Current Noise through a Kondo Quantum Dot in a SU(N) Fermi Liquid State

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The current noise through a mesoscopic quantum dot is calculated and analyzed in the Fermi liquid regime of the SU(N) Kondo model. The results connect the Johnson-Nyquist noise to the shot noise for an arbitrary ratio of voltage and temperature, and show that temperature corrections are sizable in the usual experiments. For the experimentally relevant SU(4) case, quasiparticle interactions are shown to increase the shot noise.

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The Kondo effect,[1] i.e., the screening of a local spin by coupling to conduction electrons, is a paradigm for strongly correlated systems as it exhibits sophisticated many-body correlations with a quite simple model. Its tunable realization in mesoscopic quantum dots, semiconductors, or carbon nanotubes has triggered a renewed interest[2] in Kondo physics probed by transport measurements. In the ground state, the dot spin is screened by a cloud of conduction electrons and forms a singlet. The low energy properties of the remaining electrons are described by a local Fermi liquid theory.[3] In this theory, electrons are scattered elastically by the singlet in a similar way as by a resonant level. Electrons also interact through polarization of the spin singlet. The ratio of elastic to inelastic scattering is fixed by universality; i.e., the Kondo temperature $T_K$ is the only scale that governs low energy properties of the model. This description applies to SU(2) symmetry but also more generally to SU(N). In that case, both the spin and orbital degrees of freedom are screened by delocalized electrons in the reservoirs. In particular, the SU(4) case has recently become the subject of extensive investigation. Various experimental settings have been proposed[4,5] and its realization has already been reported in vertical quantum dots[6] and carbon nanotubes.[7]

A promising experimental tool to study Kondo physics is current noise measurement. These experiments are technically challenging in the Kondo regime notwithstanding recent progress[8]. Noise can probe out of equilibrium properties of the model. This has not been much investigated so far since only a few theoretical methods[9–11] apply to the out of equilibrium situation in comparison with the equilibrium case. In particular, the shot-noise at low temperature could provide information on the statistics of charge transfer. Interestingly, a picture has emerged recently[12,13] for the SU(2) Kondo effect at low energy where scattering events of two electrons lead to an effective charge of $5/3e$ in the backscattering current. This emphasizes the strong role of interactions for charge transfer even in the vicinity of a Fermi liquid fixed point.

The heating due to voltage polarization and the decoupling of phonons to electrons at low energy implies that the temperature of electrons is never really small in practical situations. It is therefore highly desirable to have a theory that holds at finite temperature. The purpose of this Letter is to provide a general analysis for the zero-frequency current noise $S \equiv 2 \int dr \langle \Delta \hat{I}(r) \Delta \hat{I}(0) \rangle$ in the SU(N) Fermi liquid regime. $\hat{I}$ is the current operator. Our expressions are valid at low temperature $T$ and voltage $V$ and for any value of $V/T$. They interpolate between the shot-noise $T \ll eV$ and the Johnson-Nyquist $eV \ll T$ limits where the fluctuation-dissipation relation to the conductance is recovered. Note that the zero-temperature limit has been addressed recently[14]. Three uncorrelated processes were identified and interpreted for the shot noise ($T \ll eV$). The first two are backscattering of charges $e(2T_0 - 1)$ and $2e(2T_0 - 1)$, $T_0$ being the transmission at vanishing energy. They give the charge $5/3e$ derived in Refs.[12,13] for SU(2). The third process increases available electron-hole pairs leading to an enhancement of the partition noise.

We consider a quantum dot attached to two leads with source and drain. Only a single, possibly degenerate, energy level participates to transport. While the dot spin usually has SU(2) symmetry, a higher SU(N) symmetry can be achieved if orbital degeneracy of the dot level takes place, resulting in a larger N-component pseudospin. Below the strong Coulomb energy $U$, a SU(N) Kondo model is obtained coupling antiferromagnetically the spin of the dot with lead electrons. We assume here that the dot has exactly one electron. Below the Kondo temperature $T_K (\ll U)$, the dot spin forms a SU(N) singlet with lead electrons. This requires, however, that any orbital index is conserved during lead-dot tunneling processes[15]. Experiments currently investigate Kondo SU(4)[6,7]. In a double-dot structure the orbital index discriminates the two dots; in carbon nanotubes it originates from the $K - K'$ orbital degeneracy of the graphene band structure.

In the regime of energies (temperature and voltage) smaller than $T_K$, the dot spin is frozen out in the singlet configuration and Fermi liquid theory applies to lead electrons.[3] We first detail elastic scattering. The low energy form of the dot Green’s function[1] results in an (s-wave)
electron phase shift [16], \( \delta_{\text{el}}(\epsilon) = \arctan[\Gamma/(\epsilon_K - \epsilon)] \), i.e., the same as for a resonant level of energy \( \epsilon_K \) and half-width \( \Gamma \), \( \epsilon \) being measured from the Fermi level. Expanding to second order in \( \epsilon \) leads to

\[
\delta_{\text{el}}(\epsilon) = \delta_0 + \frac{\alpha_1}{\Gamma} \epsilon + \frac{\alpha_2}{\Gamma^2} \epsilon^2, 
\]

with \( \alpha_1 / T_K = \sin^2(\delta_0)/\Gamma \) and \( \delta_0 = \delta_{\text{el}}(0) = \pi/N \), the phase shift at zero energy, is imposed by the Friedel sum rule [16]. The value of \( \alpha_1 \approx 1/N \) depends on the precise definition of the Kondo temperature and is therefore not universal. In contrast, the ratio \( \alpha_2 / \alpha_1^2 = \cos \delta_0 \) is universal. For SU(2), \( \alpha_2 = 0 \), and there is no \( \epsilon^2 \) dependence in contrast to SU(N) (\( N > 2 \)) as noted by Ref. [14].

Electron interaction—through polarization of the spin singlet—is written below on the basis of scattering states that include the phase shift Eq. (1).

Close to the SU(2) unitary limit, Ref. [17] proposes to write the current as \( \hat{I} = I_s - \hat{I}_{\text{BS}} \) with \( I_s = 2e^2 / \pi V \) the unitary contribution and \( \hat{I}_{\text{BS}} \) the backscattering current. Current and shot noise can then be derived but not the noise at finite temperature, since correlations between \( I_s \) and \( \hat{I}_{\text{BS}} \) are not included. In particular, the Johnson-Nyquist relation to the conductance is not recovered. We use here a more general approach where the current operator is expanded over a convenient basis of scattered states. Elastic scattering is then easily described. We separate the lead electron field into its symmetric and antisymmetric parts \( \psi_s(x) = \psi_s(x) + \psi_a(x) \). The \( x \) axis is oriented from left (source) to right (drain). We consider the case where the dot is coupled symmetrically to the two leads. In that case, only the symmetric part \( \psi_s \) (s wave) is modified by the Kondo coupling [2]. Therefore we can expand over eigenstates of the free problem (we write \( \sum_k \equiv \int \frac{dk}{2\pi} \) throughout)

\[
\psi_a(x) = \sum_k \frac{e^{i(k_{K} + k)x} - e^{-i(k_{K} + k)x}}{\sqrt{2}} a_k, 
\]

where \( a_k \) annihilates an antisymmetric mode with energy \( \epsilon_k = \hbar v_F k \). For symmetric modes of energy \( \epsilon_k \), electrons are reflected at \( x = 0 \) with dephasing \( 2\delta_{\text{el}}(\epsilon_k) \); see Eq. (1). For \( \epsilon < \epsilon_F \), they read \( (e^{i(k_{K} + k)x} - e^{-i(k_{K} + k)x})/\sqrt{2} \). Left (right) scattering states are obtained as (anti)symmetric combination of the symmetric and antisymmetric modes. \( \delta_{\text{el}} = 0 \) (\( \delta_{\text{el}} = \pi/2 \)) corresponds to totally reflected (transmitted) scattering states. Following Ref. [18], we write the symmetric part for \( x < 0 \) as

\[
\psi_s(x) = \frac{1}{\sqrt{2}} \left[ e^{i(k_{K} + k)x} b(x) - e^{-i(k_{K} + k)x} \tilde{b}(x) \right], 
\]

\( b(x) \) describes incoming (outgoing) waves. Writing \( b(x) = \sum_k b_k e^{ikx} \) defines an extension of \( b(x) \) to \( x > 0 \), which has nothing to do with the physical \( x > 0 \) half-space. Physically, it corresponds to unfolding the outgoing part of the incoming wave to the positive axis. The \( S \) matrix relates \( b(x) \) and \( \tilde{b}(x) \) through the boundary condition \( \tilde{b}(x) = S b(-x) = \sum_k e^{2i\delta_0(\epsilon_k)} e^{-ikx} b_k \). Collecting Eqs. (2) and (3), we obtain a compact expression for the symmetrized current passing through the dot

\[
\hat{I} = \frac{e}{2\nu\hbar} \left[ a^+(x) b(x) - a^+(x-\nu S b(-x) + \text{H.c.} \right], 
\]

with \( a(x) \equiv \sum_k e^{ikx} a_k \) and arbitrary \( x < 0 \). \( \nu = 1/(\hbar v_F) \) is the density of state for 1D fermions moving along one direction. This expression can be generalized straightforwardly to spinful fermions.

A dc bias is applied between electrodes, \( \mu_L = -\mu_R = eV/2 \) where symmetric capacitive coupling is assumed. This defines the population of left and right scattering states with annihilation operators \( c_{L,R,k} = (b_k \pm a_k)/\sqrt{2} \) such that \( \langle a^+_k a_{k'} \rangle = \langle b^+_k b_{k'} \rangle = \delta(k - k')f_{aa}(k) \) and \( \langle b^+_k a_{k'} \rangle = \delta(k - k')f_{ab}(k) \) with the notations \( f_{aa}(k) = \frac{1}{2} \sum_{\pm} f(\epsilon_k \pm eV/2) \) and \( f_{ab}(k) = \frac{1}{2} \sum_{\pm} f(\epsilon_k + eV/2) \) (\( f \) is the Fermi distribution). Elastic current and noise are obtained in a straightforward manner using Eq. (4) and reproduce Landauer-Büttiker [19] formulas with the energy dependent transmission

\[
\mathcal{T}(\epsilon) = \mathcal{T}_0 + \sin^2 \delta_0 \frac{\alpha_1 \epsilon}{T_K} + \left( \frac{\alpha_2 \sin 2\delta_0 + \alpha_1^2 \cos 2\delta_0}{T_K} \right)^2, 
\]

with \( \mathcal{T}_0 \approx \sin^2(\delta_0) \). The lowest order is given by (\( R_0 = 1 - \mathcal{T}_0 \))

\[
S_0 = \frac{2Ne^2}{h} \left[ \mathcal{T}_0 R_0 eV \cosh \left( \frac{eV}{2T} \right) + 2T \mathcal{T}_0 \right]. 
\]

Corrections due to elastic terms \( \sim \alpha_{1/2} \) can be written using

\[
\mathcal{W}_1 = (T/2)[(eV)^2 + 4(\pi T)^2]/3, 
\]

\[
\mathcal{W}_2 = (eV/12) \cosh(eV/2T)[(eV)^2 + 4(\pi T)^2], 
\]

\[
\mathcal{A}_1 = \alpha_1^2 [\cos 2\delta_0 + 2 \sin^2(2\delta_0) - 1] + \alpha_2 \sin 2\delta_0 (1 - \cos 2\delta_0), 
\]

and \( \mathcal{A}_2 = \alpha_1^2 \cos 4\delta_0 + \alpha_2^2 \sin 4\delta_0 \).

\[
\delta S_\epsilon = \frac{2Ne^2}{\hbar T_K^2} \left( \mathcal{W}_1 \mathcal{A}_1 + \mathcal{W}_2 \mathcal{A}_2 \right). 
\]

We now detail electron interactions within the Fermi liquid picture. As explained by Nozières [3], scattering off the frozen singlet results in electron phase shifts \( \delta_{\text{el}}(\epsilon, n_{\sigma'}) \) depending on the energy \( \epsilon \) and other spin \((\sigma')\) distributions \( n_{\sigma'} \). Assuming analyticity, a low energy expansion in \( \epsilon \) and \( n_{\sigma'} \) can be written

\[
\delta_{\sigma} = \delta_{\text{el}}(\epsilon) - \sum_{\sigma' \sigma \epsilon} \frac{\phi_{\sigma} \phi_{\sigma'} e^{2i\delta_{\text{el}}(\epsilon)}}{\epsilon T_K}, 
\]

with the phenomenological coefficients \( \alpha_1 \), \( \alpha_2 \) [see Eq. (1)], \( \phi_{\sigma_1} \), \( \phi_{\sigma_2} \). As the Kondo resonance position is determined relative to the Fermi energy, these coefficients are not independent [3]; one finds \( \alpha_1 \approx (N - 1) \phi_1 \) and \( \alpha_2 \approx (N - 1) \phi_2 \), in accordance with conformal field theory arguments [18,20]. Universality is recovered as \( \alpha_1 \) sets all other coefficients.

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The inelastic part of Eq. (7) is obtained from

\[
H_{\text{int}} = \frac{\phi_1}{\pi v^2 T_K} \sum_{\alpha<\alpha'} \epsilon_{\alpha} b_{\alpha}(0) b_{\alpha'}(0) b_{\alpha'}(0) + \frac{\phi_2}{\pi v^2 T_K} \sum_{\alpha<\alpha',(k)} \frac{\epsilon_{\alpha} + \epsilon_{k}}{2} b_{\alpha,k} b_{\alpha',k}^\dagger b_{\alpha',k}^\dagger b_{\alpha',k}:
\]

where \(\::\) denotes normal order. Inelastic corrections are calculated perturbatively. Since scattering states annihilated by \(c_{L/R}\) are eigenstates of the Hamiltonian for \(H_{\text{int}} = 0\), they form a convenient basis for a perturbation calculation in \(H_{\text{int}}\) in the Keldysh framework. The corresponding diagrams for the noise are shown Fig. 1. They are obtained by connecting current operator vertices to interaction four-point vertices. Internal lines involve only \(b\) fermions, whereas external current vertices mix \(a\) and \(b\) fermions; see Eq. (4). Noninteracting Green’s functions are combinations of \(L/R\) thermal distributions, with \(G_{ab} = G_{ba}\), \(G_{aa} = G_{bb}\), \(G_{ab}^{\dagger}(\omega, k) = 2i\pi \delta(\omega - \epsilon_k) f_{ab}(k)\) for all Keldysh indices \((\alpha, \beta)\) and, for instance, \(G_{bb}^{\dagger}(\omega, k) = 2i\pi \delta(\omega - \epsilon_k) f_{bb}(k)\), \(G_{bb}^{\dagger}(\omega, k) = 2i\pi \delta(\omega - \epsilon_k) \times [f_{bb}(k) - 1]\). Consistency in the calculation requires one to stop at quadratic order in \(\max(eV, T)/T_K\) for inelastic corrections. Therefore only \(\delta S_0\) is kept in Eq. (1) for second order Figs. 1(b)–1(f), while \(\sim \alpha_1\) corrections are included in Fig. 1(a). Moreover, the second term in the Hamiltonian Eq. (8) enters solely Fig. 1(a).

The inelastic part of Eq. (7) is exactly included in the scattering states. It is therefore not necessary here to use an elastic Hamiltonian [21] in contrast to Refs. [10,12,13]. We have nevertheless checked that the two approaches give coinciding results. To write results in a concise way, we define the prefactor \(S_{\rho} \equiv 2(e^2/h)N(N - 1)(\phi_1/T_K)^2\) and the functions \(G_1 = e^{\phi_1}(eV)^2 + (\pi T)^2\), \(G_2 = e^{\phi_1}(eV)^2 + 4(\pi T)^2\), \(G_3 = T(2eV)^2 + \frac{1}{2}(\pi T)^2\), \(F_1 = \coth(eV/T)\), \(F_2 = \coth(eV/2T)\). Collecting Figs. 1(c)–1(f), we find the contribution

\[
\delta S_{\rho,1} = S_{\rho}[G_1(2\cos 4\delta_0 + 2)F_1 - 2\sin^2(2\delta_0)F_2] \\
+ G_22F_2\cos(2\delta_0) + \frac{2}{3} 2\sin^2(2\delta_0)\pi T^3 \\
+ G_3\sin^2(\delta_0)\cos 2\delta_0).
\]

Figure 1(b) gives

\[
\delta S_{\rho,2} = 2S_{\rho}(N - 1)T_0R_0(eV)^2\left[\frac{eV}{2} \coth\left(\frac{eV}{2T}\right) + T\right]
\]

The last Fig. 1(a) gives a mixed contribution with elastic and inelastic parts,

\[
\delta S_m = -S_{\rho}(eV)^2T_0R_0\left(\frac{4\alpha_1}{\phi_1} + \frac{T_0}{R_0}\phi_2\phi_1^{-}\right)
\]

To summarize, the total noise reads \(S = S_0 + \delta S_0 + \delta S_{\rho,1} + \delta S_{\rho,2} + \delta S_m\) where the different terms are given in Eqs. (5), (6), and (9)–(11). This is the central result of this Letter.

At low voltage \((eV \ll T)\), the Johnson-Nyquist relation is verified, i.e., \(S = 4kT\) where \(G = \partial I/\partial V\) is the linear conductance. It is interesting to study more specifically the shot noise (at \(T = 0\)). In contrast with the SU(2) case, shot noise does not vanish at the Fermi liquid fixed point. For \(T_0 \rightarrow \infty\), \(S = S_0 + 2(eV/h)e2N\tau_0 R_0\). Hence this term dominates at low energy; elastic and inelastic contributions come only as corrections to \(S_0\).

The inelastic term Eq. (8) describes scattering of 0, 1, or 2 electrons from left scattering channel \((L)\) to right \((R)\) or vice versa [12]. At zero temperature, three processes are relevant: (i) \((L, L) \rightarrow (R, R)\), (ii) \((L, R) \rightarrow (R, R)\), (iii) \((L, L) \rightarrow (R, L)\) with rates

\[
\Gamma_1 = N(N - 1)\frac{eV}{h}\frac{\phi_1^2}{48}\left(\frac{eV^2}{T_K}\right),
\]

for (ii), (iii), and \(\Gamma_2 = 8\Gamma_1\) for (i). Figure 1(a) does not contribute at zero temperature. Equation (10) gives

\[
\delta S_{\rho,1} = 8\Gamma_1(e^*)^2 + 2\Gamma_2[(2e^*)^2 - 8\tau_0 R_0 e^2],
\]

where \(e^* = e(1 - 2T_0)\). The first two terms can be identified as two uncorrelated backscattering events with effective charges \(e^*\) and \(2e^*\). This picture is in agreement with the inelastic current correction that we find \(\delta I_1 = 4\Gamma_1 e\cos 2\delta_0 + \Gamma_2 e\cos 2\delta_0\). The third term in Eq. (12) is a reduction of the partition noise. It is balanced by Fig. 1(b) that adds a positive contribution to the partition noise [22]

\[
\delta S_{\rho,2} = \frac{2e^2}{h} N(N - 1)\left(\frac{\phi_1}{T_K}\right)^2 T_0 R_0 (eV)^3.
\]

Gathering this term and the last one of Eq. (12), we find an
FIG. 2. Generalized Fano factor $F$ from Eq. (14) as a function of $eV/T$ (full lines) for SU(2) and SU(4). Dotted lines are shot-noise limits.

overall increase of the partition noise due to Fermi liquid interactions. This can be interpreted as an increase in the density of states of $(L)$ electron–$(R)$ hole pairs—involving in the partition noise—due to repulsive electron interactions.

To analyze the effect of temperature, we define a generalized experimentally relevant Fano factor

$$F(eV/T) = \frac{1}{2e} \frac{S(V, T) - S_0(V, T) - 4T \frac{\delta I(V, T)}{I(V, T)}}{\delta I(V, T)} \quad (14)$$

with $\delta I(V, T) = I(V, T) - I_0(V)$, $I_0(V) = (Ne^2V/h)T_0$ is the current for $T_K \rightarrow \infty$. $T_K$ disappears in Eq. (14) and $F$ becomes a universal function of the ratio $eV/T$, shown Fig. 2 for SU(2) and SU(4). In the shot-noise limit, $F = -5/3$ for SU(2) in accordance with Refs. [12,13], and $F = 2/3$ for SU(4). $F = -1$ is obtained if only elastic terms are present for $N = 2, 4$. For larger $N > 4$, $F$ converges towards a universal increasing function where only elastic terms contribute and with $F = 1$ in the shot-noise limit. In the case of SU(4), noise is suppressed by elastic terms but enhanced by inelastic ones. The overall effect is a noise increase and a positive $F$—noise and current corrections are both positive—in contrast with SU(2). The convergence to the shot-noise limit is quite slow; see Fig. 2. For SU(4), $F$ can even change sign, depending on the temperature, due to the negative $\delta S_m$, Eq. (13), competing with other terms at intermediate temperatures. It clearly shows that temperature corrections are important in a large window of experimentally accessible parameters.

To conclude, we have determined the current noise through a quantum dot in the Kondo Fermi liquid regime. The result holds for general SU(N) symmetry and arbitrary $V/T$. We find, in particular, that temperature corrections are important for comparison with experiments. For the SU(4) case, electron interactions are shown to enhance the noise, yielding a positive effective Fano factor in contrast with SU(2). We stress that our approach is not restricted to the present problem but could be applied to mesoscopic systems where elastic scattering is accompanied by weak inelastic processes.

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[21] This can be written as $H_{el} = -\frac{1}{\beta e^2} \sum_{k',\sigma} \left( \alpha_1 \epsilon_{k',\sigma} + \alpha_2 \epsilon_{k,\sigma} \right) b_{k,\sigma}^{\dagger} b_{k,\sigma}$ with $\epsilon_{k,\sigma} = \hbar v_F (k + k')$
[22] Results for the shot noise all agree with Ref. [14].