How to choose under social influence ?

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Project ELICCIR supported by the program
« Complex Systems in Social and Human Sciences » (CNRS-French Ministry of Research)

motivation

- in a simple market model
  - a single good
  - heterogeneous customers
  - social interactions

- price fixed by a monopolist

- many equilibria may exist, due to the social effects
  - which equilibrium is reached depends on the customers' coordination
    - monopolist : needs to anticipate the customers equilibrium to fix the price that optimizes his profit

- question :
  - can the agents reach collectively the Pareto-optimal equilibrium ?

plan

- market model with a single good and externalities
    - S.N. Durlauf (2001) A framework for the study of individual behaviour and social interactions
    - W.A. Brock & S.N. Durlauf (2000) Interactions-based models
    - J.-P. Nadal, D. Phan, M.B. Gordon, J. Vannimenus (2003) Multiple equilibria in a monopoly market with heterogeneous agents and externalities

- equilibrium properties and collective states
    - multiple solutions
    - customer's phase diagram

- repeated game framework : learning by the customers
    - Experience Weighted Attraction (EWA) learning scheme (Camerer, 2003)
    - results for different EWA learning models

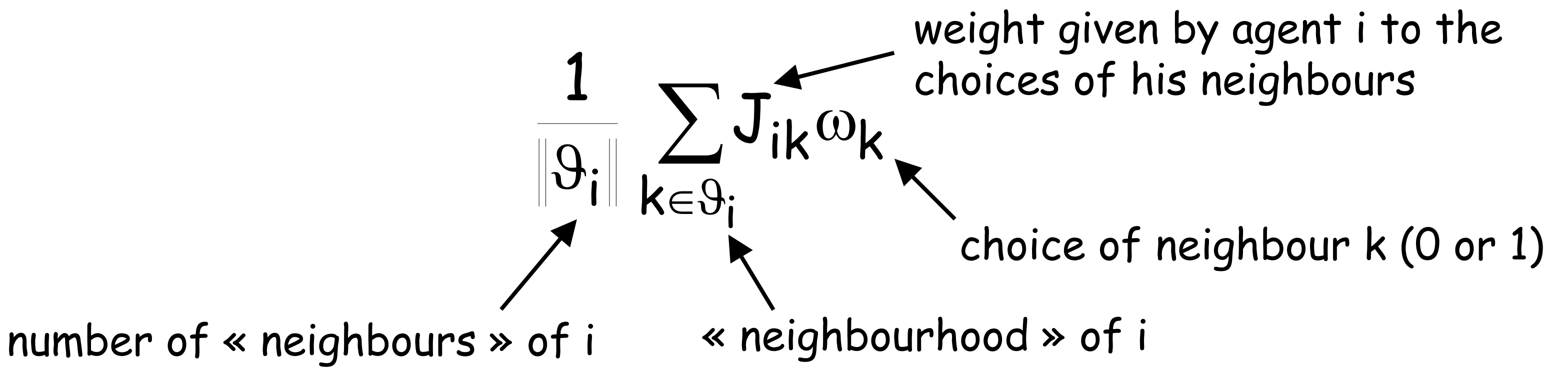
the market model

- a single good at a price P fixed by a monopolist
- a population of N agents
  - each agent i has to make a binary choice : to buy ($\omega_i=1$) or not ($\omega_i=0$) one unit

- each customer's willingness to pay is the sum of two terms :
  - an idiosyncratic term $H_i = H + \theta_i$ randomly distributed in the population

$$\left\{\begin{array}{l} H : \text{mean value of the distribution} \\ \theta_i : \text{deviation with respect to the mean, of pdf } f(\theta_i) \end{array}\right.$$

  - a social influence term : a weighted sum of the choices of other agents



$$\frac{1}{\|\vartheta_i\|}\sum_{k\in\vartheta_i} J_{ik}\omega_k$$

weight given by agent i to the choices of his neighbours

choice of neighbour k (0 or 1)

number of « neighbours » of i

« neighbourhood » of i

simplifying hypothesis

- strategic complementarity : $J_{ik} > 0$

  making the same choice as the others is advantageous

- homogeneous social influence ($J_{ik}=J$):

$$J \frac{1}{\|\vartheta_i\|} \sum_{k \in \vartheta_i} \omega_k = J\,\eta_i$$

weight of neighbours' choices

fraction of i's neighbours that adopt

- global neighbourhood and large N :

$$\eta_i = \frac{1}{N-1} \sum_{\substack{k=1 \\ (k \neq i)}}^{N} \omega_k \cong \frac{1}{N} \sum_{k=1}^{N} \omega_k \equiv \eta$$

= fraction of buyers

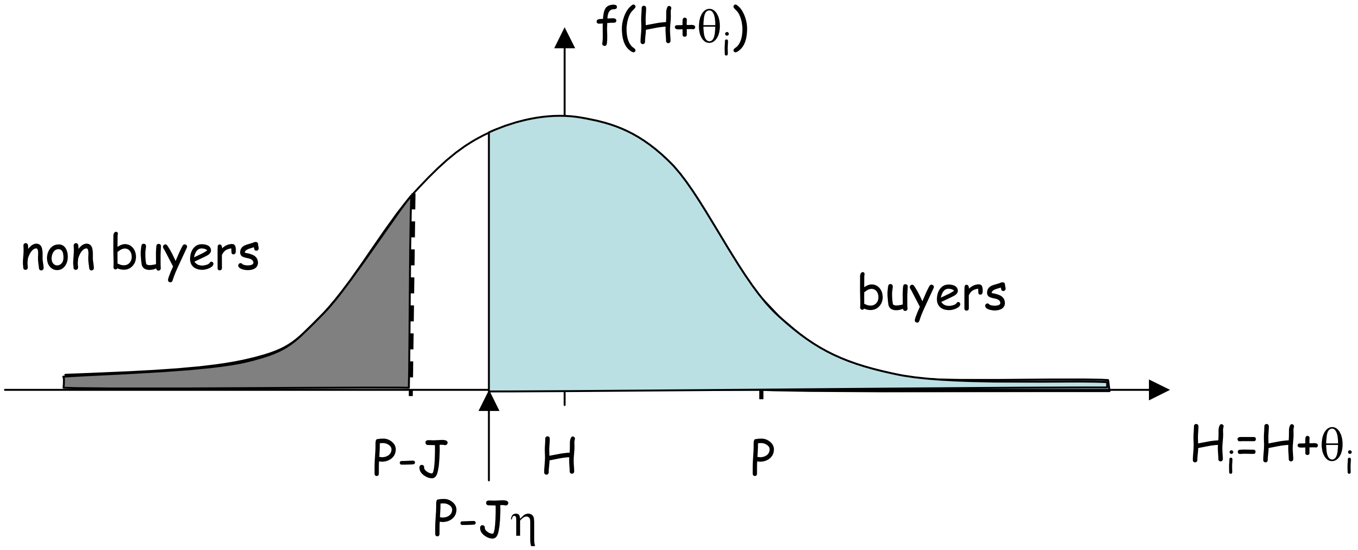
η insensitive to fluctuations:

single agents cannot influence individually the collective term $J\eta$

collective effects of the social influence

- individual i's choice : buy if $V_i - P = H + \theta_i + J\,\underset{\substack{\uparrow \\ \text{fraction of buyers}}}{\eta} - P > 0$
- fraction of buyers : if $\eta = 0 \Rightarrow$ for $H + \theta_i > P \;: \omega_i = 1$

  if $\eta = 1 \Rightarrow$ for $H + \theta_i < P - J \;: \omega_i = 0$



- Nash equilibrium : $\eta = \int_{P-J\eta}^{\infty} f(H+\theta)\,d\theta$

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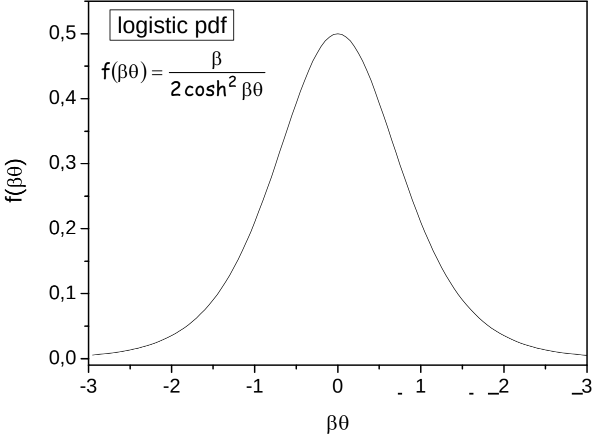
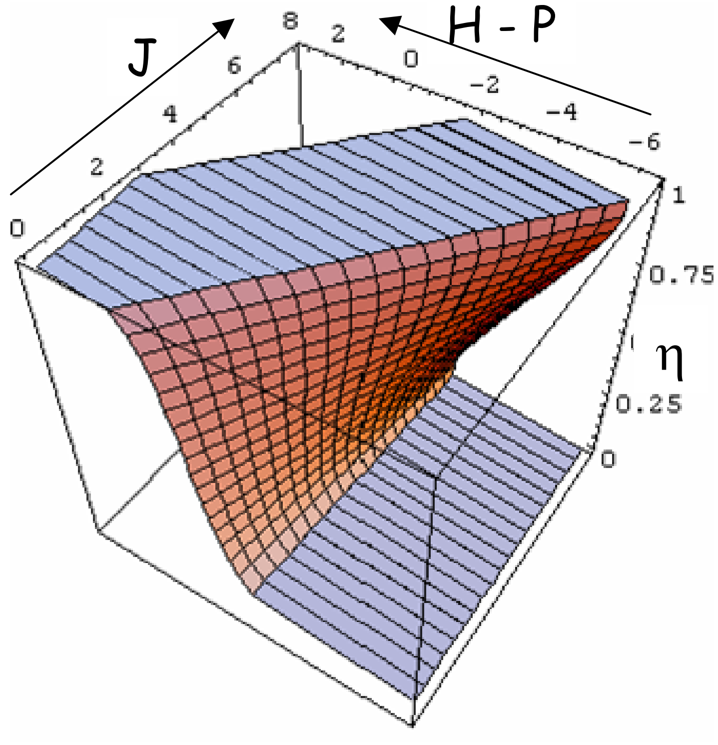
logistic distribution

$$f(\theta) = \frac{\beta}{2\cosh^2 \beta\theta}$$

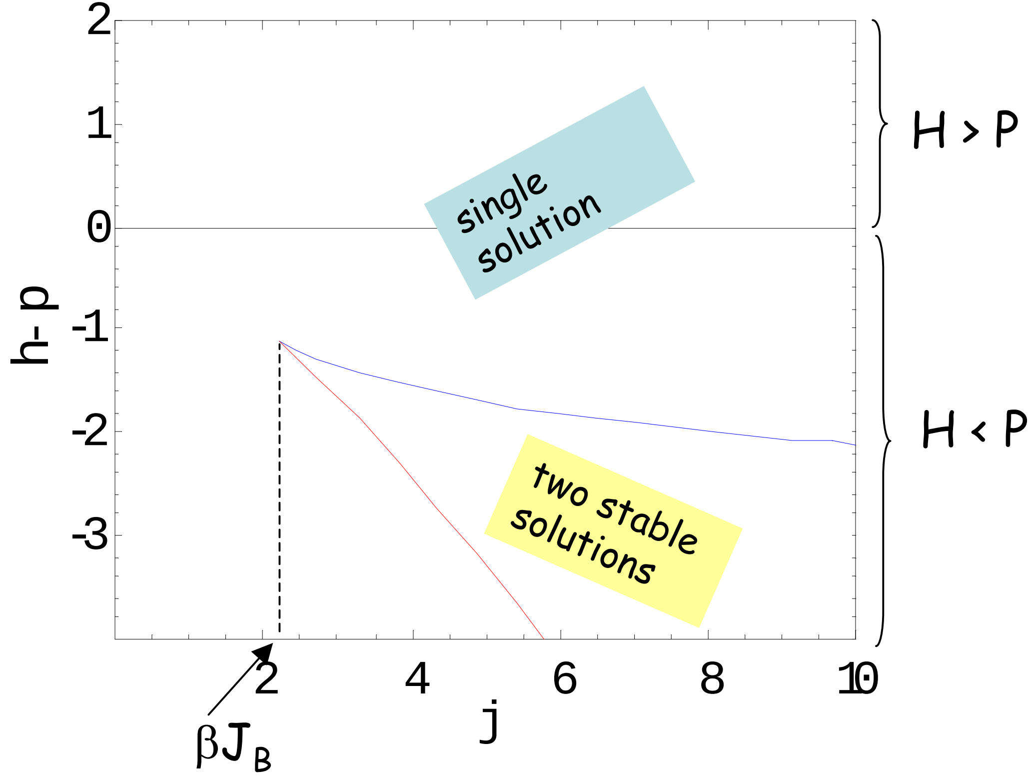


fraction of buyers



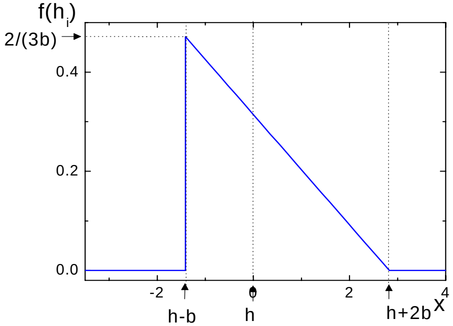
customers

phase diagram

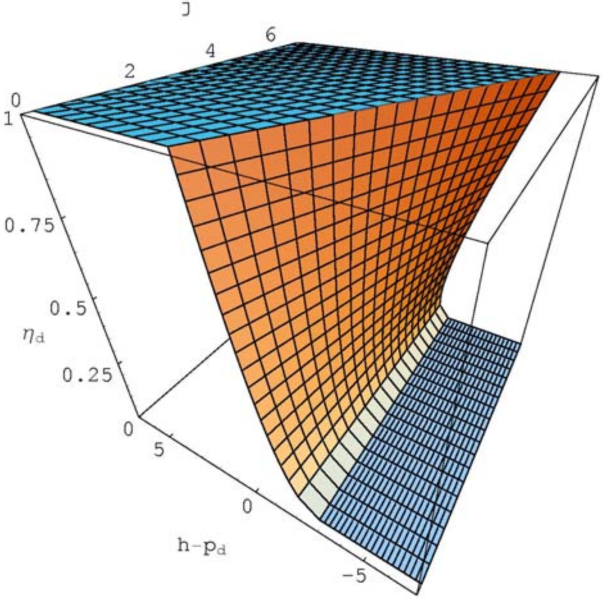


triangular distribution

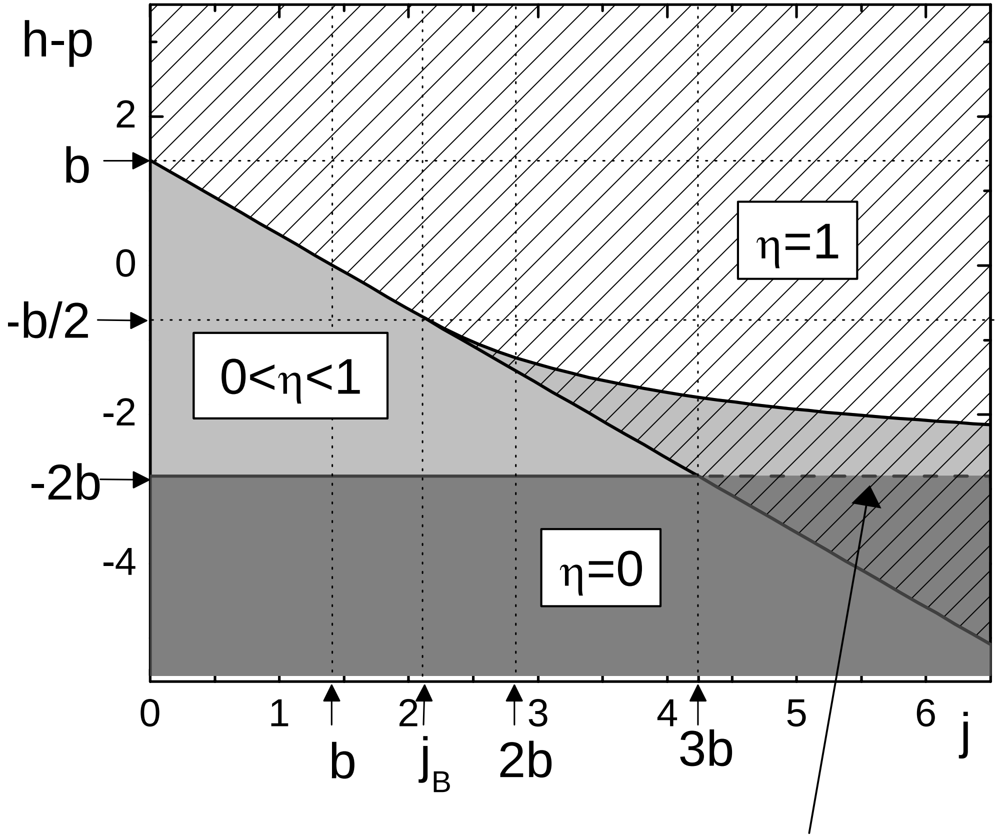
distribution :



fraction of buyers



customers phase diagram :



coexistence
of 2 solutions

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hypothesis

- the demand adapts to the price faster than the time scale of price revision:

  price is assumed to be fixed during the customers' learning.

- each agent must decide whether to buy or not
  - under imperfect information (he doesn't know the decisions of the others)
  - and incomplete information (he doesn't know his actual payoff)

- based on the "attraction" of buying or not buying

- attraction values are updated (learned) based on the actual fractions of buyers

modelisation of the learning dynamics

- At each time step each agent i makes a binary decision :

$$\mathrm{Proba}[\omega_i(t)=1]=f(\Delta_i(t)), \quad \text{with} \quad \Delta_i(t) \equiv A_i^1(t)-A_i^0(t)$$

relative attraction for buying

- best response:

$$\omega_i(t)=\begin{cases}1 & \text{if} \quad \Delta_i(t)-P>0 \\ 0 & \text{if} \quad \Delta_i(t)-P<0\end{cases}$$

- trembling hand:

$$P[\omega_i(t)=1]=1-\varepsilon(\Delta_i(t)-P)$$

$$\text{If} \quad \text{logistic}: \; P[\omega_i(t)=1]=\frac{1}{1+\exp-\beta[\Delta_i(t)-P]}$$

- Learns his relative attraction [Camerer: Experience Weighted Attractions] from the observation of the actual fraction of buyers $\eta(t)$:

$$\Delta_i(t)=H_i+J\hat{\eta}_i(t)$$

$\hat{\eta}_i(t)=$ agent i's estimate of η

$$\hat{\eta}_i(t+1)=\hat{\eta}_i(t)+\mu(t+1)\{[\delta+(1-\delta)\omega_i(t)]\,\eta(t)-\hat{\eta}_i(t)\}$$

$$\mu(t+1)=(1-\kappa)\frac{\mu(t)}{\mu(t)+\phi}+\kappa(1-\phi)$$

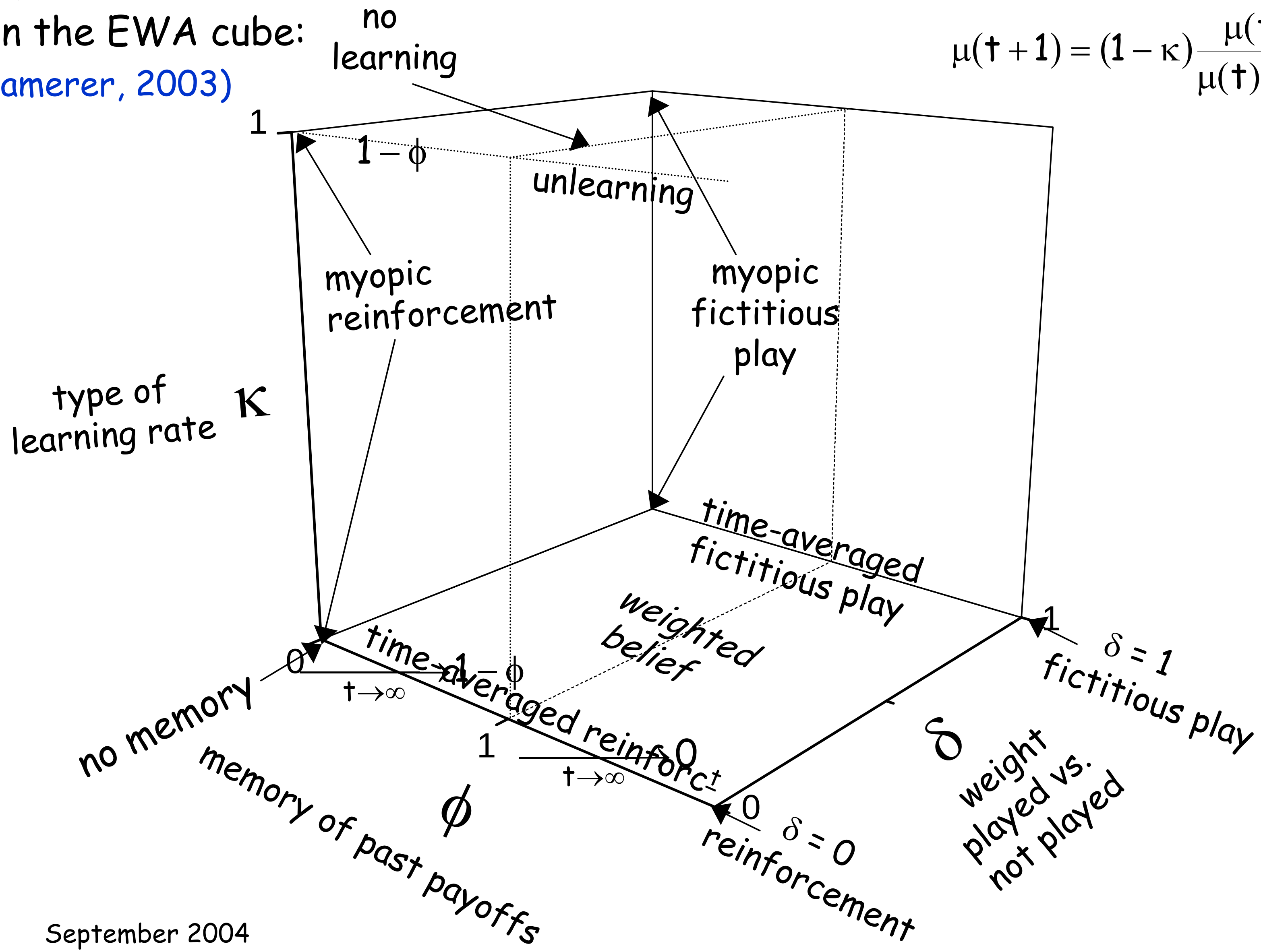
Experience Weighted Attraction

- updating attractions in the EWA cube:

(Camerer, 2003)

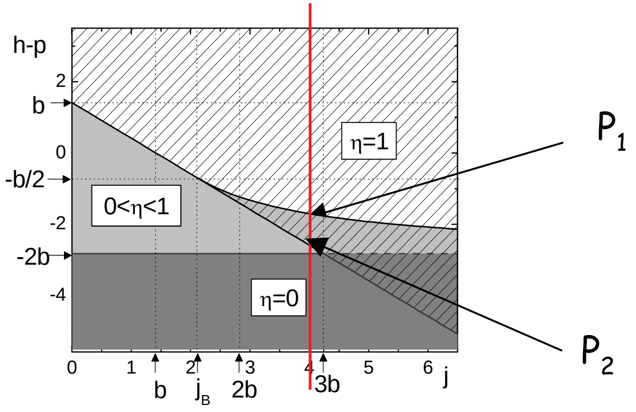
$$\hat{\eta}_i(t+1) = \hat{\eta}_i(t) + \mu(t+1)\{[\delta + (1-\delta)\omega_i(t)]\eta(t) - \hat{\eta}_i(t)\}$$

$$\mu(t+1) = (1-\kappa)\frac{\mu(t)}{\mu(t)+\phi} + \kappa(1-\phi)$$

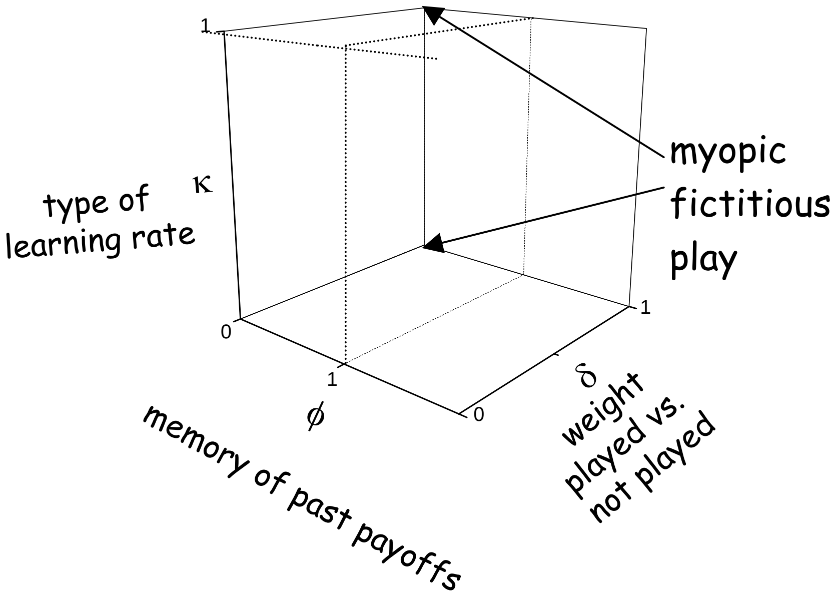


simulations

phase diagram
for the
triangular distribution :



J=4

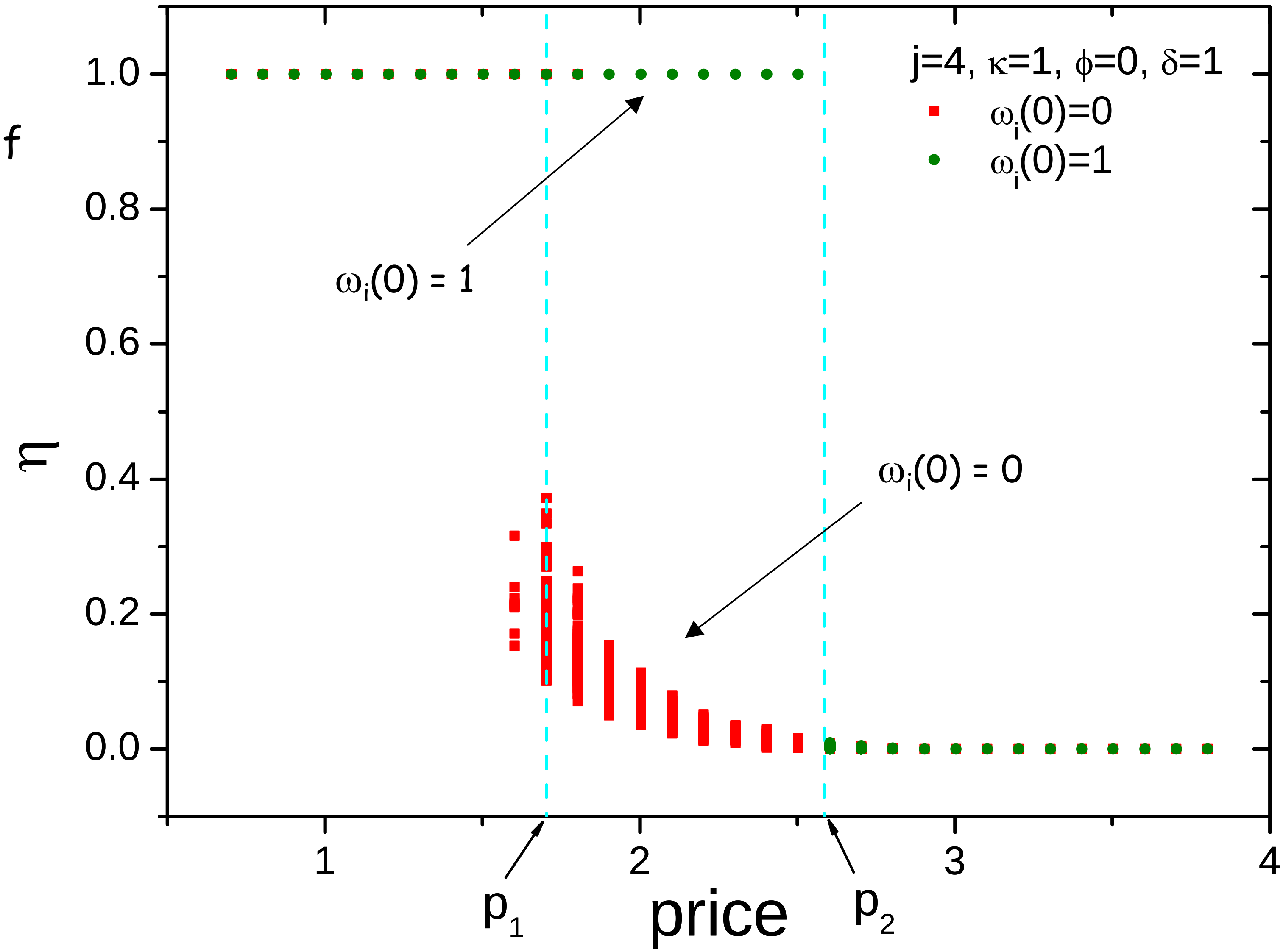


$$\mu(t+1)=1$$

$$\begin{aligned}\hat{\eta}_i(t+1)&=\hat{\eta}_i(t)+\mu(t+1)\left[\eta(t)-\hat{\eta}_i(t)\right]\\&=\eta(t)\end{aligned}$$

myopic fictitious play

fraction of buyers



September 2004

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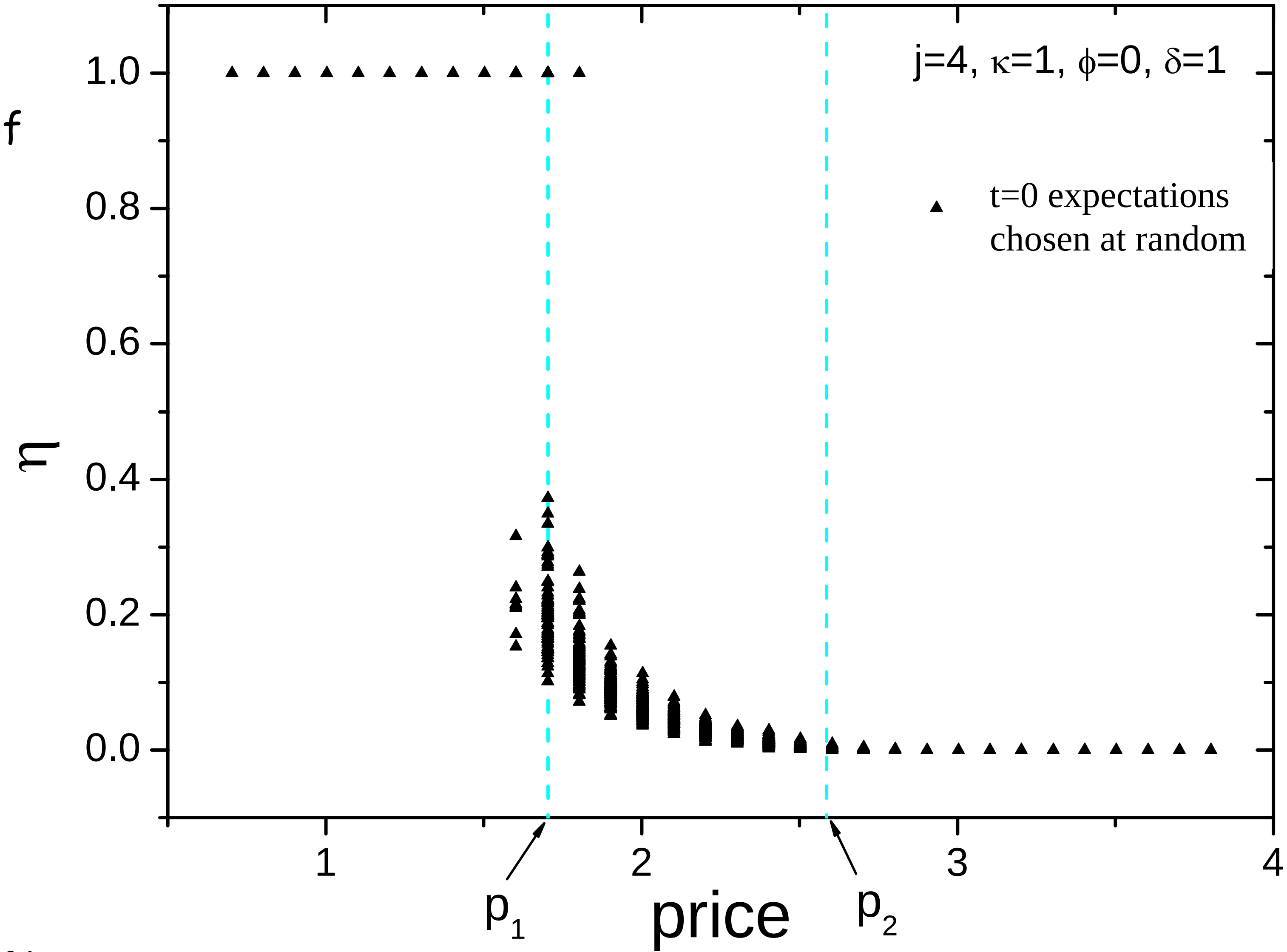

discussion

- asymptotics for $\phi < 1$
  - if $\delta > 0$
    - buyers : $\hat{\eta} \rightarrow \eta$
    - for non buyers: $\hat{\eta} \rightarrow \delta\,\eta < \eta$ attractions are underestimated
  - discontinuity at $\Delta = P$

- asymptotics for $\phi > 1$
  - the time decay of $\mu(t)$ may hinder the learning process

- work in progress:
  - decision through a trembling hand: interference between ε and μ
  - analytic results
    - the learning process as a special random walk
  - learning directly the attractions (estimations of $H_i+J\eta-P$) leads to very different results