the (market) model

- a population of $N$ agents (large $N$ limit)
- each agent $i$ has to make a binary choice: to buy ($\omega_i=1$) or not ($\omega_i=0$) one unit of a single good at a posted price $P$
  (a single choice: to participate or not, at a given constant cost $P$)
- Agent $i$ (wants to) buy/participate if: $H_i + J \eta - P > 0$
  $\eta$ = fraction of buyers in the population = $(1/N) \sum_i \omega_i$
- individual willingness to pay/participate (IWP) $H_i = H + \theta_i$
  (reservation price), randomly distributed in the population:
  \[
  \begin{align*}
  H & : \text{mean value of the distribution} \\
  \theta_i & : \text{deviation with respect to the mean, of p.d.f. } f(\theta_i) \\
  \sigma^2 & : \text{variance}
  \end{align*}
  \]
- social influence: a weighted sum of the choices of other agents
  - homogeneous social influence
  - strategic complementarity $J > 0$
Nash equilibria

- Individual $i$’s choice: buy if $V_i - P = H + \theta_i + J \eta - P > 0$

- Fraction of buyers:

  \[
  \text{if } \eta = 0 \implies \text{for } H + \theta_i > P: \omega_i = 1
  \]

  \[
  \text{if } \eta = 1 \implies \text{for } H + \theta_i < P - J: \omega_i = 0
  \]

- Nash equilibrium:

  \[
  \eta = \int_{P - J \eta}^{\infty} f(H + \theta) d\theta
  \]
customers phase diagram

Exple: triangular distribution of the IWP:

\[ f(h+\theta_i) \]

\[ -\frac{2}{3b} \leq h < 1 \]

phase diagram:

\[ h-p \]

\[ h = H/\sigma \quad p = P/\sigma \]

\[ j = J/\sigma \]

coexistence of 2 solutions
\[ f(\theta) = \frac{\beta}{2 \cosh^2 \beta \theta} \]

\[ \eta = \text{fraction of buyers} \]

\[ h - p = \beta(H - P) \]

customers phase diagram

single solution

two stable solutions

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customer’s phase diagram

smooth mono-modal distribution of the IWP

Figure 4: Demand phase diagram on the plane $(j = J/\sigma, \delta = (H - P)/\sigma)$, for a smooth IWP distribution (here the logistic). In the shaded region the demand presents multiple Nash equilibria. Outside this region, the demand is a single valued function of $j$ and $\delta$. 

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• alternative representation

Figure 5: Demand phase diagram in the plane ($\tilde{\sigma} = \sigma/J$, $\tilde{\delta} = (H-P)/J$), for a smooth IWP distribution (here the logistic). Inside the dashed region the demand presents multiple Nash equilibria. Outside this region, the demand is a single valued function of $\tilde{\sigma}$ and $\tilde{\delta}$. 

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Customer's phase diagram

- bi-modal distribution
Customer’s phase diagram

- multimodal distribution: example bi-modal case

Figure 12: Phase diagram (aggregate demand) for the case of the smooth bimodal pdf shown on figure 11.