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Bound States of Three and Four Resonantly Interacting Particles¶

I. V. Brodsky[a], A. V. Klaptsov[b], M. Yu. Kagan[a], R. Combescot[c], and X. Leyronas[c]

[a] *Kapitza Institute for Physical Problems, Russian Academy of Sciences, Moscow, 119334 Russia*
e-mail: kagan@kapitza.ras.ru

[b] *Russian Research Centre Kurchatov Institute, Moscow, 123182 Russia*

[c] *Laboratoire de Physique Statistique, Ecole Normale Superieure, 75231 Paris Cedex 05, France*

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We present an exact diagrammatic approach for the problem of dimer–dimer scattering in 3D for dimers being a resonance bound state of two fermions in a spin-singlet state, with corresponding scattering length a_F. Applying this approach to the calculation of the dimmer–dimer scattering length a_B, we recover exactly the already known result $a_B = 0.6\, a_F$. We use the developed approach to obtain new results in 2D for fermions and bosons. Namely, we calculate bound state energies for three *bbb* and four *bbbb* resonantly interacting bosons in 2D. For the case of resonance interaction between fermions and bosons, we exactly calculate bound state energies of the following complexes: two bosons plus one fermion *bbf*, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and three bosons plus one fermion *bbbf*. © 2005 Pleiades Publishing, Inc.

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1. INTRODUCTION

The physics of ultracold Fermi gases has been the subject of intensive investigations in recent years. In particular, the possibility of experimental observation of the crossover from the BCS to BEC limit due to Feshbach resonance is being actively discussed. In the vicinity of the resonance, the scattering length is very large, being positive on one side of the resonance and negative on the other side. In the limit of the positive scattering length for two fermions, the formation of weakly bound dimers consisting of two different fermions becomes energetically favorable. Far from the resonance on the positive side, a weakly interacting gas of these composite bosons exists. In this paper, we present a diagrammatic approach to an exact solution for dimmer–dimer elastic scattering, assuming that the (positive) scattering length greatly exceeds the characteristic radius r_0 of interaction between atoms (the so-called resonance approximation). As was first shown by Skorniakov and Ter-Martirosian [1], in the case of the three-body fermionic problem, the scattering length of a fermion on a weakly bound dimer is determined by a single parameter, the two-body scattering length a_F, and is equal to $1.18a_F$ in the zero-range approximation for the interatomic potential. The same situation holds in the case of the fermionic four-body problem, where the dimmer–dimer scattering amplitude is fully determined by the value of a_F.

In the first study of the crossover problem by Haussmann [2], the scattering length of composite bosons a_B was found in the lowest order (Born approximation) and is equal to $2a_F$. Later, Pieri and Strinati [3], using a diagrammatic approach, greatly improved this result and found that in the ladder approximation the scattering length of composite bosons is approximately equal to $0.75a_F$. However, the ladder approximation, strictly speaking, is not valid, because it misses an infinite number of diagrams, which give a contribution of the same order of magnitude as those taken into account. Recently, Petrov, Salomon, and Shlyapnikov [4] have found the exact value of the scattering length of composite bosons $a_B = 0.6a_F$. They solved the Schrödinger equation using the well-known method of pseudopotentials. Below we show an exact solution of the scattering problem of two weakly bound dimers using the diagrammatic approach in the resonance approximation.

We use the developed approach to obtain new results for two different systems in the 2D case. Namely, we consider first a system of resonantly interacting bosons. We calculate exactly the three boson *bbb* and four boson *bbbb* bound state energies in this case. We also apply our approach to the 2D system of bosons resonantly interacting with fermions. Here we calculate exactly the bound state energies for the following complexes: two bosons plus one fermion *bbf*, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and three bosons plus one fermion *bbbf*.

This paper is a natural continuation of our previous results, where we predicted the possibility of two-fermion *ff* [5, 6] and two-boson *bb* [7] pairing, as well as composite fermion creation *fb* [8] in resonantly inter-

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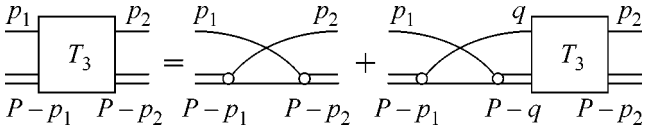

Fig. 1. Graphic representation of the equation for the full dimmer–fermion scattering vertex T_3.

acting ($a \gg r_0$) 2D Fermi–Bose gases and Fermi–Bose mixtures.

2. THREE-PARTICLE SCATTERING

As a preliminary exercise, we will rederive the result of Skorniakov and Ter-Martirosian for dimer–fermion scattering length using the diagrammatic method [9].

Following Skorniakov and Ter-Martirosian in the presence of the weakly bound resonance level $-|E_b|$ in a two-particle cross section, we can limit ourselves to the zero-range interaction potential between fermions. A two-fermion vertex can be approximated by a simple one-pole structure, which reflects the presence of the s-wave resonance level in a spin-singlet state:

$$\begin{aligned} T_{2\alpha\beta;\,\gamma\delta}(P) &= T_2(P)(\delta_{\alpha,\gamma}\delta_{\beta,\delta} - \delta_{\alpha,\delta}\delta_{\beta,\gamma}) \\ &= T_2(P)\chi(\alpha,\beta)\chi(\gamma,\delta), \end{aligned} \tag{1}$$

$$T_2(P) = \frac{4\pi}{m^{3/2}}\frac{\sqrt{|E_B|} + \sqrt{\mathbf{P}^2/4m - E}}{E - \mathbf{P}^2/4m + |E_B|},$$

where $P = \{E, \mathbf{P}\}$, E is the total energy and $\mathbf{P}$ is the total momentum of incoming particles, m is the fermionic mass, $|E_B| = 1/ma_F^2$. Indices α, β and γ, δ denote spin states of incoming and outgoing particles. The function $\chi(\alpha, \beta)$ stands for the spin singlet state, $\chi(\alpha, \beta) = \delta_{\alpha,\uparrow}\delta_{\beta,\downarrow} - \delta_{\alpha,\downarrow}\delta_{\beta,\uparrow}$.

The simplest process that contributes to dimer–fermion interaction is the exchange of a fermion. We will denote it as Δ_3. Its analytical expression is

$$\Delta_{3\alpha,\beta}(p_1, p_2; P) = -\delta_{\alpha,\beta}G(P - p_1 - p_2), \tag{2}$$

where $G(p) = 1/(\omega - \mathbf{p}^2/2m + i0)$ is a bare fermion Green's function. The minus sign on the right hand side of Eq. (2) comes from the permutation of two fermions. In order to obtain a full dimer–fermion scattering vertex T_3, we need to build a ladder from Δ_3 blocks. One can easily verify that the spin projection is conserved in every order of T_3 and thus $T_{3\alpha,\beta} = \delta_{\alpha,\beta}T_3$. An equation for T_3 will have the diagrammatic representation shown in Fig. 1, and in analytical form it is written as

$$\begin{aligned} T_3(p_1, p_2; P) &= -G(P - p_1 - p_2) \\ &- i\sum_q G(P - p_1 - q)G(q)T_2(P - q)T_3(q, p_2; P), \end{aligned} \tag{3}$$

where $\sum_q \equiv \int d^3q d\Omega/(2\pi)^4$. We can integrate out the frequency Ω in Eq. (3) by closing the integration contour in the lower half-plane, since both $T_2(P - q)$ and $T_3(q, p_2; P)$ are analytical functions in this region. Moreover, if we are interested in the low-energy s-wave dimmer–fermion scattering length a_3, we can safely put $P = \{E, \mathbf{P}\} = \{-|E_b|, 0\}$ and $p_2 = 0$. The full vertex T_3 is connected with a_3 by the following relation:

$$\left(\frac{8\pi}{m^2 a_F}\right)T_3(0, 0; -|E_b|) = \frac{3\pi}{m}a_3. \tag{4}$$

Introducing a new function $a_3(\mathbf{k})$ according to the formula

$$\left(\frac{8\pi}{m^2 a_F}\right)T_3(\{k^2/2m, \mathbf{k}\}, 0; -|E_b|) = \frac{3\pi}{m}a_3(\mathbf{k}) \tag{5}$$

and substituting it in Eq. (3), we obtain the Skorniakov–Ter-Martirosian equation for the scattering amplitude:

$$\begin{aligned} \frac{3/4a_3(\mathbf{k})}{\sqrt{m|E_B|} + \sqrt{3k^2/4 + m|E_B|}} &= \frac{1}{k^2 + m|E_b|} \\ - 4\pi\int\frac{a_3(\mathbf{q})}{q^2(k^2 + q^2 + \mathbf{kq} + m|E_b|)}&\frac{d^3q}{(2\pi)^3}. \end{aligned} \tag{6}$$

Solving this equation, one obtains the well-known result for dimer–fermion scattering length $a_3 = a_3(0) = 1.18a_F$.

3. DIMER–DIMER SCATTERING

Now we can proceed to the problem of dimer–dimer scattering. This problem was previously solved by Petrov *et al.* [4] by studying the Shrödinger equation for a 4-fermion wave function.

Inspired by the work of Petrov *et al.* [4] we are looking for a special vertex, which describes an interaction of two fermions constituting a first dimer with a second dimer as a single object. An obvious candidate for this vertex would be the sum of all diagrams with two fermionic and one dimer incoming line. It would be natural to suppose that these diagrams should have the same set of outgoing lines—two fermionic and one dimmer. However, in this case there will be a whole set of disconnected diagrams contributing to our sum that describes the interaction of a dimer with only one fermion. As was pointed out by Weinberg [10], one can construct a good integral equation of the Lippmann–Schwinger type only for a connected class of diagrams. Thus far, we have been forced to give attention to the vertex $\Phi_{\alpha\beta}(q_1, q_2; p_2, P)$ corresponding to the sum of all diagrams with one incoming dimer line, two incoming fermionic lines, and two outgoing dimer lines (see Fig. 2). This vertex $\Phi_{\alpha\beta}(q_1, q_2; p_2, P)$ is rather straight-

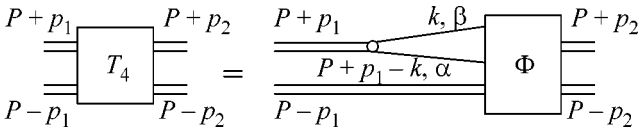

Fig. 2. Vertex Φ represents the full dimer–dimer scattering matrix T_4 with one dimer line being cut.

forwardly related to the standard dimer–dimer scattering vertex $T_4(p_1, p_2; P)$:

$$T_4(p_1, p_2; P) = \frac{i}{2}\sum_{k;\,\alpha,\beta}\chi(\alpha,\beta)G(P+p_1-k) \times G(k)\Phi_{\alpha\beta}(P+p_1-k, k; p_2, P). \quad (7)$$

Note that, by definition, in any order of interaction Φ contains only connected diagrams.

The spin part of the vertex $\Phi_{\alpha,\beta}$ has the simple form $\Phi_{\alpha,\beta}(q_1, q_2; P, p_2) = \chi(\alpha, \beta)\Phi(q_1, q_2; P, p_2)$. A diagrammatic representation of the equation on Φ is given in Fig. 3. One can assign some a physical meaning to the processes described by these diagrams. The diagram of Fig. 3a represents the simplest exchange process in dimmer–dimer interaction. The diagram of Fig. 3b accounts for the more complicated nature of a "bare" (irreducible by two dimer lines) dimmer–dimer interaction. Finally, the diagram in Fig. 3c allows for multiple dimer–dimer scattering via a bare interaction. The last term in Fig. 3 means that we should add another three diagrams analogous to Figs. 3a, 3b, and 3c, but with two incoming fermions (q_1 and q_2) exchanged. The analytical equation for the vertex Φ can be written as

$$\Phi(q_1, q_2; p_2, P) = -G(P-q_1+p_2)G(P-q_2-p_2) - i\sum_k G(k)G(2P-q_1-q_2-k)T_2(2P-q_1-k) \times \Phi(q_1, k; p_2, P) + \frac{1}{2}\sum_{Q,k} G(Q-q_1)G(2P-Q-q_2) \times T_2(2P-Q)T_2(Q)G(k)G(Q-k)\Phi(k, Q-k; p_2, P) + (q_1 \longleftrightarrow q_2). \quad (8)$$

Since we are looking for s-wave scattering length, we can put $p_2 = 0$ and $P = \{0, -|E_B|\}$. At this point we have a single closed equation on the vertex Φ in the momentum representation, which, we believe, is analogous to the equation of Petrov *et al*. in the coordinate representation. To make this analogy more prominent, we have to exclude frequencies from the equation. However, this exclusion is rather cumbersome and we leave it for a more extended publication.

The dimer–dimer scattering length is proportional to the full symmetrized vertex $T_4(p_1, p_2; P)$:

$$\left(\frac{8\pi}{m^2 a_F}\right)^2 T_4(0, 0; -2|E_b|, 0) = \frac{2\pi(2a_B)}{m}. \quad (9)$$

If one skips the second term in Eq. (8), i.e., omits the diagram in Fig. 3b, one arrives at the ladder approximation of Fieri and Strinati. The exact equation (8) corresponds to the summation of all diagrams. We have calculated the scattering length in the ladder approximation and the scattering length derived from the exact equation and obtained $0.78a_F$ and $0.6a_F$, respectively. Thus, our results in the ladder approximation are in agreement with the results of Pieri *et al*. [3] and in the general form with the results of Petrov *et al*. [4]. Note also that our approach allows one to find the dimmer–dimer scattering length in the 2D case (this problem was previously solved by Petrov *et al*. [11]).

Finally, we would like to mention that our results allow one to find the fermionic Green's function, chemical potential, and sound velocity as a function of a_F in the case of the dilute superfluid bose gas of dimers at low temperatures. The problem of dilute superfluid bose gas of di-fermionic molecules was solved by Popov [12] and later deeply investigated by Keldysh and Kozlov [13]. Those authors managed to reduce the gas problem to a dimer–dimer scattering problem in vacuum but were unable to express the dimmer–dimer scattering amplitude in a single two-fermion parameter. A direct combination of our results with those ones of Popov, Keldysh, and Kozlov allows one to get all the thermodynamic values of a dilute superfluid resonance gas of composite bosons. However, a more interesting subject for the application of our results is the high-temperature expansion for the thermodynamic potential and sound velocity in the temperature region $T \sim T_* \sim |E_B|$, where composite bosons start to appear.

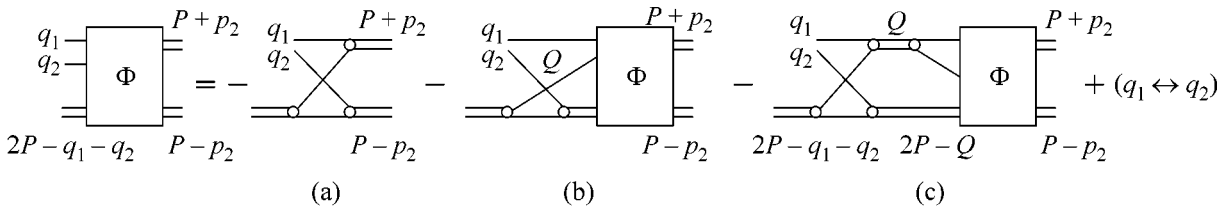

Fig. 3. Graphic representation of the equation on function Φ describing dimer–dimer scattering.

4. NEW RESULTS IN THE 2D CASE

As was first shown by Danilov [14] (see also the paper of Minlos and Fadeev [15]) in the 3D case, the problem of three resonantly interacting bosons cannot be solved in the resonance approximation. This statement stems from the fact that in the case of identical bosons the homogeneous part of Skorniakov–Ter-Martirosian equation (6) has a nonzero solution at any negative energies. The physical meaning of this mathematical artifact was elucidated by Efimov, who showed that a two-particle interaction leads to the appearance of an attractive $1/r^2$ interaction in a three-body system. Since, in the attractive $1/r^2$ potential in 3D, a particle falls to the center, short-range physics is important and one cannot replace the exact pair interaction by its resonance approximation.

In contrast, in the case of the 2D problem, the phenomenon of the particle falling to the center is absent, and one can utilize the resonance approximation [16]. Therefore, it is possible to describe three- and four-particle processes in terms of the two-particle binding energy $|E_B| = 1/ma^2$ only (below, for simplicity we will assume that all particles under consideration have the same mass m). We will omit the problem of composite particles\ scattering and will mainly concentrate on the problem of the binding energies of the complexes of three and four particles.

As in the 3D case, the cornerstone in the diagrammatic technique is the two-particle resonance scattering vertex T_2. For two resonantly interacting particles with total mass $2m$, it can be written in 2D as

$$T_2(P) = -\frac{4\pi}{m}\frac{\alpha}{\ln(\{\mathbf{P}^2/4m - E\}/|E_B|)}, \qquad (10)$$

where we introduce the factor $\alpha = \{1, 2\}$ in order to take into account whether or not two particles are indistinguishable. It is $\alpha = 2$ for the case of resonance interaction between identical bosons and $\alpha = 1$ for the case of resonance interaction between fermion and boson or for the case of two distinguishable bosons.

4.1. Three Particles in 2D

We start with a system of three resonantly interacting identical bosons—*bbb*—in 2D. The equation for the dimmer–boson scattering vertex T_3, which describes interaction of three bosons, has the same diagrammatic form as shown in the Fig. 1; however the rules of its analytical notation are changed. It can be written as

$$T_3(p_1, p_2; P) = G(P - p_1 - p_2) + i\sum_q G(P - p_1 - q) \times G(q)T_2(P - q)T_3(q, p_2; P), \qquad (11)$$

where $\sum_q \equiv \int d^3q\, d\Omega/(2\pi)^3$, $P = \{0, E\}$, and one should put $\alpha = 2$ for the two-particle vertex T_2 in Eq. (10). The opposite signs in Eq. (3) for fermions and Eq. (11) for bosons are due to the permutational properties of the particles involved: an exchange of fermions results in a minus sign, while an analogous exchange of bosons brings no extra minus. Finally we note that the three-particle s-wave (s-wave channel of boson–dimer scattering) binding energies E_3 correspond to the poles in $T_3(0, 0; -|E_3|)$ and, consequently, at energies $E = E_3$ the homogeneous part of Eq. (11) has a nontrivial solution. Solving Eq. (11), we find that a complex of three identical bosons has two s-wave bound states $E_3 = 16.52E_B$ and $E_3 = 1.267E_B$ in accordance with the previous results of Bruch and Tjon [16, 17].

Let us now consider a complex—*fbb*—consisting of one fermion and two bosons. As noted above, we consider bosons and fermions with equal masses $m_b = m_f = m$. We assume that a fermion–boson interaction U_{fb}, characterized by the radius of interaction r_{fb}, yields a resonance two-body bound state with an energy $E = -|E_B|$. At the same time, a boson–boson interaction U_{bb}, characterized by the interaction radius r_{bb} does not yield a resonance. Hence, if we are interested in low-energy physics, the only relevant interaction is U_{fb}, and we can ignore the boson–boson interaction U_{bb}, the latter would give small corrections on the order of $|E_B|mr_{bb}^2 \ll 1$ at low energies. In order to determine three-particle bound states, one has to find poles in the dimmer–boson scattering vertex T_3. Since we neglect boson–boson interaction U_{bb}, the vertex T_3 is described by the same diagrammatic equation of Fig. 1 as in the problems of three bosons. The analytical form of this equation also coincides with Eq. (11), with the minor correction that the resonance scattering vertex T_2 now corresponds to the interaction between a boson and a fermion and, therefore, we should put $\alpha = 1$ in Eq. (10) for T_2. Solving the equation for T_3, we find that the *fbb* complex has only one s-wave bound state with the energy $E_3 = 2.39E_B$.

Note that a complex—*bff*—consisting of a boson and two spinless identical fermions (or a complex $bf_\uparrow f_\downarrow$ of a boson and spin ↑ and spin ↓ fermions) with the resonance interaction U_{fb} does not have any three-particle bound states.

4.2. Four Particles in 2D

After solving the above three-particle problems, we may proceed to complexes consisting of four particles. First we will consider four identical resonantly interacting bosons *bbbb* [18]. Any two bosons would form a stable dimer with a binding energy $E = -|E_B|$. We are going to find a four-particle binding energy as the energy of an s-wave bound state of two dimers. Gener-

ally speaking, bound states could emerge in channels with larger orbital momenta; however, this question will be the subject of further investigations. To find the binding energy, we should examine the analytical structure of the dimer–dimer scattering vertex T_4 and find its poles. The set of equations for T_4 has the same diagrammatic structure as those shown in Figs. 2 and 3. The analytical expression of the first equation can be written as

$$T_4(p_1, p_2; P) = \frac{i}{\alpha}\sum_k G(P + p_1 - k)G(k) \times \Phi(P + p_1 - k, k; p_2, P), \quad (12)$$

and the equation for the vertex Φ is

$$\begin{aligned}\Phi(q_1, q_2; p_2, P) &= G(P - q_1 + p_2)G(P - q_2 - p_2) \\ &+ i\sum_k G(k)G(2P - q_1 - q_2 - k)T_2(2P - q_1 - k) \\ &\times \Phi(q_1, k; p_2, P) \\ &- \frac{1}{2\alpha}\sum_{Q,k} G(Q - q_1)G(2P - Q - q_2) \\ &\times T_2(2P - Q)T_2(Q)G(k)G(Q - k)\Phi(k, Q - k; p_2, P) \\ &+ (q_1 \longleftrightarrow q_2),\end{aligned} \quad (13)$$

where T_2 should be taken from Eq. (10) and one should put $\alpha = 2$ for the case of identical resonantly interacting bosons. Solving the above equations for the poles of T_4 as a function of the variable $P = \{0, E\}$, we found 2 bound states for the *bbbb* complex (see table). Certainly, for the validity of our approximation we should have $|E_4| \ll 1/mr_0^2$. For the case of four bosons *bbbb*, this means that $194|E_b| \ll 1/mr_0^2$ and, hence, $a/r_0 \gg \sqrt{194}$. This case can still be realized in the Feshbach resonance scheme.

The case of a four-particle complex $bf_\uparrow bf_\downarrow$ consisting of resonantly interacting bosons and fermions is described by the same equations (12, 13) but with the parameter $\alpha = 1$. In this case, we found two bound states; they are also listed in the table.

In order to obtain bound states of the *fbbb* complex, one has to find energies $P = \{0, E\}$ corresponding to nontrivial solutions of the following homogeneous equation:

$$\begin{aligned}\Phi(q_1, q_2; p_2, P) &= i\sum_k G(k)G(2P - q_1 - q_2 - k) \\ &\times T_2(2P - q_1 - k)\Phi(q_1, k; p_2, P) + (q_1 \longleftrightarrow q_2).\end{aligned} \quad (14)$$

This equation corresponds to the diagrams of Fig. 3b. We found a bound state of the *fbbb* complex with energy $E_4 = 4.10E_B$.

Bound states of resonantly interacting particles in 2D

System	Relative[1] interaction	Number of bound states	Energy (in E_B)[2]	α[3]
bbb	U_{bb}	2	1.267, 16.52	2
fbb	U_{fb}	1	2.39	1
fbbb	U_{fb}	1	4.10	1
$bf_\uparrow bf_\downarrow$	U_{fb}	2	2.84, 10.64	1
bbbb	U_{bb}	2	24, 194	2

[1] Interaction that yields resonance scattering. All other interactions are negligible.
[2] $m = m_b = m_f$.
[3] The indistinguishability parameter in Eq. (10).

Finally, we summarize the results concerning binding energies of three and four resonantly interacting particles in 2D in the table. Note that all our calculations correspond to the case of particles with equal masses $m_f = m_b = m$, though they can be easily generalized to the case of different masses.

5. CONCLUSIONS

For the problem of resonantly interacting fermions in 3D, we developed an exact diagrammatic approach that allows the dimmer–dimer scattering length to be found. We apply the developed approach to obtain new results in the 2D case. Namely, we exactly calculate the binding energies of the following complexes: three bosons *bbb*, two bosons plus one fermion *bbf*, three bosons plus one fermion *bbbf*, two bosons plus two fermions $bf_\uparrow bf_\downarrow$, and four bosons *bbbb*.

Our investigations enrich the phase diagram of ultracold Fermi–Bose gases with resonance interaction. They serve as an important step for future calculations of the thermodynainic properties and the spectrum of collective excitations in different temperature and density regimes. Note that, in purely bosonic models in 2D or in the Fermi–Bose mixtures in the case of prevailing density of bosons $n_B > n_F$, creation of larger complexes consisting of 5, 6, or more particles is also possible. In fact, here we are dealing with the macroscopic phase separation (with the creation of large droplets). The radius of this droplet R_N for N bosons in 2D is estimated in [18] on the basis of a variational approach. Note that even for $N = 5$ the exact calculation of the bound state energies requires huge computational capability, which is why it was not performed by us.

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