ESPCI
ÉCOLE SUPÉRIEURE DE PHYSIQUE ET DE CHIMIE INDUSTRIELLES

Gulliver
UMR 7083
d'une échelle à l'autre...

From mean field to three-dimensional spin glasses (A journey through the numerics)

Florent Krząkała

Thanks to
O.C. Martin,
T. Jörg,
J.P. Bouchaud,
H. Katzgraber,
and others...

Find these slides on my webpage:
www.pct.espci.fr/~florent/

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A starting point to understand glasses, aging, out-of-equilibrium dynamics (and other fundamental questions of modern physics).

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Few rigorous results in finite dimension

- cf: Newman & Stein lectures
- Self-averaging: Variance of the Free energy grows as volume (Aizenman-Wehr 90')
- No proof of phase transition in any finite dimension !
- Many controversies. The 10^6\$ question: are finite-dimensional model "meanfieldish" ?

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Perform Numerical Simulations

In order to get a feeling “what’s going on...”

Some “accepted” results from the numerics:

- Free-energy distribution is Gaussian with variance N in finite dimension

  J.-P. Bouchaud, **FK**, O.C. Martin PRE 02’

- Spin glass phase transition for d>=3 for Ising Spins (Heisenberg still subject of debate !)

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Some which are not “accepted” yet:

- The phase diagram in presence of a field or a magnetic bias
- Presence or absence of Temperature chaos
- Nature of the spin glass phase

The questions (in this seminar)

- **Is there a spin glass phase under an applied magnetic field ?**
- **Is there temperature/disorder chaos ?**

A Physicist Strategy

Compute (and discuss) the prediction for (diluted) mean field systems
Discuss heuristic arguments

Compare with numerical simulations of small systems in finite dimension

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Answering (at the heuristic level) the long-standing question: does 3d model are "mean-field like" ?

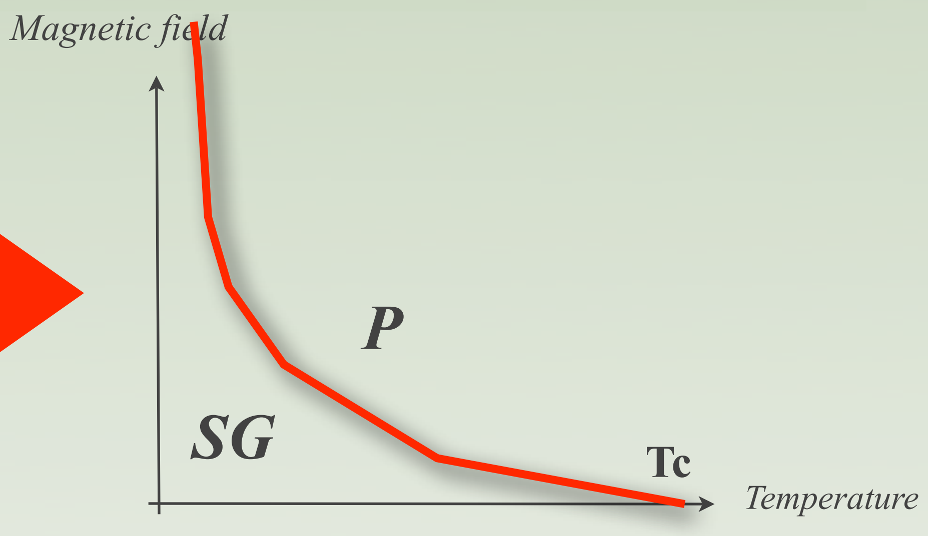
Ordering of the Ising Spin Glass under an applied magnetic field

T. Jörg, H. Katzgraber & **FK** 07, *in preparation*

Mean field Ising Spin glass under a field

SK: Almeida & Thouless '78:

A Spin glass phase exists under Tc
for low enough magnetic field
The critical value of the field diverges at T=0



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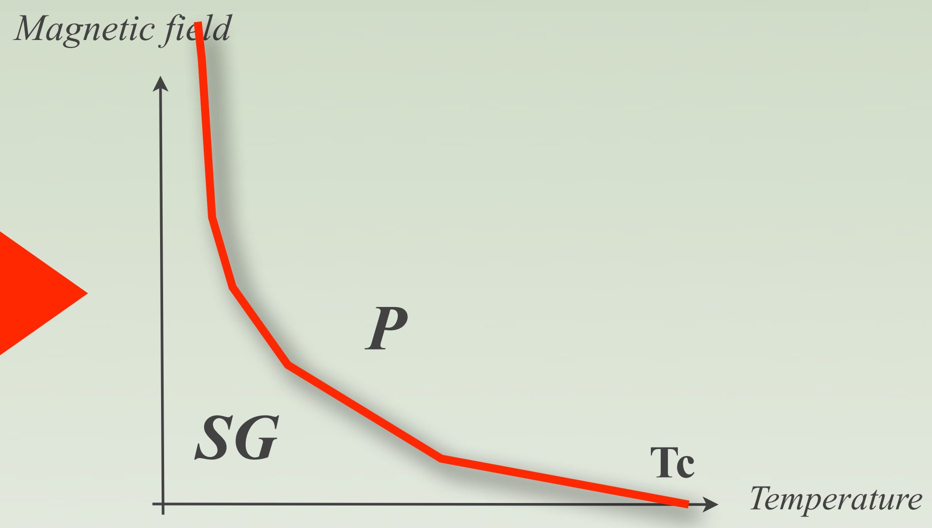
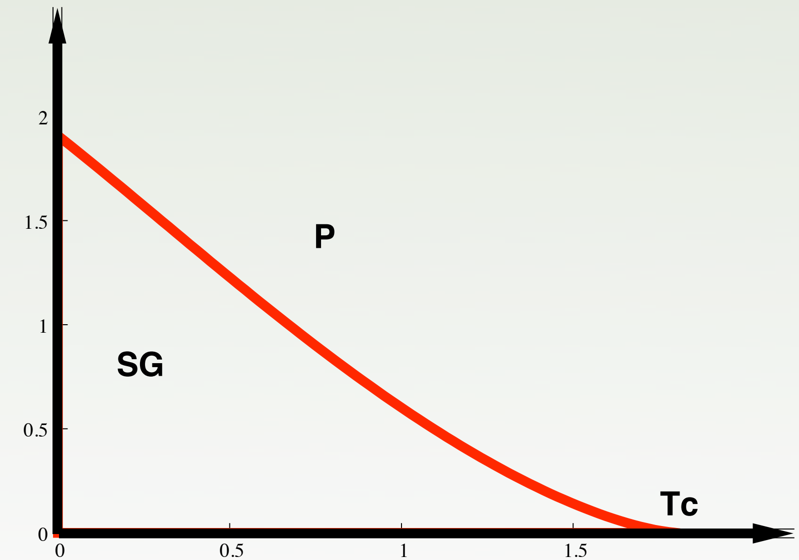


Diluted model: Jörg, Katzgraber & FK 07:

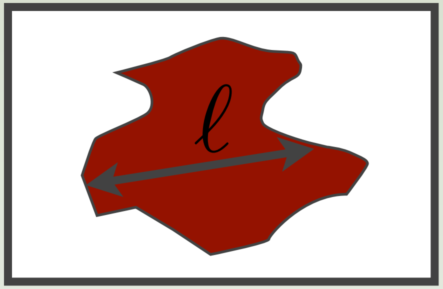
A similar phase diagram with a finite $h_c(T=0)$
6-connectivity regular graph with Gaussian couplings



Finite dimensional Spin glass under a field

Finite dimension Mc Millian '84:

Imrie-Ma argument: ***the ground state is highly unstable under a field***



Spin glass phase: *existence of large low energy excitations*

Consider the ground state and an excitation of size ℓ *with excess energy* $E = A\ell^{\theta}$

With a field h, we have now $E = A\ell^{\theta} \pm Bh\ell^{d/2}$

If $\theta < d/2$ *the ground state is unstable for large size.*

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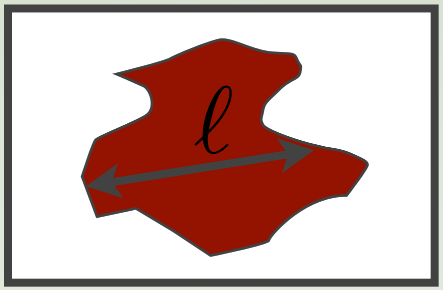

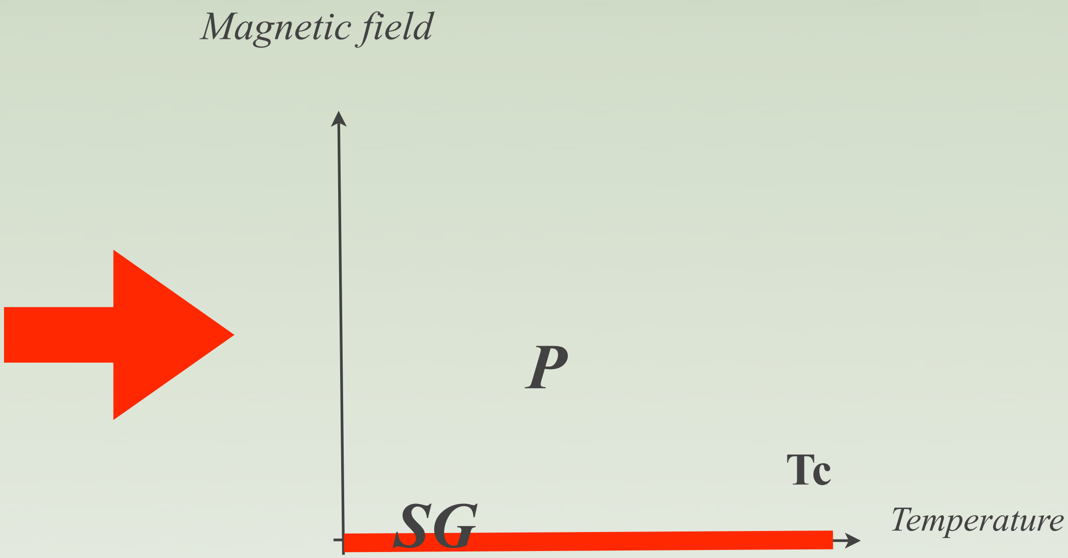

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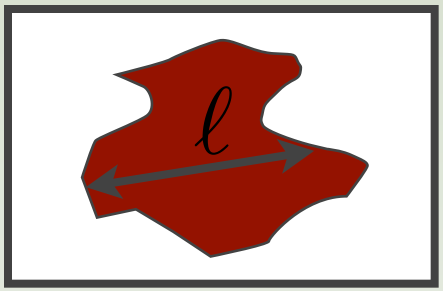

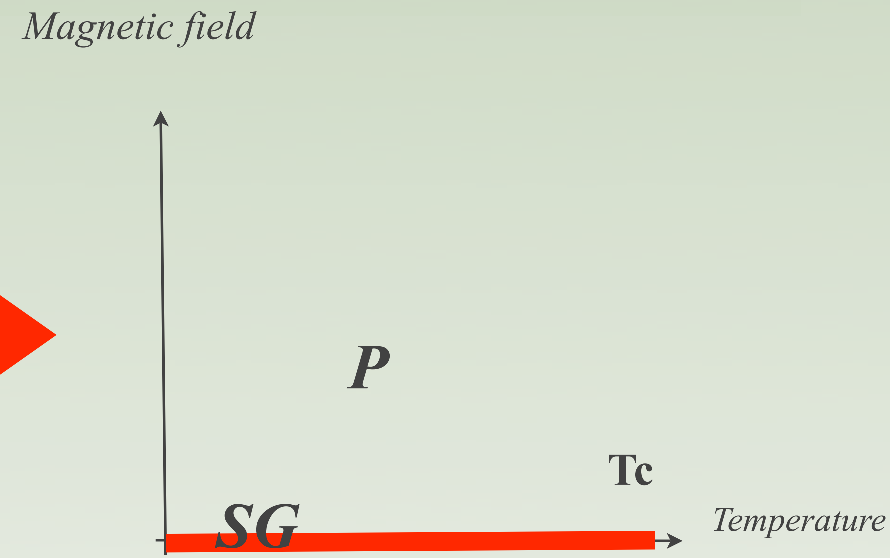

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Is the Imrie-Ma argument convincing ?

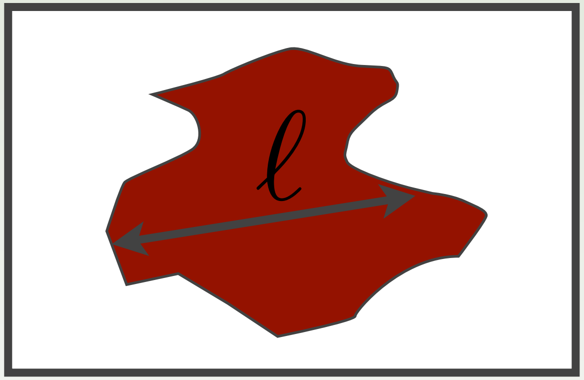
(even for a physicist...)

- It applies equally well to the spin glass on a Bethe Lattice where there **is** an AT line !
- The same argument is used with temperature variation to suggest **Temperature Chaos** but **not the absence of a spin glass phase** in temperature !

A modified Imrie-Ma argument

- Consider a large sample with a magnetization by spin M under a field H

- Question: are there large excitations with low energies ?
- Fliping a cluster of size ℓ :
  - Costs an energy $E_{field} \propto MH\ell^d$
  - That we try to compensate by a gain from the boundaries: $E_{J_{ij}} \propto \ell^{d_s}$
- For large “droplet”, the field energy dominates and is <u>large !</u>



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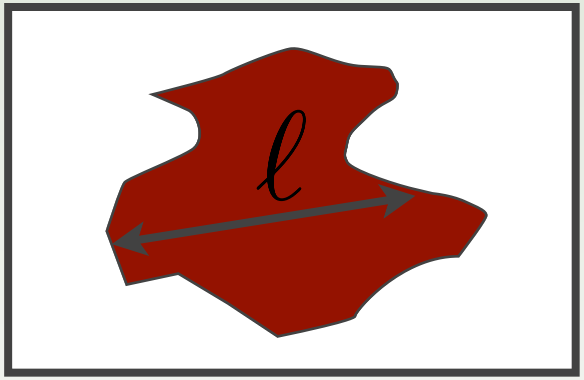
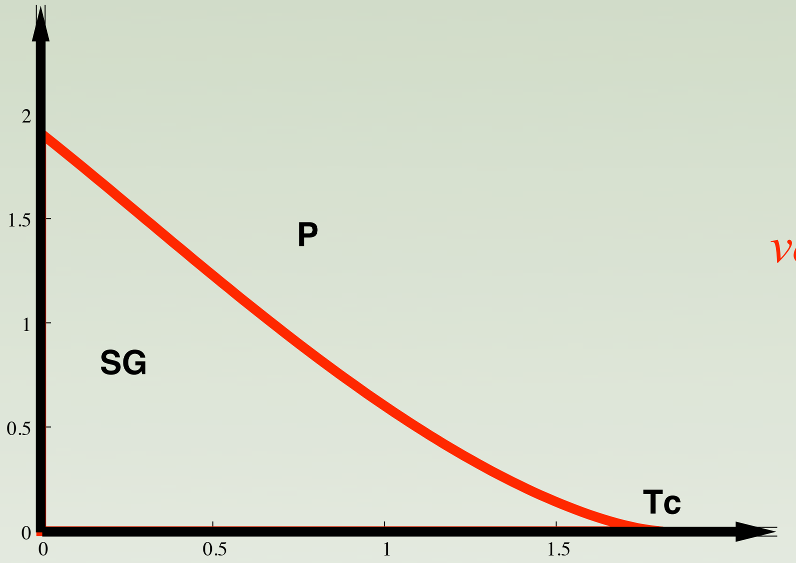


It is very hard to have low energy large-scale excitation under a field

The controversy



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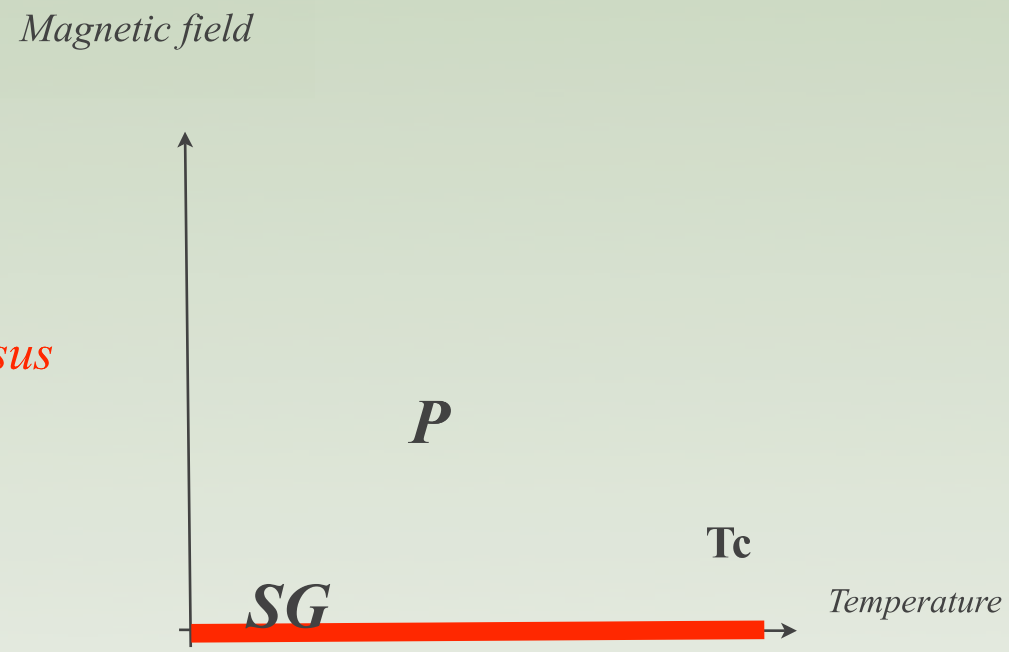

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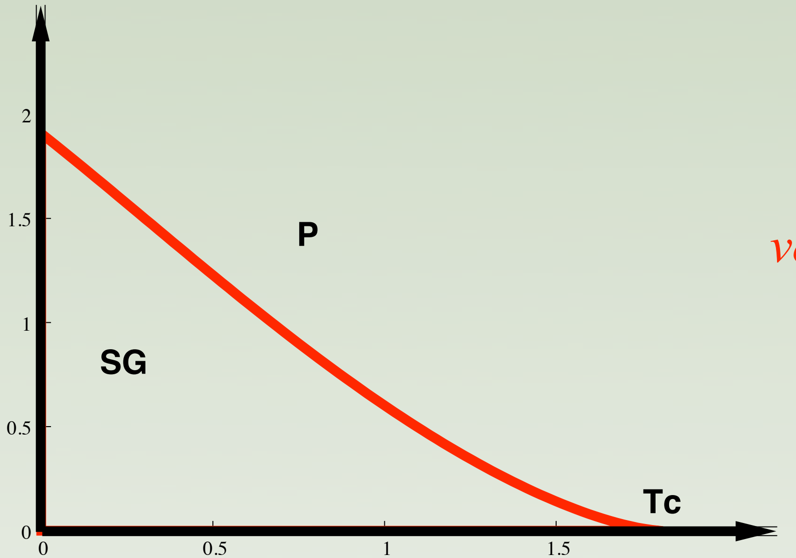

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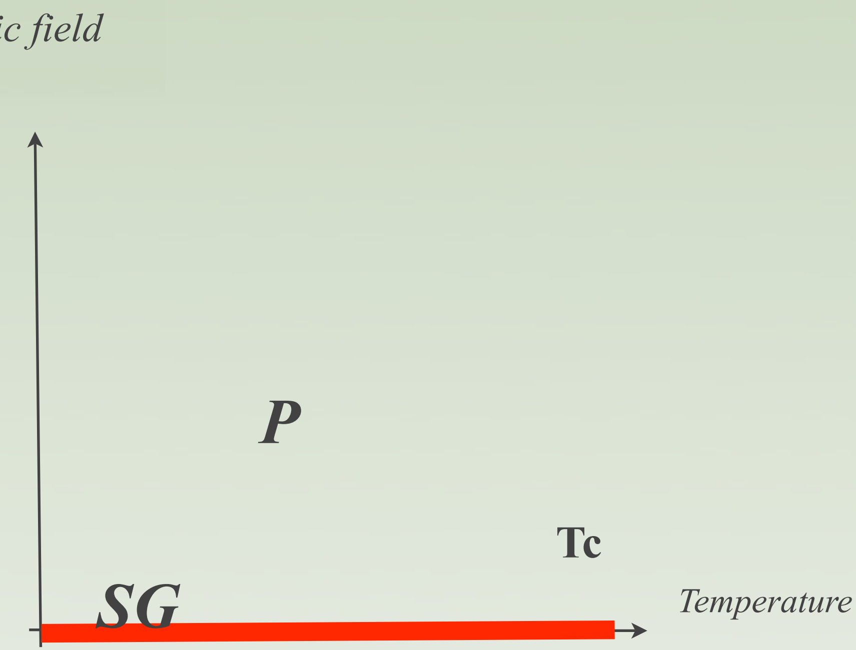

Monte-Carlo simulations

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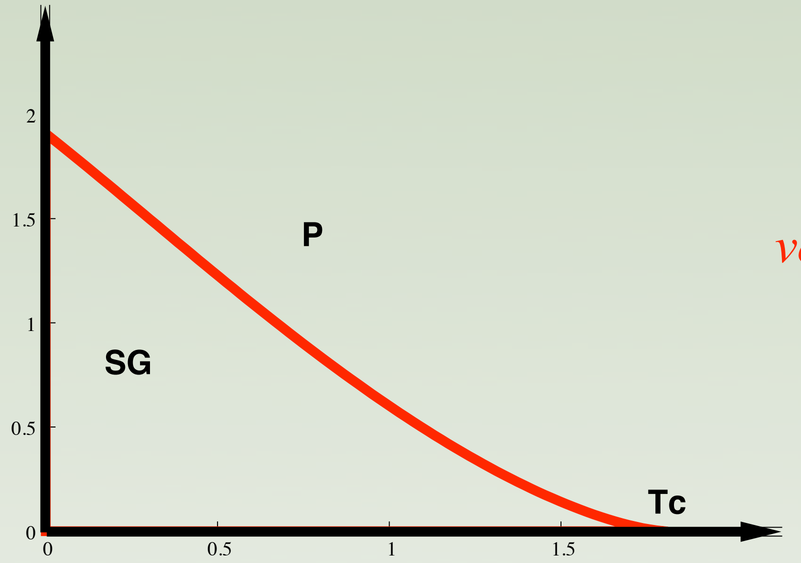

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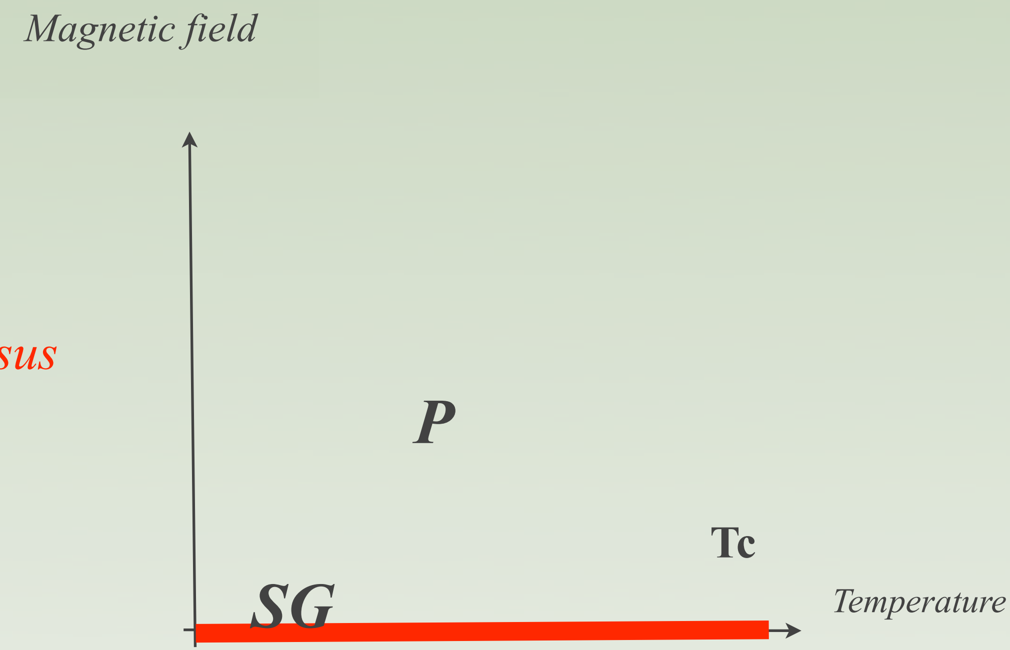
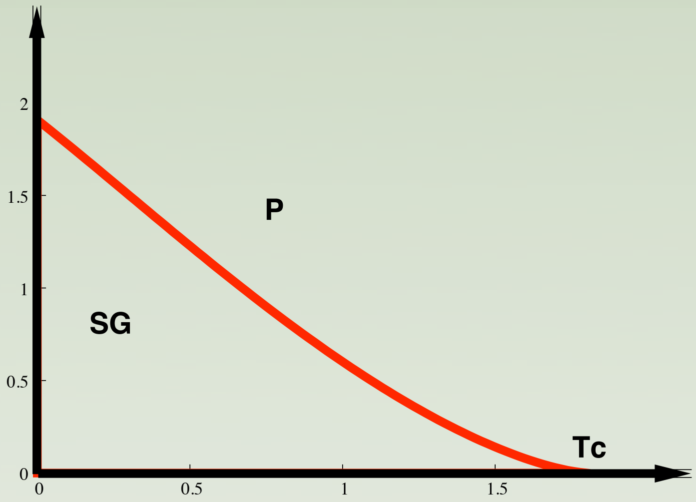


Monte-Carlo simulations

- Generating a representative set of configurations (with the proper weight) for small systems
- Using Metropolis dynamics and parallel tempering (mixing different replicas at different temperatures)
- Question 1: Are we able to see the AT line for random graphs ?
- Question 2: What do we observe on 3d lattices ?

Spin glass transition on random graph



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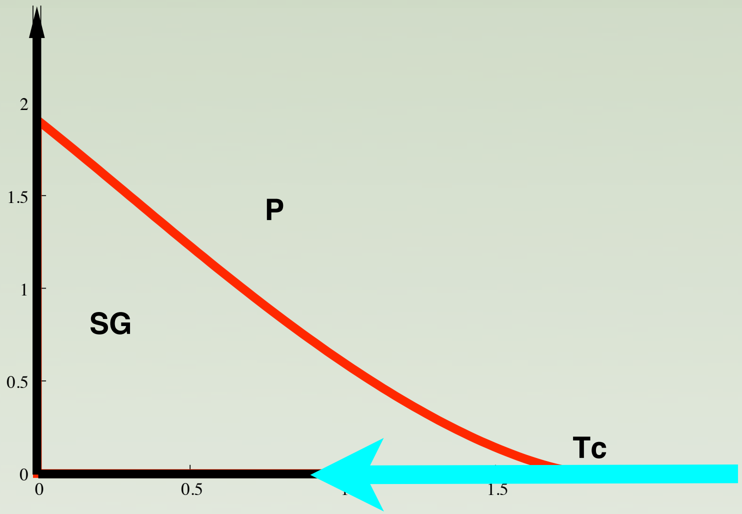

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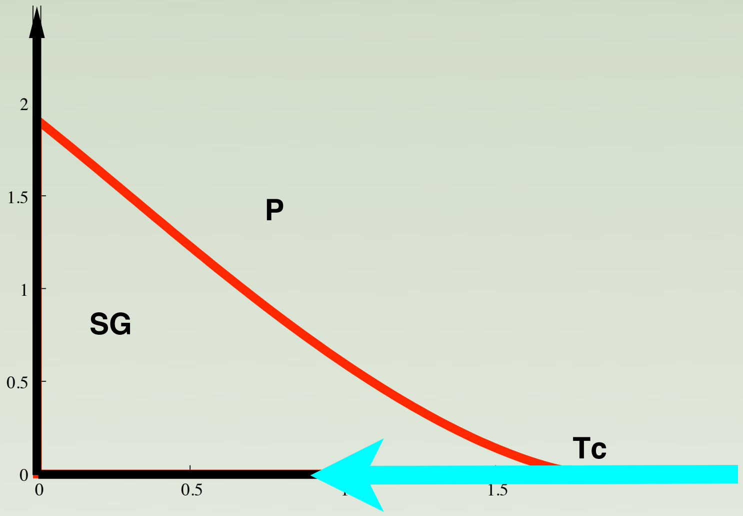

Compute the spin glass susceptibility

$$\frac{\chi_{SG}}{N} = < q^2 > - < q >^2$$

From replica theory, we expect that at the transition

$$\chi_{SG} \propto N^{1/3}$$

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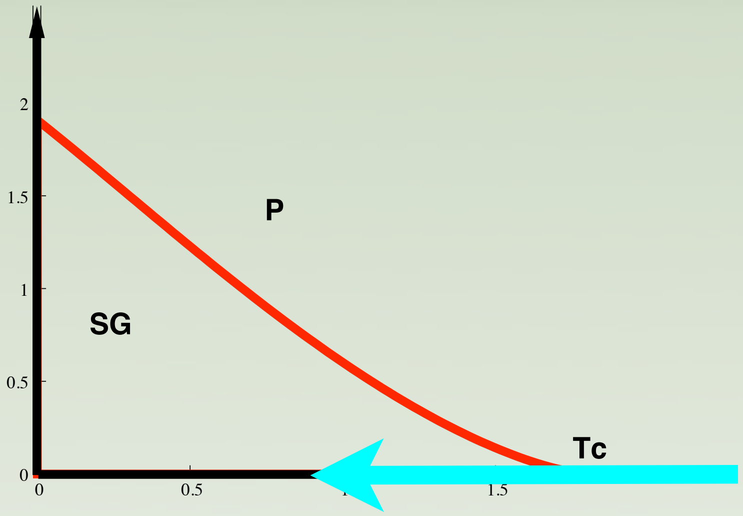

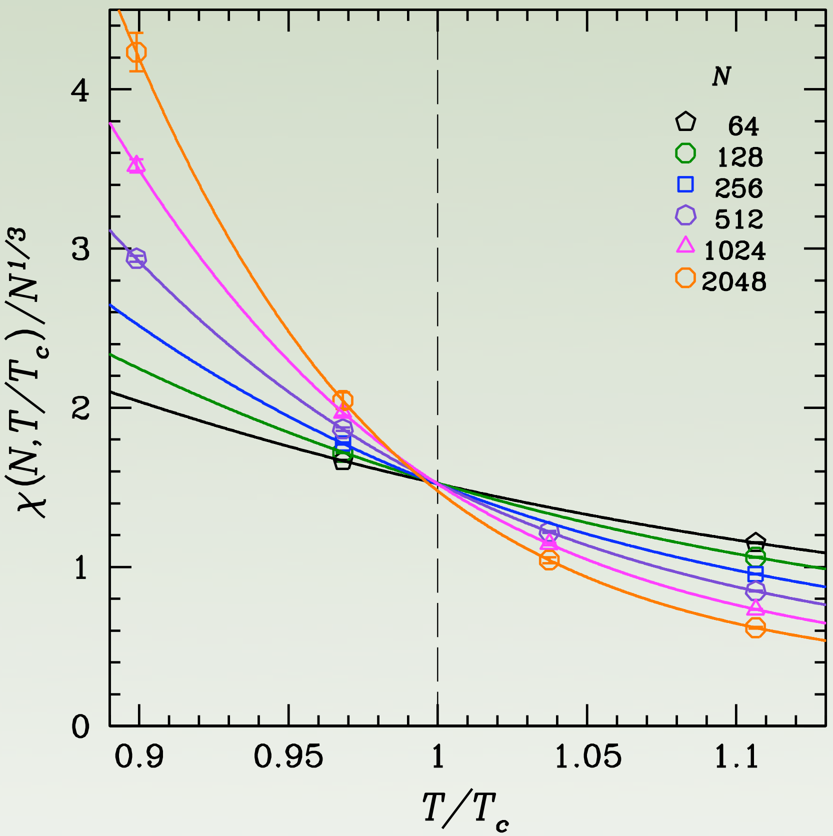

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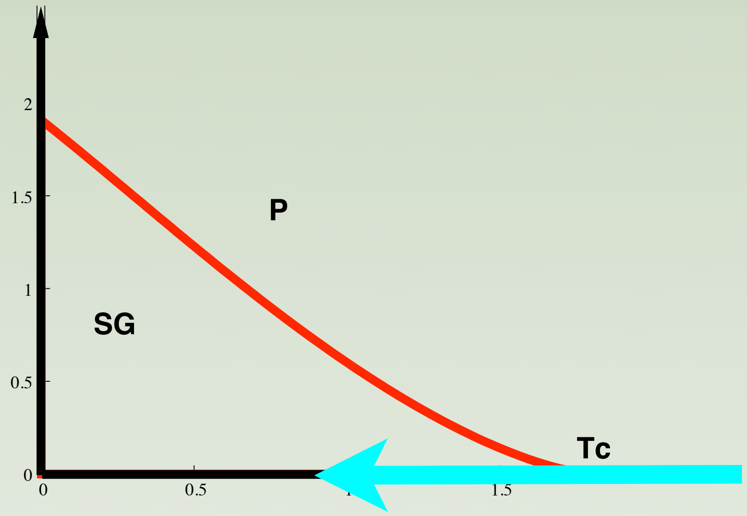

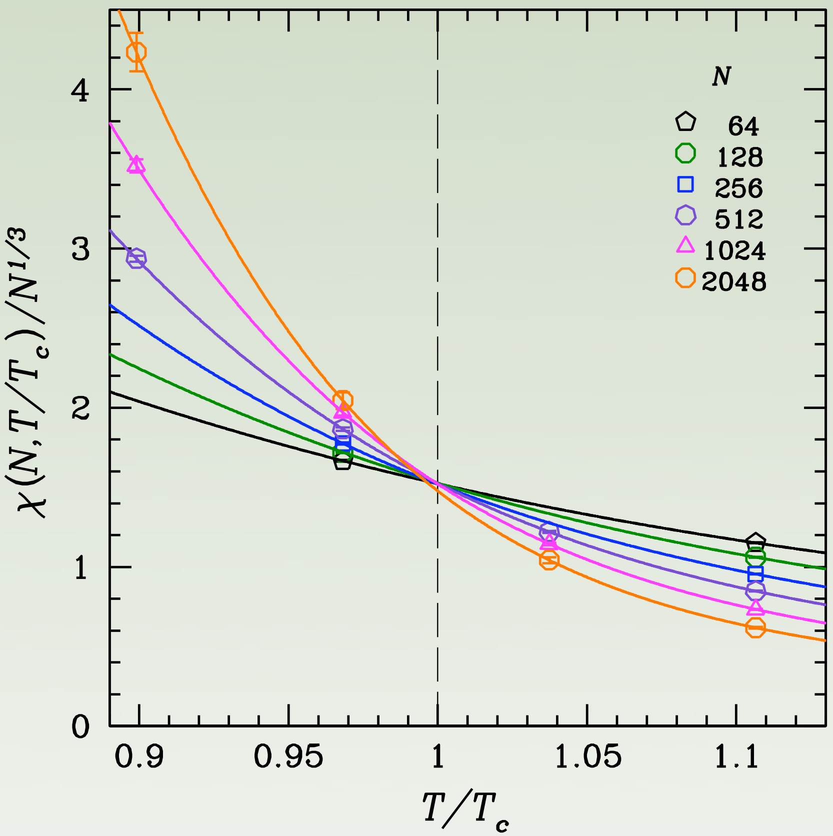

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"Accord parfait" between simulation and theory

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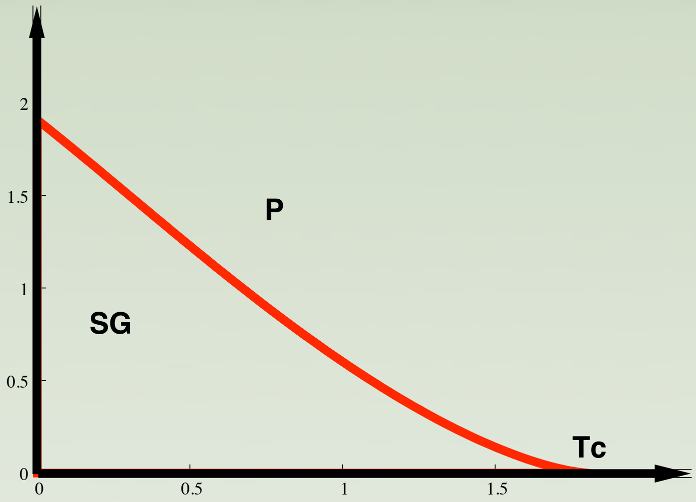

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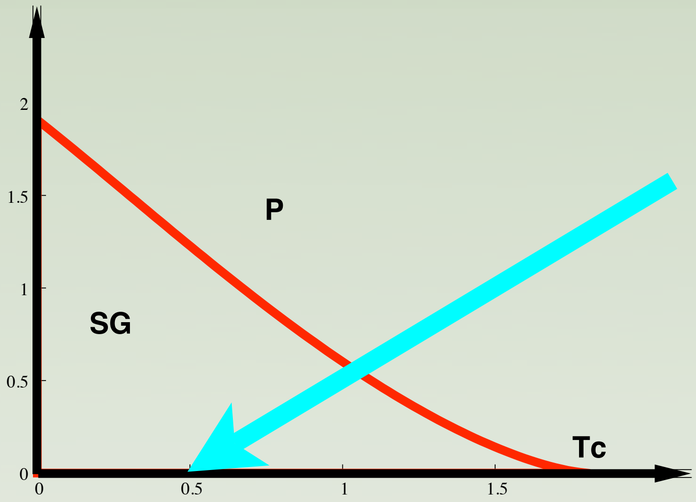

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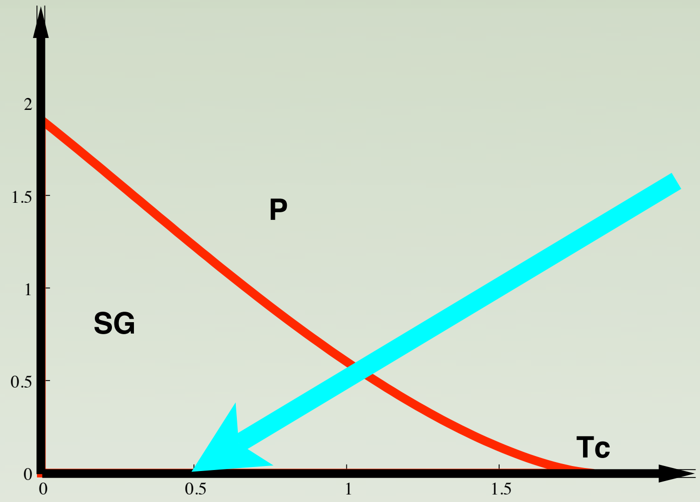

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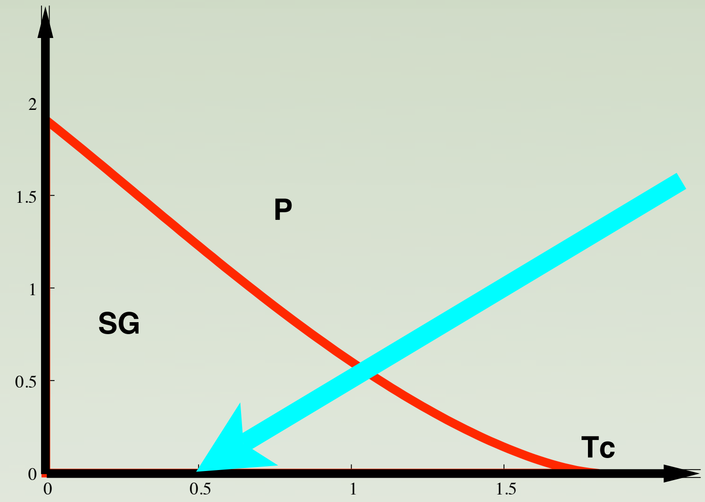

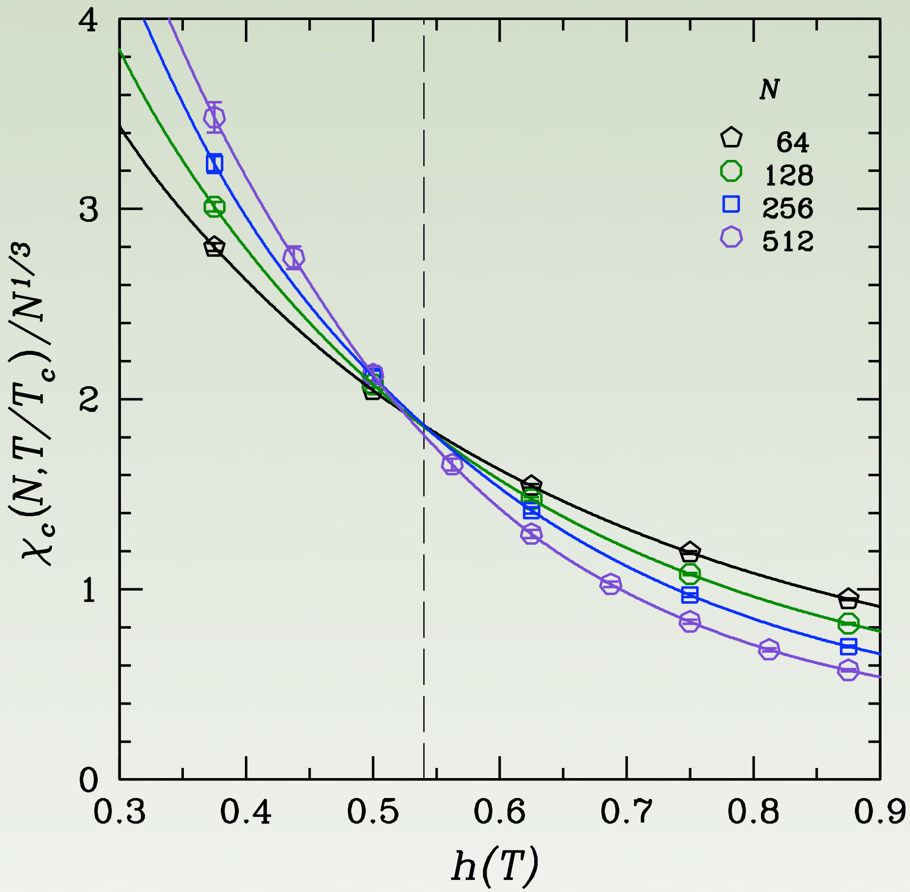

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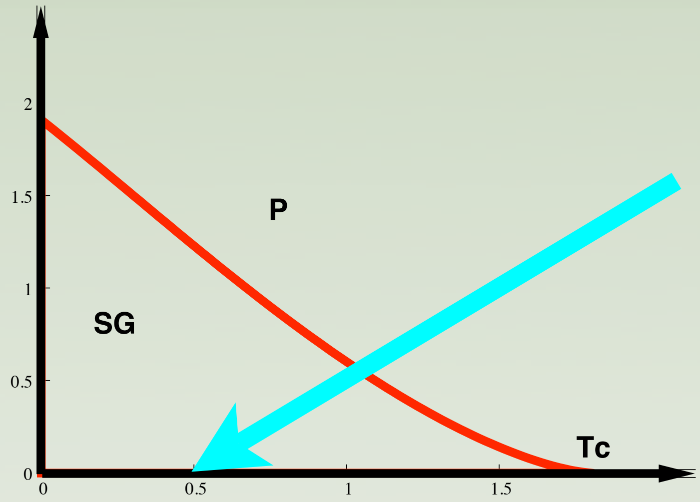

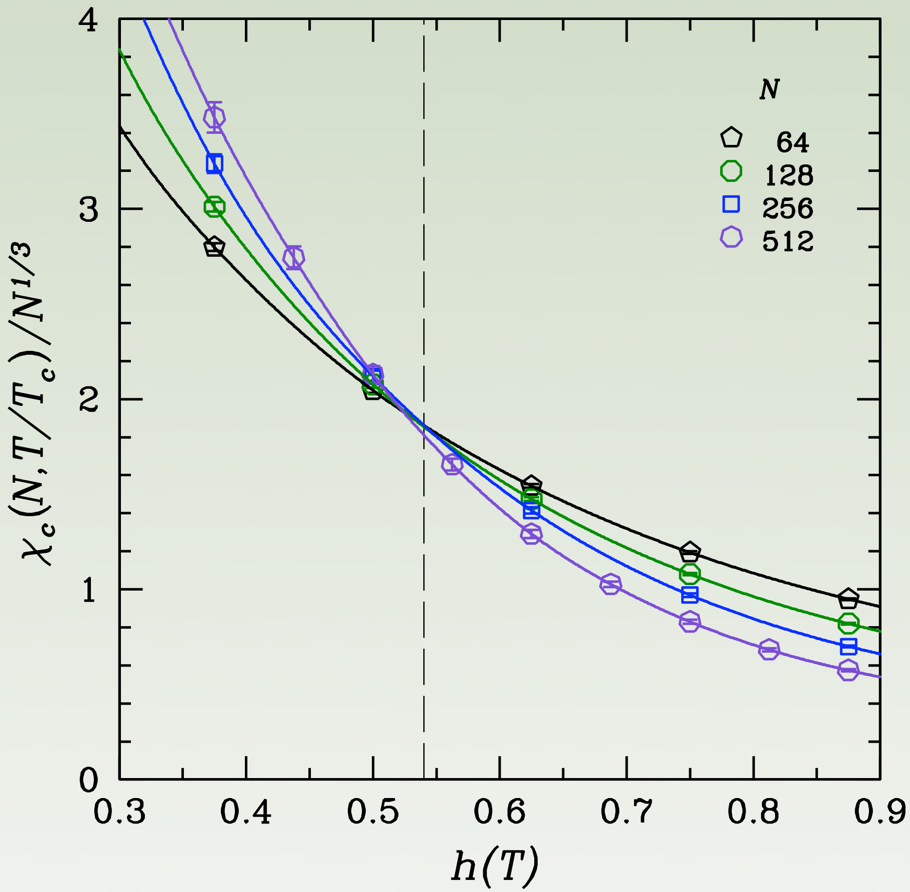

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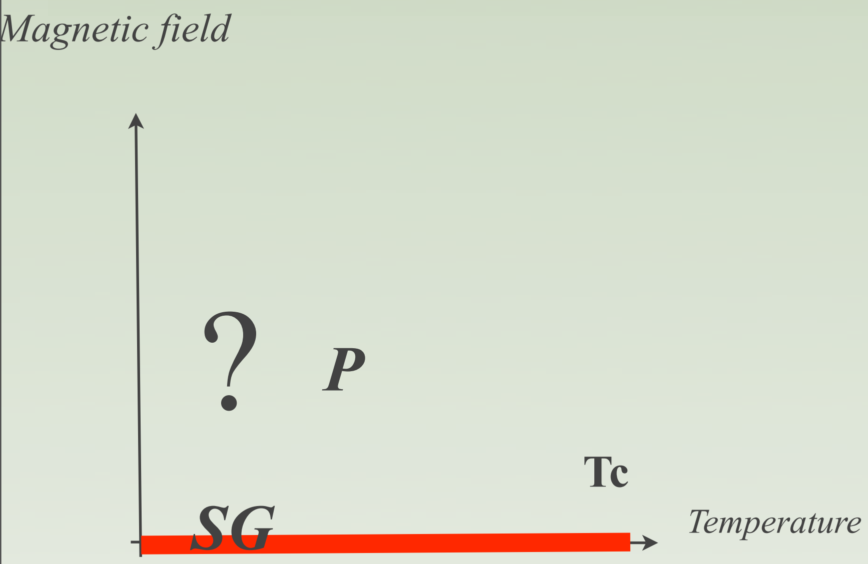
"Accord parfait" between simulation and theory again !
Simulations are able to see the AT line when it is there !

Spin glass transition in 3d



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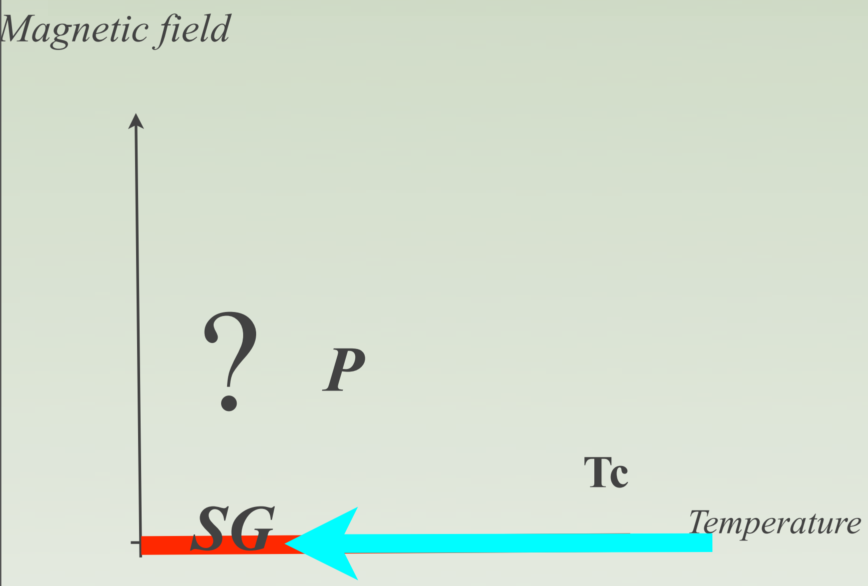

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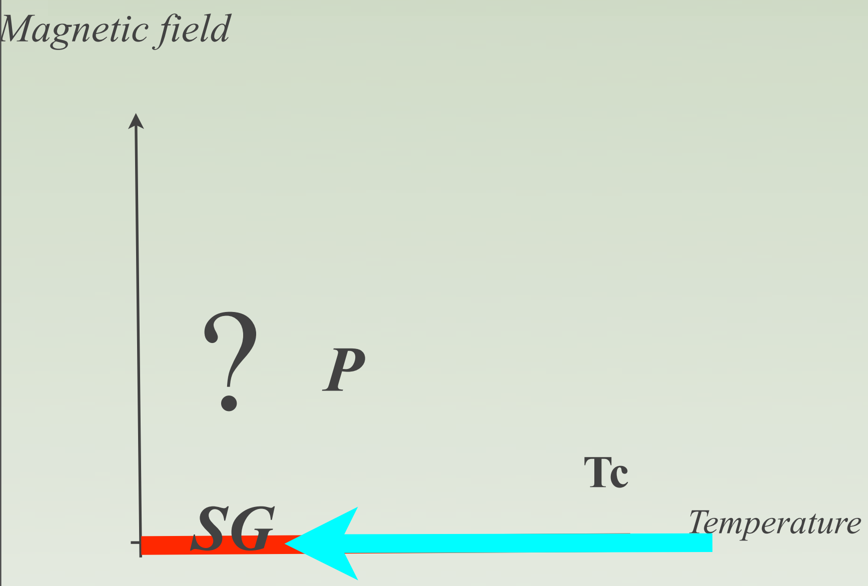


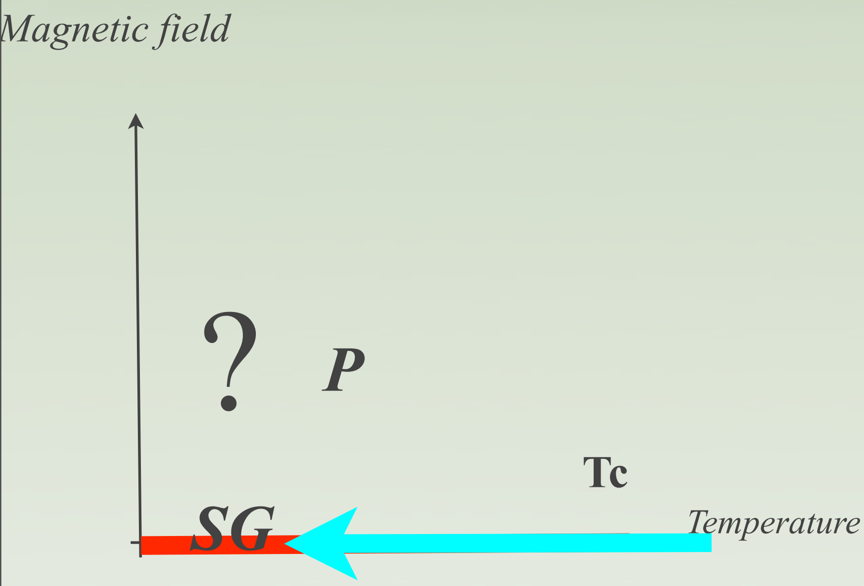
Compute the spin glass correlation length

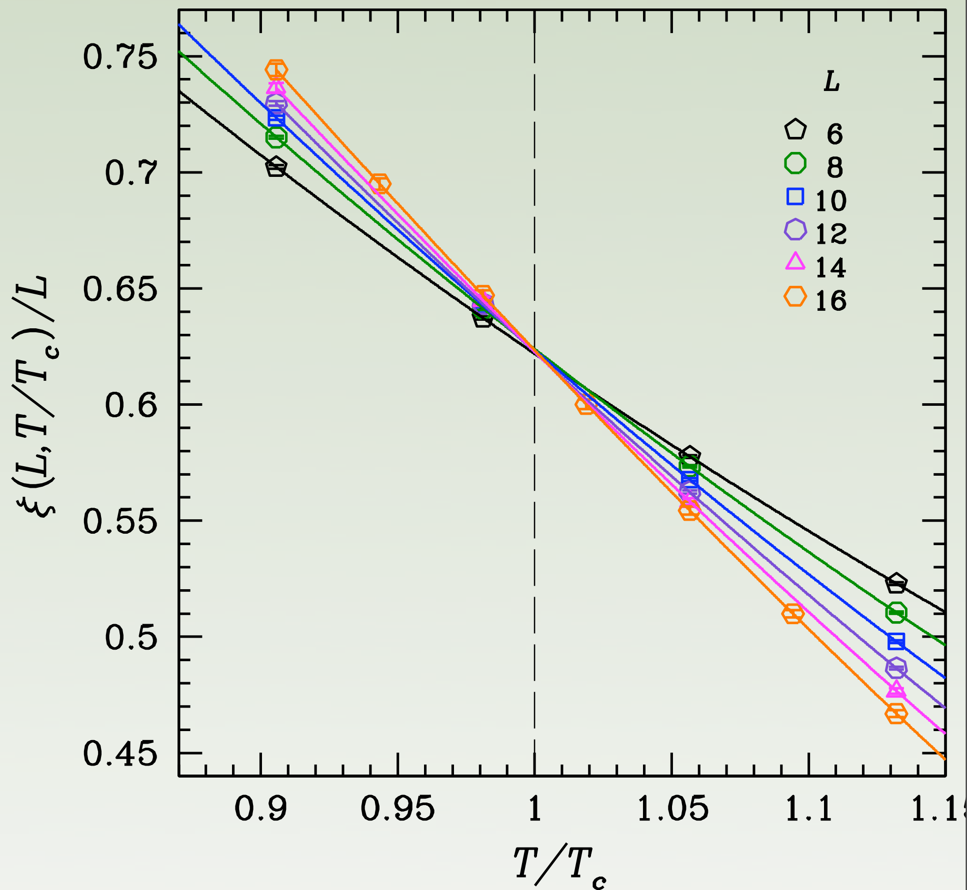
$$< SS_r >_c^2 \propto e^{-r/\xi}$$

From usual theory of phase transition, we expect that at the transition

$$\xi(L, T = T_c) \propto L$$

Spin glass transition in 3d





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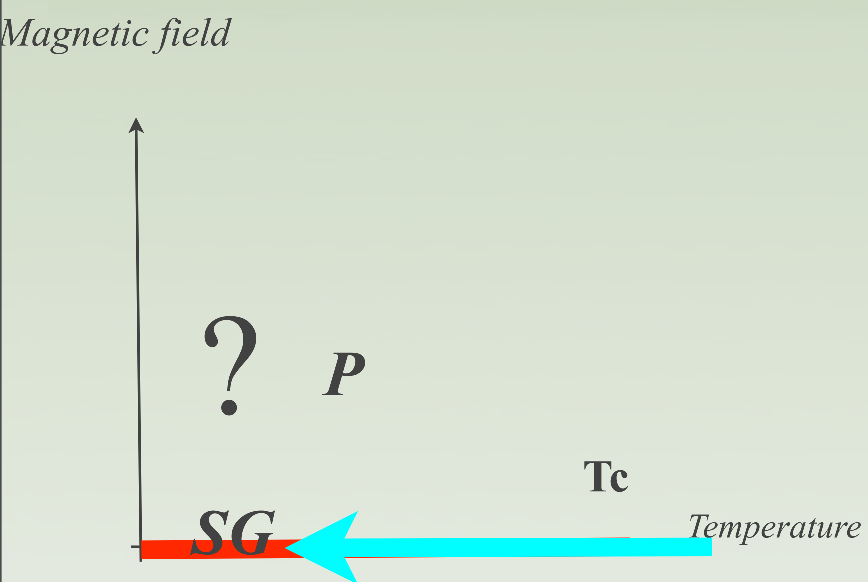

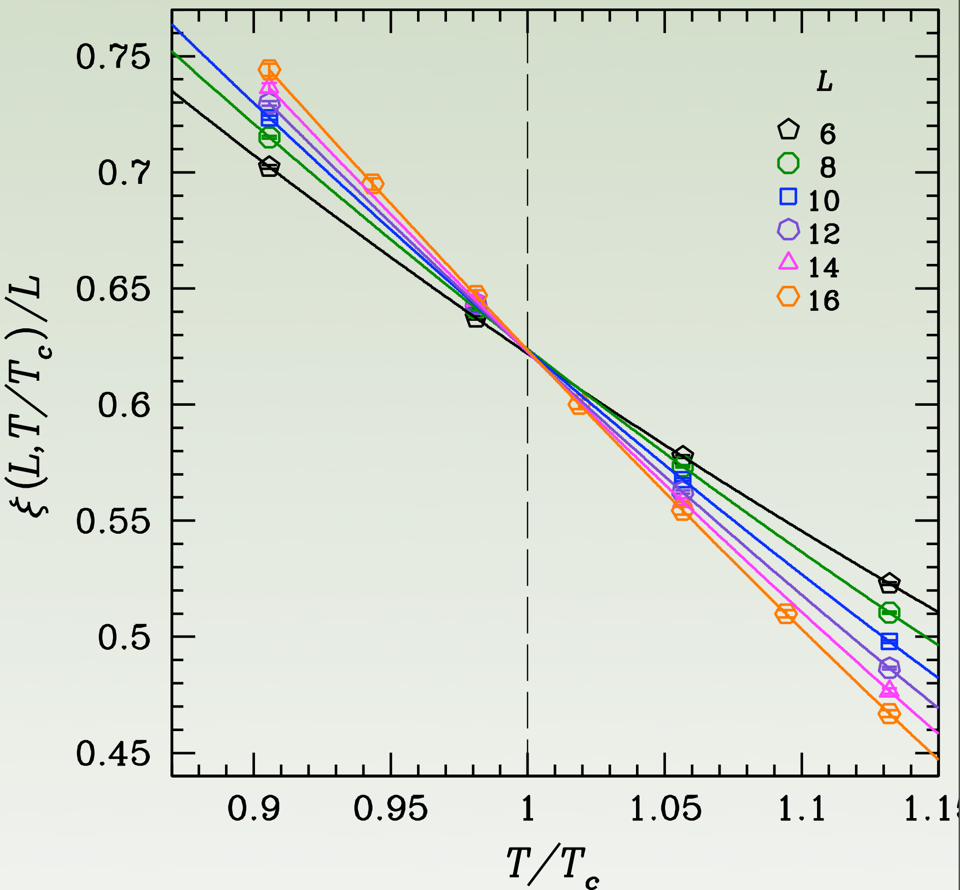

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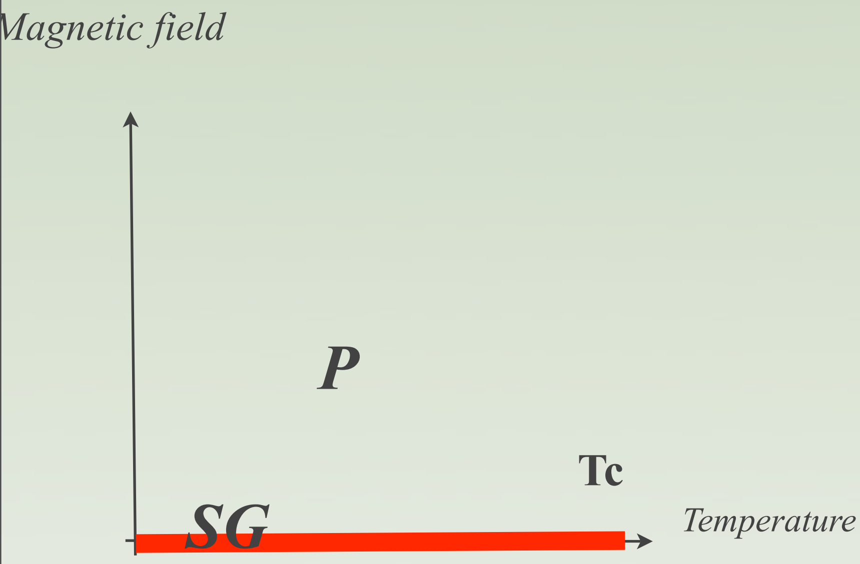
Good (and convincing) evidence for the presence of a SG transition

Spin glass in field in 3d spin glasses ?



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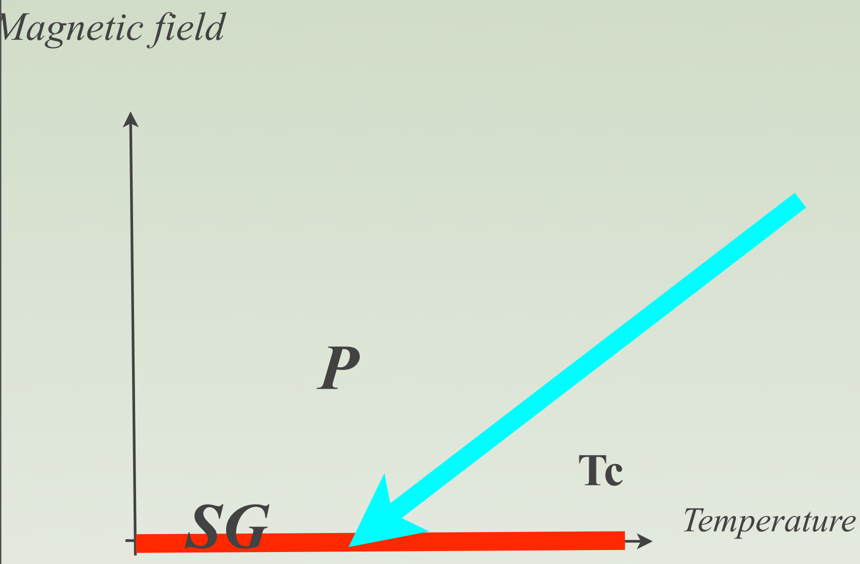

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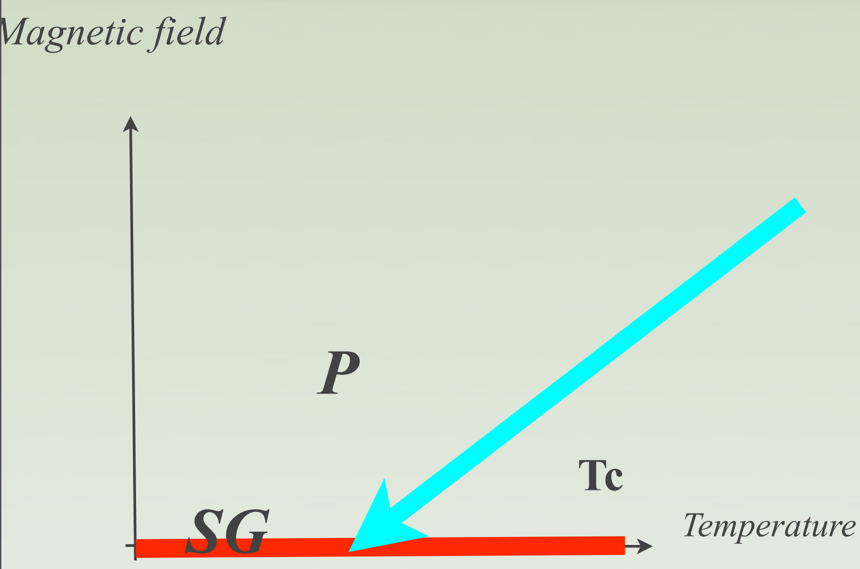

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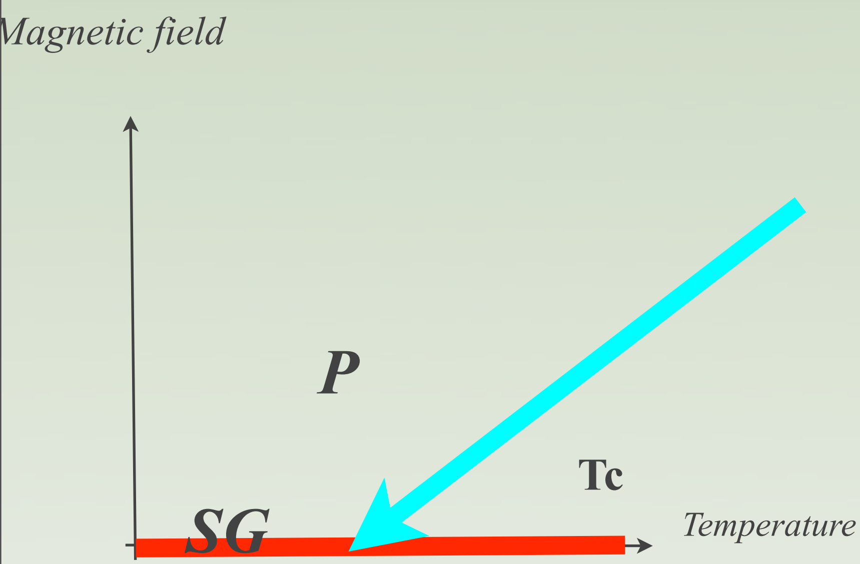

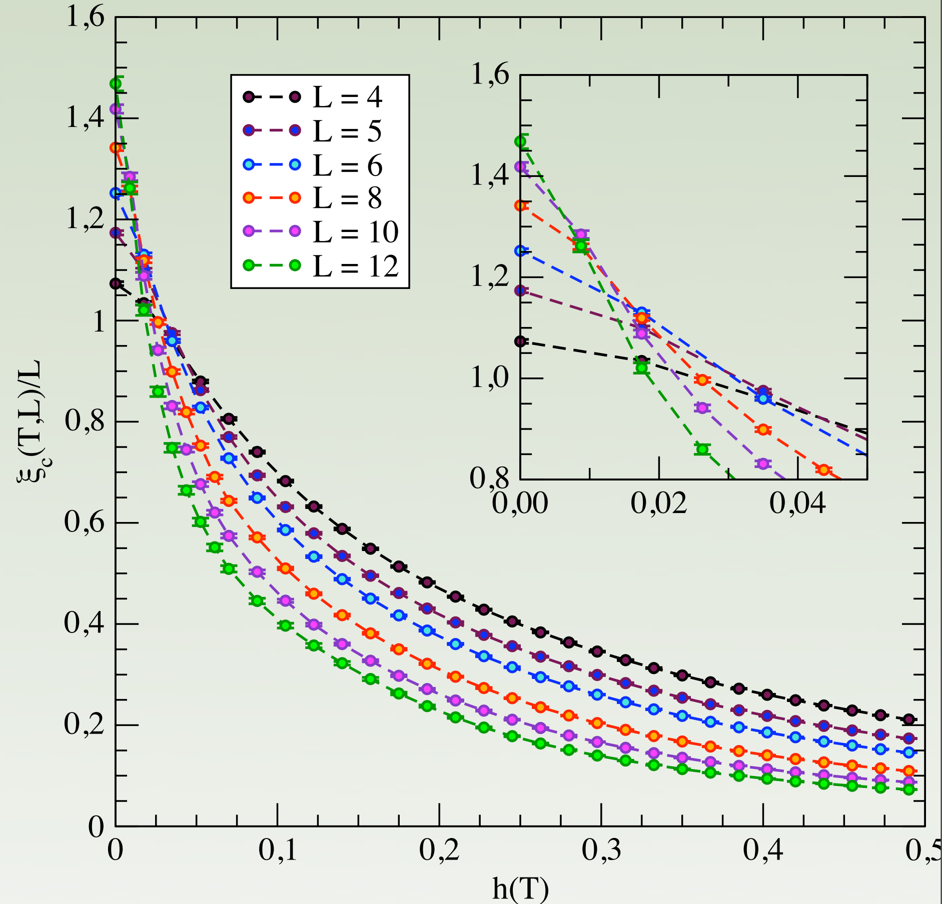

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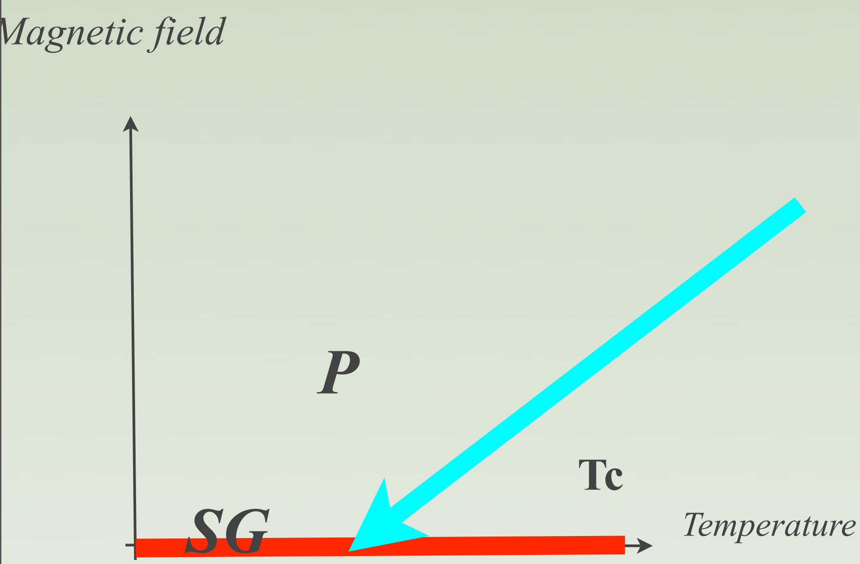

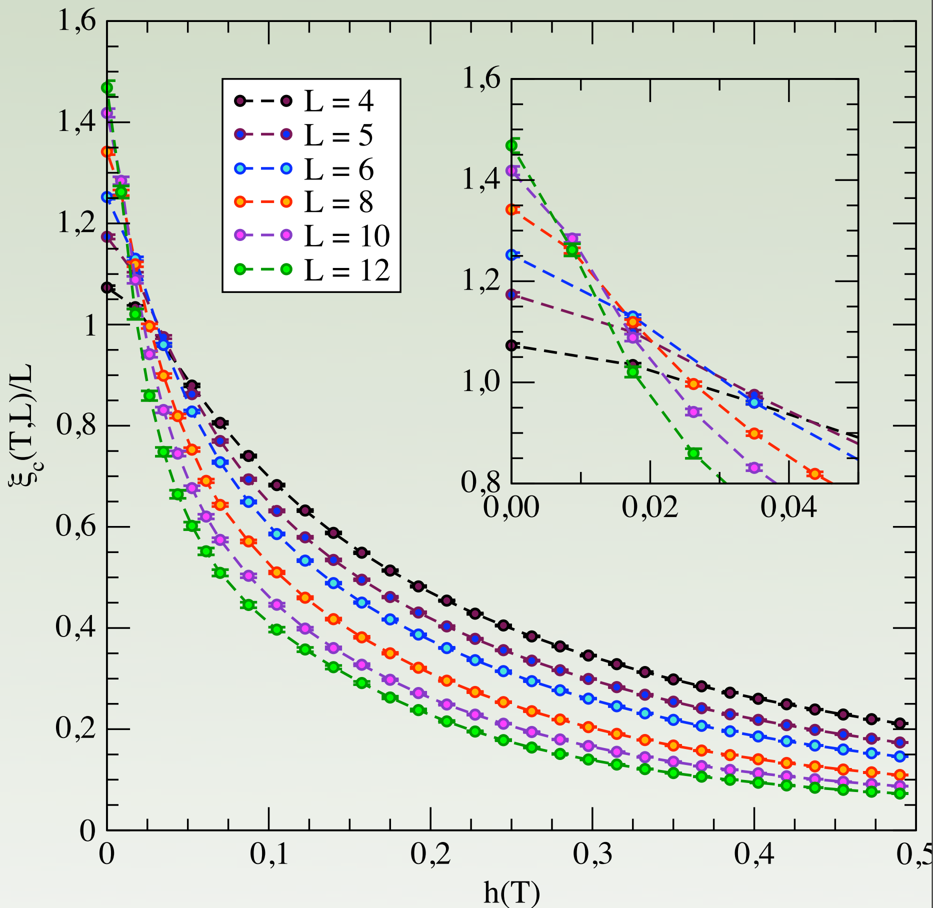

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Good (and convincing) evidence for the absence of a SG transition in field

Temperature and Disorder chaos

FK & O. Martin, EPJ B 01
FK & J.-P. Bouchaud, EPL 05
H. Kaztgraber & **FK**, PRL 07

Random energies strike back:The Re-Rem

Let us consider 2^N *"states" with free energies* $F_i=E_i-TS_i$

$$\rho_E(E_i) = \frac{\exp\left(-\frac{E_i^2}{N}\right)}{\sqrt{N\pi}} \quad \rho_S(S_i) = \frac{\exp\left(-\frac{S_i^2}{N^{2\alpha}}\right)}{\sqrt{N^{2\alpha}\pi}}$$

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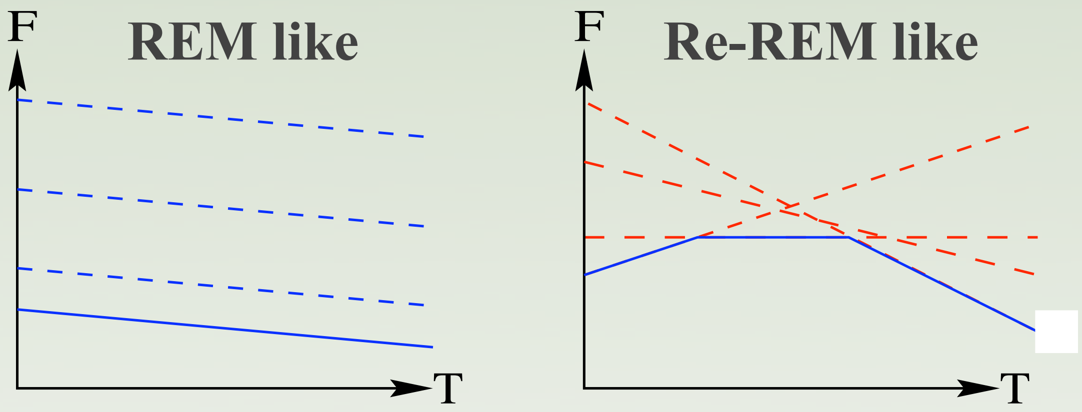

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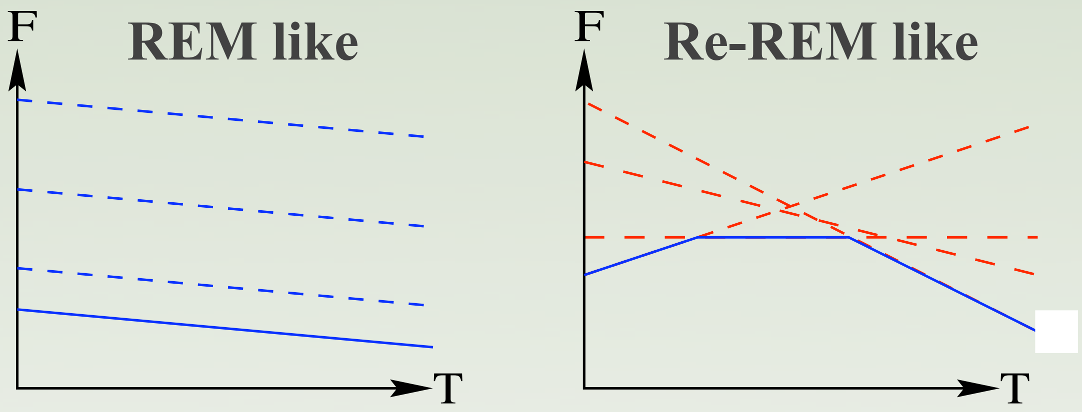

Temperature chaos arises from level crossings !

Free energies being random Gaussian, the model is solvable with a mapping to the REM:

$\alpha < 0.5$ | $\alpha = 0.5$ | $\alpha > 0.5$

$$T_c = \frac{1}{2\sqrt{\ln 2}} \qquad T_c = \frac{1}{\sqrt{4\ln 2 - 1}} \approx 0.75 \qquad T_c \to \infty$$

F non-extensive

Level crossings in the Re-Rem

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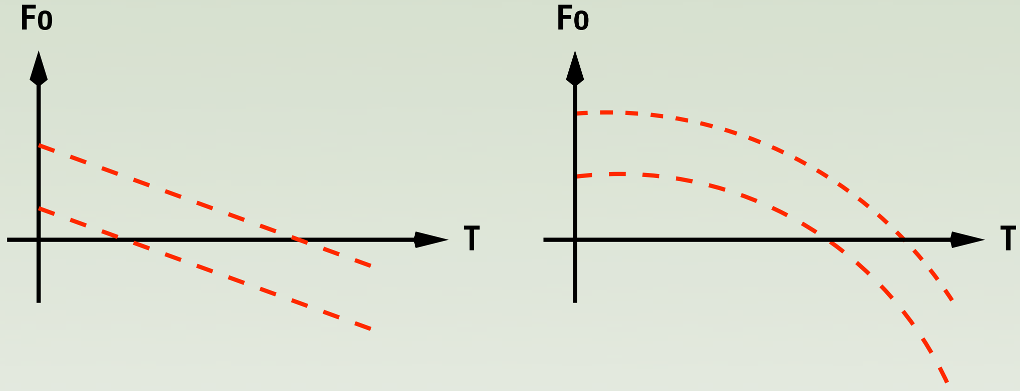
Free-energy of the lowest state is

$$F_0 \approx -\sqrt{2N \log 2}\sigma_f = -N\sqrt{\log 2(1 + T^2 N^{2\alpha - 1})}$$



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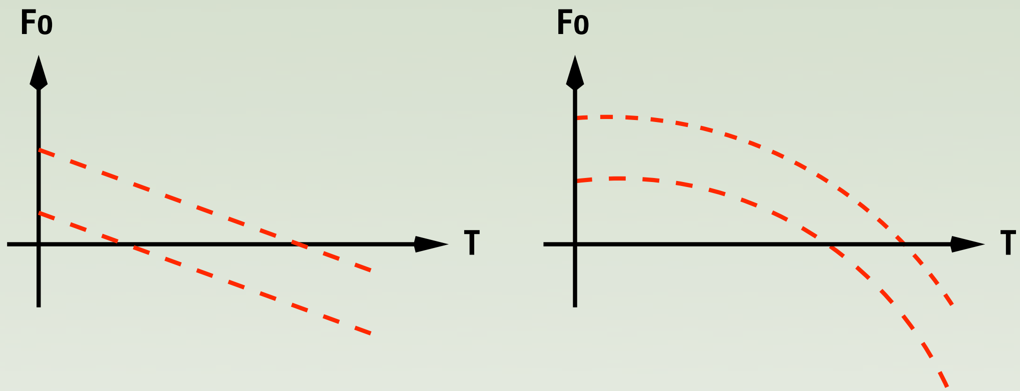
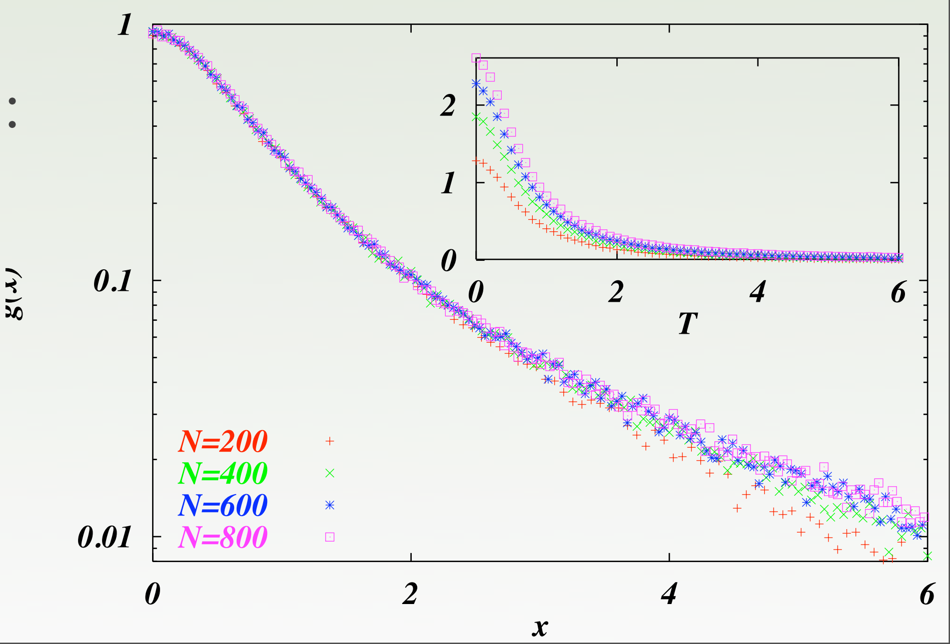


Number of lowest states crossings :

$$\mathcal{N}_N(T,T+\delta T) = N^\alpha \delta T g(\frac{T}{N^{0.5-\alpha}})$$

High quality numerical determination of g(x) (up to 2^{800} levels simulated)



The Re-Rem

- States to state fluctuations induces extensive level crossing
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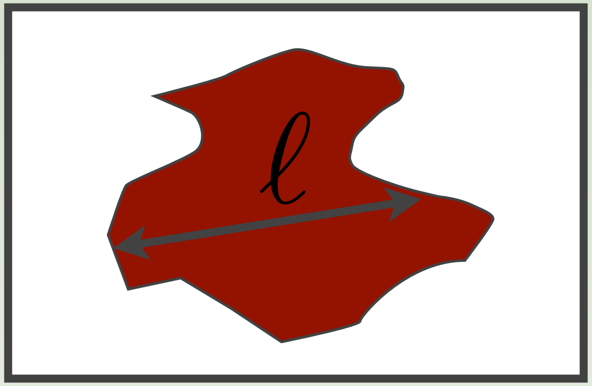
But what about finite dimensions ?

The Imrie-Ma argument again

- **Consider a small change in disorder**

$$J_{ij} \to J'_{ij} = \frac{J_{ij} + x_{ij}\Delta J}{\sqrt{1+\Delta J^2}},$$



Spin glass phase: *existence of large low energy excitations*

Consider the ground state and an excitation of size ℓ *with excess energy* $E = A\ell^{\theta}$

Changing the couplings, we have now $E = A\ell^{\theta} \pm B\Delta J \ell^{d_s/2}$

If $\theta < d_s/2$ *the ground state is unstable for large size.*

McKay, Berker & Kirkpatrick, PRL 82'
Fischer-Huse PRB 86'
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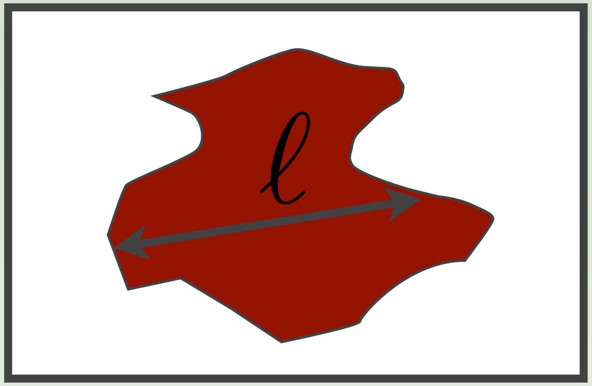

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"Chaotic" length scale: $$\xi_{chaos} \propto \Delta J^{-\frac{1}{d_s/2-\theta}}$$

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Disorder chaos

Ground states computations:
Two copies with couplings J and J'

$$J_{ij} \to J'_{ij} = \frac{J_{ij} + x_{ij}\Delta J}{\sqrt{1+\Delta J^2}},$$

"Chaotic" length $\Delta J^{-1/\xi}$

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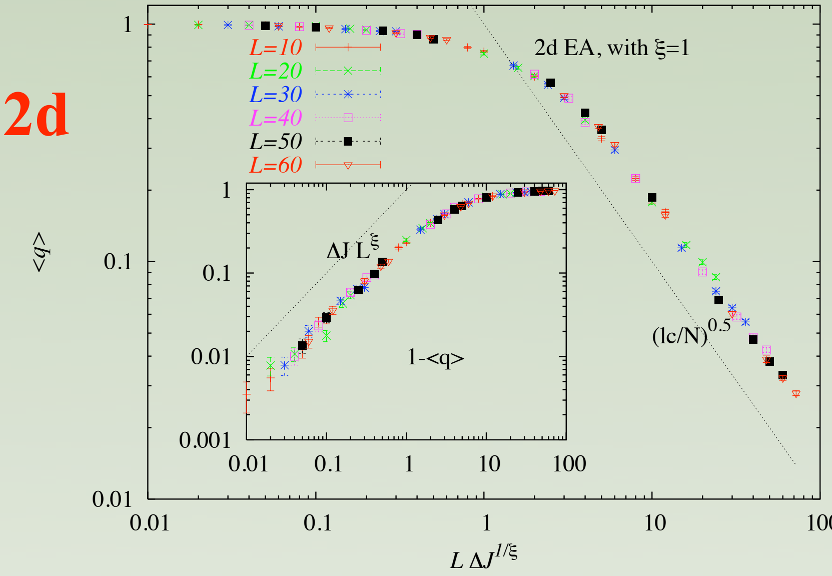

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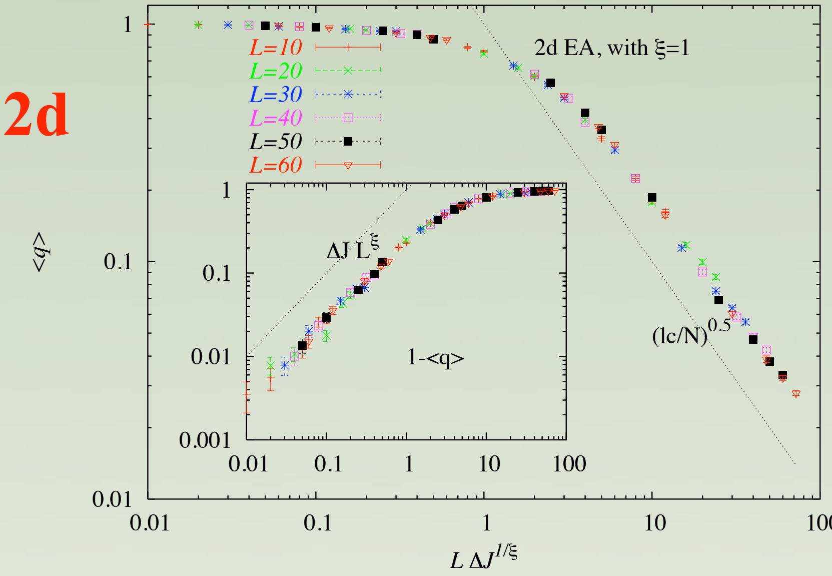

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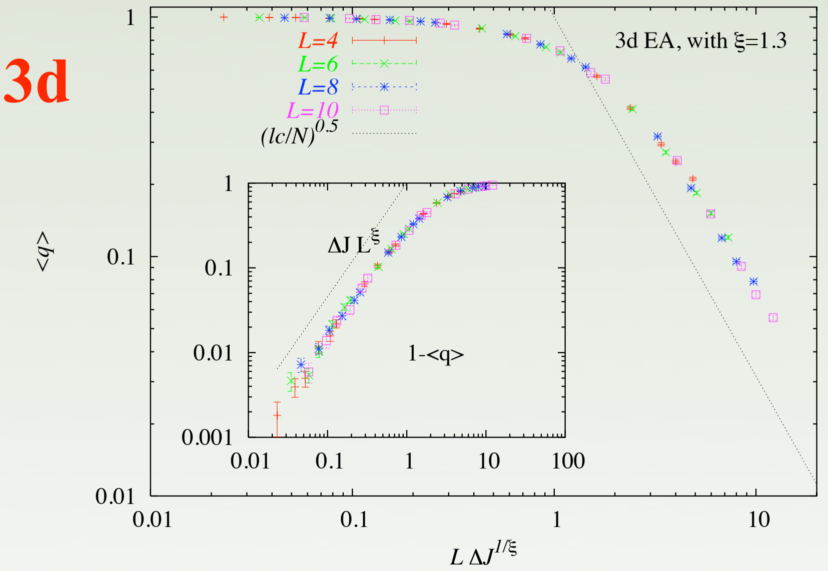

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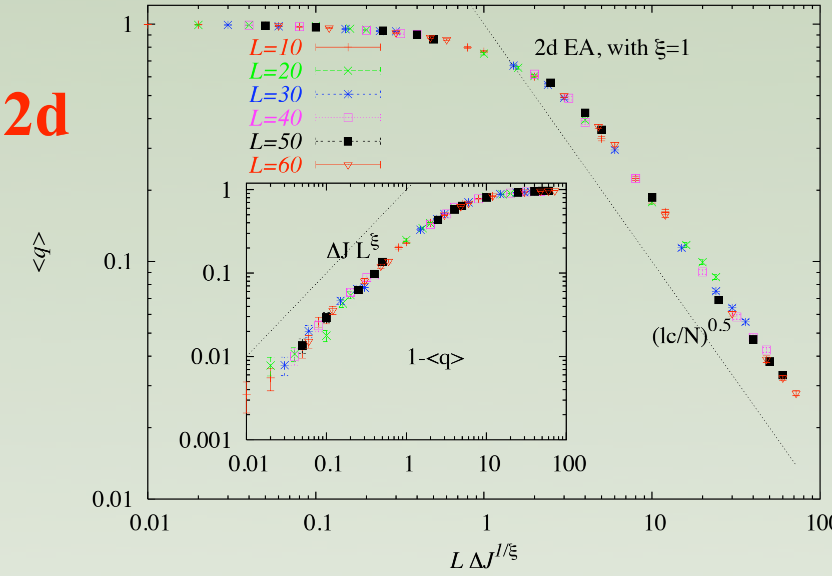

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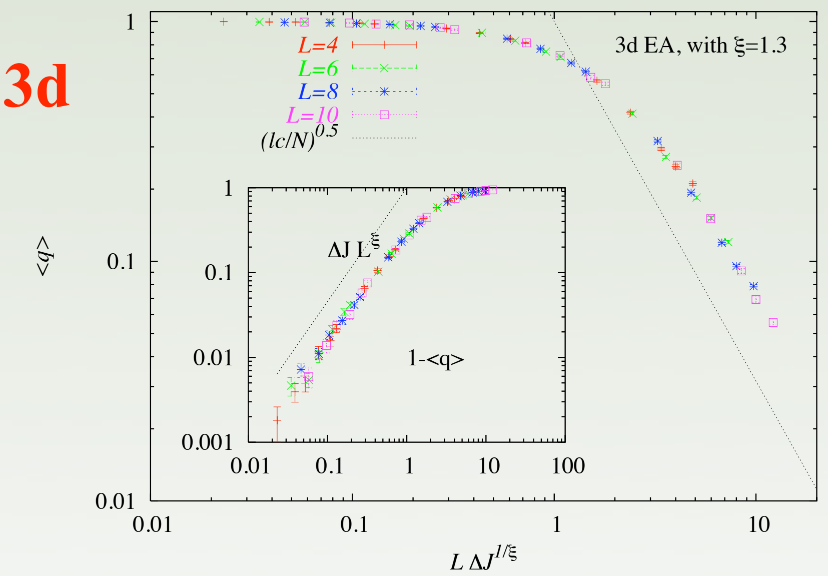
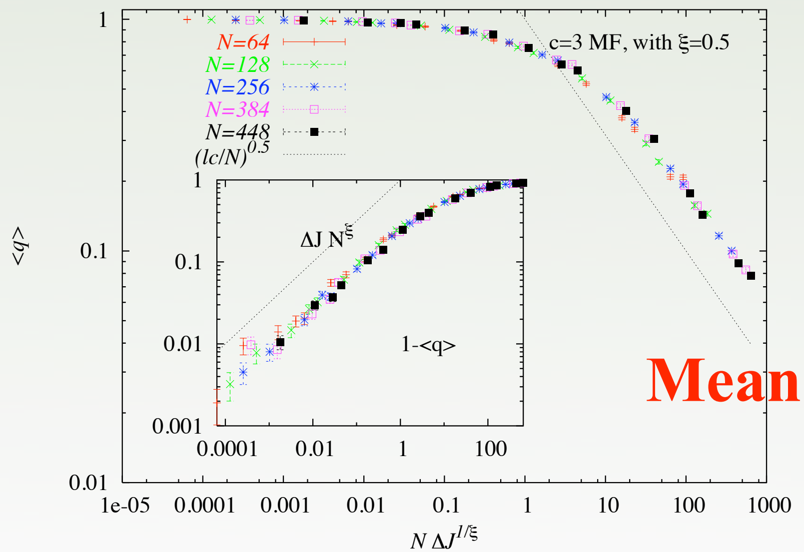




Mean field

The Imrie-Ma argument over and over again

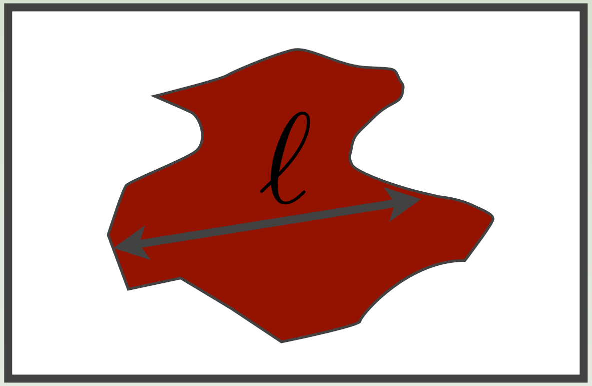
- **Consider a small change in temperature T+dT**

Spin glass phase: *existence of large low energy excitations*
Consider two temperatures and the free energy of one "droplet"

$$F(T_1) = \gamma(T_1)\ell^{\theta}$$

According to the droplet picture, the energy is almost T-independent

$$F(T_2) \approx F(T_1) + T_1 S(T_1) - T_2 S(T_2).$$



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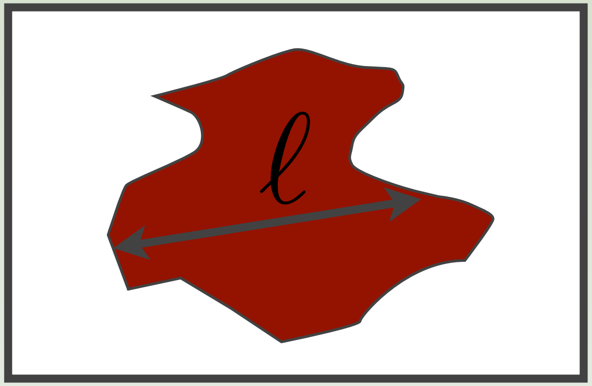

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$$\ell_c = \left(\frac{\gamma(T_1)}{T_2\ \sigma(T_2) - T_1\ \sigma(T_1)}\right)^{1/\zeta} \quad \text{with} \quad \zeta = \frac{d_s}{2} - \theta.$$

"Chaotic" length scale: $\ell_c(T_1, T_2) \propto \left(T_2^{3/2} - T_1^{3/2}\right)^{-1/\zeta}.$

Temperature and disorder chaos in 3d

Monte Carlo simulations
Two copies with couplings J and J'

$$J_{ij} \to J'_{ij} = \frac{J_{ij} + x_{ij}\Delta J}{\sqrt{1+\Delta J^2}},$$

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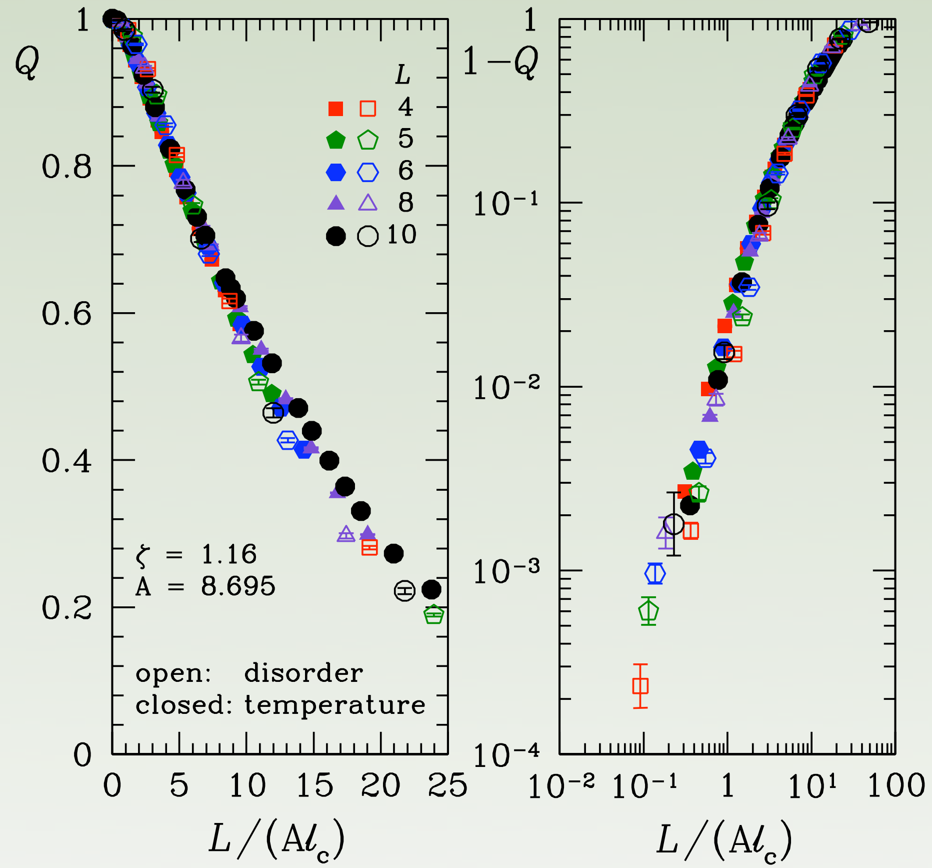

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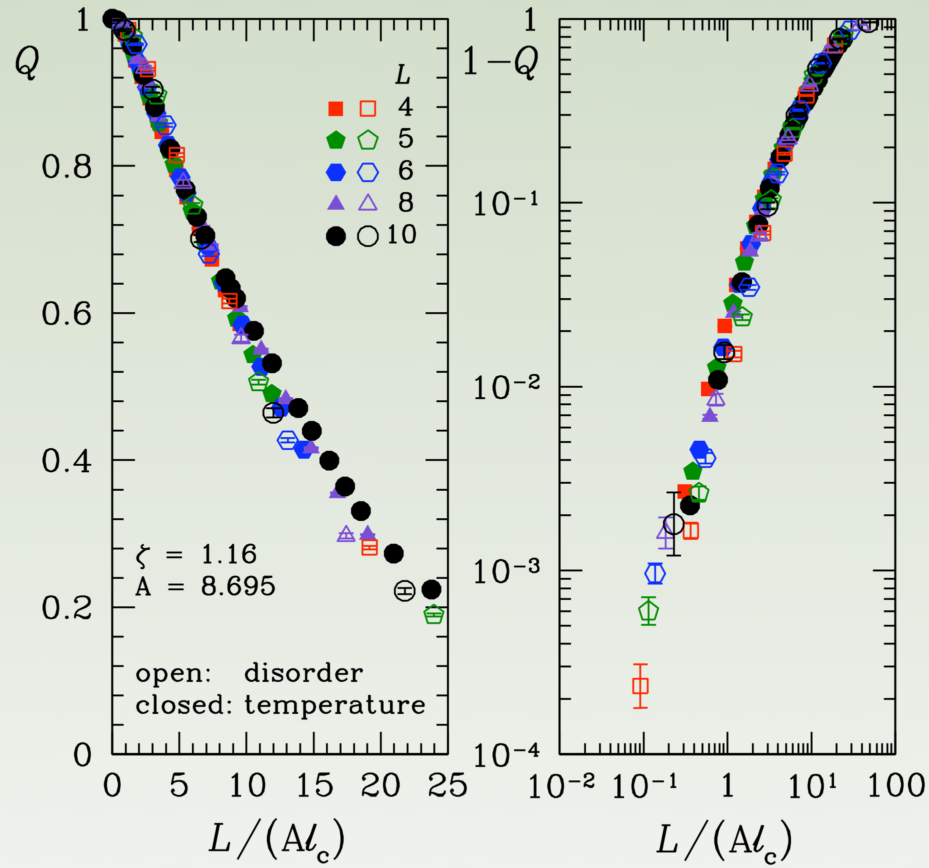

The two functions might even be the same !
(up to a rescaling factor)

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Final remark: A simple case where the mean field picture applies

Book on spin glasses

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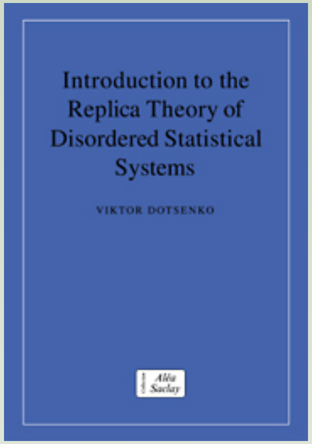

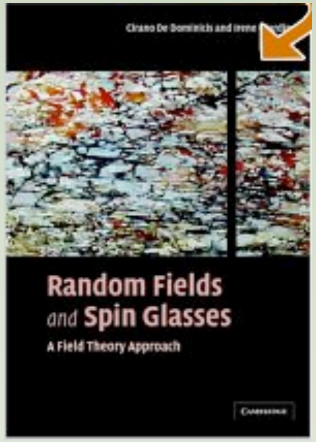

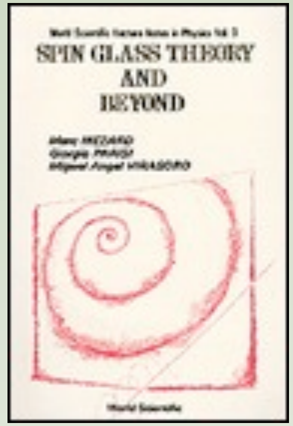

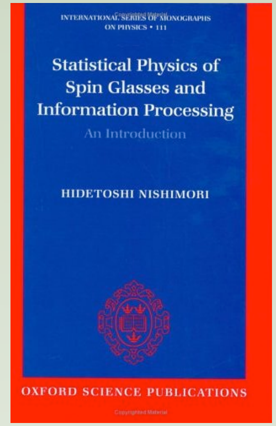

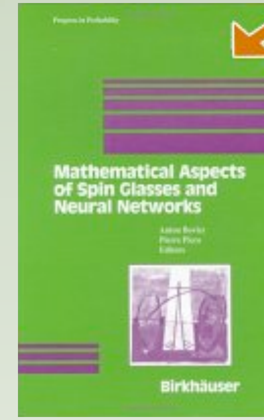

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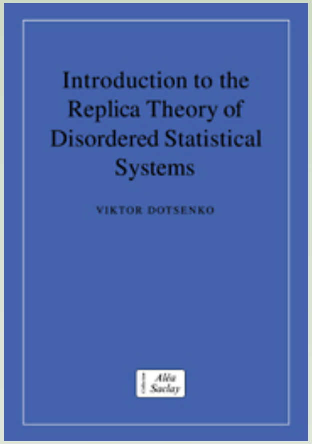

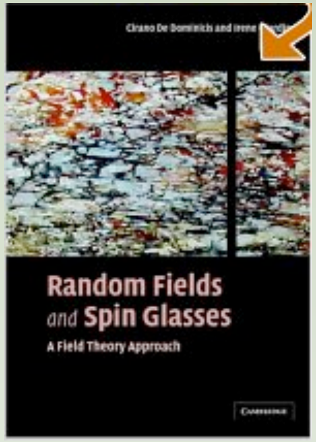

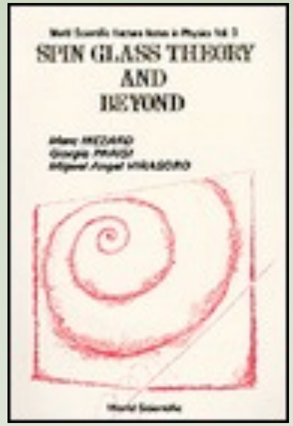

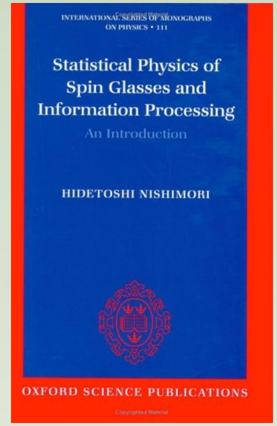

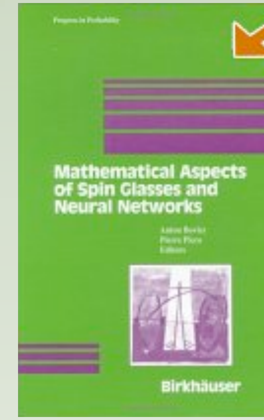

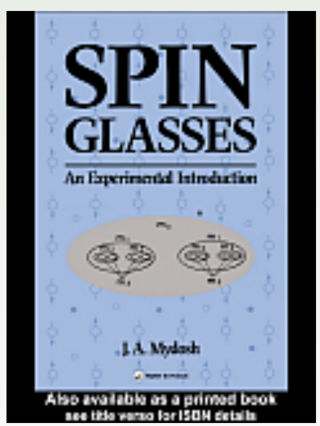

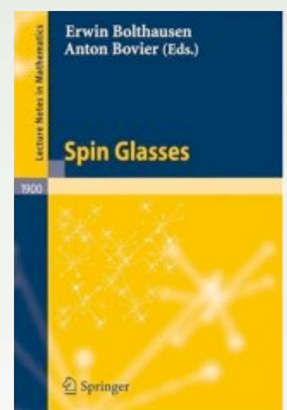

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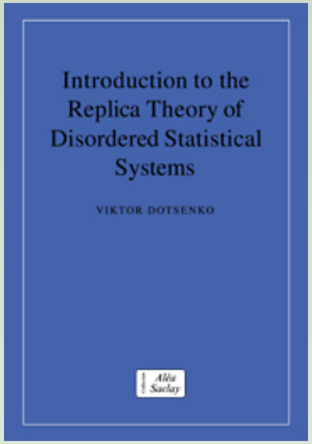
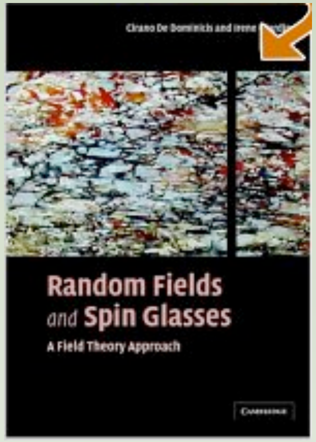
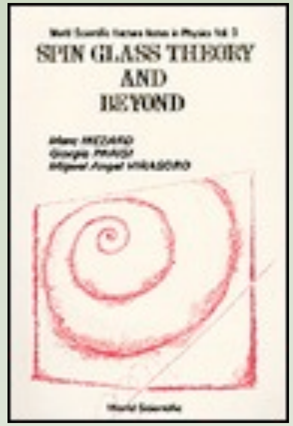
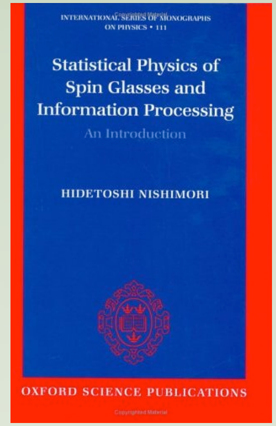
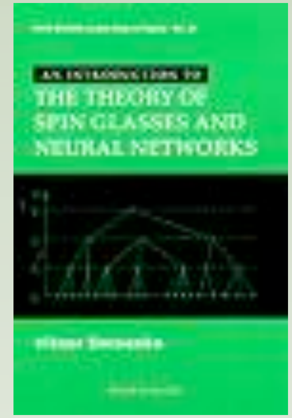
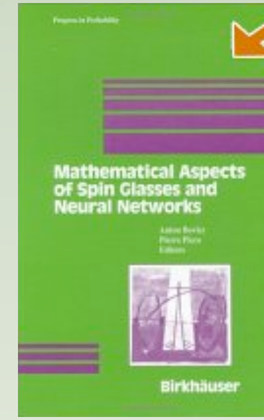




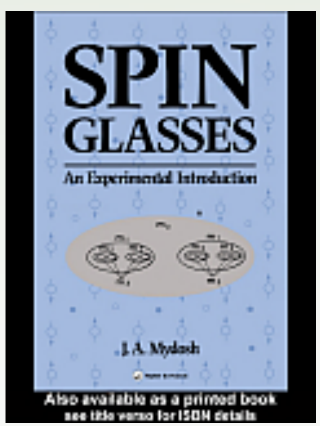


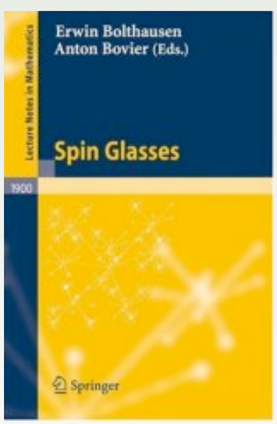






Many states....





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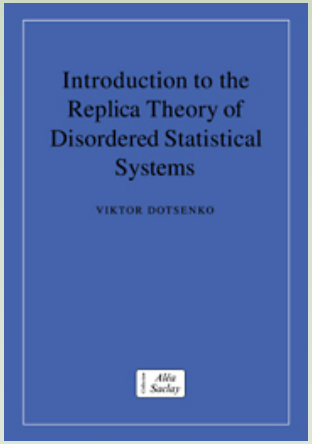
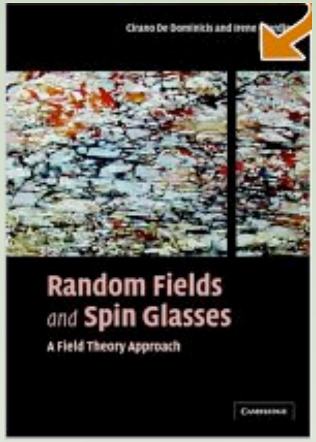
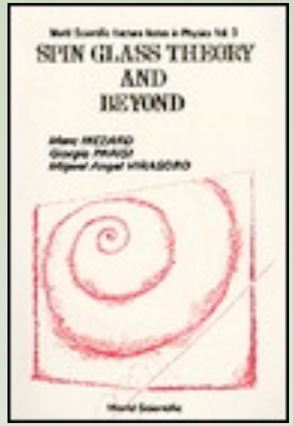
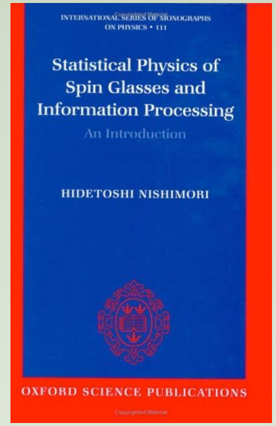
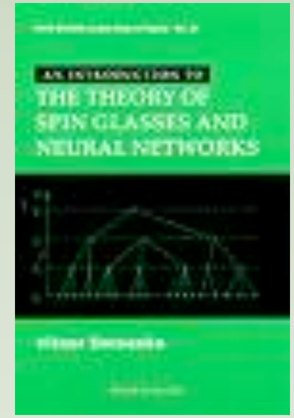
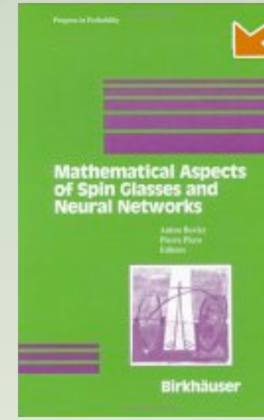






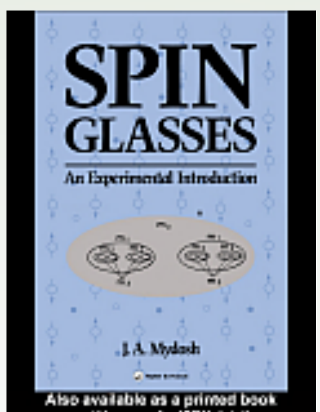


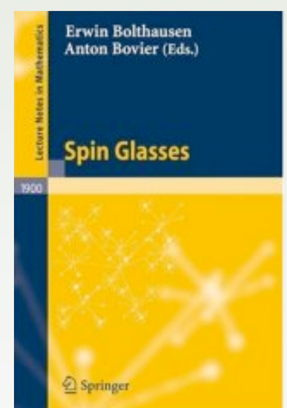




Many states....

...Non trivial overlap (in titles)...





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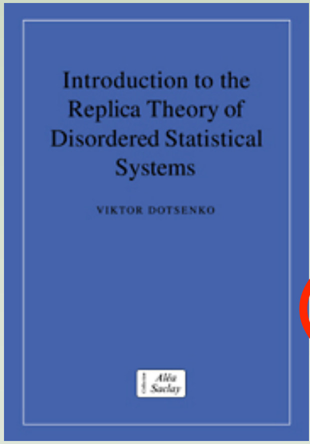
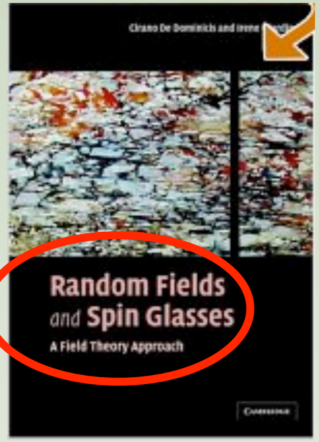
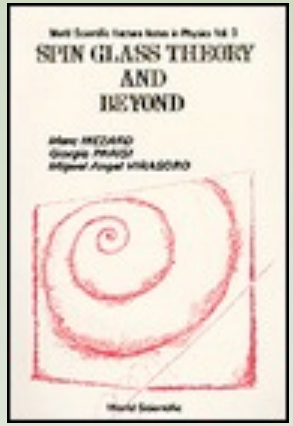
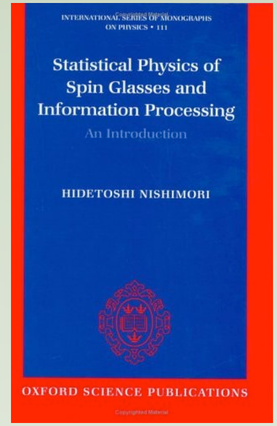
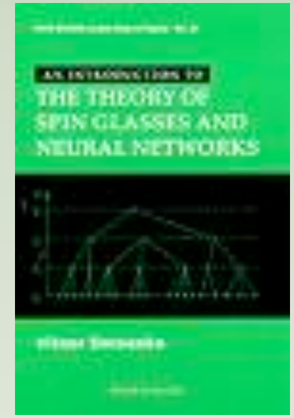
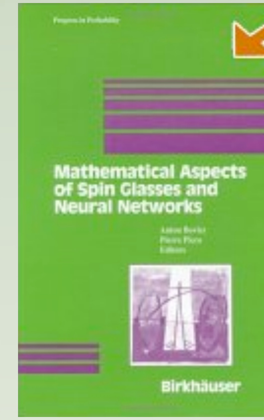






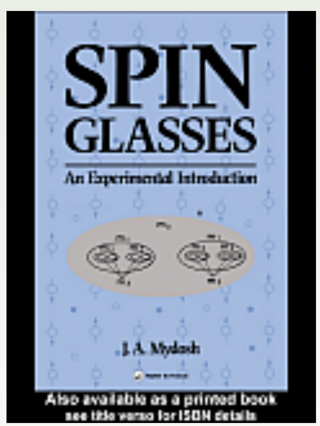


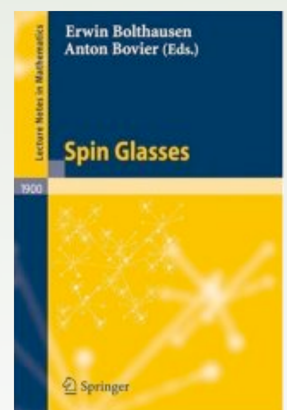




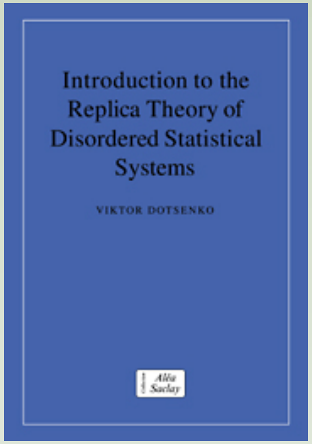
Many states....

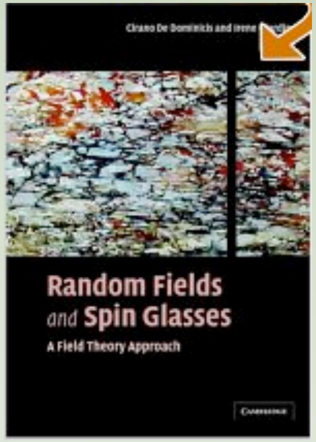
...Non trivial overlap (in titles)...

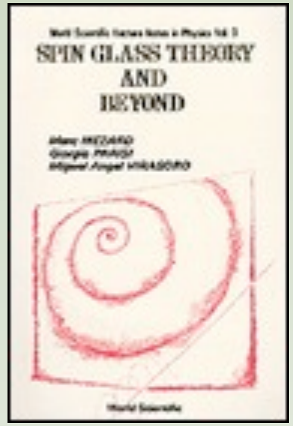


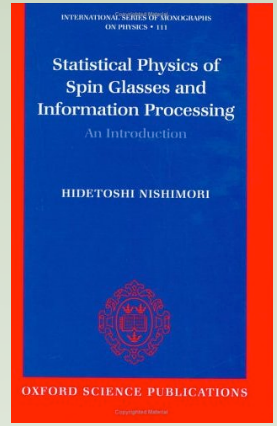


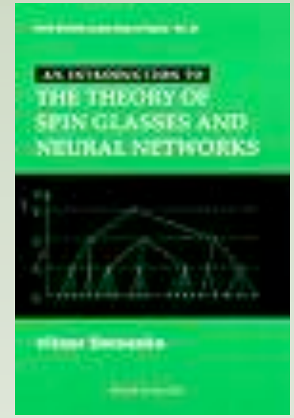
Book on spin glasses

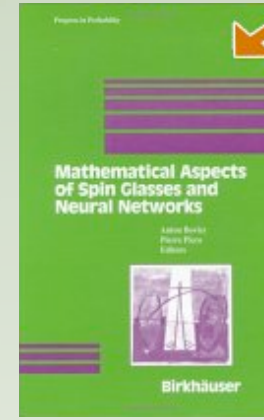












Many states....

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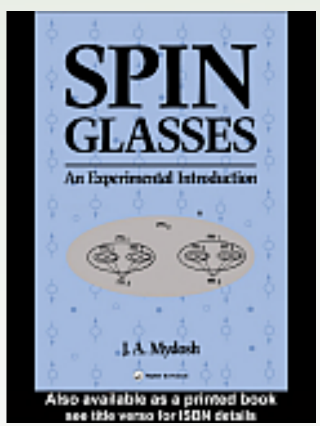

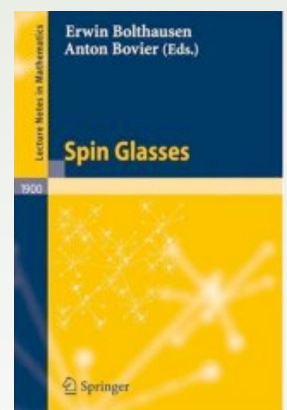

Book on spin glasses

Many states....

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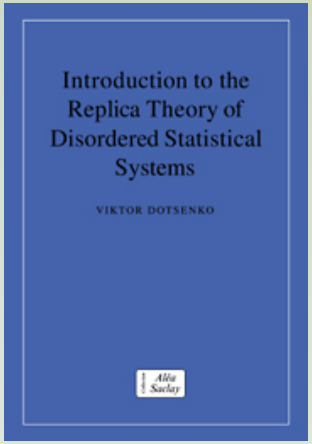
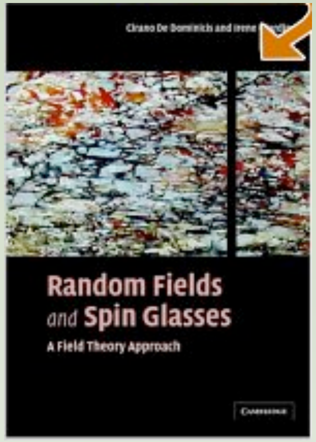
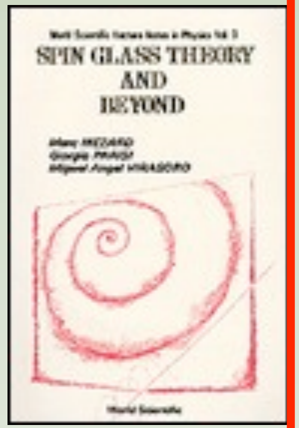
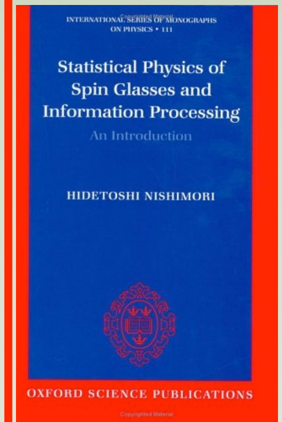
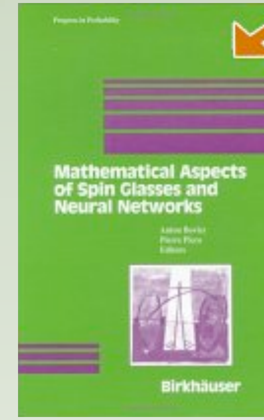
Book on spin glasses











Many states....

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...and clustering properties !