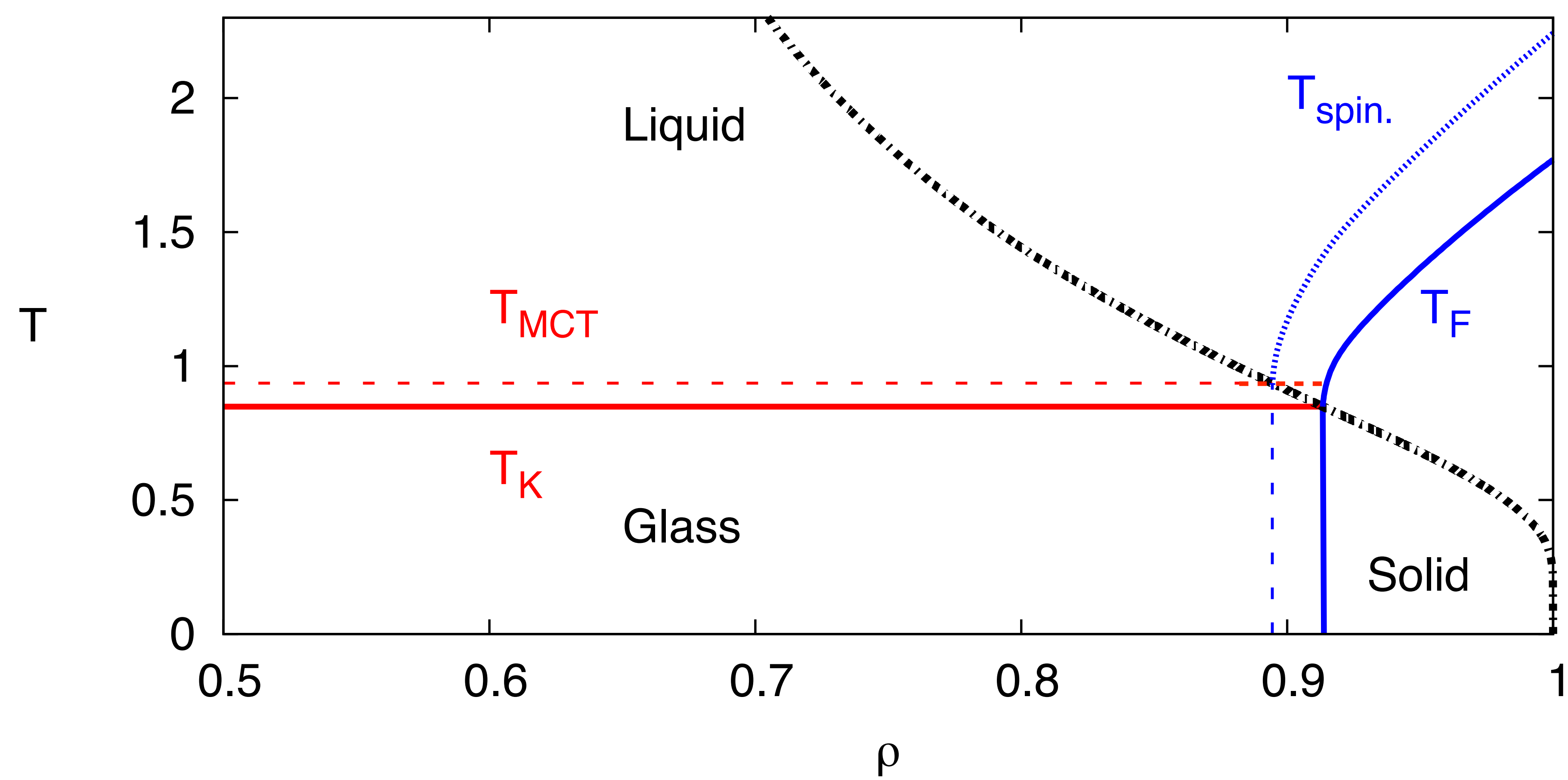
The mean-field p-spin model

$$\mathcal{H} = -\sum_{ijk} J_{ijk} S_i S_j S_k$$

Multi-spin interaction models

J=1 with prob. ρ and J=-1 with prob. (1-ρ)

Ex: Bethe lattice, c=5



Starting point of the Random-First-Order Theory

The mean-field p-spin model

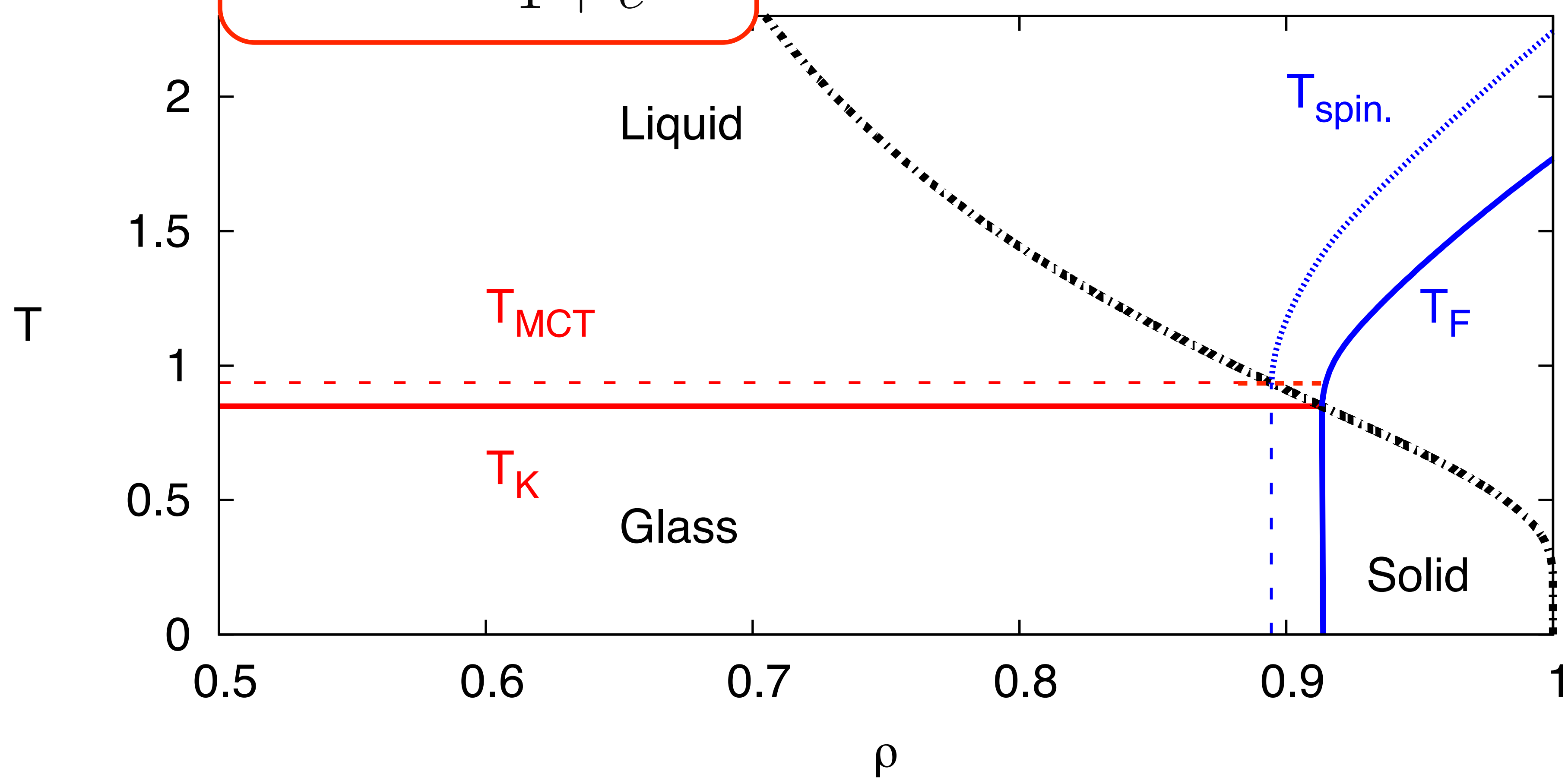
$$\mathcal{H} = -\sum_{ijk} J_{ijk} S_i S_j S_k$$

Multi-spin interaction models

J=1 with prob. ρ and J=-1 with prob. (1-ρ)

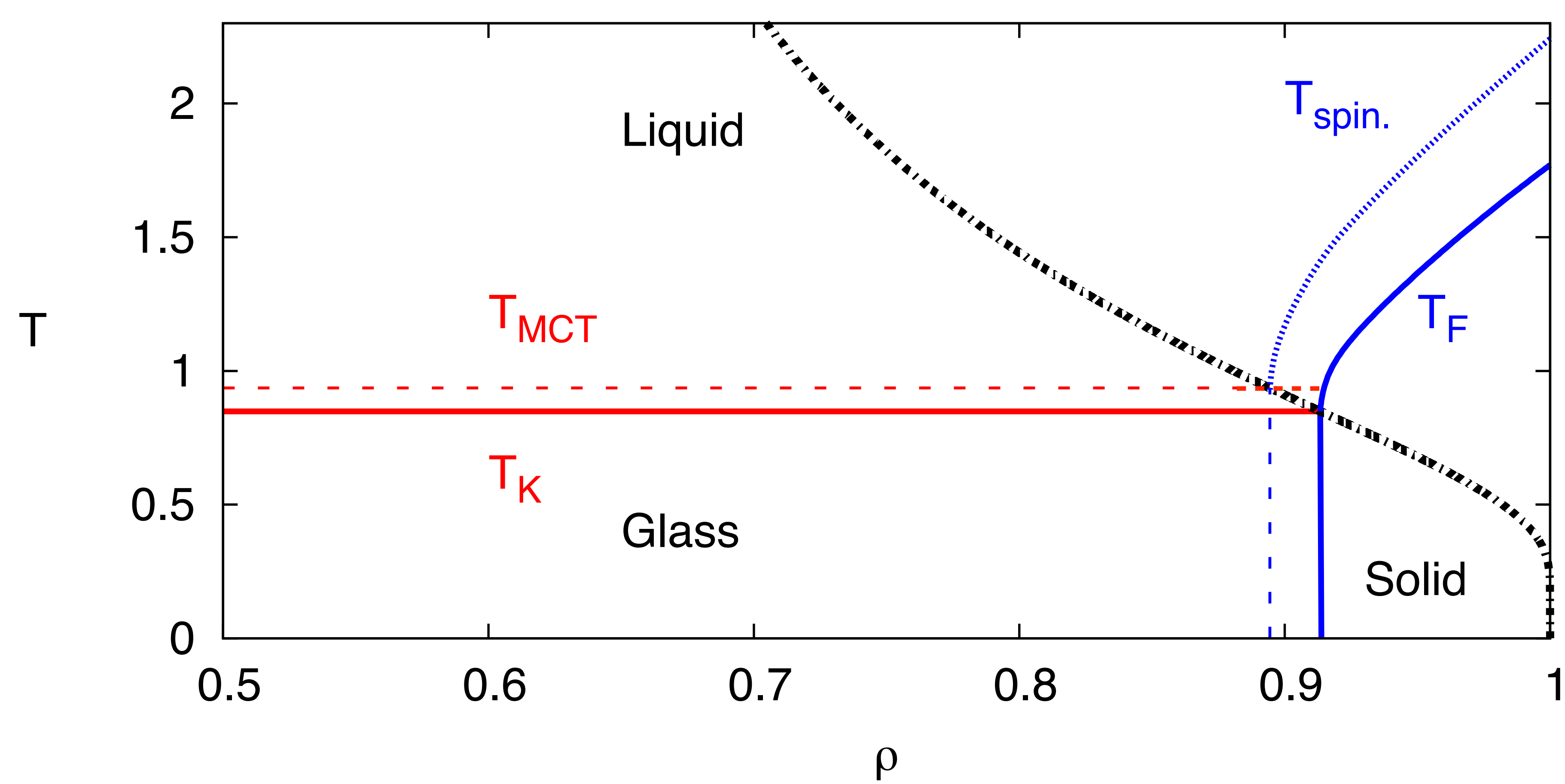
Ex: Bethe lattice, c=5

$$\rho_{NL}(\beta) = \frac{1}{1+e^{-2\beta}}$$

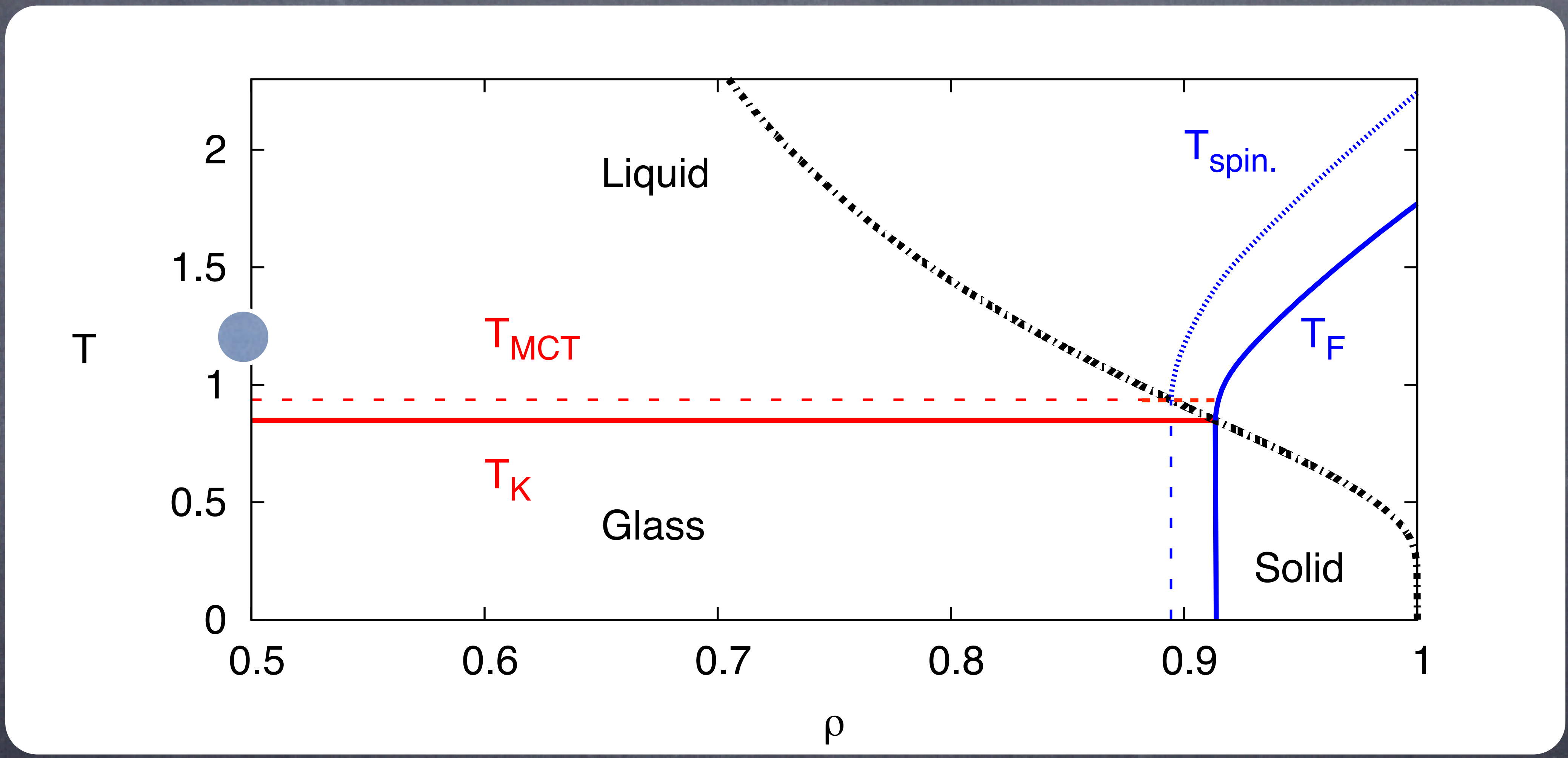


On the **Nishimori line**, a gauge symmetry allows to compute many quantities and to derive many identities

A new (and powerful) result on the NL

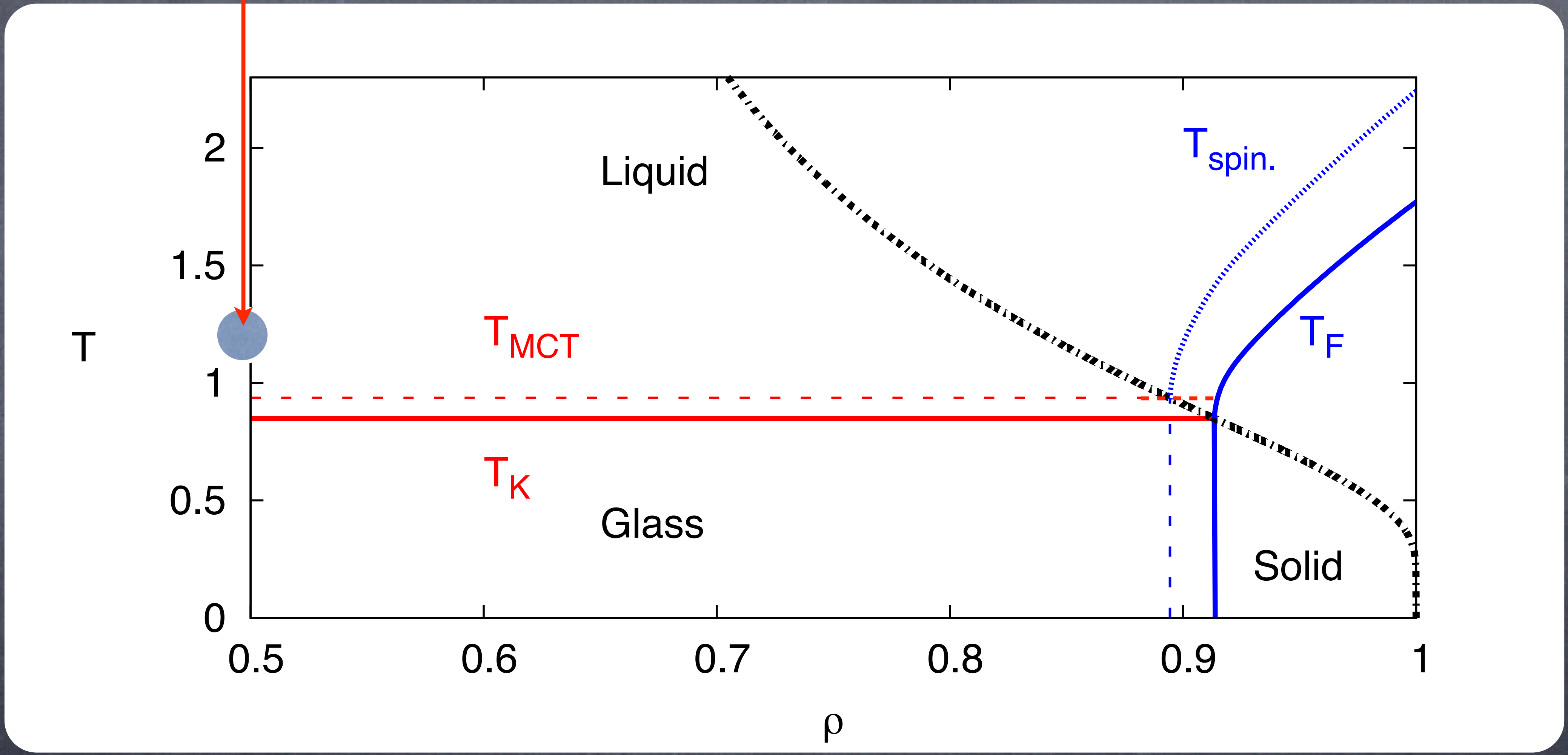


A new (and powerful) result on the NL



A new (and powerful) result on the NL

$C^{SG}_{eq}(t)$

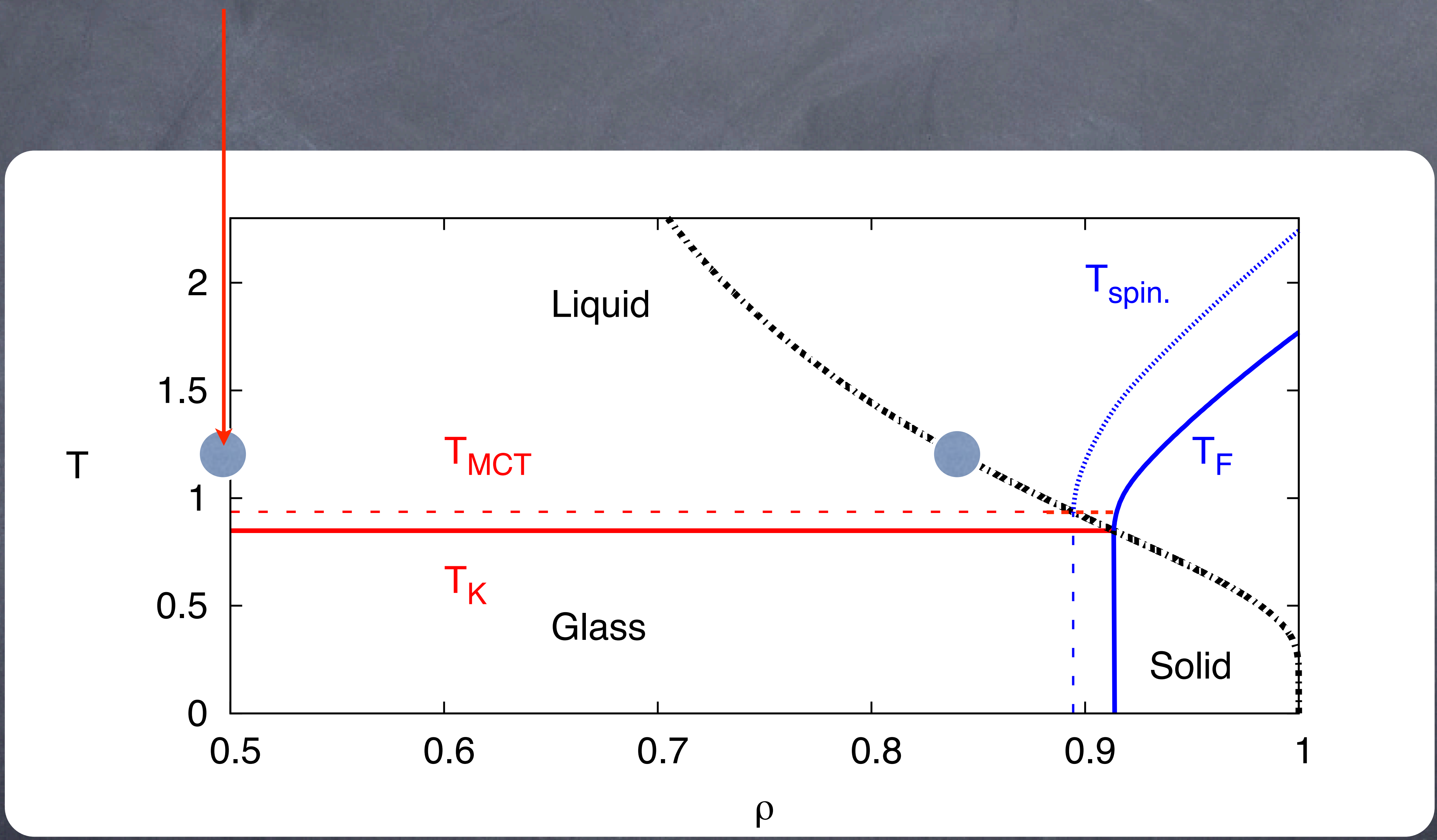


Equilibrium correlation function

$$C_{eq}(t) = \lim_{t_w \to \infty} \frac{1}{N} \sum_i S_i(t_w) S_i(t_w + t)$$

A new (and powerful) result on the NL

$C_{eq}^{SG}(t)$

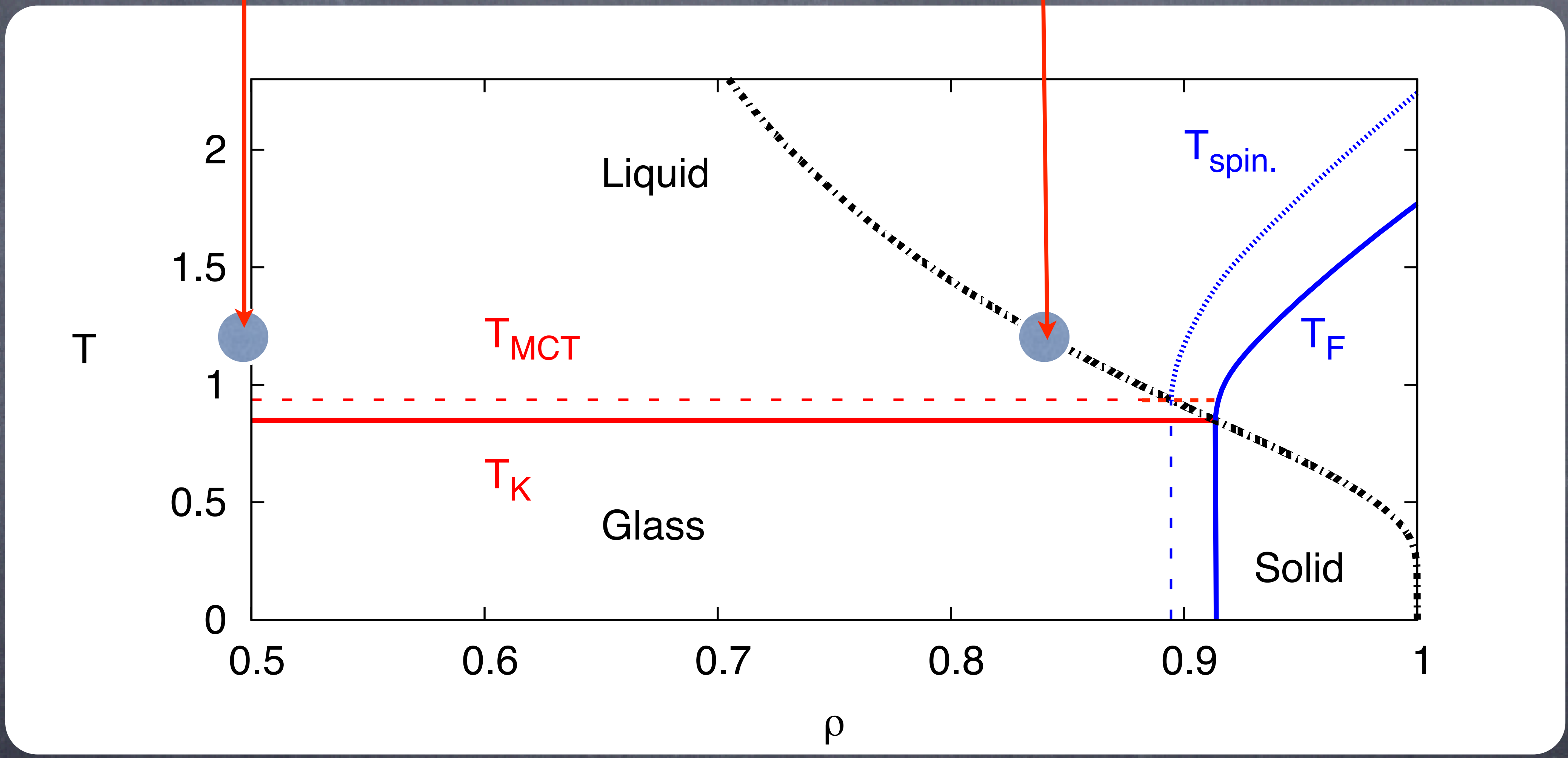


Equilibrium correlation function

$$C_{eq}(t) = \lim_{t_w \to \infty} \frac{1}{N} \sum_i S_i(t_w) S_i(t_w + t)$$

A new (and powerful) result on the NL

$C_{eq}^{SG}(t)$ $m^{NL}(t)$



Equilibrium correlation function

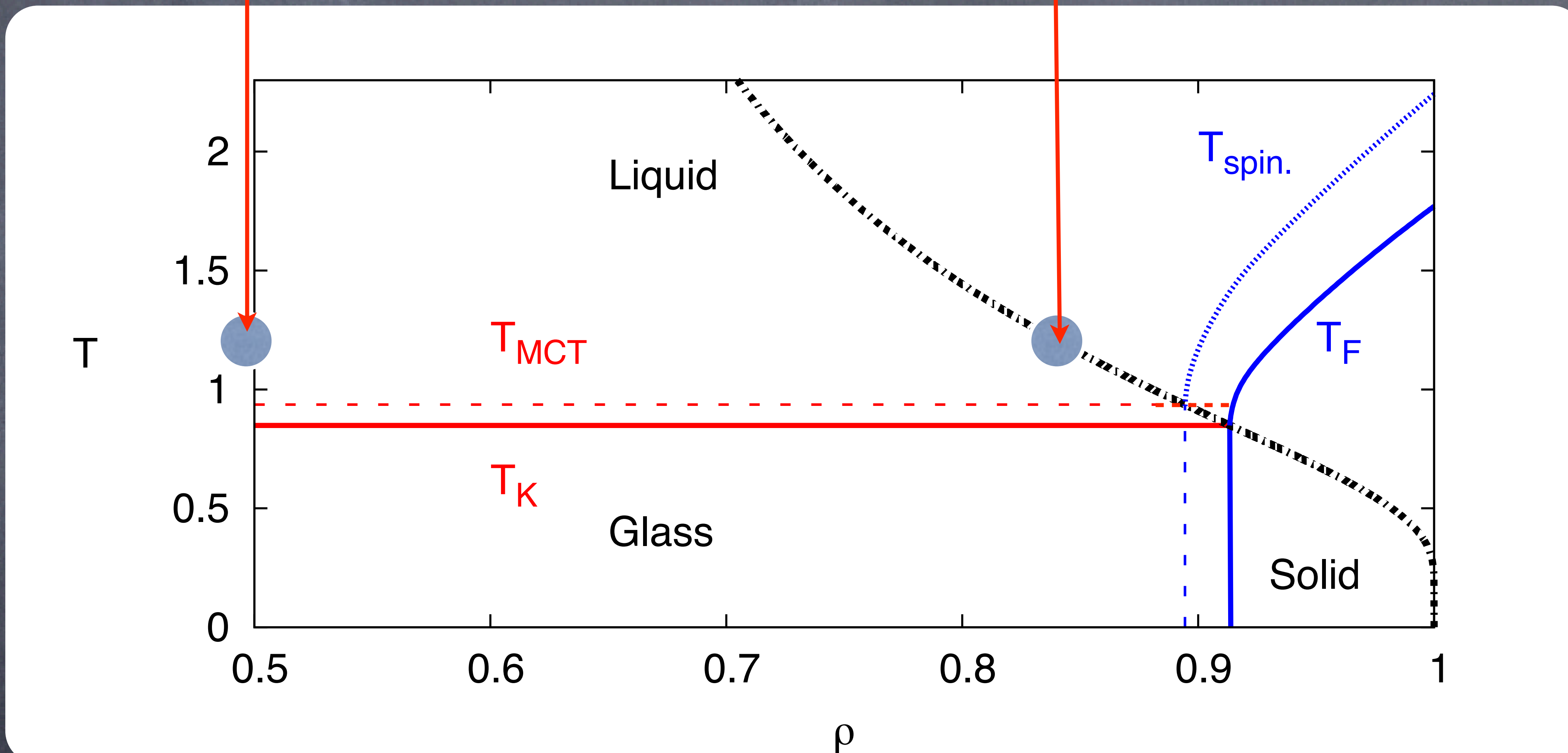
$$C_{eq}(t) = \lim_{t_w \to \infty} \frac{1}{N} \sum_i S_i(t_w) S_i(t_w + t)$$

Magnetization starting from the fully ordered state

$$m(t) = \frac{1}{N} \sum_i S_i(t), \quad \text{with } m(0) = 1$$

A new (and powerful) result on the NL

$$C_{eq}^{SG}(t) = m^{NL}(t)$$



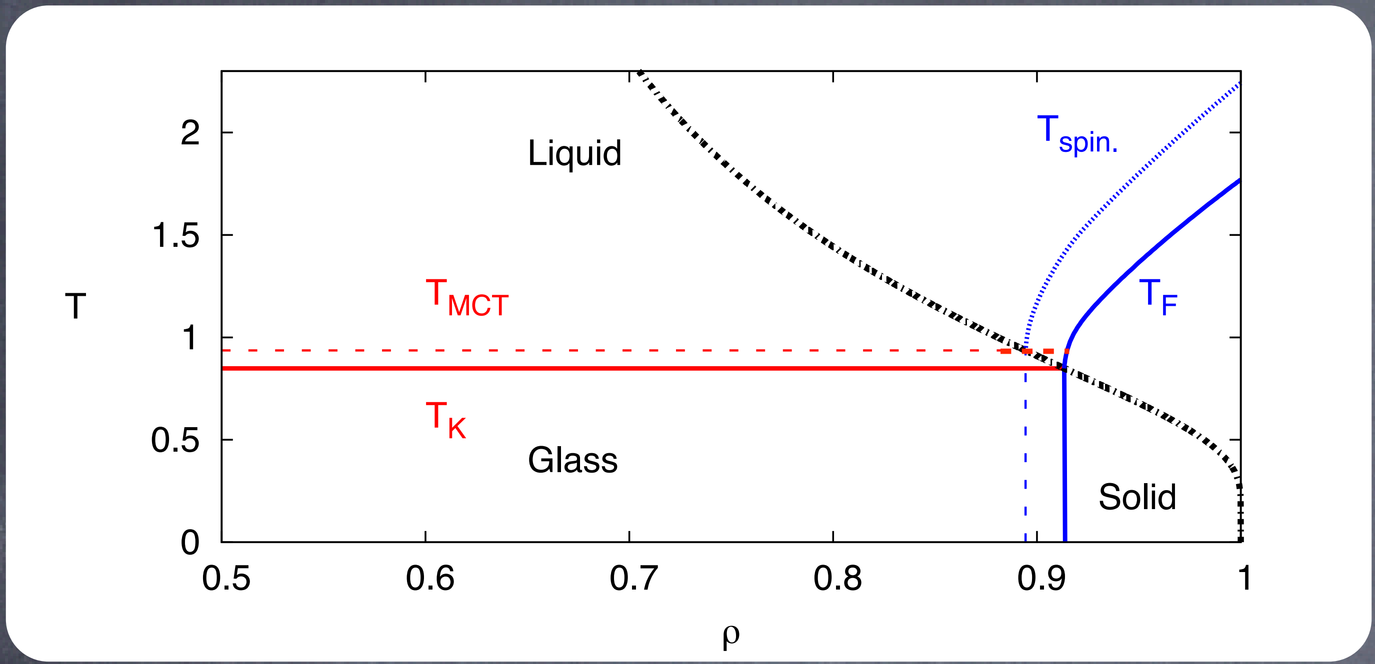
Equilibrium correlation function

$$C_{eq}(t) = \lim_{t_w \to \infty} \frac{1}{N} \sum_i S_i(t_w) S_i(t_w + t)$$

Magnetization starting from the fully ordered state

$$m(t) = \frac{1}{N} \sum_i S_i(t), \quad \text{with } m(0) = 1$$

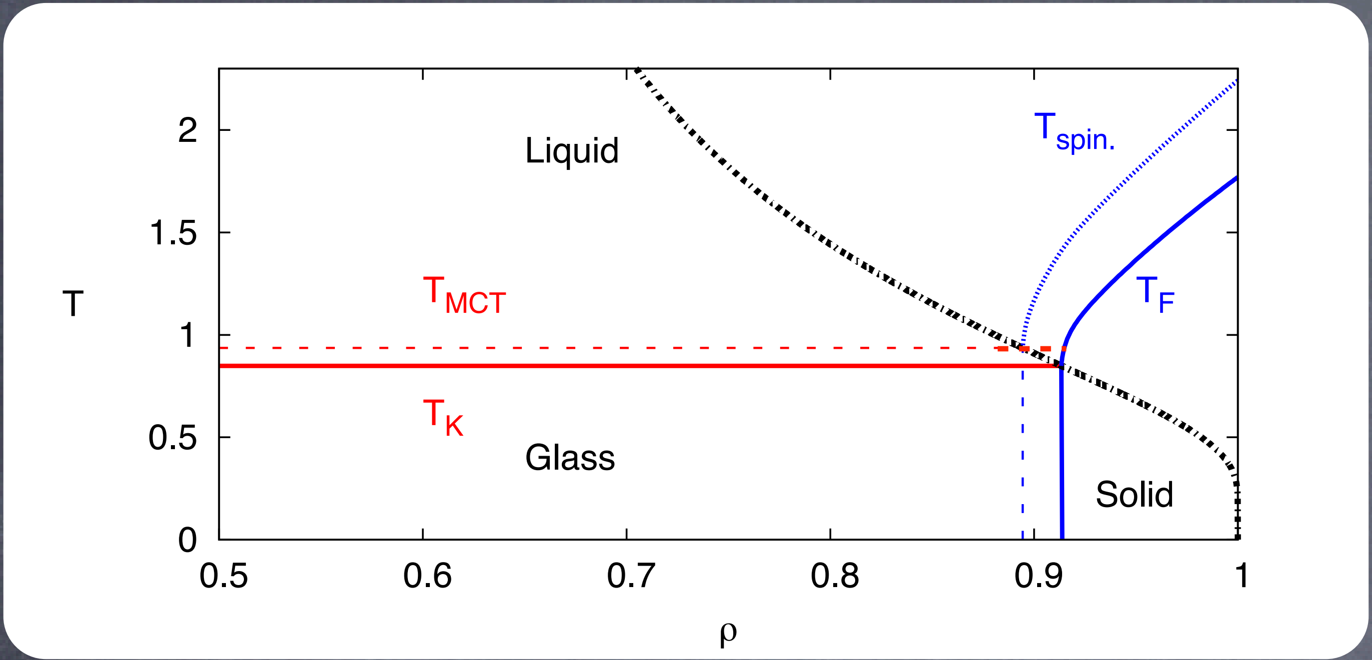
Melting=equilibrium glassy dynamics



The equilibrium time correlation is equal to the melting correlation

$$c_{\text{eq}}(t) = m_{\text{melting}}(t)$$

Melting=equilibrium glassy dynamics



The mean-field glass transition
is rigorously equivalent
to a melting problem !

The equilibrium relaxation time is equal to the melting relaxation time

$$\tau_{\text{eq}}(\beta) = \tau_{\text{melting}}(\beta)$$

The equilibrium time correlation is equal to the melting correlation

$$c_{\text{eq}}(t) = m_{\text{melting}}(t)$$

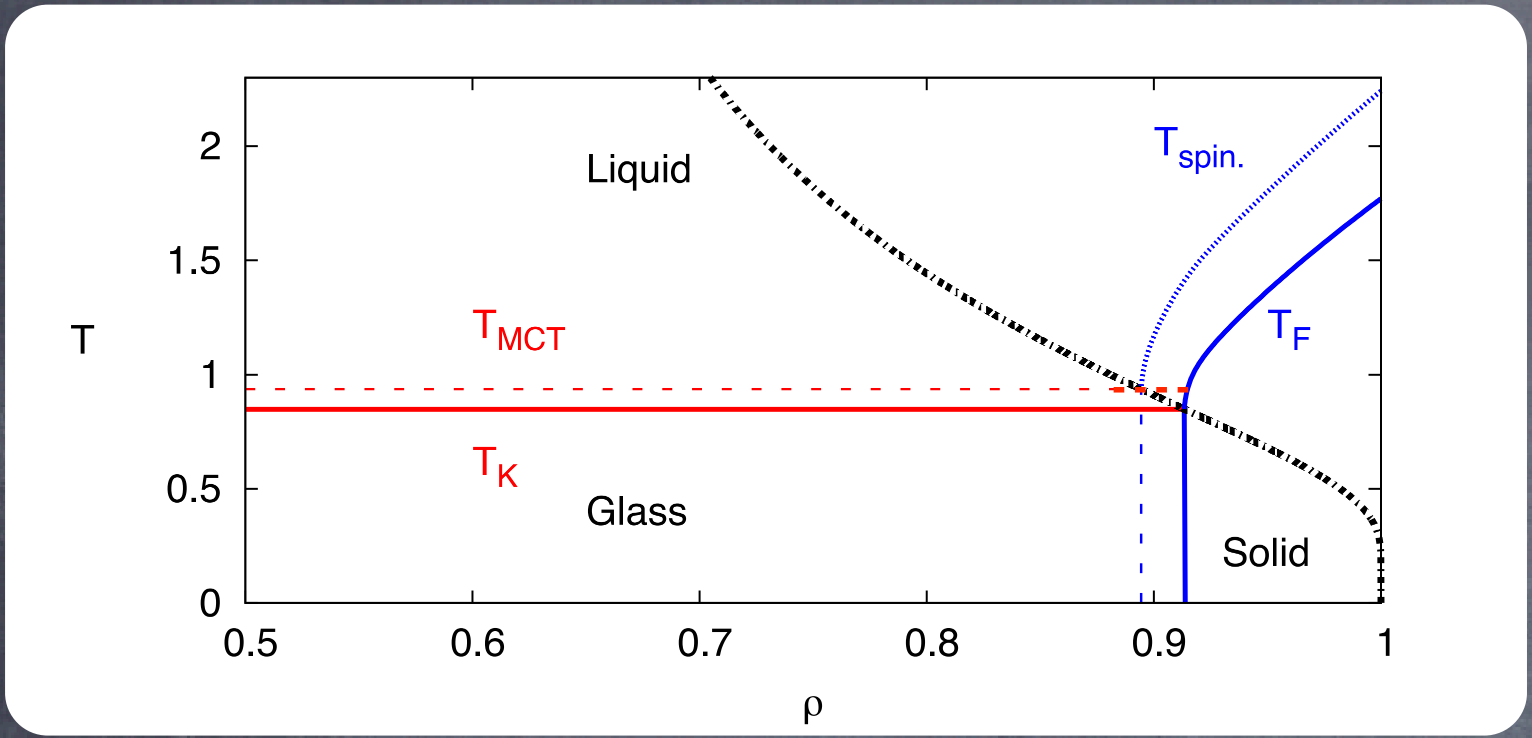
The static (point-to-set) and dynamic (heterogeneities) length scales in the are equal to the melting ones

$$\chi_4^{\text{eq}}(t) = \chi_F^{\text{melting}}(t) \qquad \ell^{\text{PTS}}(\beta) = \ell^{\text{FERRO}}(\beta)$$

The mode coupling transition-point is equivalent to the spinodal point!

Mean field model on the Nishimori line

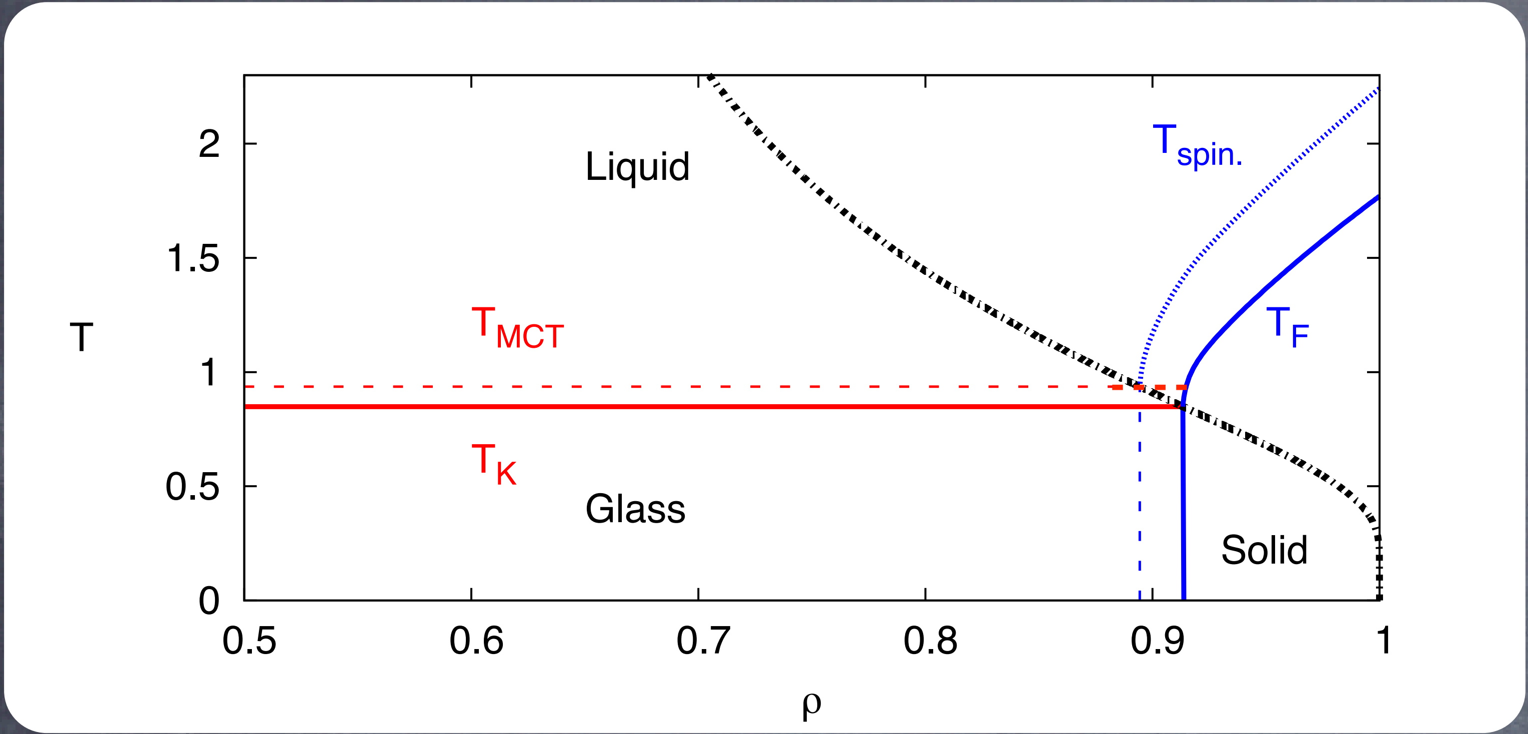
Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

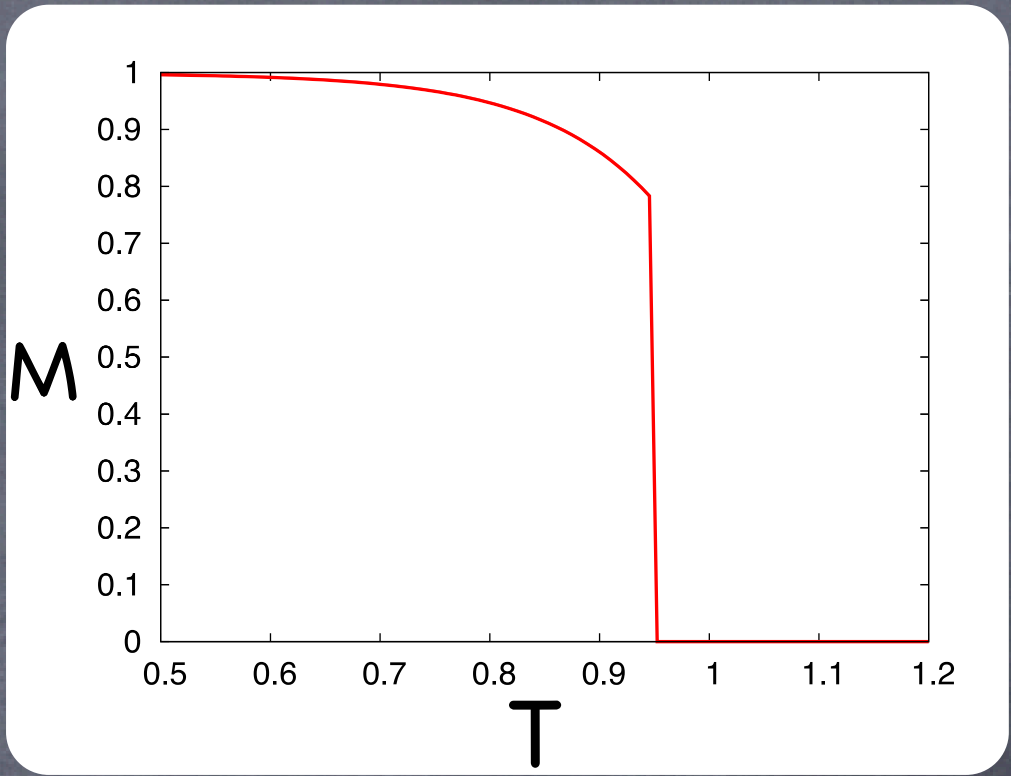


- First order ferromagnetic transition (jump in the magnetization)

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

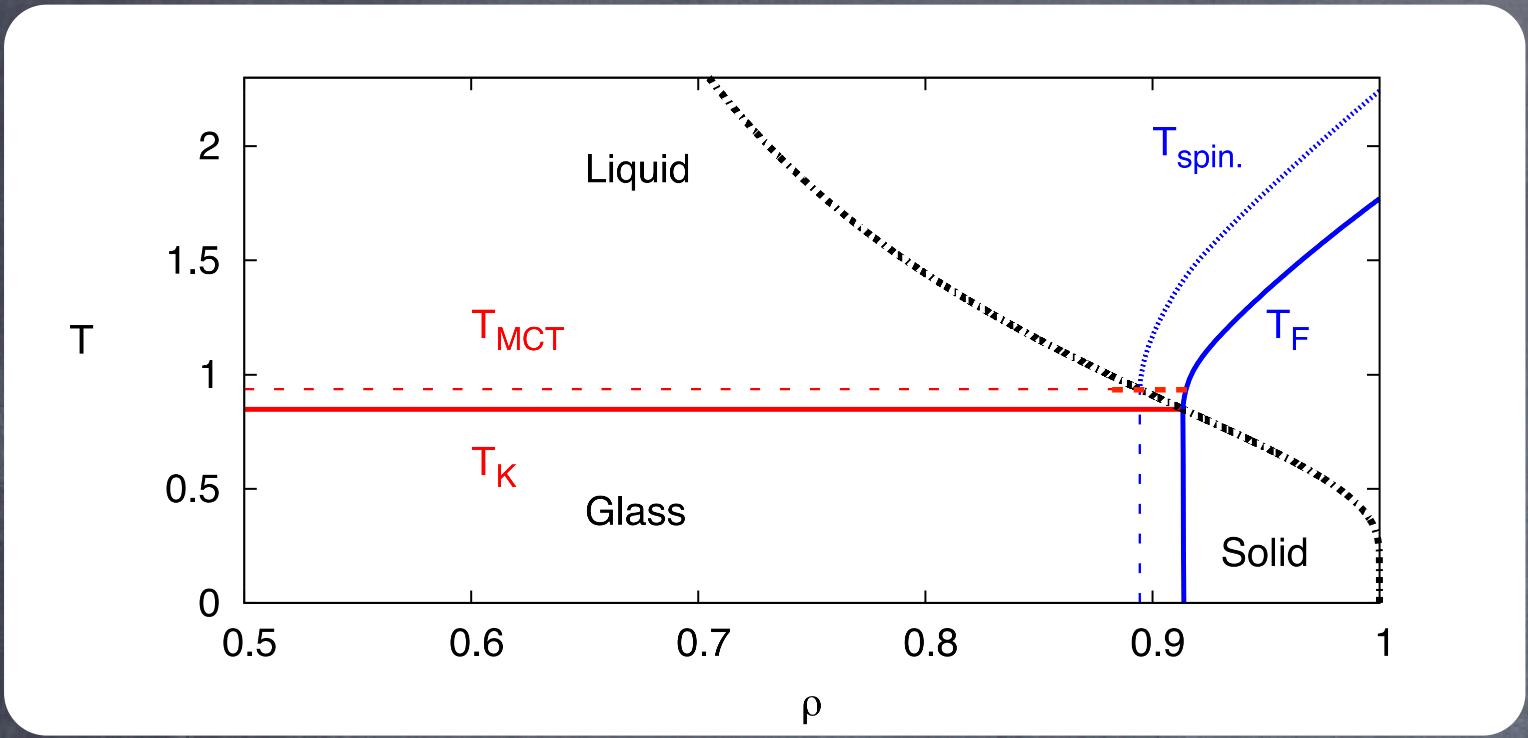


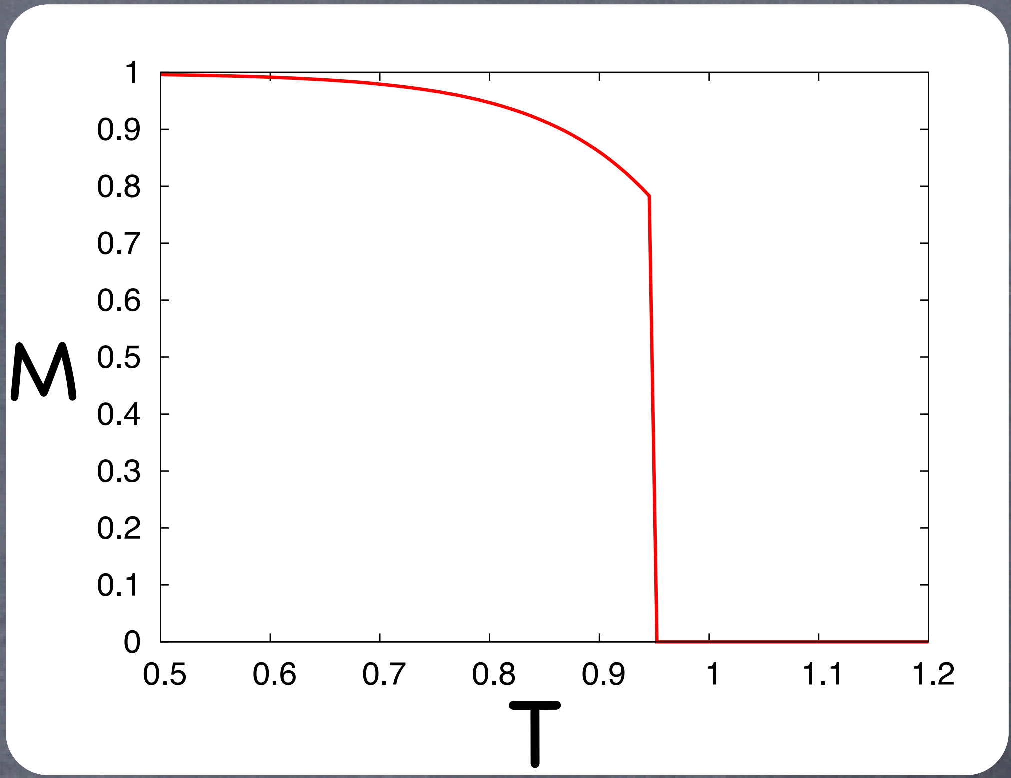


- First order ferromagnetic transition (jump in the magnetization)

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

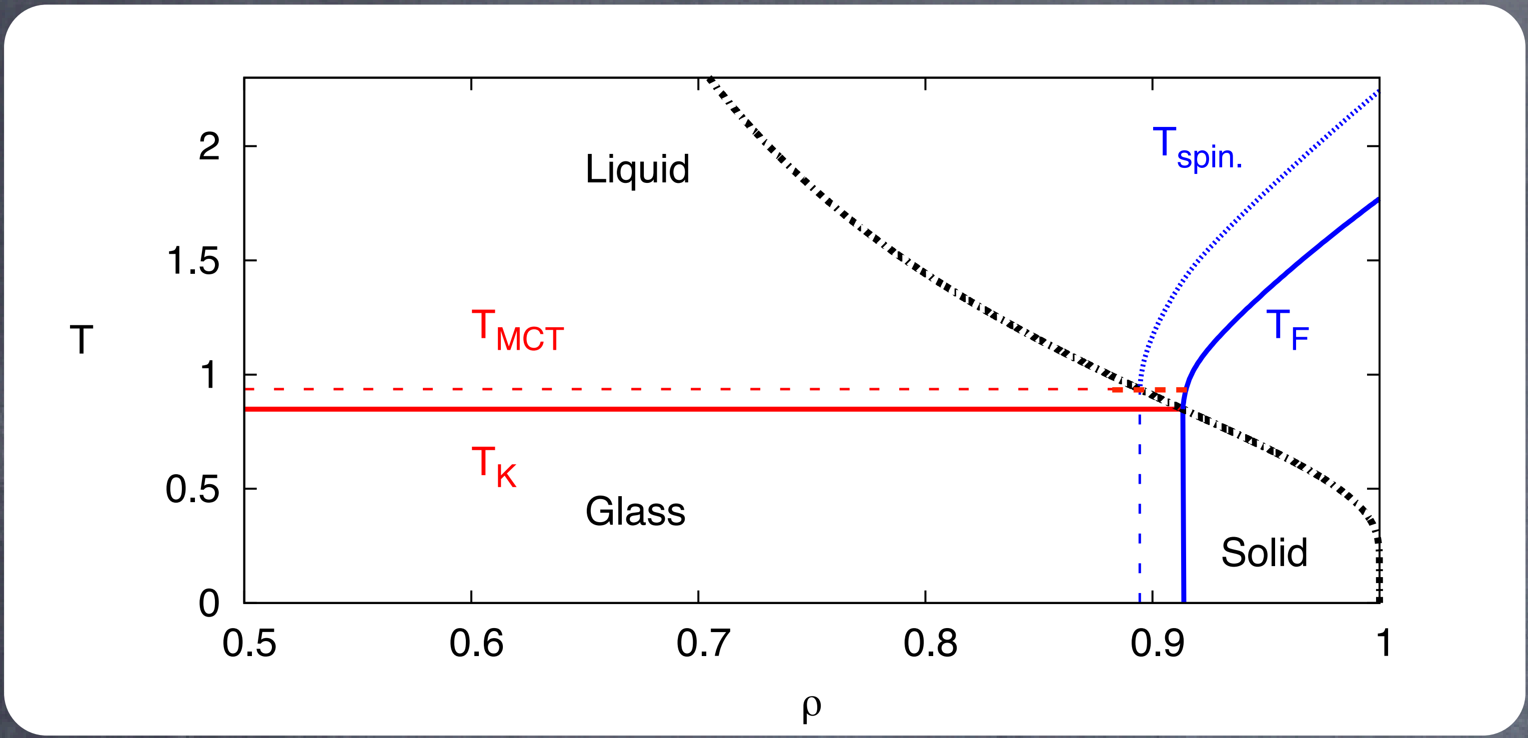


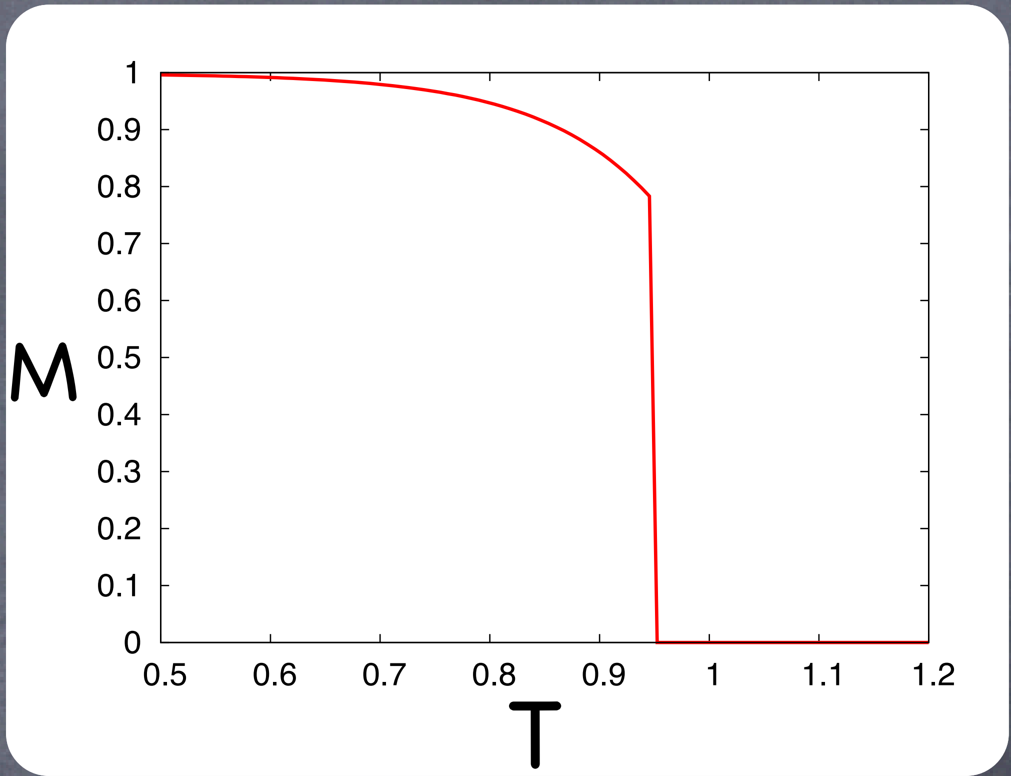


- First order ferromagnetic transition (jump in the magnetization)
- The energy is continuous and analytic at the transition

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

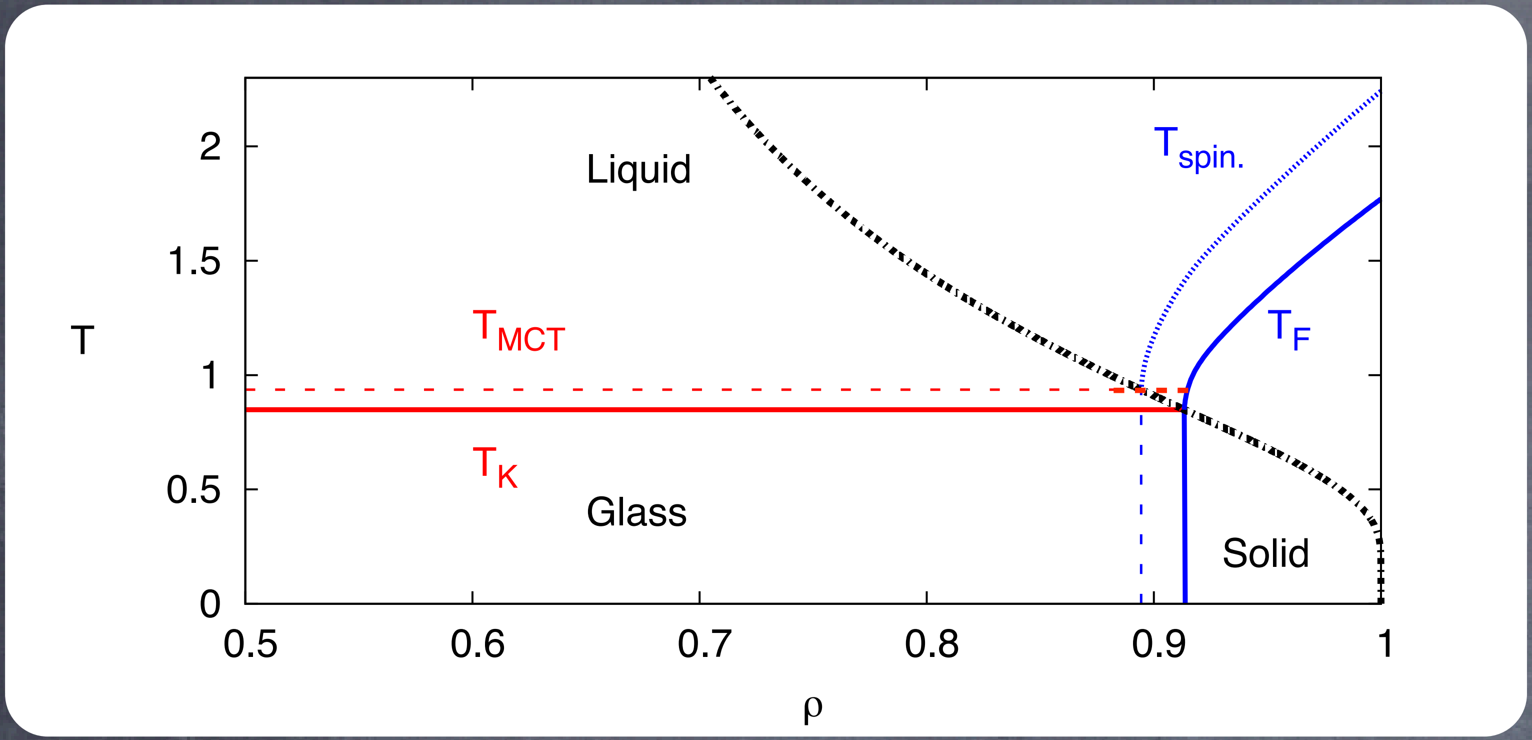


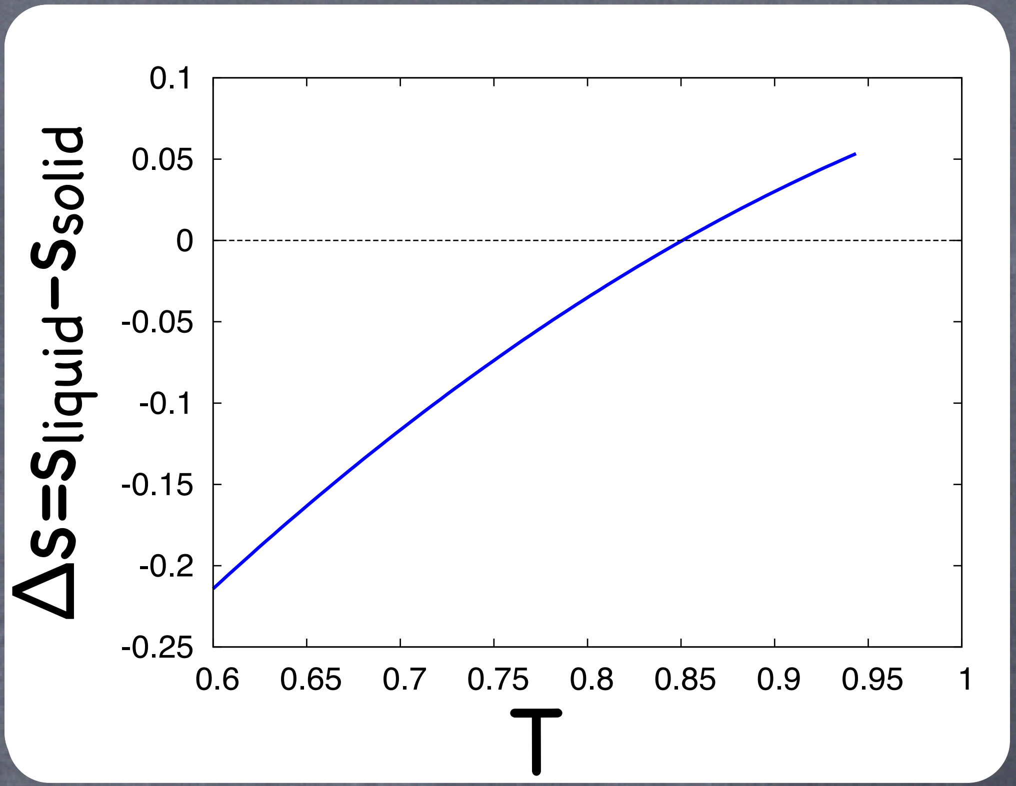


- First order ferromagnetic transition (jump in the magnetization)
- The energy is continuous and analytic at the transition
- "Entropy driven" transition: $\Delta s = s_{liquid} - s_{solid} \rightarrow 0$ at the transition

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

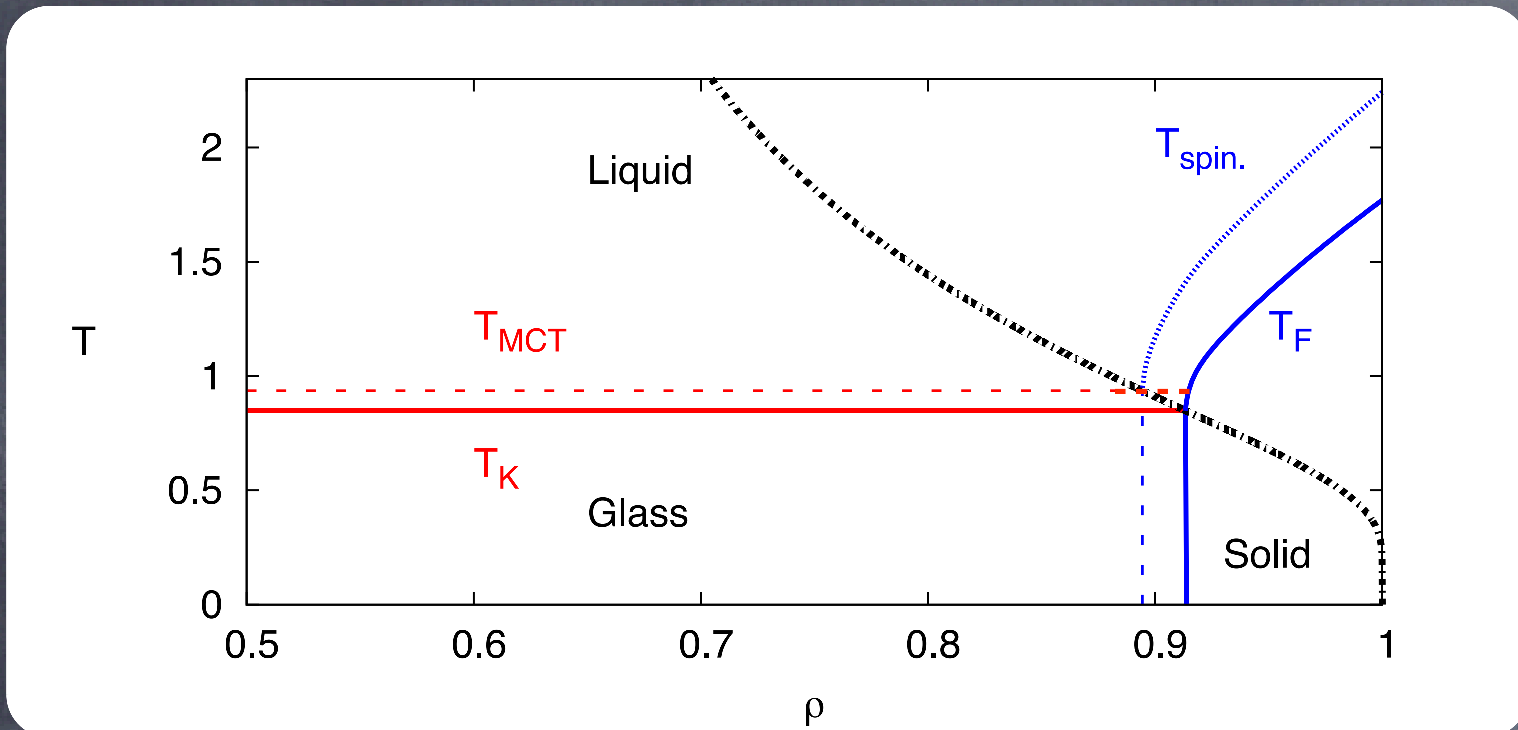


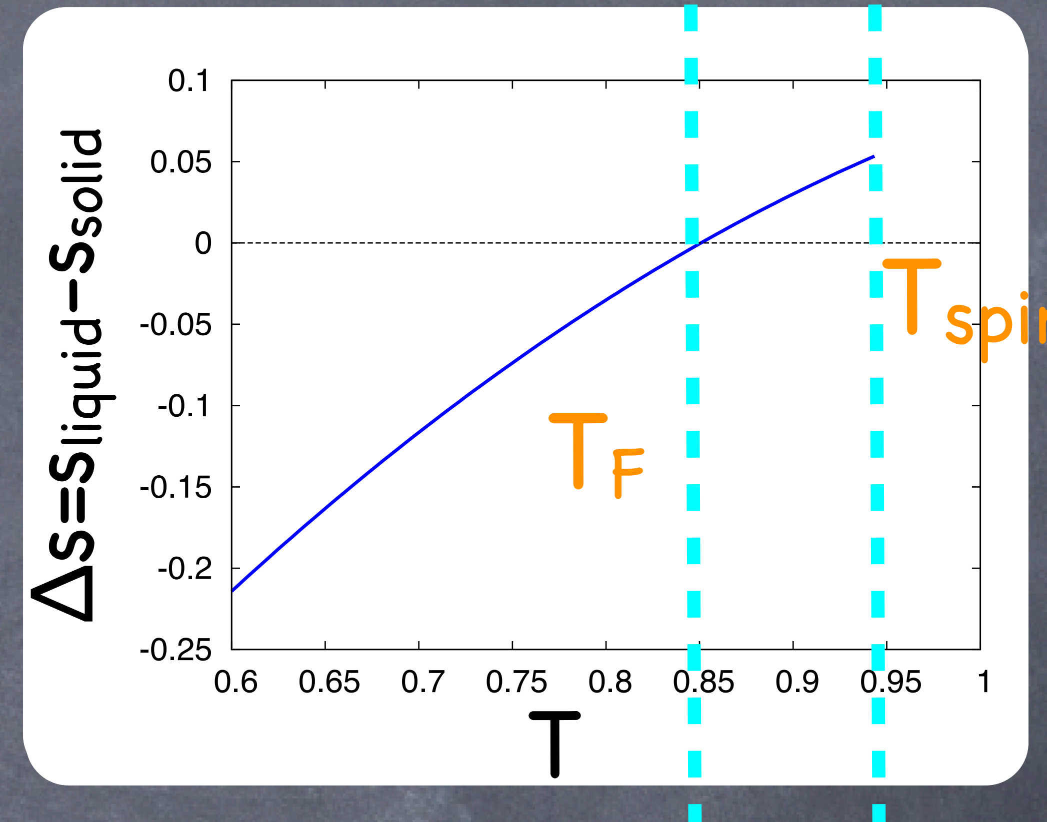


- First order ferromagnetic transition (jump in the magnetization)
- The energy is continuous and analytic at the transition
- "Entropy driven" transition: $\Delta s = s_{liquid} - s_{solid} \rightarrow 0$ at the transition

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

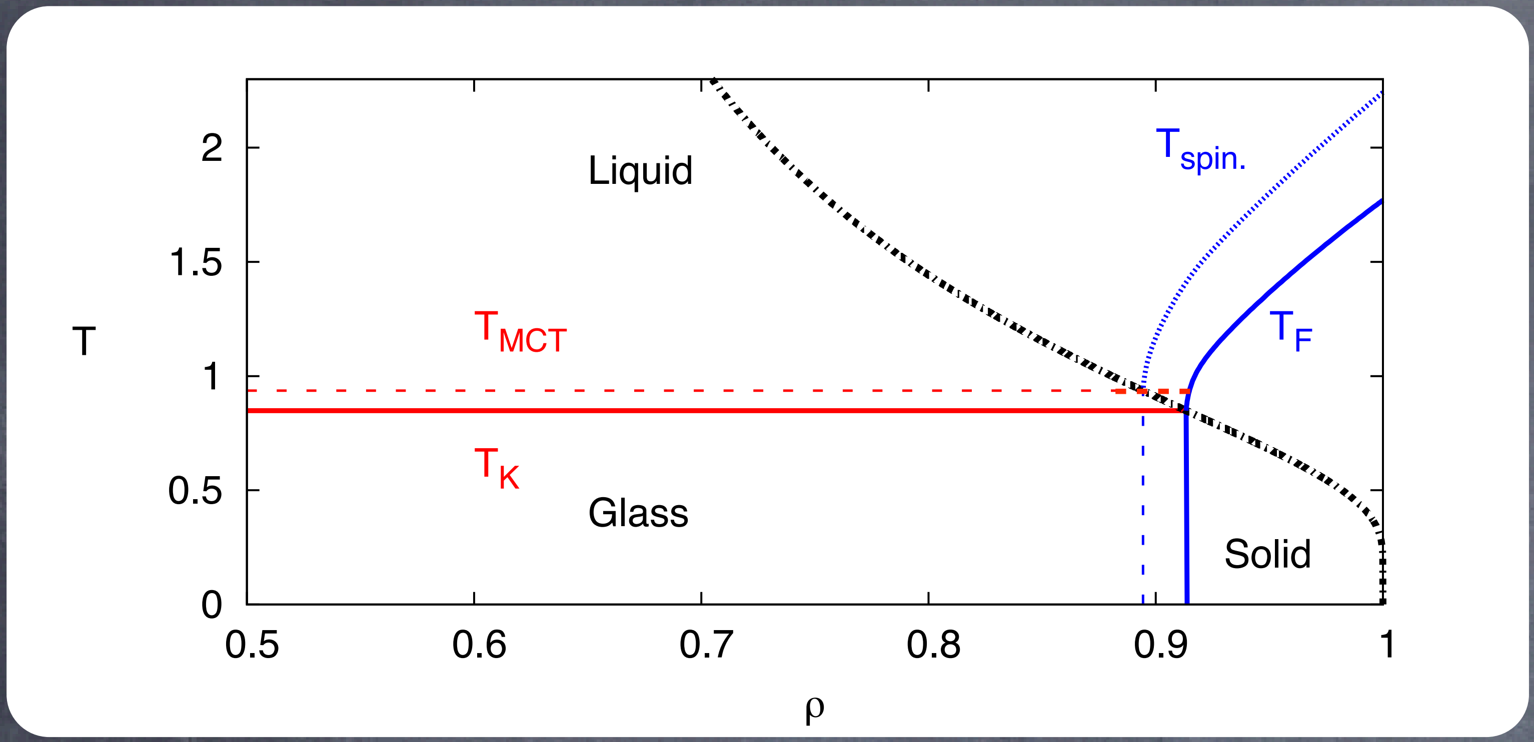


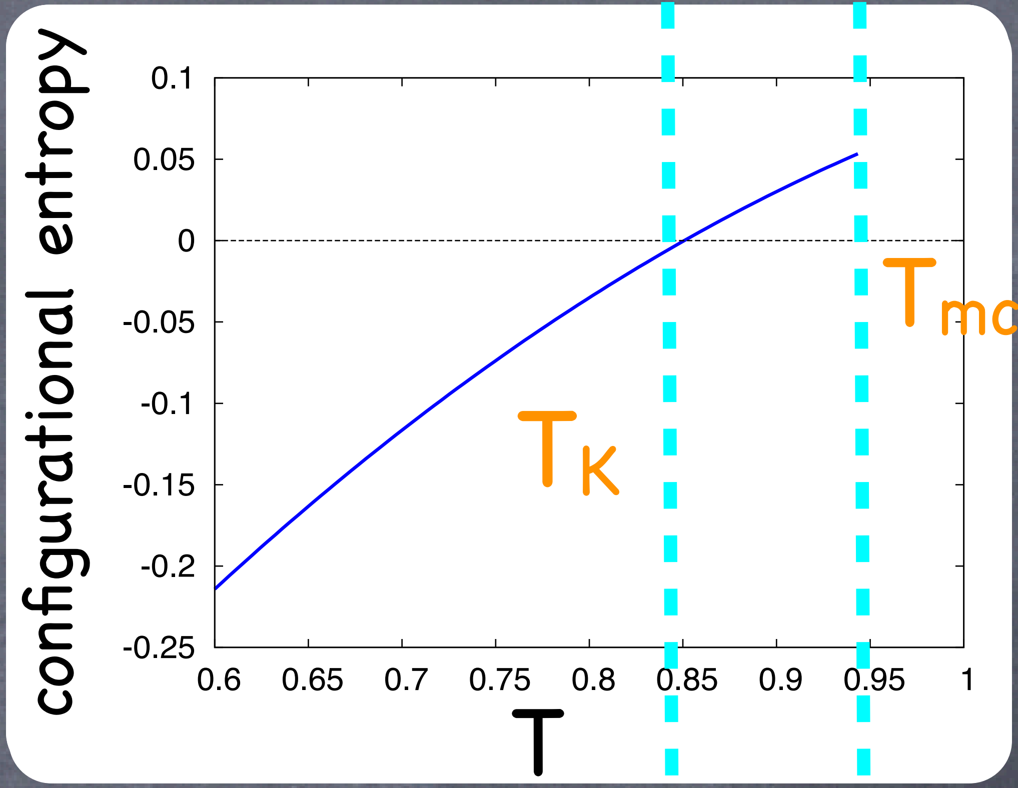


- First order ferromagnetic transition (jump in the magnetization)
- The energy is continuous and analytic at the transition
- "Entropy driven" transition: $\Delta s = s_{liquid} - s_{solid} \rightarrow 0$ at the transition

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method





- First order ferromagnetic transition (jump in the magnetization)
- The energy is continuous and analytic at the transition
- "Entropy driven" transition: $\Delta s = s_{liquid} - s_{solid} \rightarrow 0$ at the transition

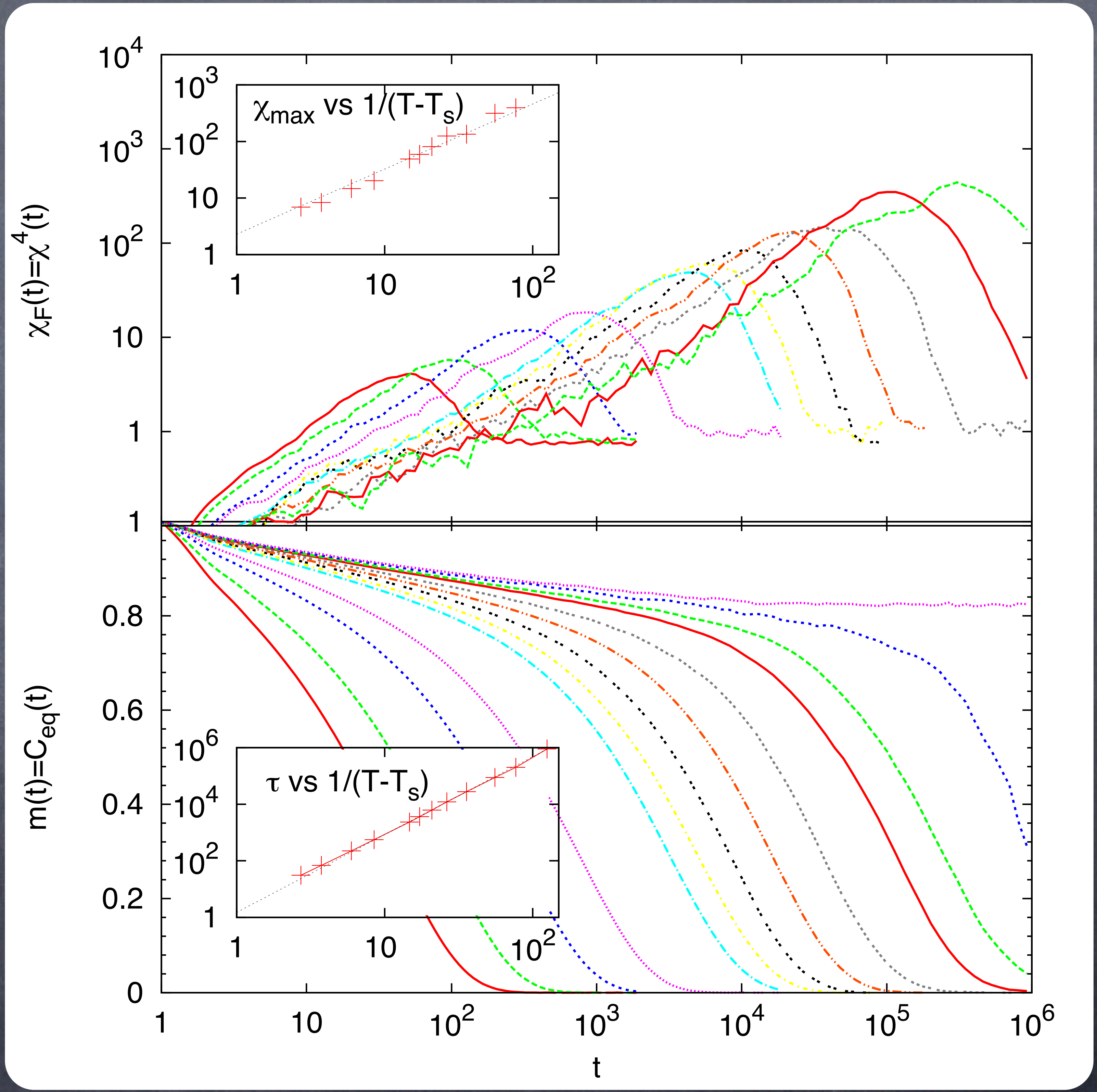
- Equilibrium dynamics along the line has a mean-glass transition (described by a mode-coupling phenomenology)
- The configurational entropy is given by $\Sigma = \Delta s = s_{liquid} - s_{solid}$

Mean field model on the Nishimori line

Bethe lattice (Regular Random graph, c=5), Solvable with the Cavity/Replica method

Dynamical heterogeneities

Correlation functions



Melting & Glassy dynamics

Melting & Glassy dynamics

- In mean field spin glasses, equilibrium glassy dynamics can be mapped to a particular melting phenomenon.

Melting & Glassy dynamics

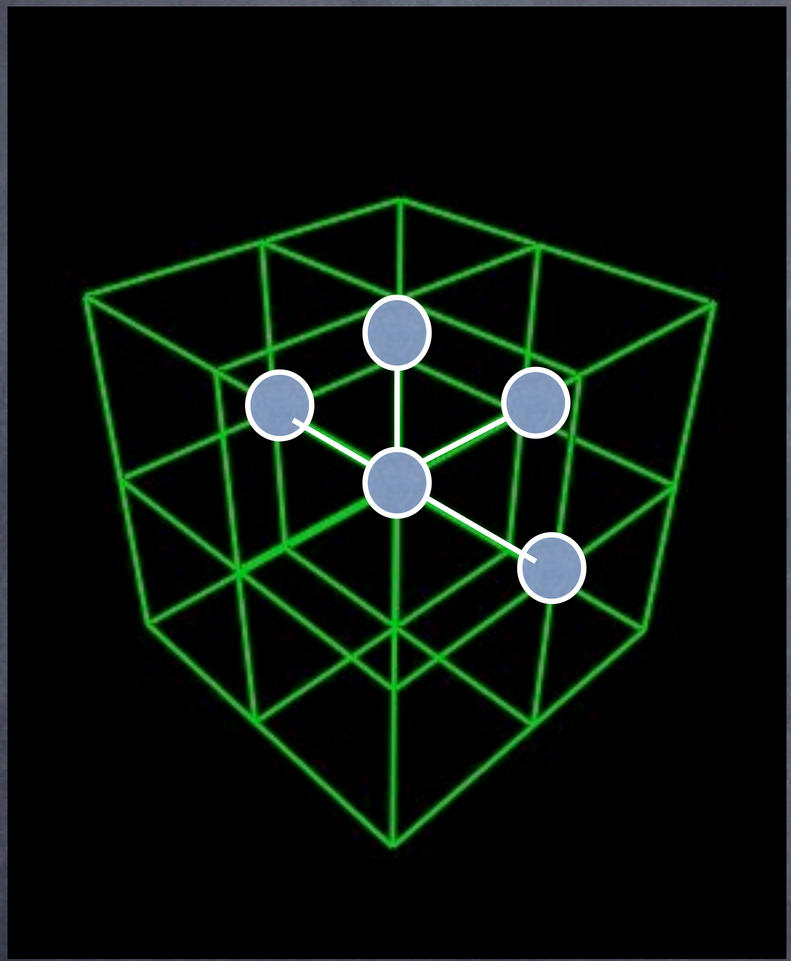
- In mean field spin glasses, equilibrium glassy dynamics can be mapped to a particular melting phenomenon.
- The Random First-Order Theory is mappable to a melting problem driven by entropy only

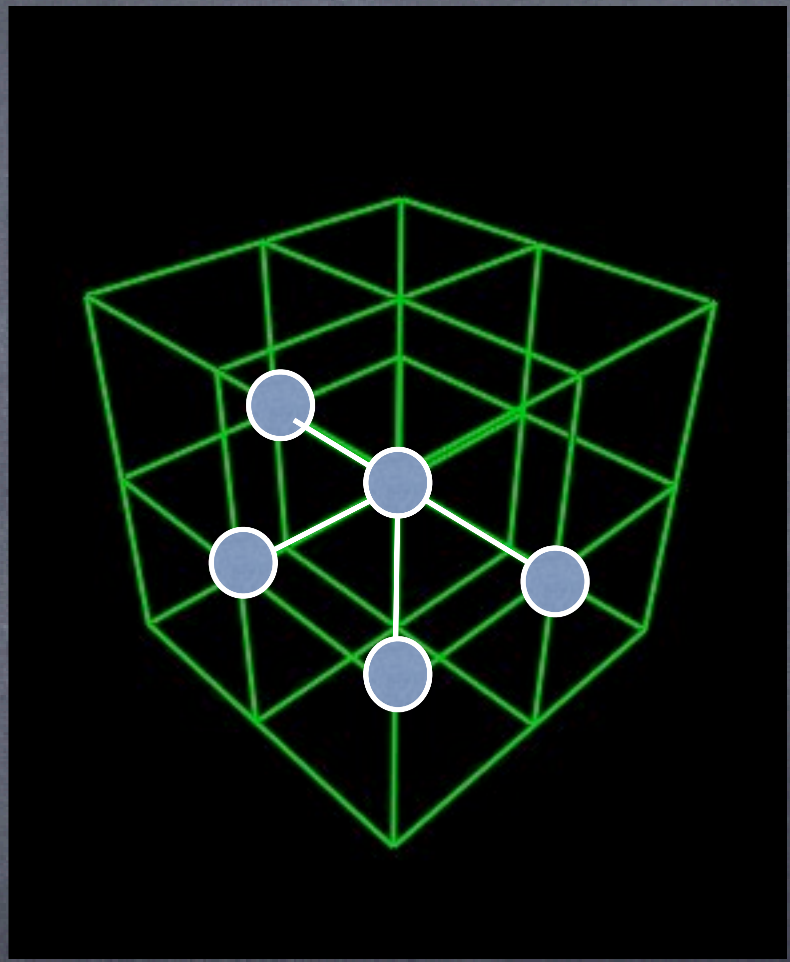
Melting & Glassy dynamics

- In mean field spin glasses, equilibrium glassy dynamics can be mapped to a particular melting phenomenon.
- The Random First-Order Theory is mappable to a melting problem driven by entropy only
- Such mapping are not limited to mean field systems and similar results can be obtained in some 3-dimensional spin models.

A 3D p-spin model on the Nishimori line

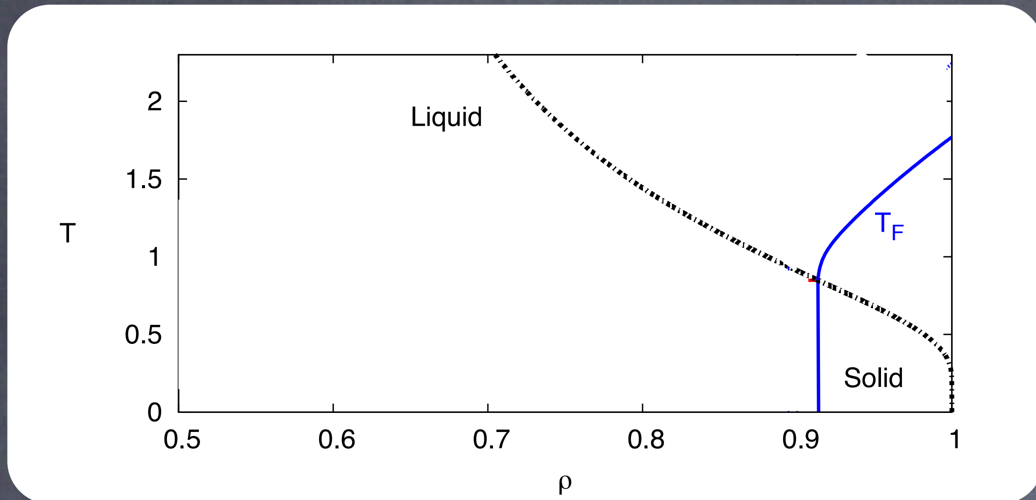
$$\mathcal{H} = -\sum_i J_i^a S_i S_{UP} S_{LEFT} S_{RIGHT} S_{BEHIND} + J_i^b S_i S_{BOTTOM} S_{LEFT} S_{RIGHT} S_{FRONT}$$



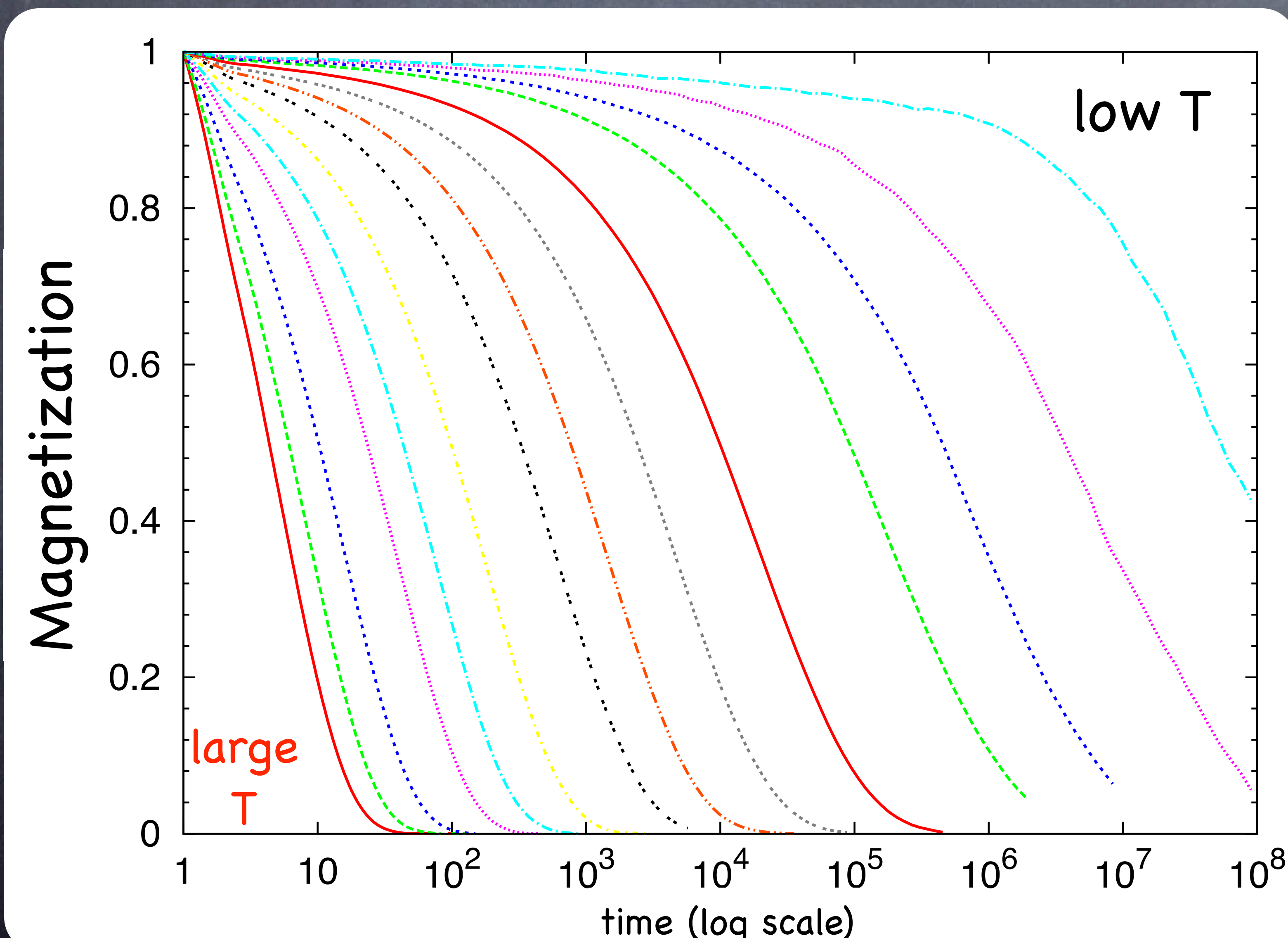


A 5-body interaction model... on the Nishimori line.

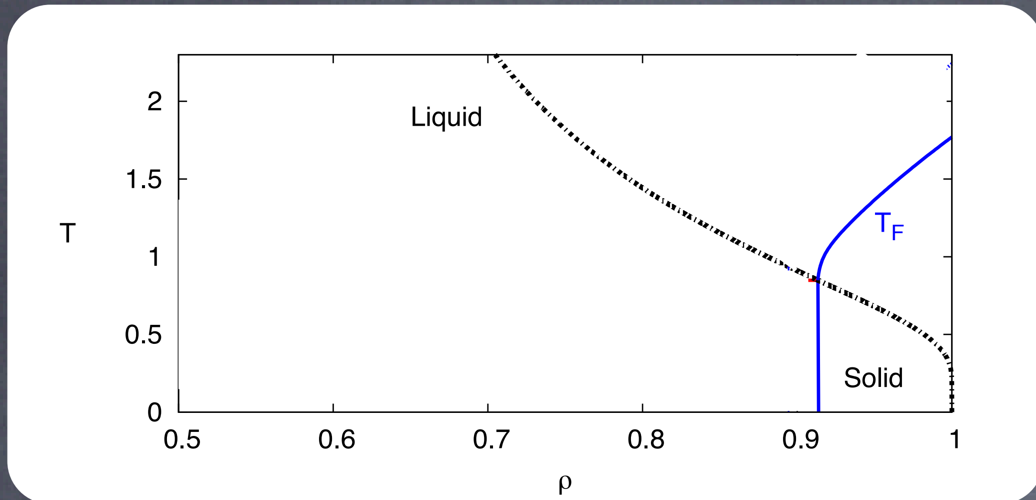
A 3D p-spin model on the Nishimori line



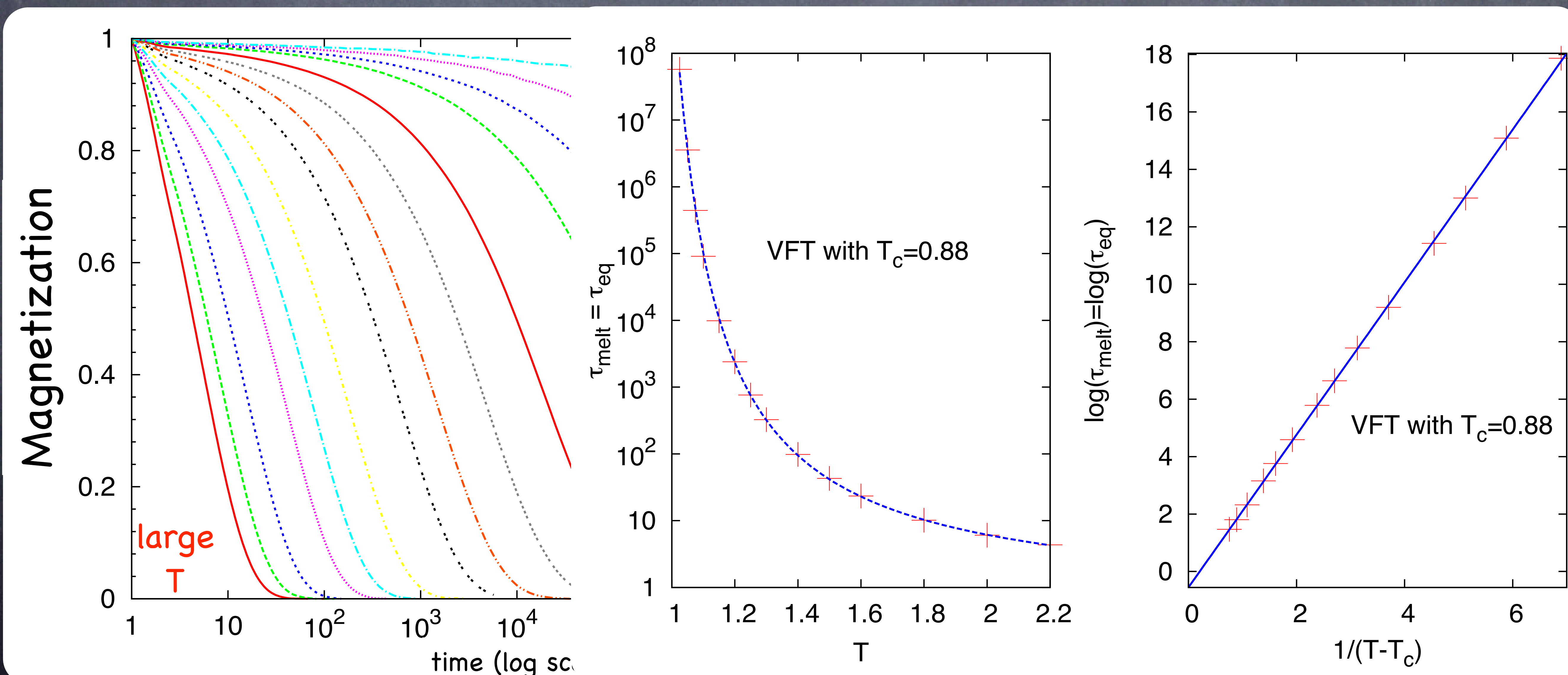
Melting Relaxation time
= Equilibrium relaxation time



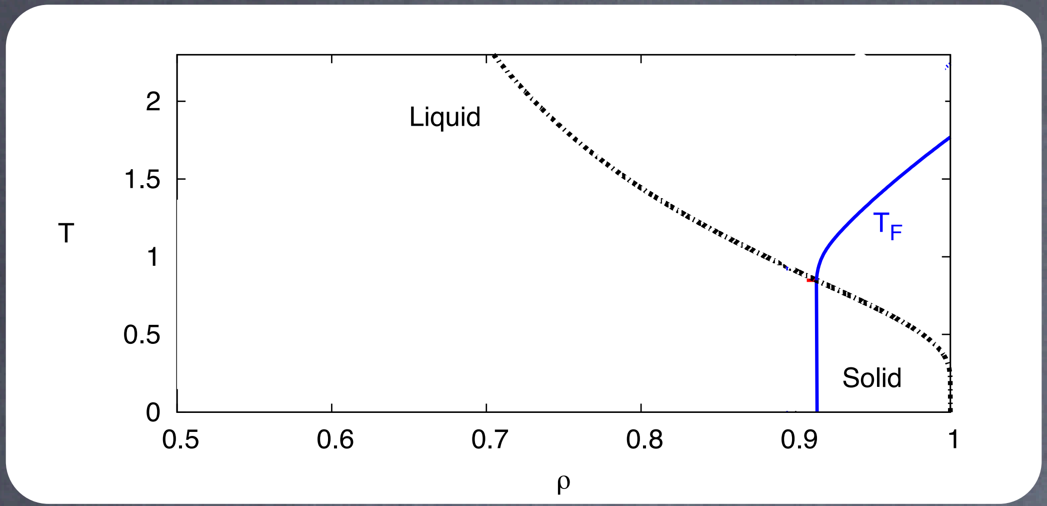
A 3D p-spin model on the Nishimori line



Melting Relaxation time
= Equilibrium relaxation time



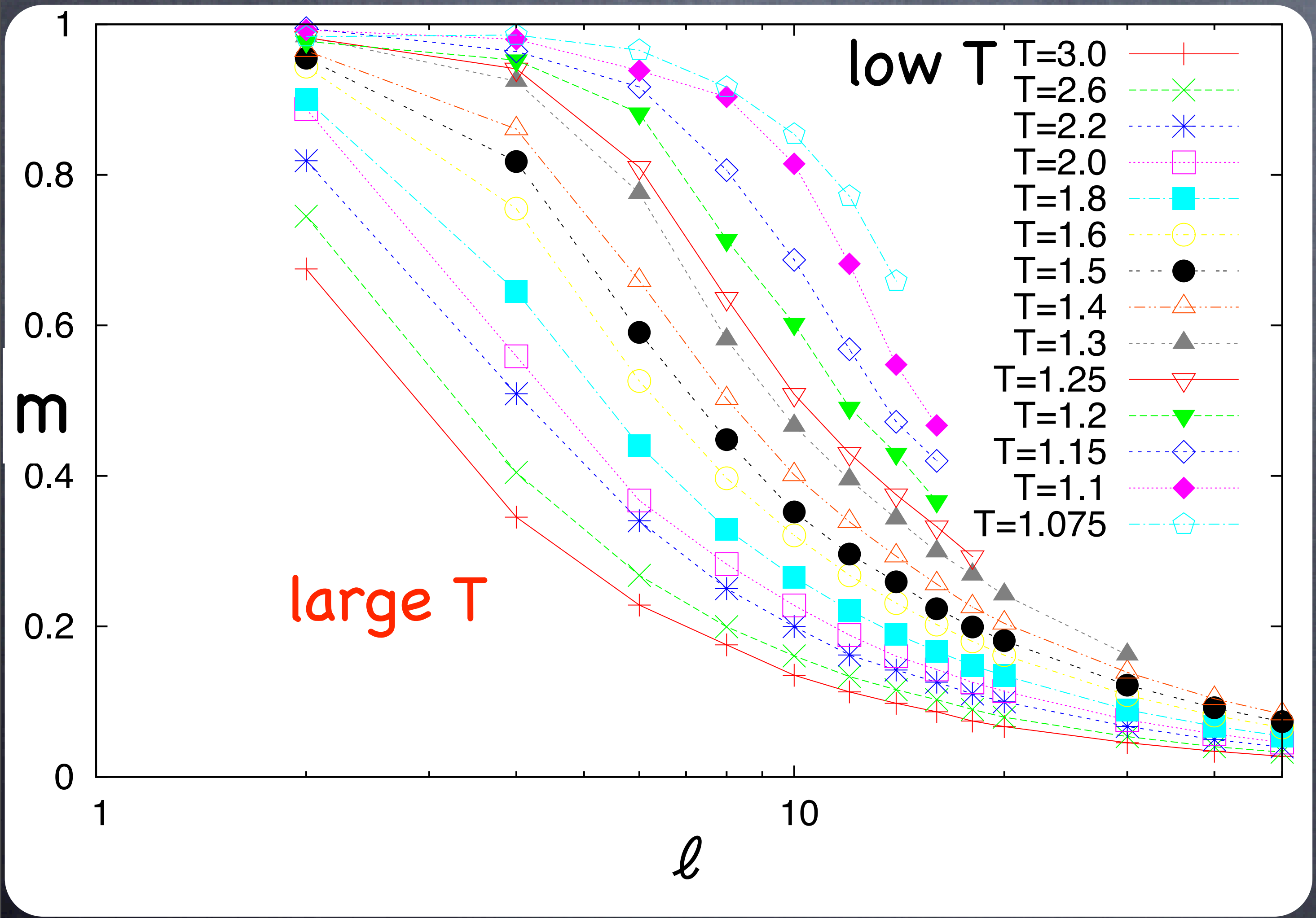
A frustrated model on a 3D Lattice

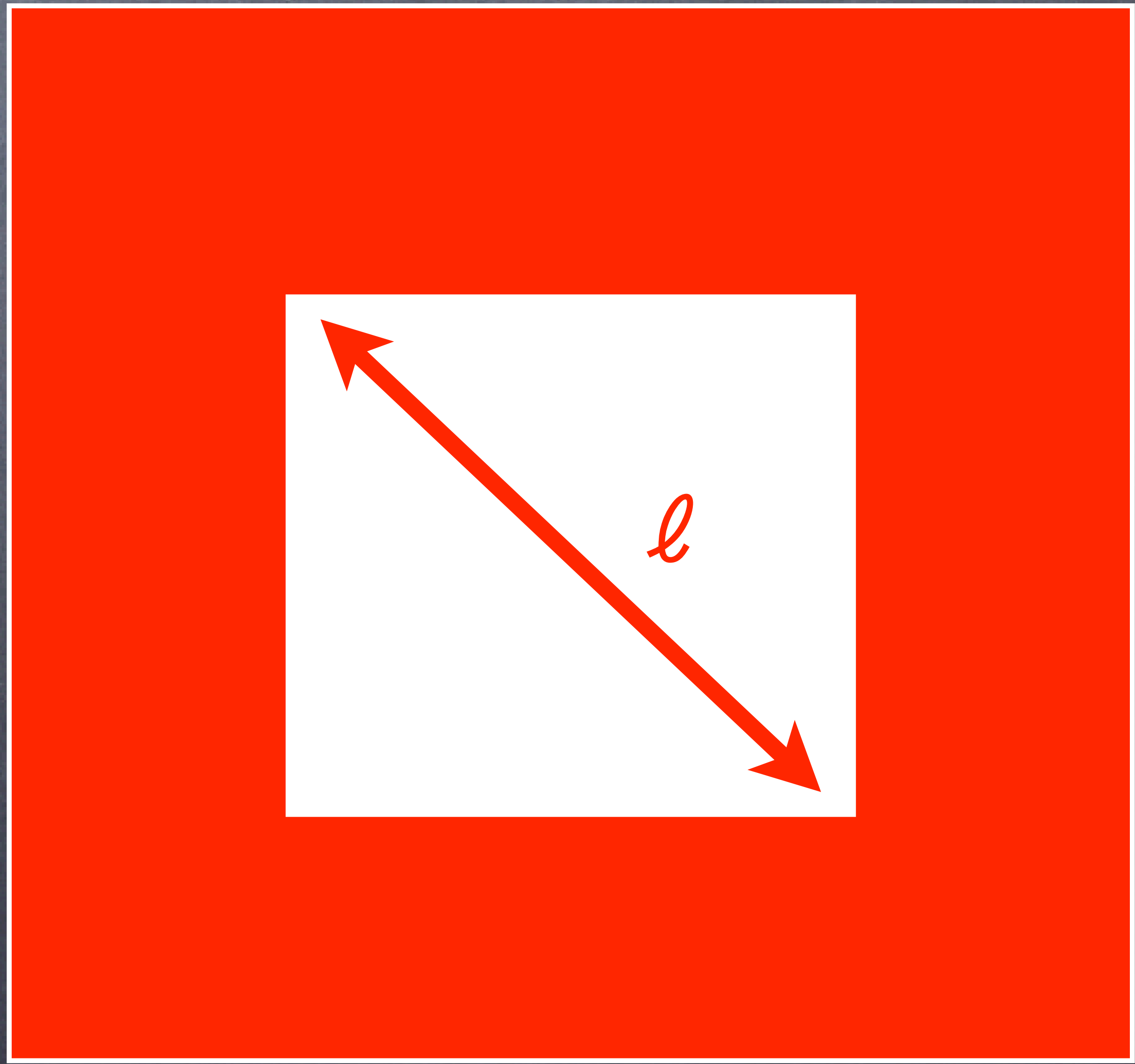


Melting dynamics

Growing of an equilibrium length scale, correlated with ordered boundaries

= Point-to-set correlations





Conclusions & perspectives

- Melting dynamics has a similar phenomenology as fragile glass formers.
- The two problems are equivalent in some models: Bulk melting in disordered spin models is in the same "universality" class as glassy dynamics!

Conclusions & perspectives

- Melting dynamics has a similar phenomenology as fragile glass formers.
- The two problems are equivalent in some models: Bulk melting in disordered spin models is in the same "universality" class as glassy dynamics!

Glass transition

- Toward a better understanding & characterization of finite dimensional effects for glasses in a standard first order setting:
  - ★ Nucleation processes ?
  - ★ Correction to mode-coupling-theory?
- Allows efficient simulations and help to rationalize the theory

Conclusions & perspectives

- Melting dynamics has a similar phenomenology as fragile glass formers.
- The two problems are equivalent in some models: Bulk melting in disordered spin models is in the same "universality" class as glassy dynamics!

Glass transition

- Toward a better understanding & characterization of finite dimensional effects for glasses in a standard first order setting:
  - ★ Nucleation processes ?
  - ★ Correction to mode-coupling-theory?
- Allows efficient simulations and help to rationalize the theory

Bulk Melting

- We should look to the melting problem with the eyes of the "glass" transitions....
  - ★ New analytical tools/Analogy ?
  - ★ Can we observed <u>experimentally</u>
    *the heterogenous dynamics...
    *the point-to-set correlation...
    *the string-like events...
    in superheated solid ?

Thank you for your attention!

BONUS

Demonstration

Step 1: A gauge symmetry

$$\mathcal{H} = -\sum_{ijk} J_{ijk} S_i S_j S_k$$

$\tau_i = \pm 1$

$S_i \to \tau_i S_i$

$J_i \to J_i \tau_i \tau_j \tau_k$

The Hamiltonian is invariant in this transformation

The dynamics is transformed in a trivial way

$$m(t) = \frac{1}{N} \sum_i S_i(t) \to \frac{1}{N} \sum_i S_i(t) \tau_i$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N} \sum_i S_i^J(t) \right]_{av}^{NL}$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N}\sum_i S_i^J(t)\right]_{av}^{NL} = \sum_J \prod_{klm} P(J_{klm}) \frac{1}{N}\sum_i S_i^J(t)$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N} \sum_i S_i^J(t) \right]_{av}^{NL} = \sum_J \prod_{klm} P(J_{klm}) \frac{1}{N} \sum_i S_i^J(t)$$

$$P(J) = \rho\delta(J-1) + (1-\rho)\delta(J+1)$$

$$\rho(\beta) = 1 - \frac{1}{1+e^{2\beta}}$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N} \sum_i S_i^J(t) \right]_{av}^{NL} = \sum_J \prod_{klm} P(J_{klm}) \frac{1}{N} \sum_i S_i^J(t)$$

$$P(J) = \rho\delta(J-1) + (1-\rho)\delta(J+1)$$

$$\rho(\beta) = 1 - \frac{1}{1+e^{2\beta}}$$

$$\Rightarrow \quad P(J_{klm}) = \frac{e^{\beta J_{klm}}}{2\cosh\beta}$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N}\sum_i S_i^J(t)\right]_{av}^{NL} = \sum_J \prod_{klm} P(J_{klm}) \frac{1}{N}\sum_i S_i^J(t)$$

$$P(J) = \rho\delta(J-1) + (1-\rho)\delta(J+1)$$

$$\rho(\beta) = 1 - \frac{1}{1+e^{2\beta}}$$

$$\Rightarrow P(J_{klm}) = \frac{e^{\beta J_{klm}}}{2\cosh\beta} \Rightarrow \prod_{klm} P(J_{klm}) = \frac{e^{\sum_{klm}\beta J_{klm}}}{2^M \cosh^M \beta}$$

Demonstration

Step 2: averaging over disorder

$$[m(t)]_{av}^{NL} = \left[\frac{1}{N}\sum_i S_i^J(t)\right]_{av}^{NL} = \sum_J \prod_{klm} P(J_{klm}) \frac{1}{N}\sum_i S_i^J(t)$$

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}}}{\cosh^M \beta} \frac{1}{N}\sum_i S_i^J(t)$$

$\Rightarrow$ $P(J_{klm}) = \dfrac{e^{\beta J_{klm}}}{2\cosh\beta}$ $\Rightarrow$ $\displaystyle\prod_{klm} P(J_{klm}) = \frac{e^{\sum_{klm}\beta J_{klm}}}{2^M \cosh^M \beta}$

Demonstration

Step 3: apply the Gauge transform

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)$$

Demonstration

Step 3: apply the Gauge transform

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)$$

$$\tau_i = \pm 1$$

$$S_i \to \tau_i S_i$$

$$J_i \to J_i \tau_i \tau_j \tau_k$$

Demonstration

Step 3: apply the Gauge transform

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)$$

$\tau_i = \pm 1$

$S_i \to \tau_i S_i$

$J_i \to J_i \tau_i \tau_j \tau_k$

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm} \tau_k \tau_l \tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t) \tau_i$$

Demonstration

Step 4: average over all possible gauges

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

Step 4: average over all possible gauges

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

$$[m(t)]_{av}^{NL} = \frac{1}{2^N} \sum_\tau \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

Step 4: average over all possible gauges

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

$$[m(t)]_{av}^{NL} = \frac{1}{2^N} \sum_\tau \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

For mean field models, as long as $T>T_K$

$$Z = 2^N \cosh \beta^M$$

Demonstration

Step 4: average over all possible gauges

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

$$[m(t)]_{av}^{NL} = \frac{1}{2^N} \sum_\tau \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

For mean field models, as long as $T>T_K$

$$Z = 2^N \cosh \beta^M$$

Demonstration

Step 4: average over all possible gauges

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

$$[m(t)]_{av}^{NL} = \frac{1}{2^N} \sum_\tau \sum_J \frac{1}{2^M} \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{\cosh^M \beta} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \sum_\tau \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{Z} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

Step 5: Final steps

This is the equilibrium correlation

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \sum_\tau \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{Z} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

Step 5: Final steps

This is the spin glass disorder average

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \sum_\tau \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{Z} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

Step 5: Final steps

This is the spin glass disorder average

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \sum_\tau \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{Z} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

Demonstration

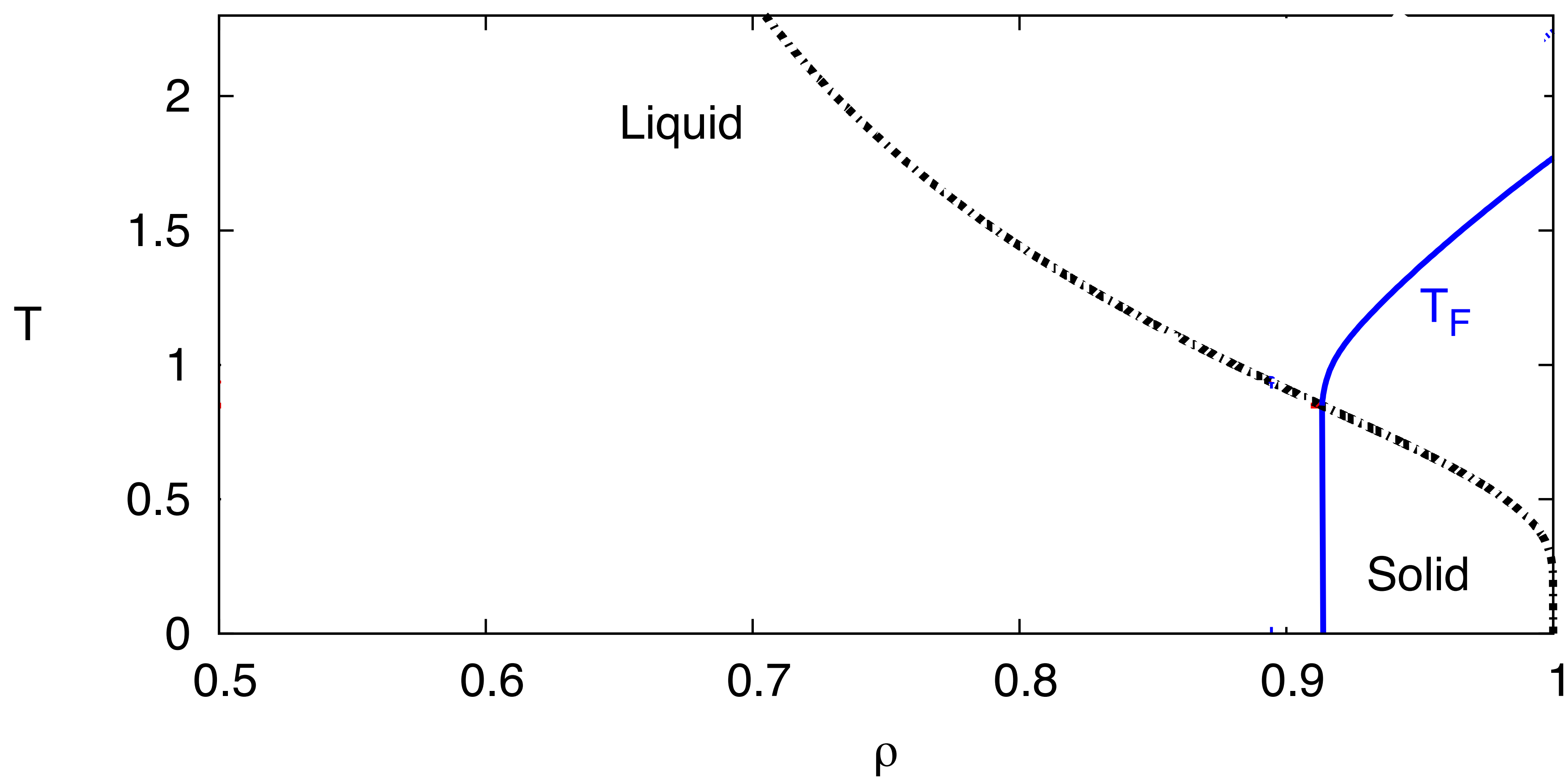
Step 5: Final steps

$$[m(t)]_{av}^{NL} = [C_{eq}(t)]_{av}^{SG}$$

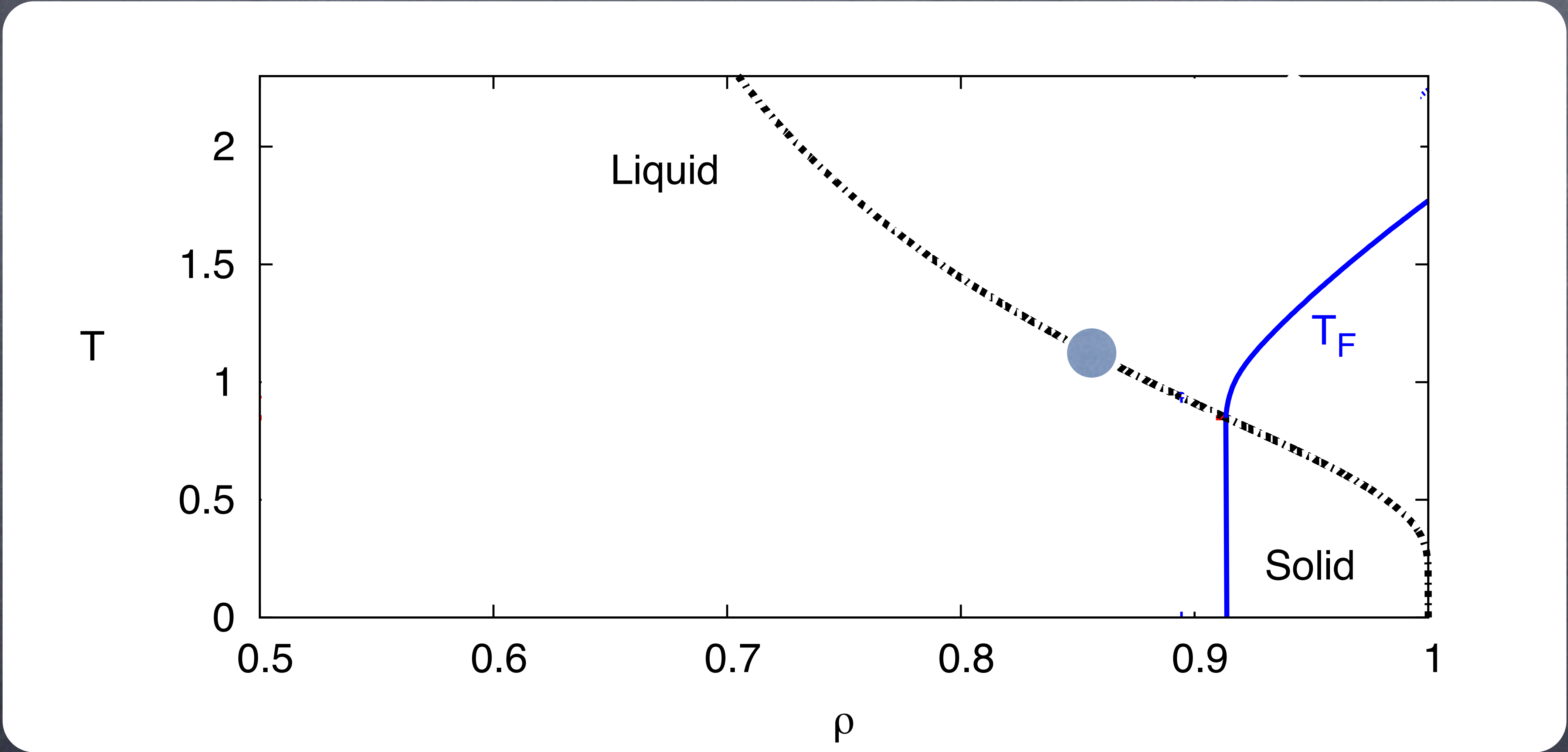
The decay of magnetization on the Nishimori line is equal to the spin glass correlation function

$$[m(t)]_{av}^{NL} = \sum_J \frac{1}{2^M} \sum_\tau \frac{e^{\sum_{klm} J_{klm}\tau_k\tau_l\tau_m}}{Z} \frac{1}{N} \sum_i S_i^J(t)\tau_i$$

P-Spin model and the Nishimori line

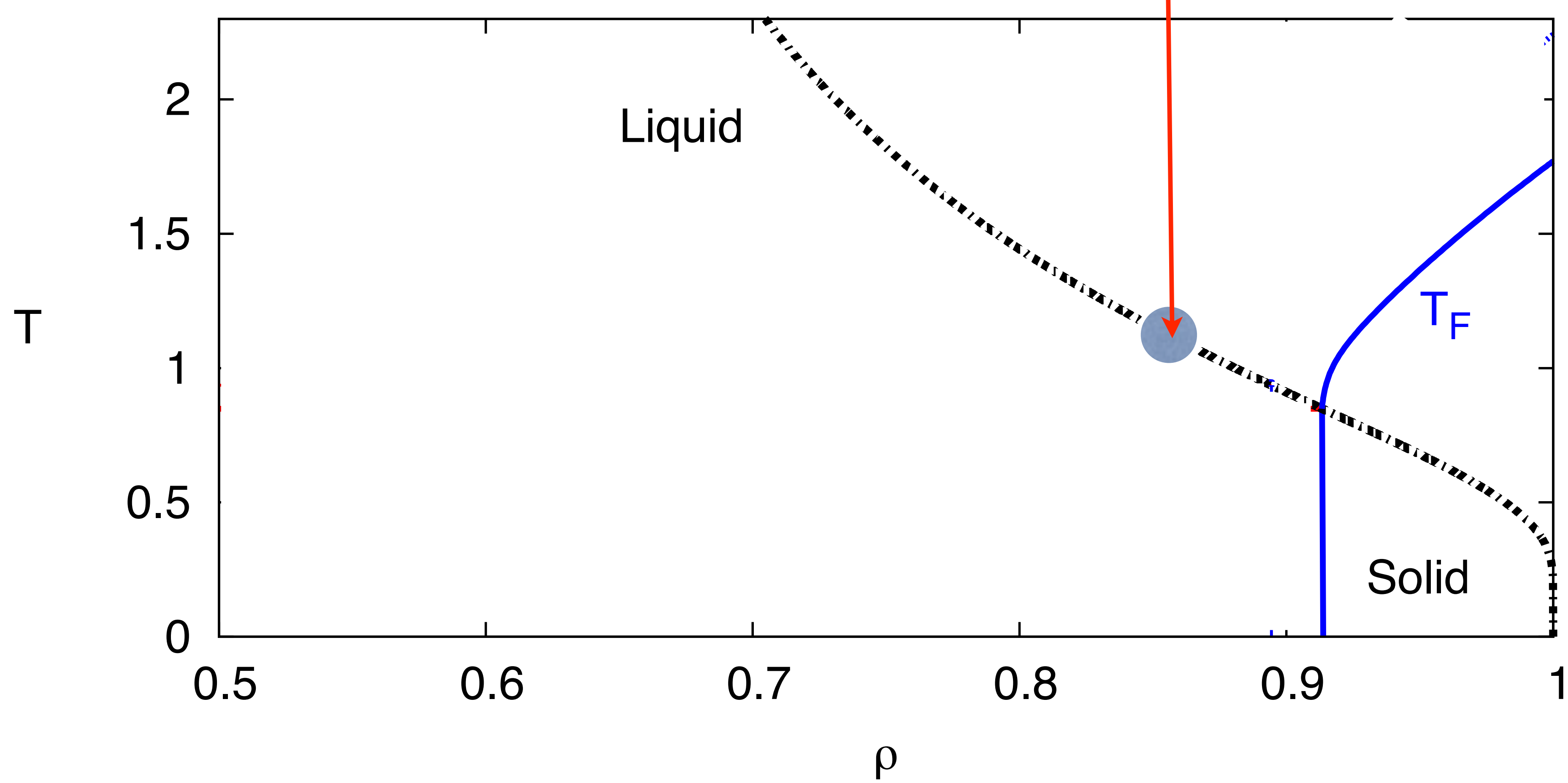


P-Spin model and the Nishimori line



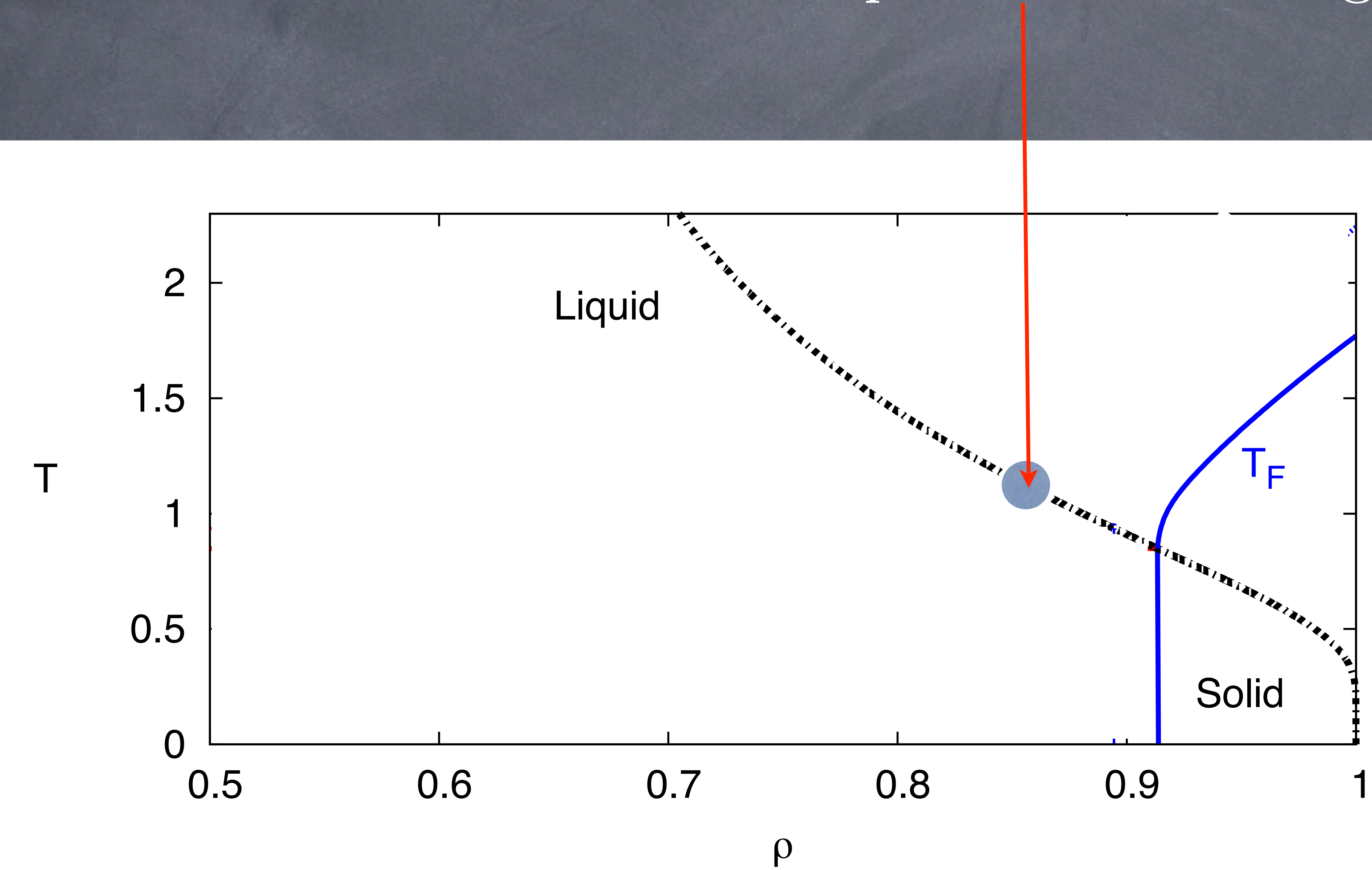
P-Spin model and the Nishimori line

$$C_{\rm eq}(t) = m_{\rm melting}(t)$$



P-Spin model and the Nishimori line

$$C_{\rm eq}(t) = m_{\rm melting}(t)$$



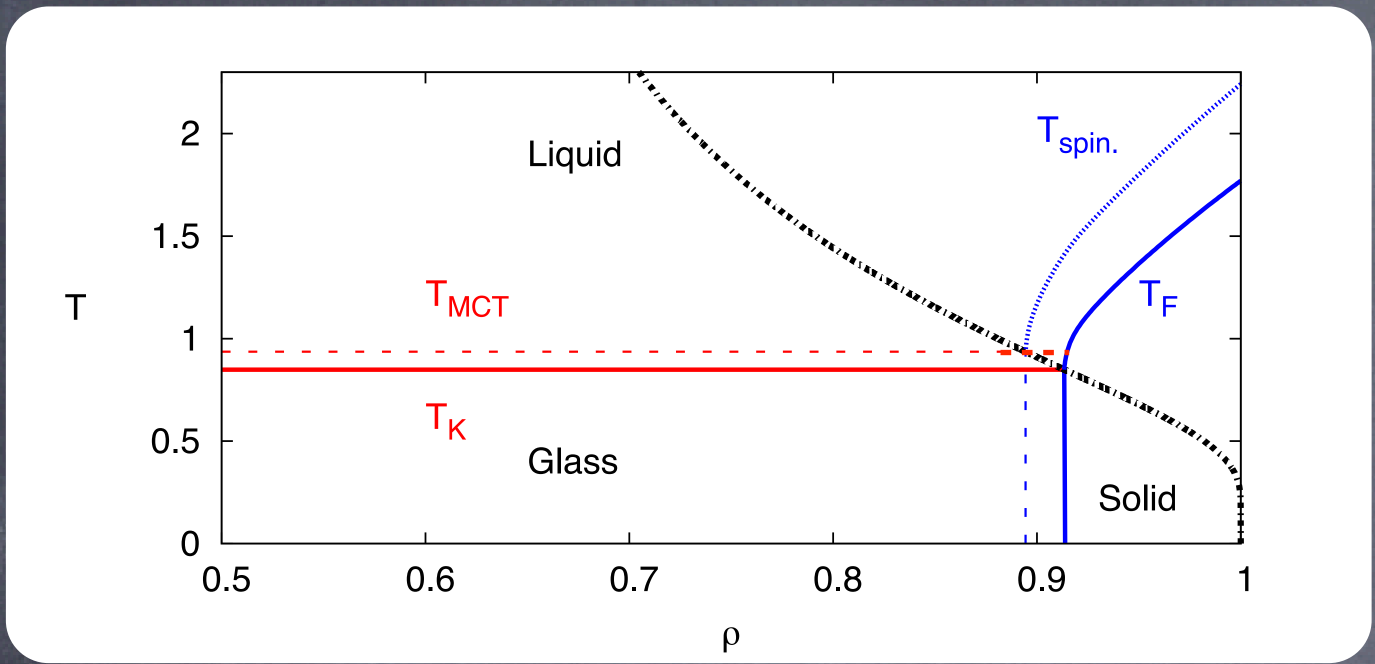
Equilibrium correlation function

$$C_{\rm eq}(t) = \lim_{t_w \to \infty} \frac{1}{N} \sum_i S_i(t_w) S_i(t_w + t)$$

Magnetization starting from the fully ordered state

$$m(t) = \frac{1}{N} \sum_i S_i(t), \quad \text{with } m(0) = 1$$

Melting=equilibrium dynamic

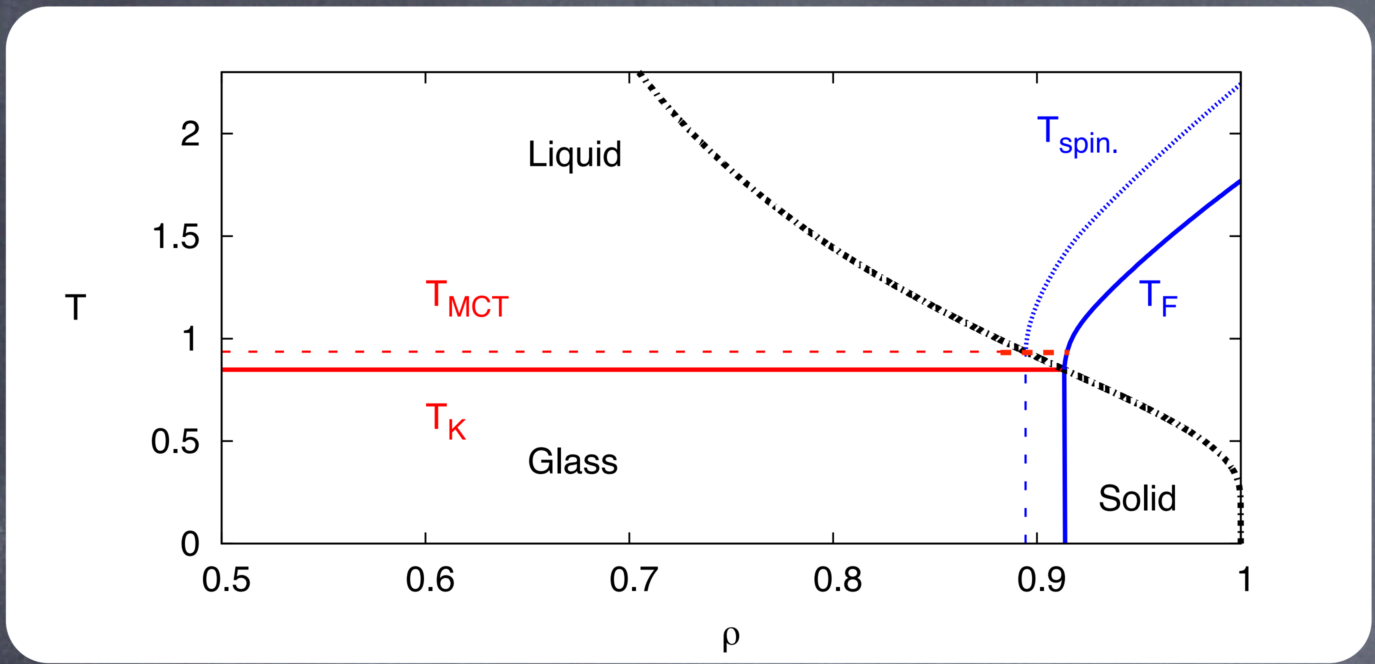


On the Nishimori line in **any** dimension

The equilibrium time correlation is equal to the melting correlation

$$c_{\text{eq}}(t) = m_{\text{melting}}(t)$$

Melting=equilibrium dynamic



On the
Nishimori line
in **any** dimension

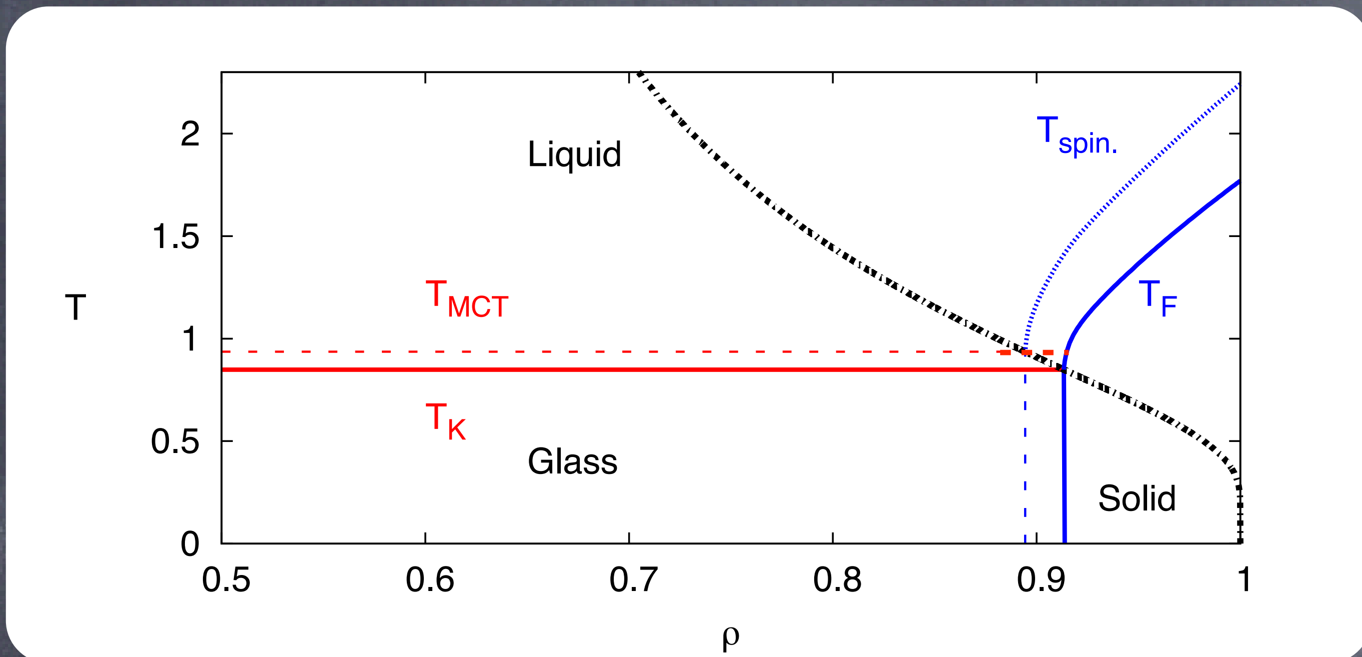
The equilibrium relaxation time is equal to the melting relaxation time

$$\tau_{\rm eq}(\beta) = \tau_{\rm melting}(\beta)$$

The equilibrium time correlation is equal to the melting correlation

$$c_{\rm eq}(t) = m_{\rm melting}(t)$$

Melting=equilibrium dynamic



On the Nishimori line in **any** dimension

The equilibrium relaxation time is equal to the melting relaxation time

$$\tau_{\mathrm{eq}}(\beta) = \tau_{\mathrm{melting}}(\beta)$$

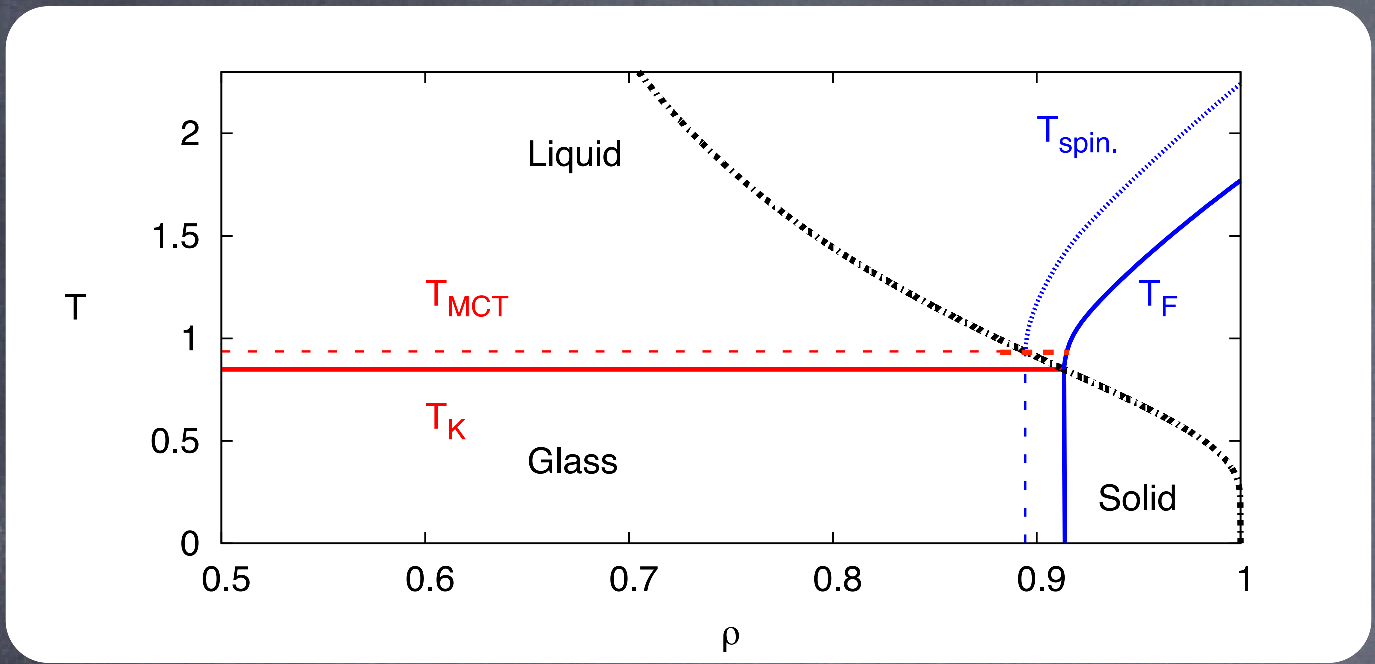
The equilibrium time correlation is equal to the melting correlation

$$c_{\mathrm{eq}}(t) = m_{\mathrm{melting}}(t)$$

The static (point-to-set) and dynamic (heterogeneities) length scales in the are equal to the melting ones

$$\chi_4^{\mathrm{eq}}(t) = \chi_F^{\mathrm{melting}}(t) \qquad \ell^{\mathrm{PTS}}(\beta) = \ell^{\mathrm{FERRO}}(\beta)$$

Melting=equilibrium dynamic



On the Nishimori line in **any** dimension

The equilibrium relaxation time is equal to the melting relaxation time

$$\tau_{\rm eq}(\beta) = \tau_{\rm melting}(\beta)$$

The equilibrium time correlation is equal to the melting correlation

$$c_{\rm eq}(t) = m_{\rm melting}(t)$$

The static (point-to-set) and dynamic (heterogeneities) length scales in the are equal to the melting ones

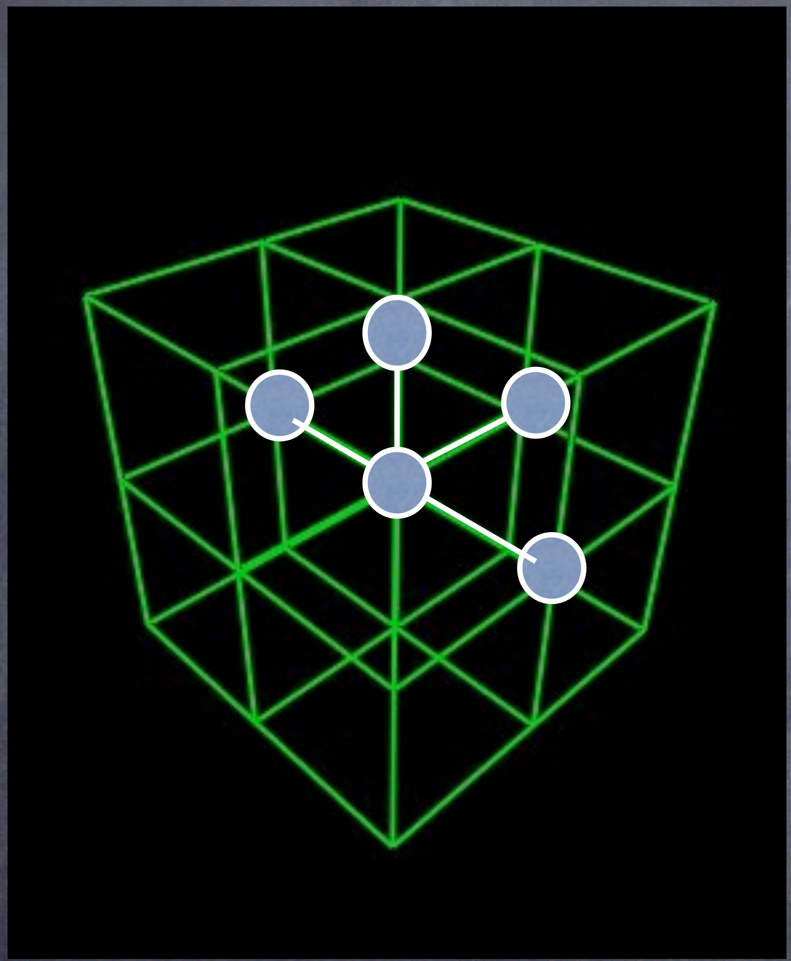
$$\chi_4^{\rm eq}(t) = \chi_F^{\rm melting}(t) \qquad \ell^{\rm PTS}(\beta) = \ell^{\rm FERRO}(\beta)$$

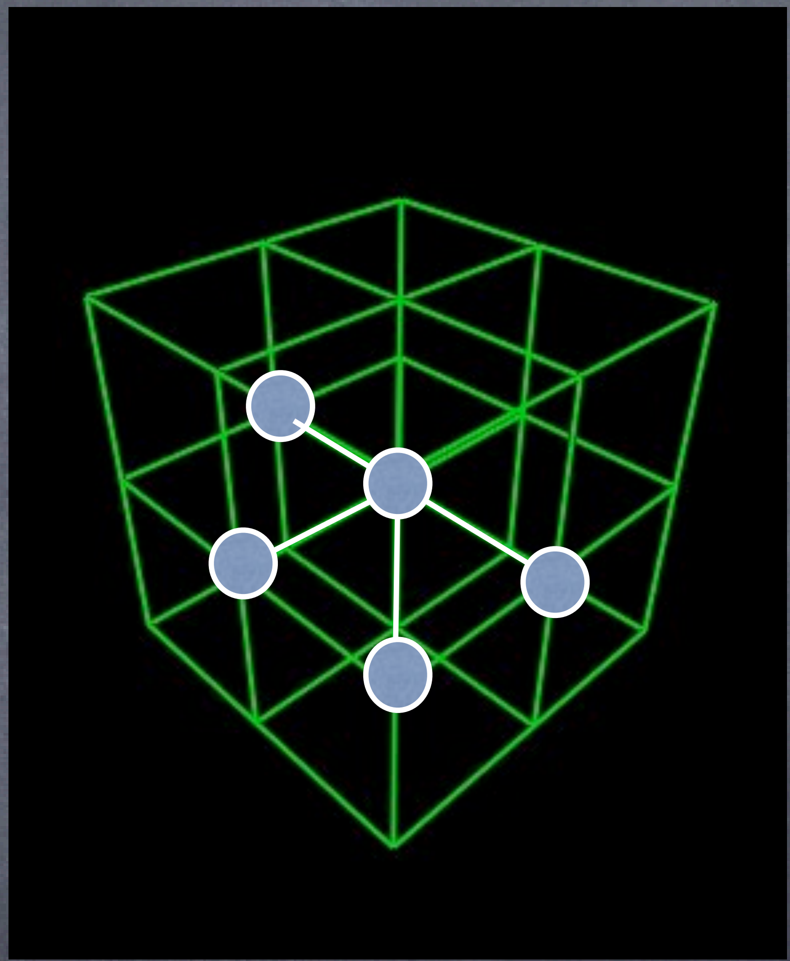
The free-energy is equal to the Franz-Parisi potential (cf. Parisi talk yesterday)

$$f(m) = f_{FP}(q)$$

A 3D p-spin model on the Nishimori line

$$\mathcal{H} = -\sum_i J_i^a S_i S_{UP} S_{LEFT} S_{RIGHT} S_{BEHIND} + J_i^b S_i S_{BOTTOM} S_{LEFT} S_{RIGHT} S_{FRONT}$$

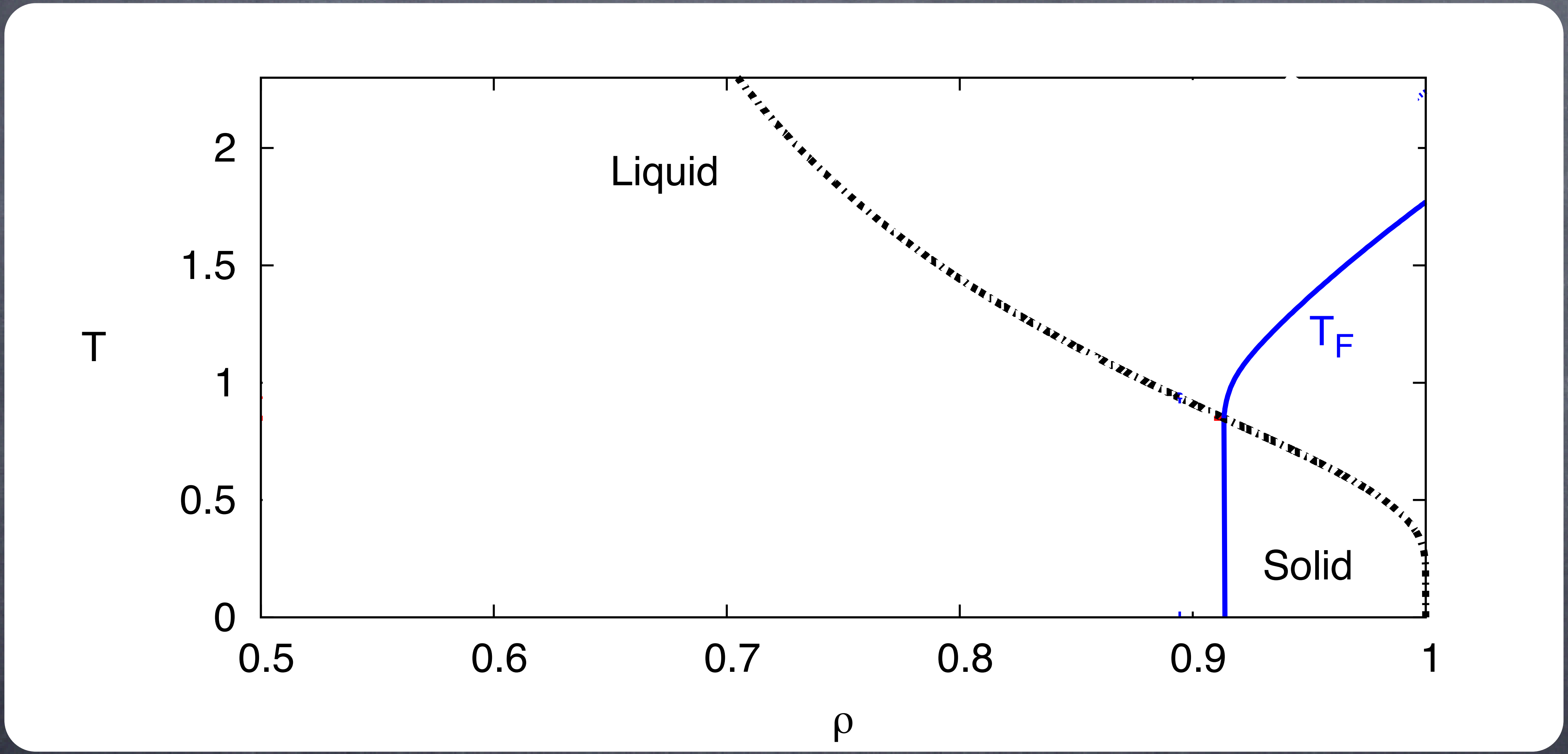




A 5-body interaction model... on the Nishimori line.

A 3D p-spin model on the Nishimori line

$$\mathcal{H} = -\sum_i J_i^a S_i S_{UP} S_{LEFT} S_{RIGHT} S_{BEHIND} + J_i^b S_i S_{BOTTOM} S_{LEFT} S_{RIGHT} S_{FRONT}$$



A 5-body interaction model... on the Nishimori line.