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plan

T. C. Schelling (70's) : « the dying seminar »

Market (and non market) model
with a single good and positive externalities

This lecture:

- The dying seminar
- Alternative formulation (~ Random Field Ising Model)
- Equilibrium properties and collective states
  - customers' phase diagram
  - empirical data
- dynamics

« The dying seminar » (T. C. Schelling) attending a conference/seminar/working group

Forum Des Sciences Cognitives
http://cognivence.risc.cnrs.fr/
cognivence@gmail.com

why coming?

- more or less interesting program…
- there might be a cost…

Registration fees: 1 578,72 € TTC

- quite importantly,

opportunity to meet and discuss with others

- some may be happy to come even almost alone

- most would be unhappy to come if they find themselves alone

- some may be happy to come only if all the others attend…

The dying seminar

- T. C. Schelling
  « Micromotives and Macrobehavior », Norton & Cy, 1978)

- N scientists are asked to participate to a seminar/working group, meeting every Saturday

- every one has his own willingness to participate:

  scientist i is willing to participate

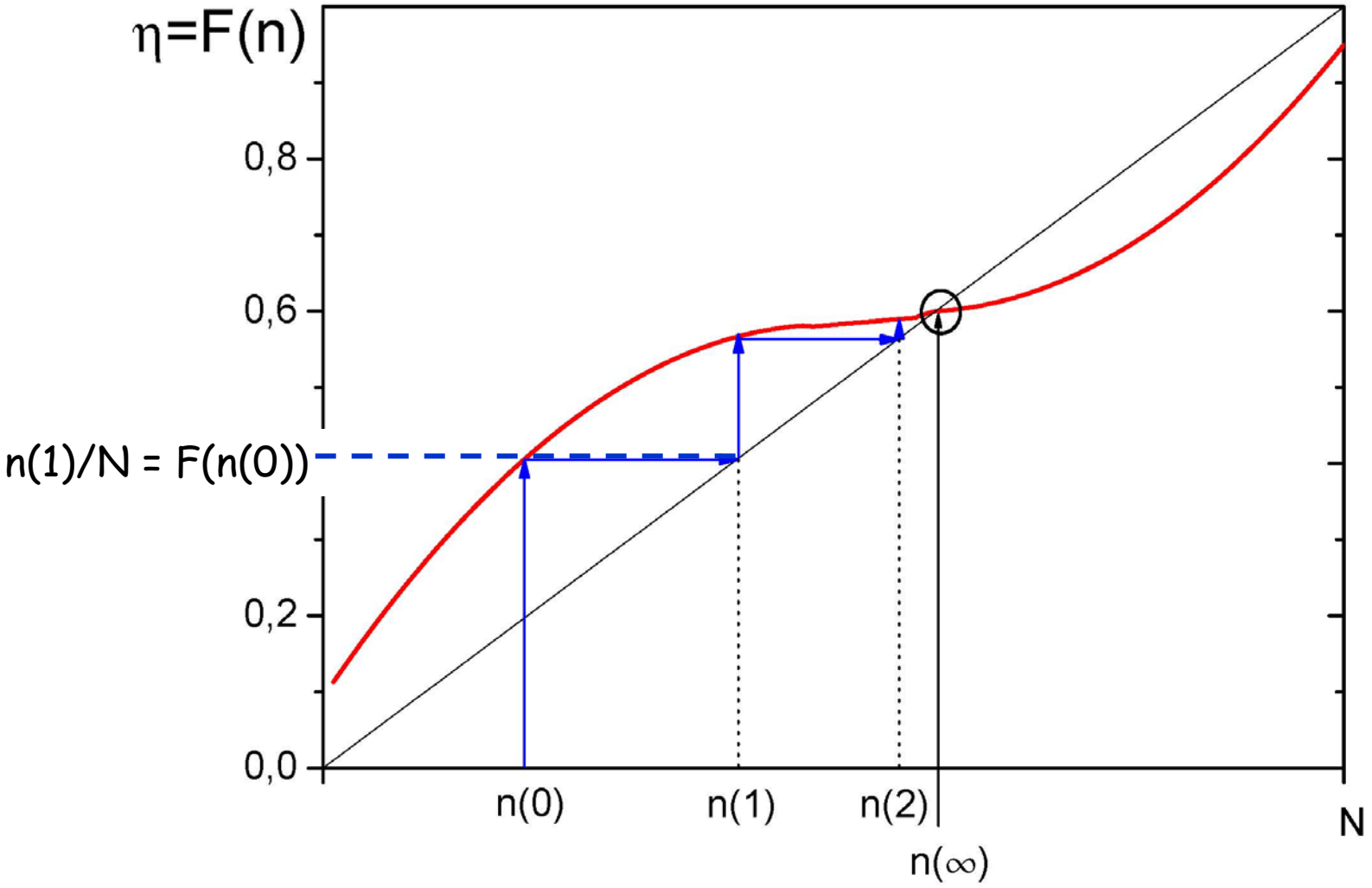
  if the number who attend is larger than n_i

- each week every one knows what was the attendance of the last Saturday

The dying seminar

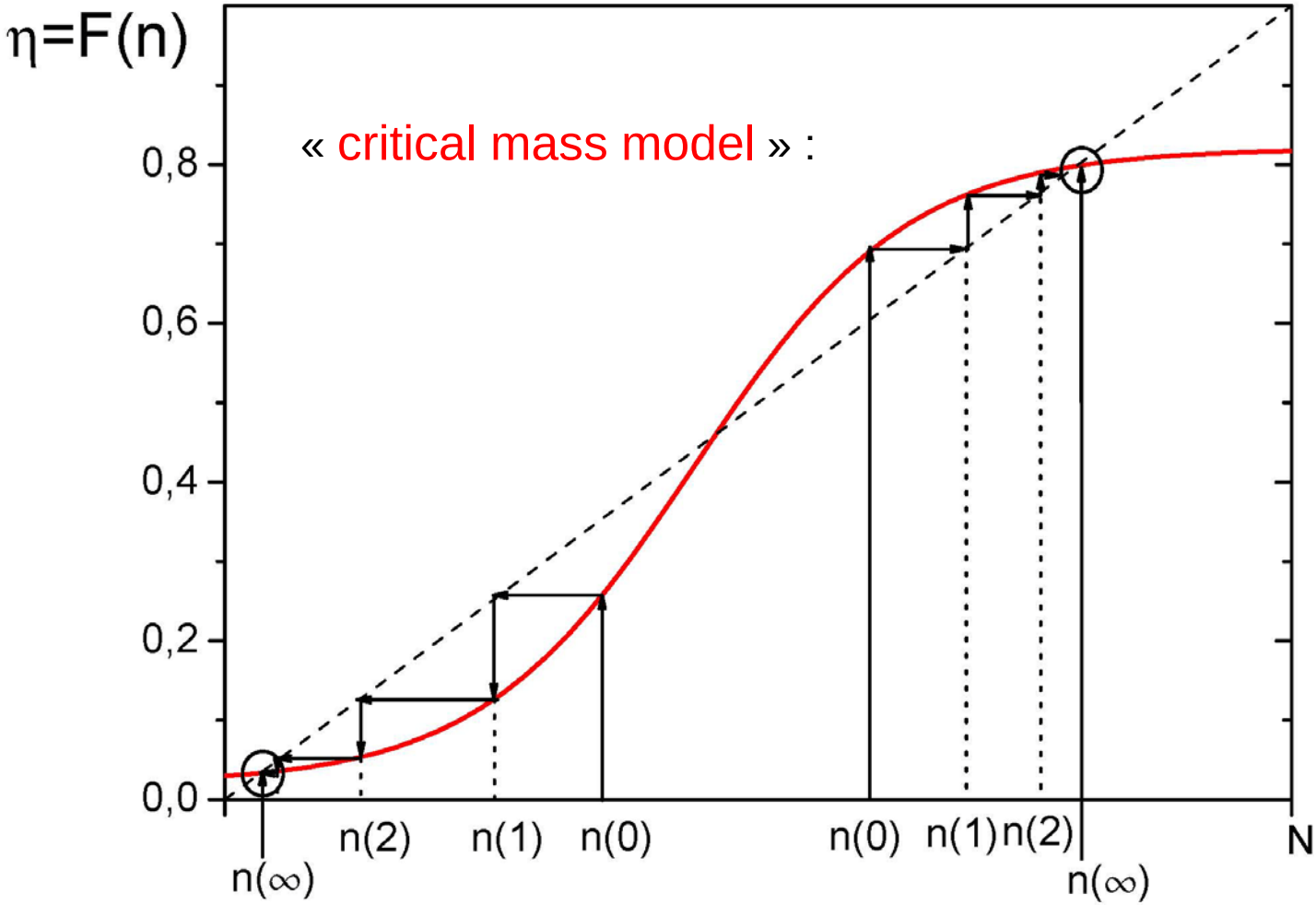
F(n) = fraction of agents who would be glad to attend if the attendence is n

= fraction of agents with $n_i \leq n$



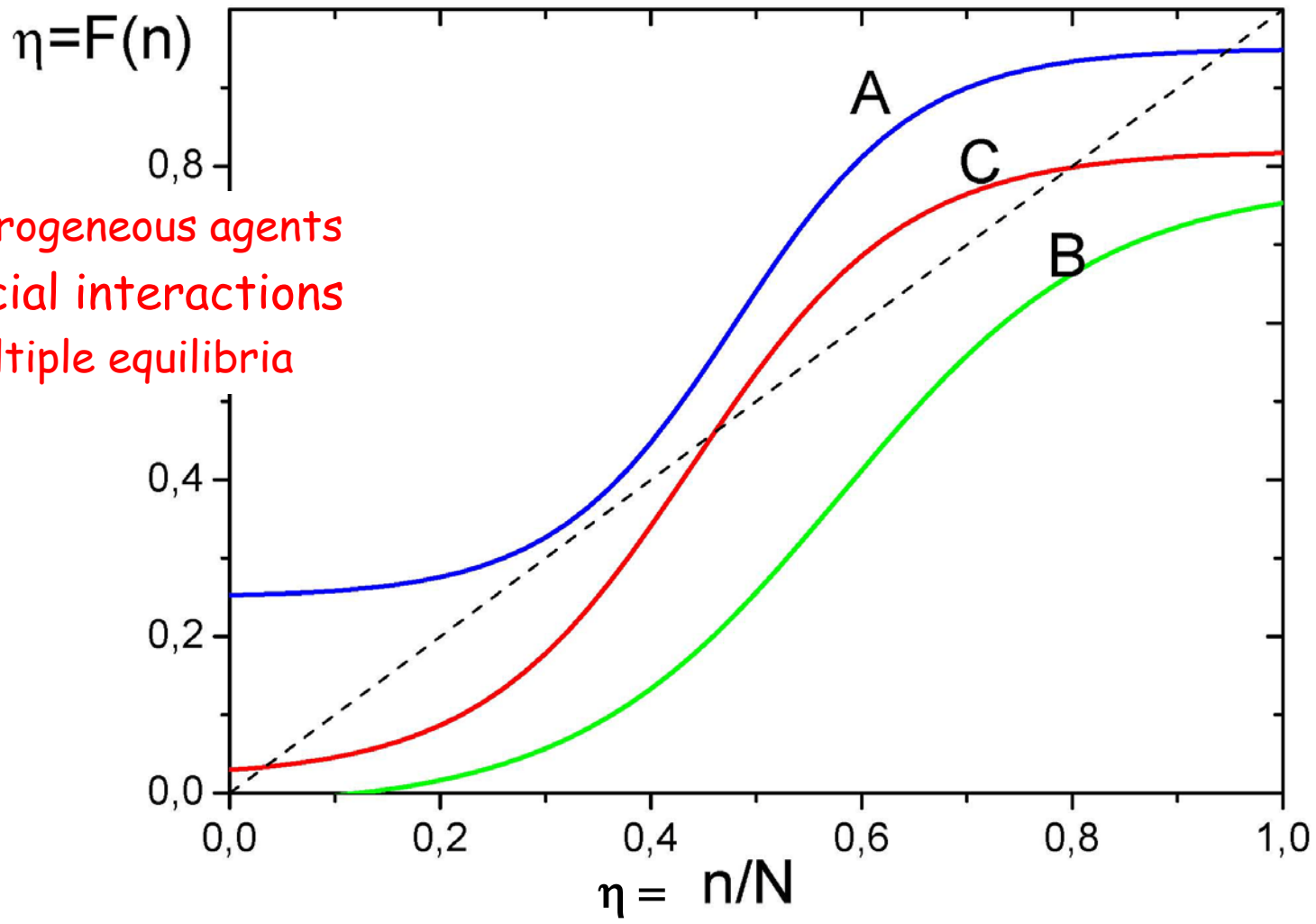
The dying seminar

F(n) = fraction of agents with $n_i \leq n$



The dying seminar

F(n) = fraction of agents with $n_i \leq n$



heterogeneous agents

+ social interactions

= multiple equilibria

alternative formulation (~ Random Field Ising Model) market context

N agents

each one has to make a binary decision:

to buy or not to buy

a single homogeneous good

at the posted price **P**

in the absence of social influence:

willingness to pay:

agent i has its reservation price H_i

agent i buys if: $P < H_i$

choice under social influence

« positive externalities »,

« bandwagon property »,

strategic complementarity

reservation price higher if others buy

agent i buys if: $P < H_i + J K_i / N$

$J > 0$

K_i = number of 'neighbours' of i who are buying

(social network)

= Dying seminar: i attends/buys if $K > n_i = N (P - H_i)/J$

= Random Field Ising Model (RFIM) at zero temperature:

$S_i = \text{sign}(H_i - P + J K_i / N)$ $\quad K_i = \sum_j (S_j + 1)/2$

reservation price higher if others buy

agent i buys if: $P < H_i + J K_i / N$

$J > 0$

K_i = number of 'neighbours' of i who are buying

(social network)

Global neighbourhood Large N limit

$K_i / N \cong$ total fraction of buyers in the population $\equiv \eta$

Heterogeneity: *« quenched disorder »*

H_i = Idiosynchratic Willingness to Pay (**IWP**)

H = population average σ^2 = variance

$H_i = H + \sigma x_i$ x_i distributed according to some pdf $f(x)$

$\langle x \rangle = 0 \quad \langle x^2 \rangle_c = 1$

$J > 0$ weight of social influence

$\text{Surplus}_i = H_i + J\,\eta - P$

agent i wants to buy iff $\text{surplus}_i > 0$

2 parameters: j, δ

$j = J/\sigma$

$h = H/\sigma$

$p = P/\sigma$

$\delta = h - p$

+ shape of f

Nash equilibria

- individual i's choice : buy iff $x_i > p - h - j\,\eta$ [$x_i = (H_i - H)/\sigma$]

  η: fraction of buyers

- equilibria?

  = states in which for every agent, his/her decision is consistent with his preferences and the collective behavior

  That is, in an equlibrium,

  for every agent i who has bought, one must have

  $x_i > p - h - j\,\eta$ (1)

  and for every agent i who has not bought, one must have

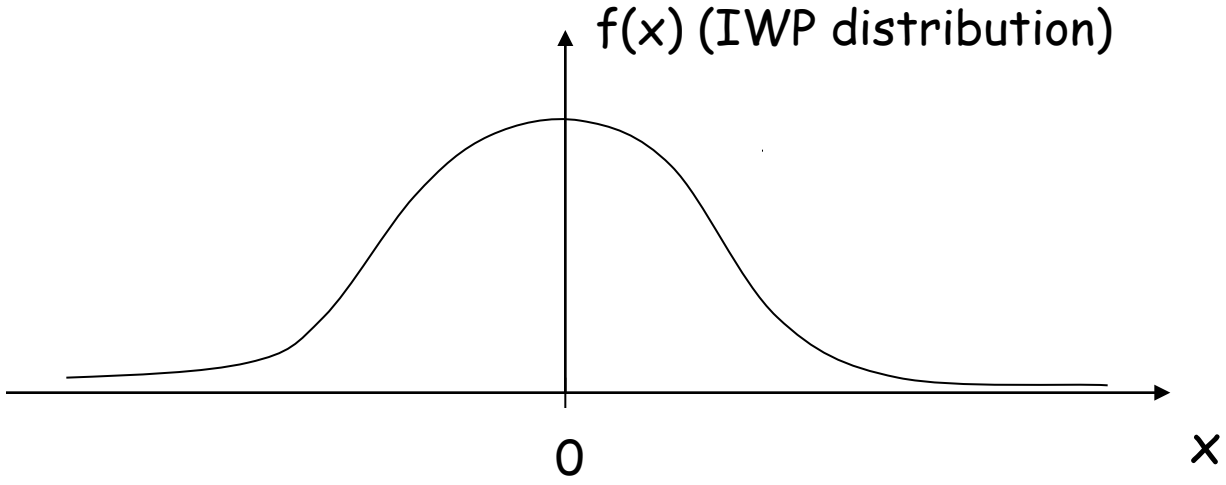
  $x_i < p - h - j\,\eta$ (2)

  where η is the actual fraction of buyers (those for which (1) is true)

Nash equilibria

- individual i's choice : buy iff $x_i > p - h - j\,\eta$ [$x_i = (H_i - H)/\sigma$]
- equilibria?

η: fraction of buyers

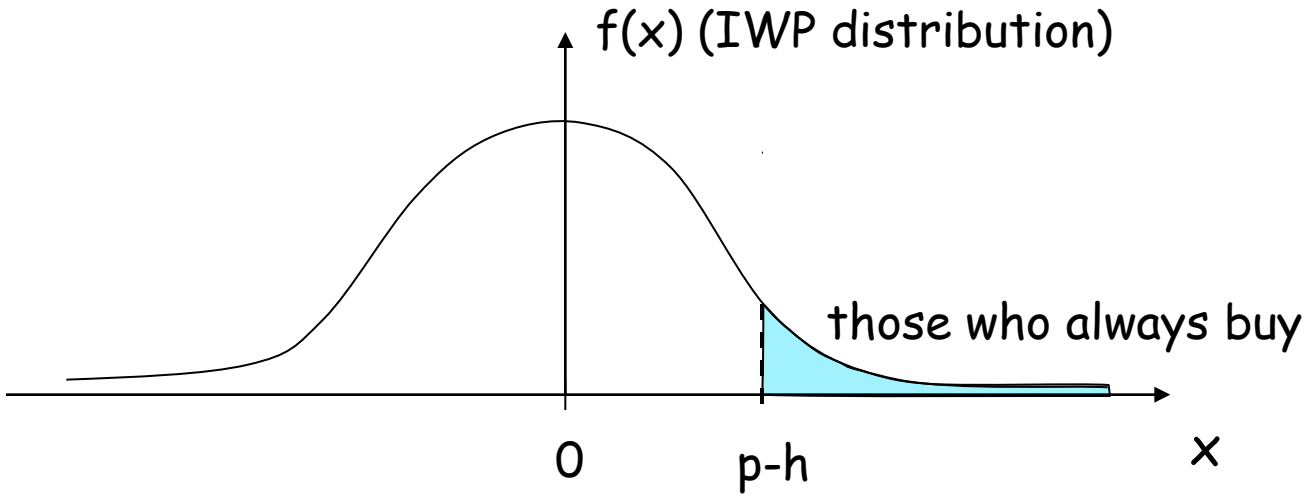


Nash equilibria

- individual i's choice : buy iff $x_i > p - h - j\,\eta$ $\quad [\, x_i = (H_i - H)/\sigma \,]$
- equilibria?

η: fraction of buyers

even if $\eta = 0$, for $x_i > p - h$, i buys



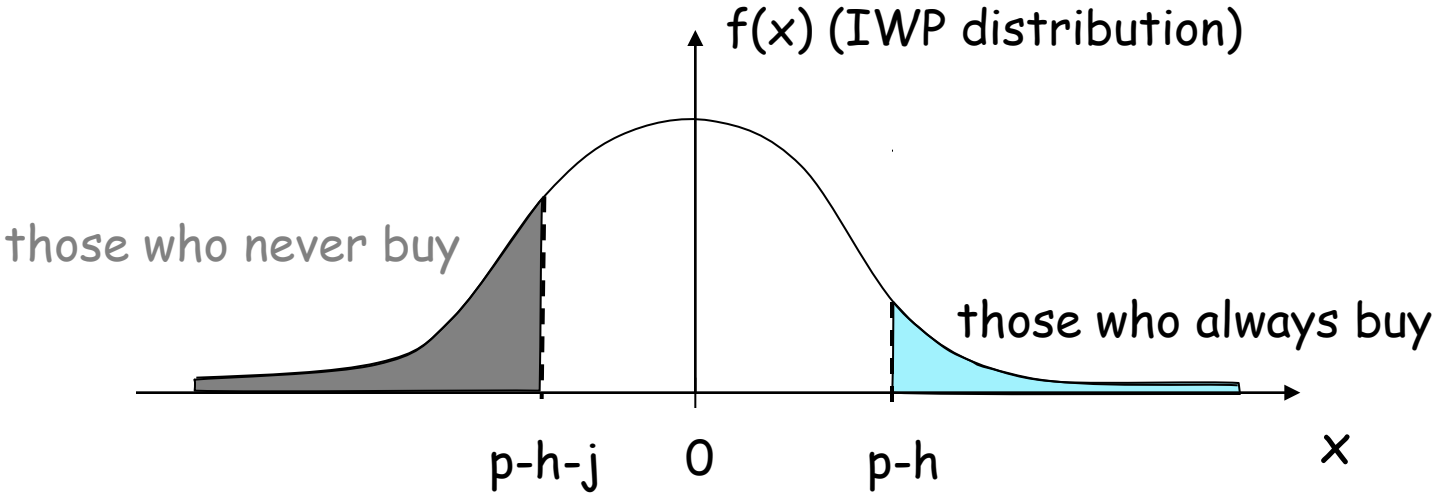
Nash equilibria

- individual i's choice : buy iff $x_i > p - h - j\,\eta$ $\quad [\, x_i = (H_i - H)/\sigma \,]$
- equilibria?

η: fraction of buyers

even if $\eta = 1$, for $x_i < p - h - j$, i does not buy

even if $\eta = 0$, for $x_i > p - h$, i buys



Nash equilibria

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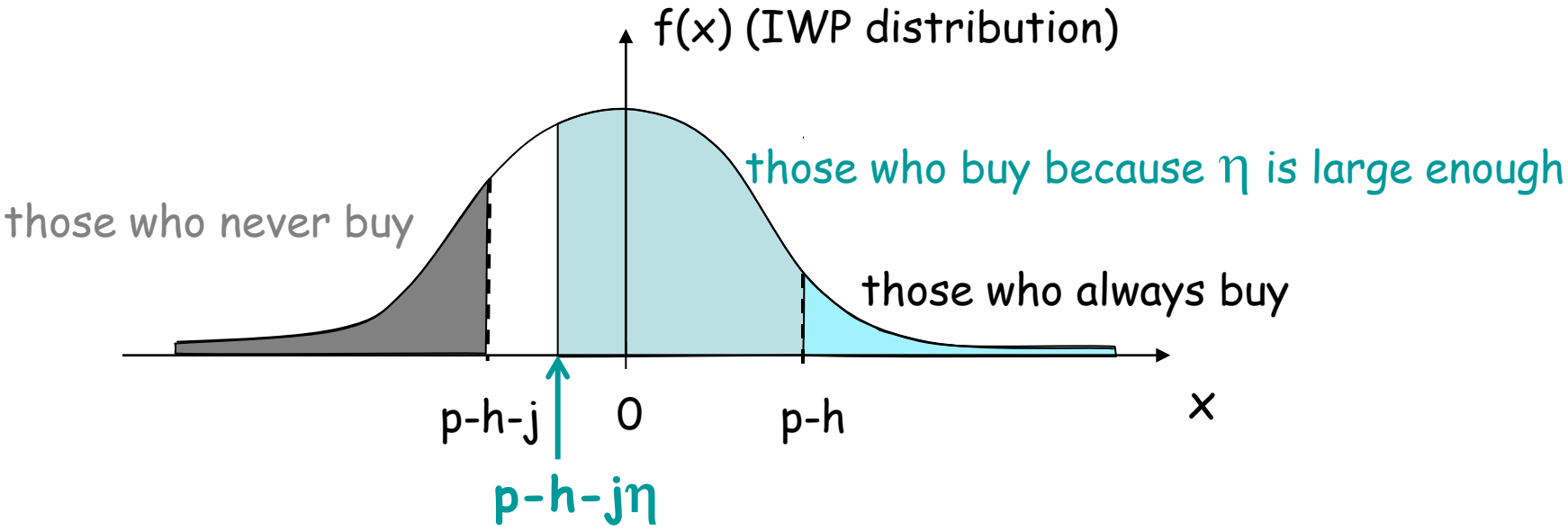

Nash equilibria

- individual i's choice : buy iff $x_i > p - h - j\,\eta$ [$x_i = (H_i - H)/\sigma$]
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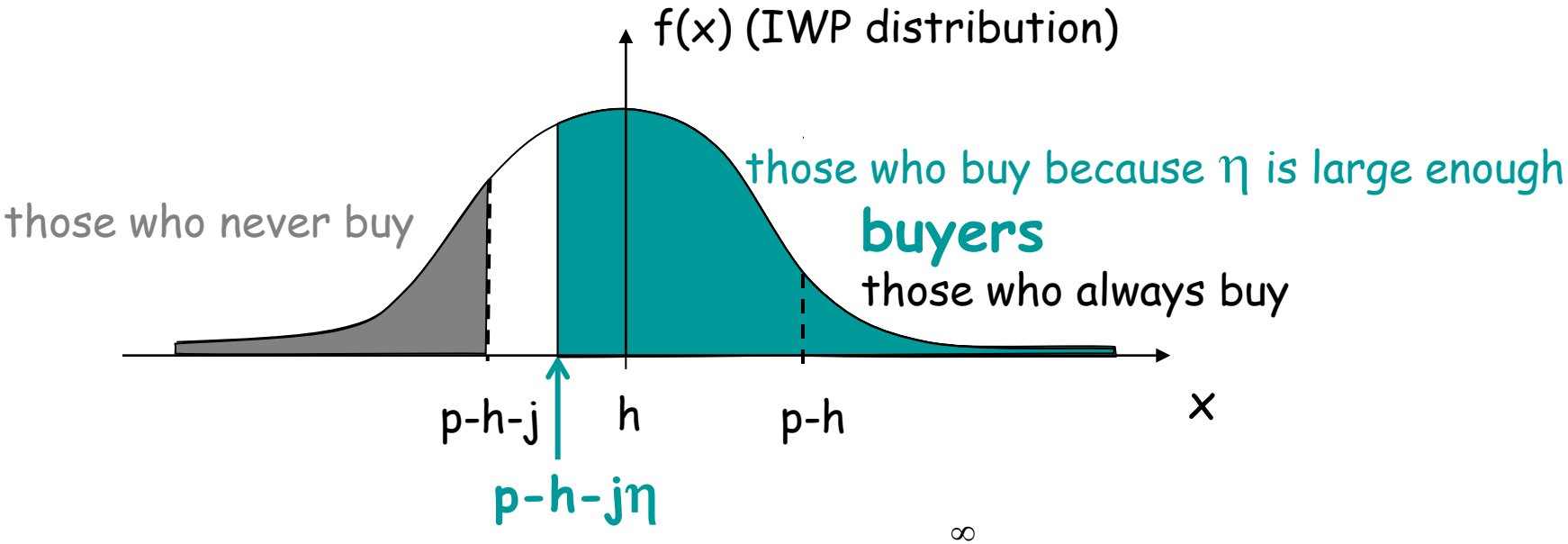


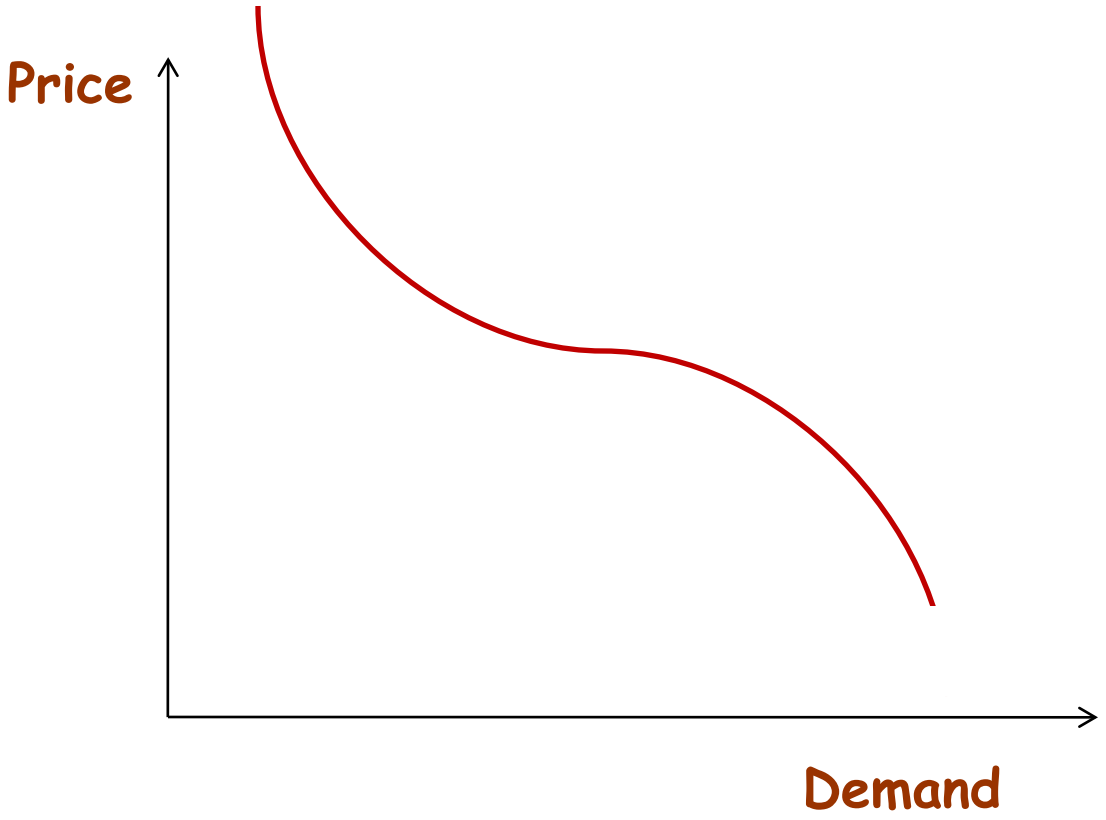
Nash equilibria : η solution of $\eta = \int_{p-h-j\eta}^{\infty} f(x)\,dx$

Demand vs. price

(at given values of **h** and **j**)

at small values of **j**
or large values of **h**

standard behaviour

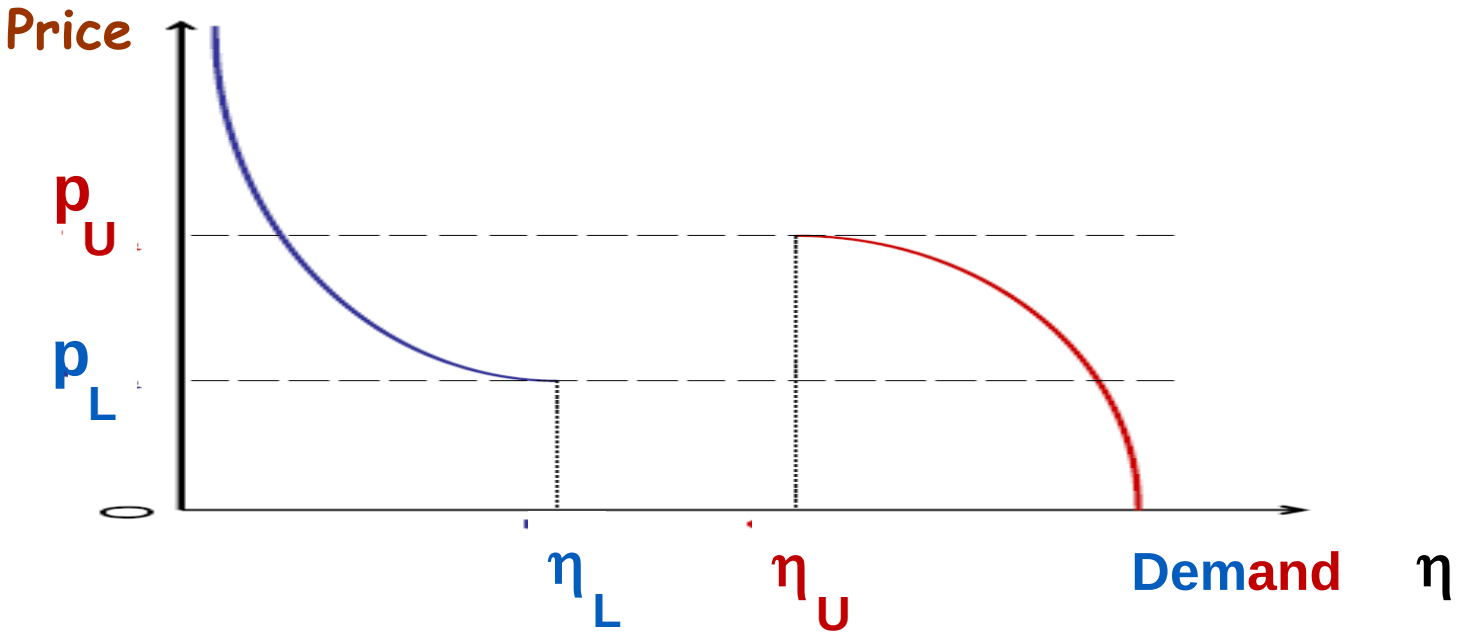


Demand vs. price

(at given values of **h** and **j**)

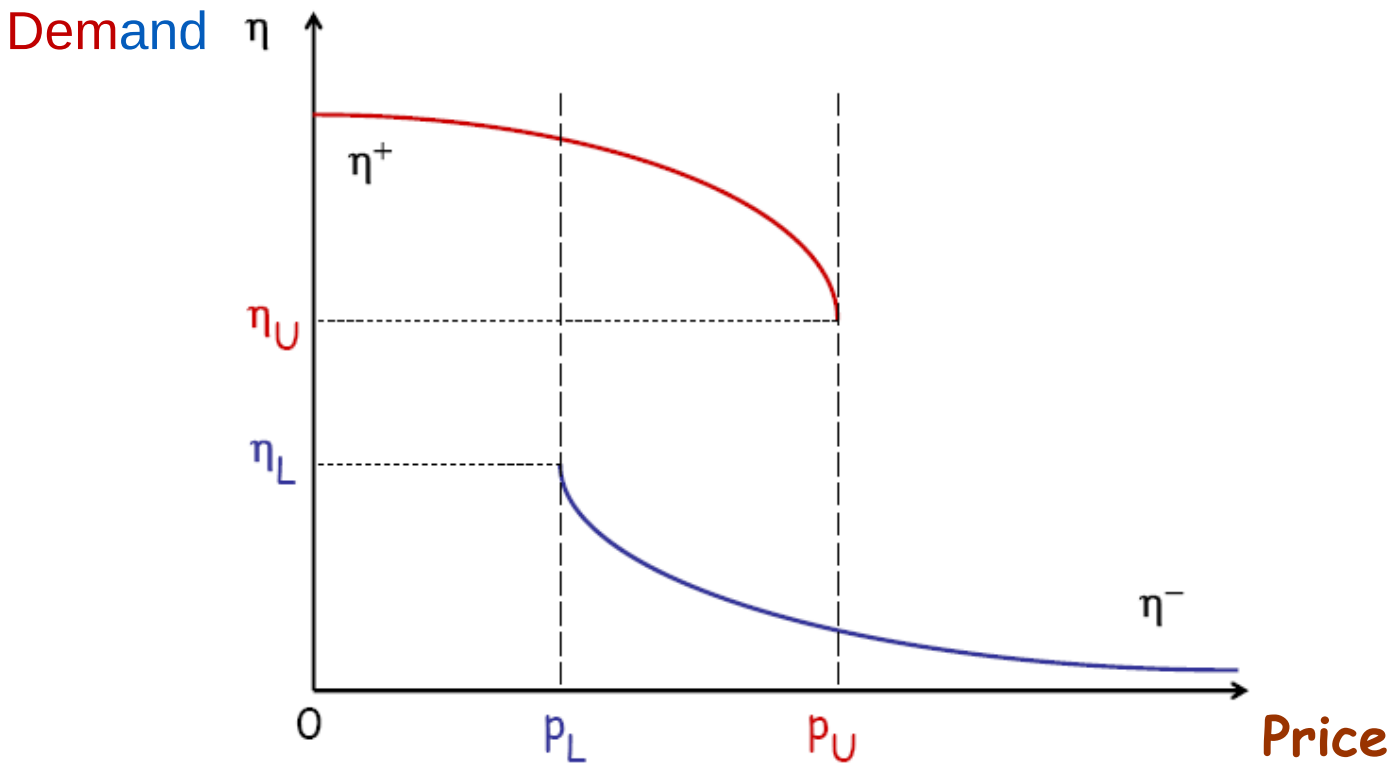
large enough values of **j**
and small values of **h**

multiple equilibria



Demand vs. price

(at given values of **h** and **j**)



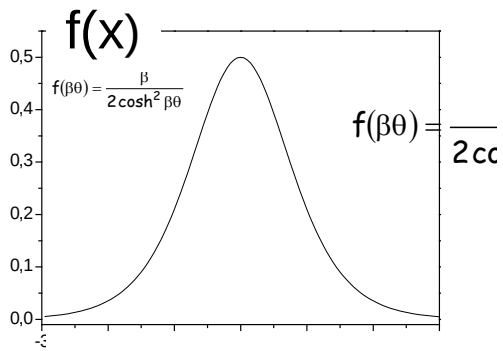
Nash equilibria

η solution of:

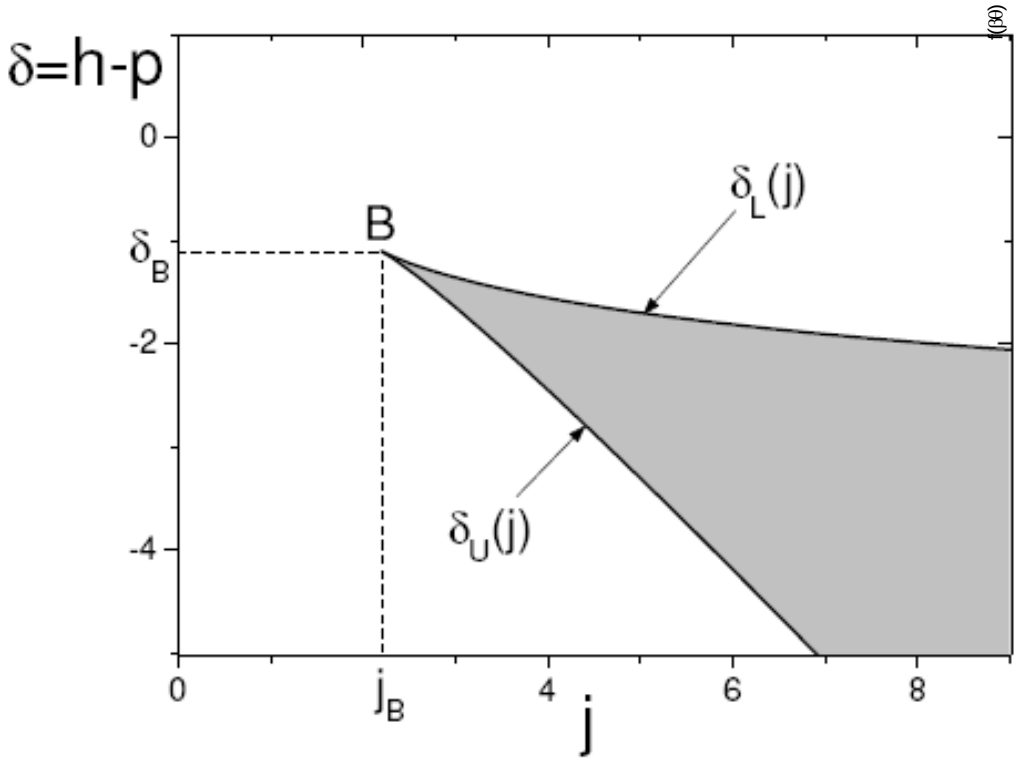
$$\eta = \int_{p-h-j\eta}^{\infty} f(x)\,dx$$

customer's phase diagram

smooth mono-modal distribution of the IWP



$x=(H_i - H)/\sigma$

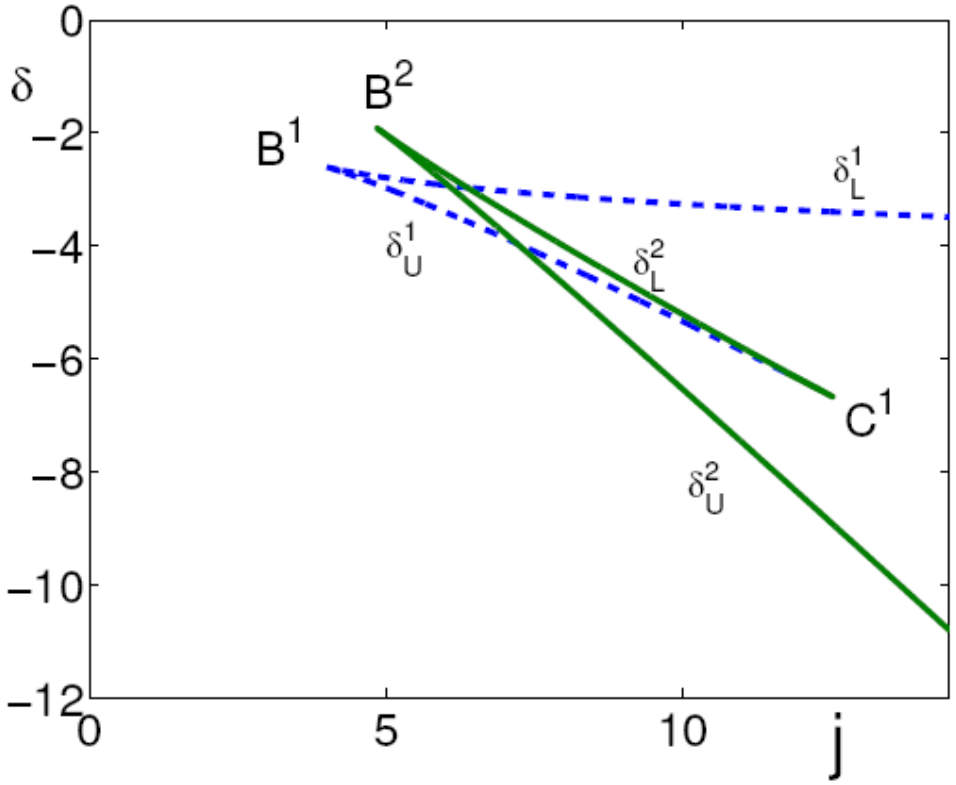


generic phase diagram for any smooth monomodal distribution

(more complex generic diagram for multimodal distributions)

Gordon et al 2009

Bimodal distribution



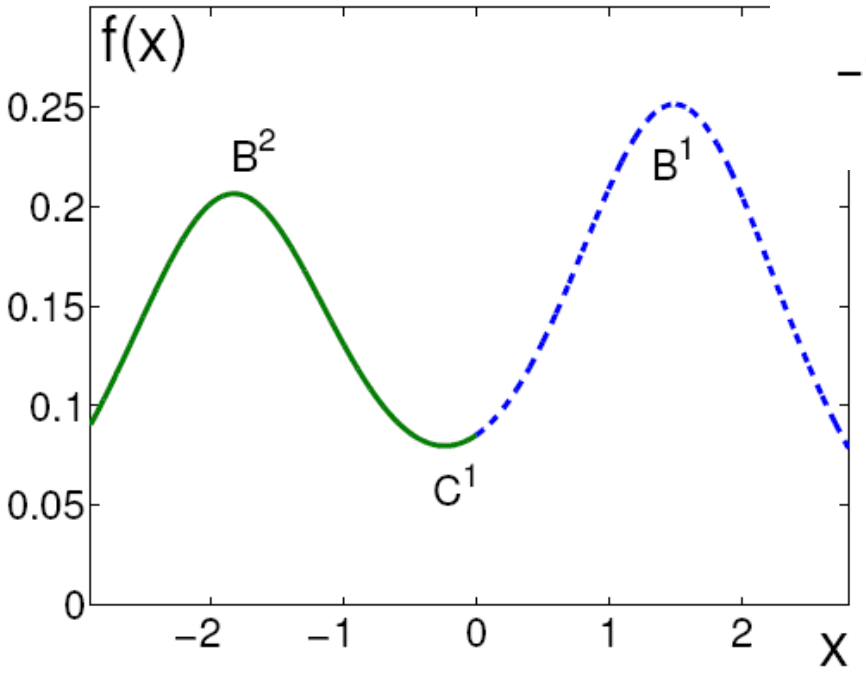
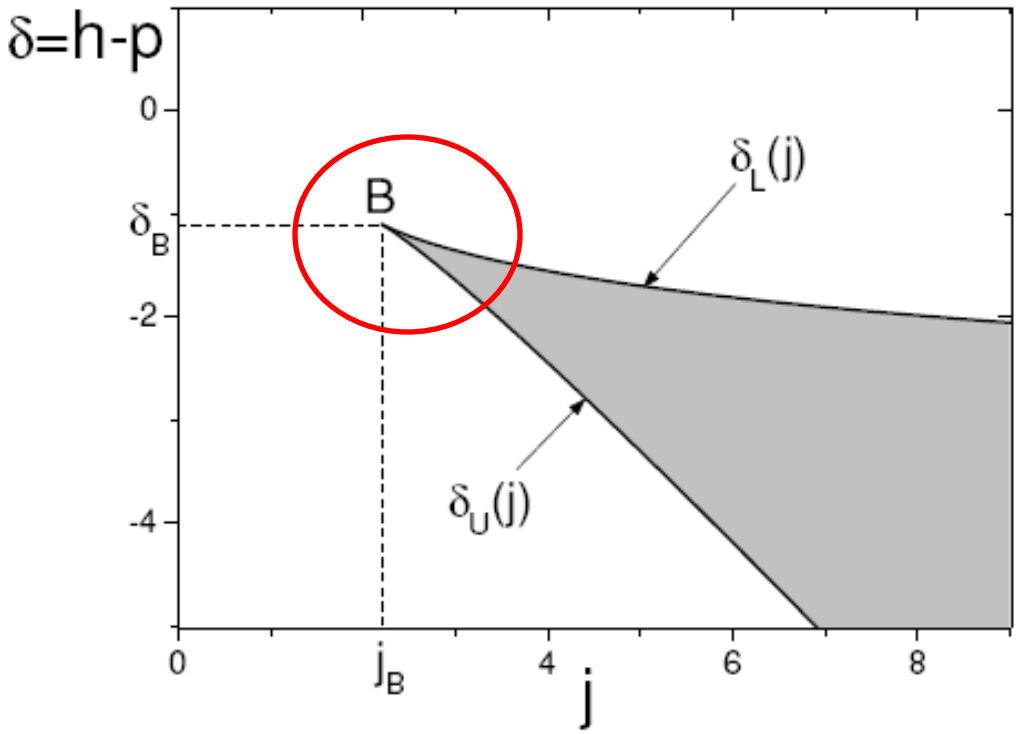


Figure 11: *An example of bimodal pdf.*

Gordon et al 2009

customer's phase diagram

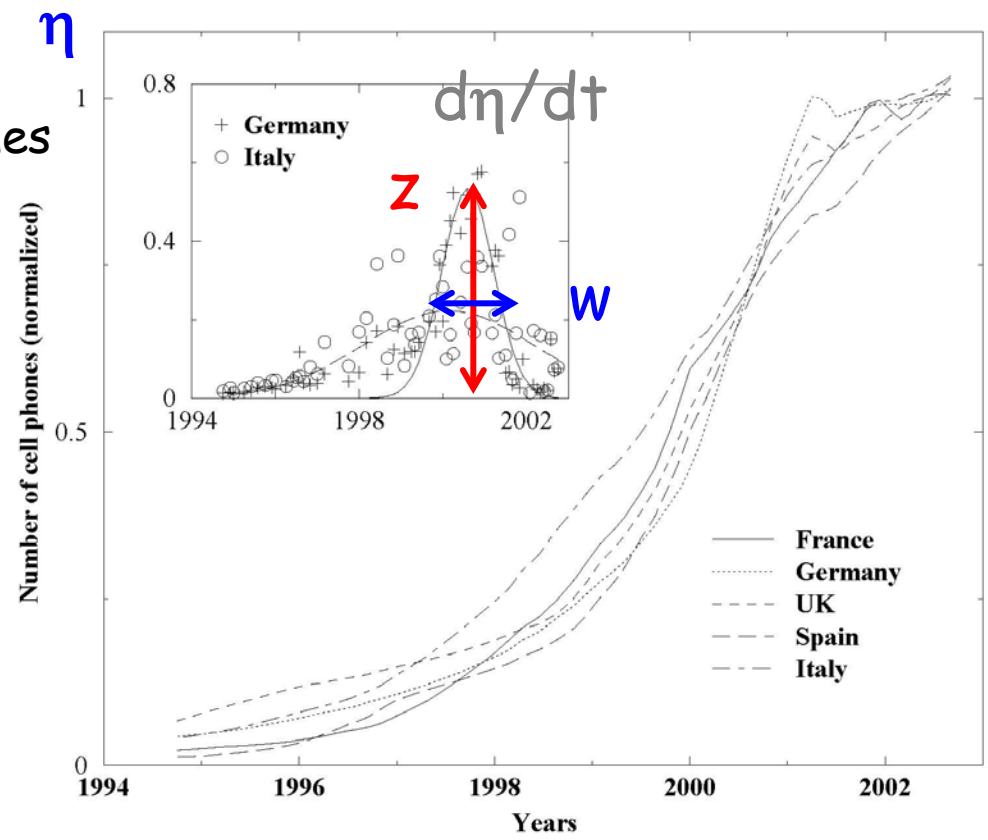
smooth mono-modal distribution of the IWP



universal properties near the singular point B

Analysis of Empirical Data

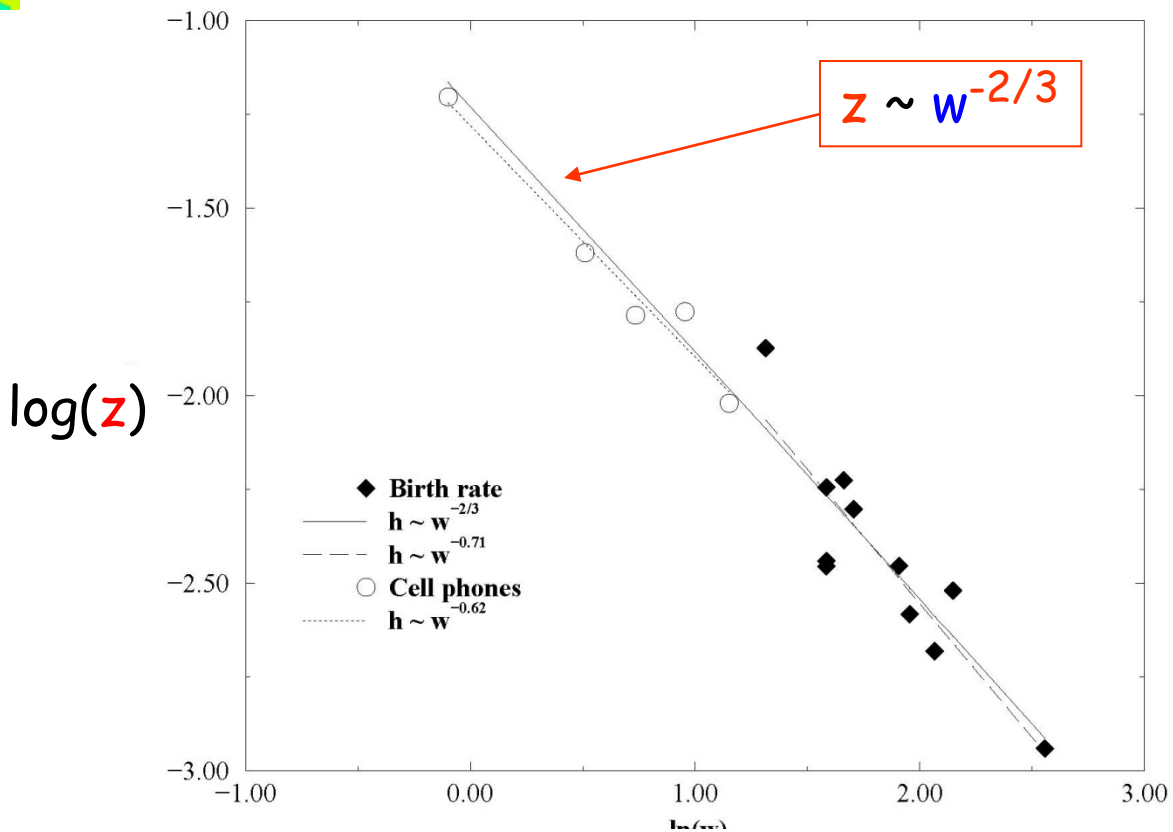
Number of cell phones



Q. Michard & J.-P. Bouchaud, 2005

« Theory of collective opinion shifts: from smooth trends to abrupt swings »

Empirical Data: scaling



Q. Michard & J.-P. Bouchaud, 2005

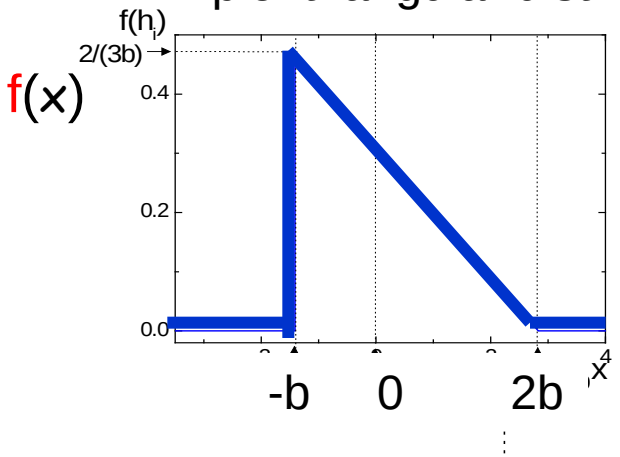
« Theory of collective opinion shifts: from smooth trends to abrupt swings »

See also: "Of Songs and Men: a Model for Multiple Choice with Herding"
C. Borghesi & J-P Bouchaud (2006)
(case of more than 2 choices)

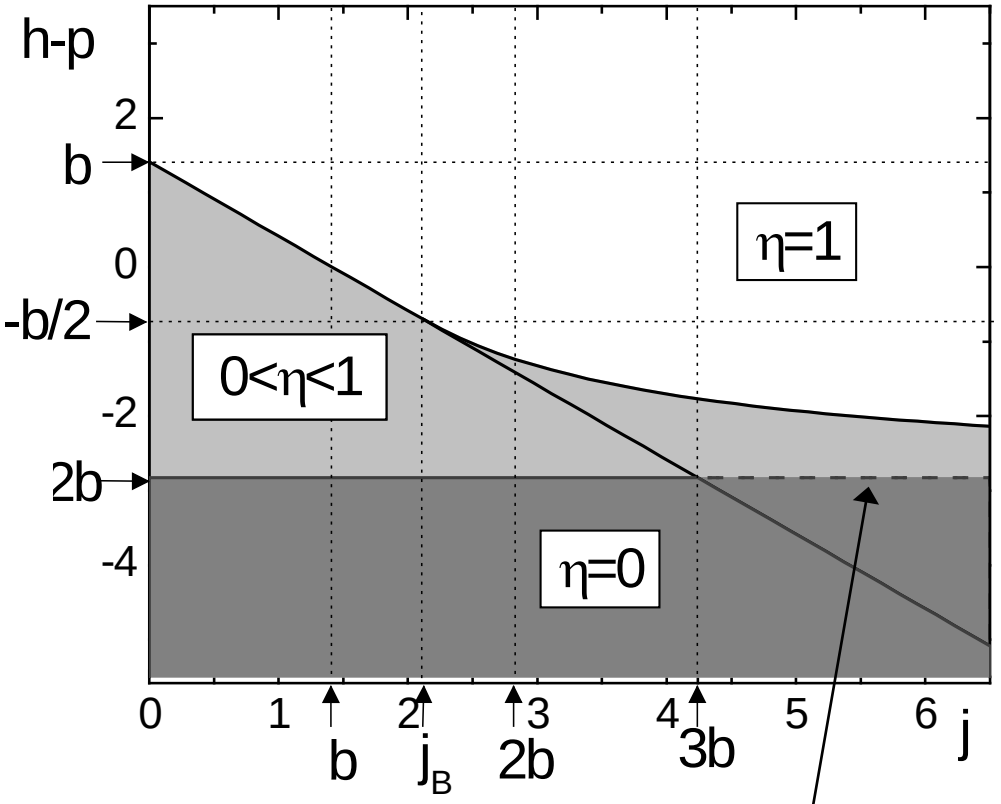
customers phase diagram

Exple: triangular distribution of the IWP:

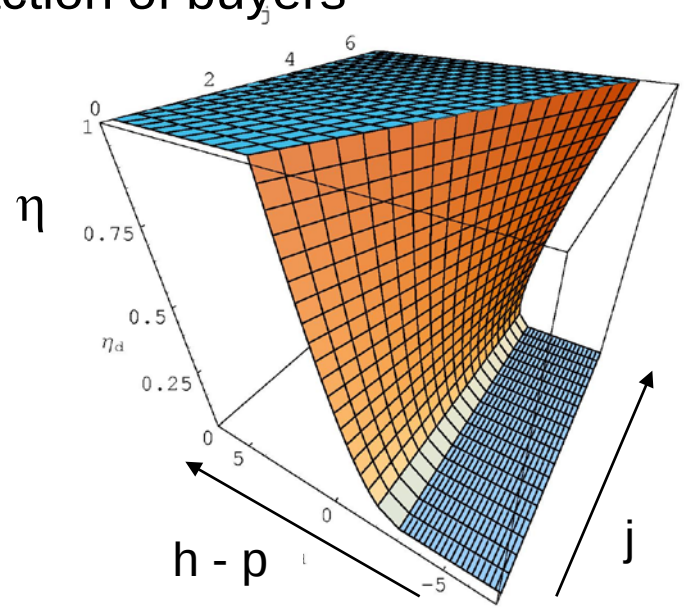
$h_i = h + x_i$



phase diagram:



fraction of buyers



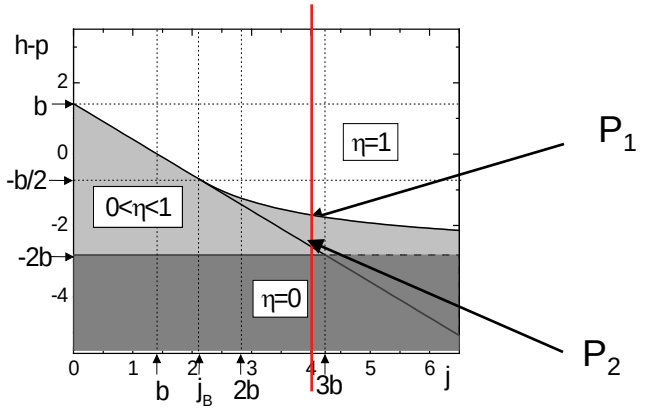
$h = H/\sigma$ $p = P/\sigma$

$j = J/\sigma$

coexistence of 2 solutions

Dynamics: simulations

phase diagram
for the
triangular distribution :

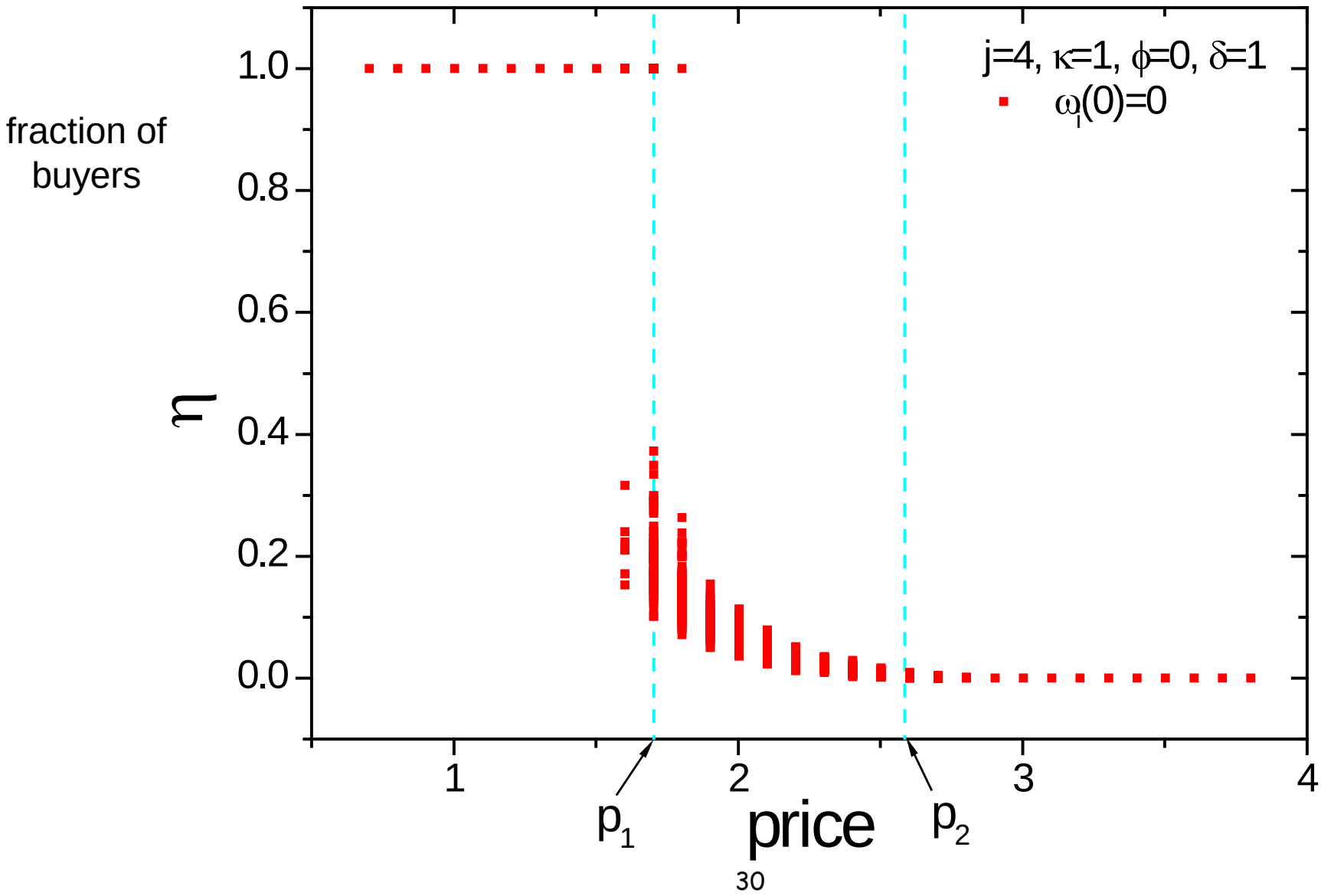


J=4

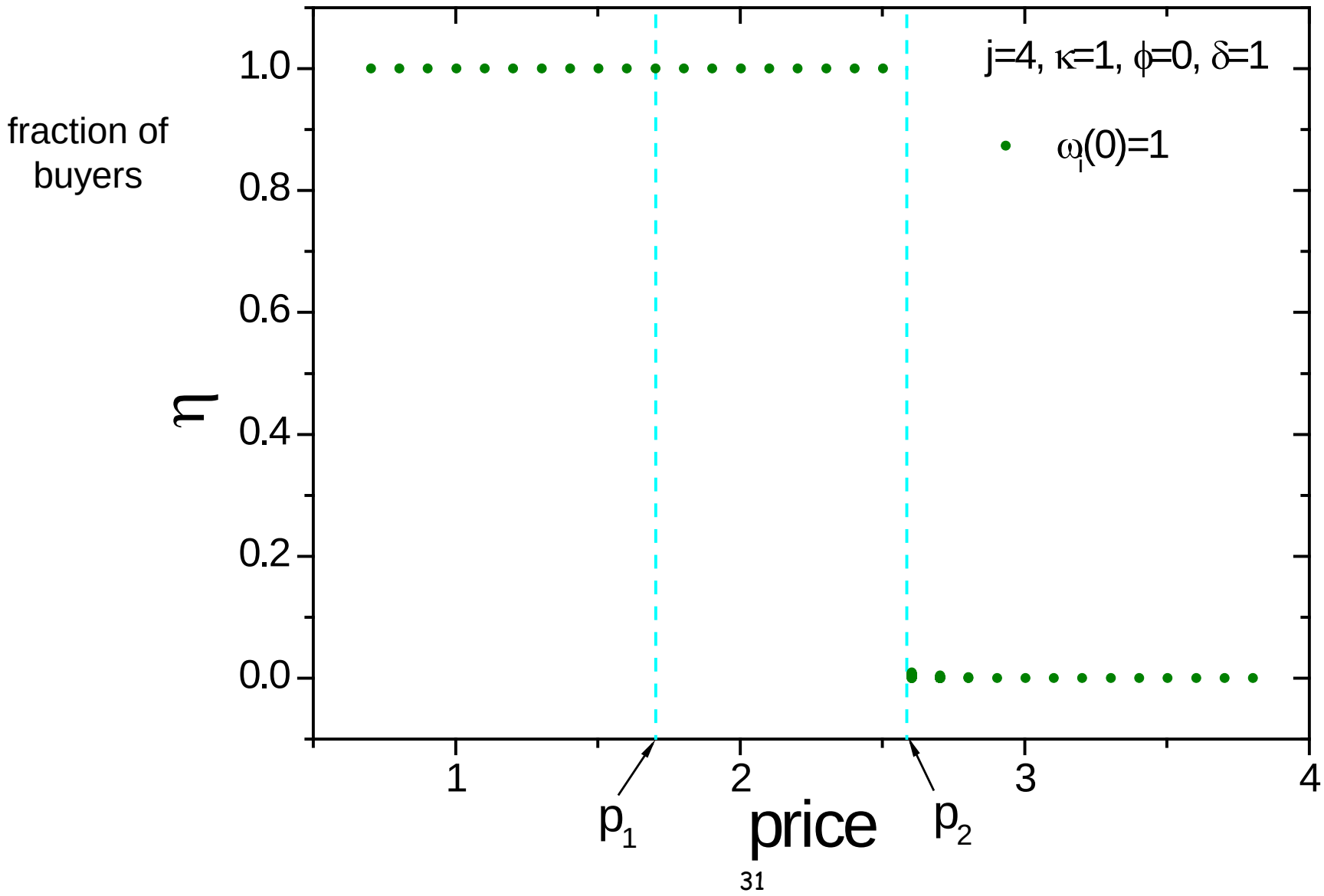
myopic fictitious play

$$\hat{\eta}_i(t+1) = \eta(t)$$

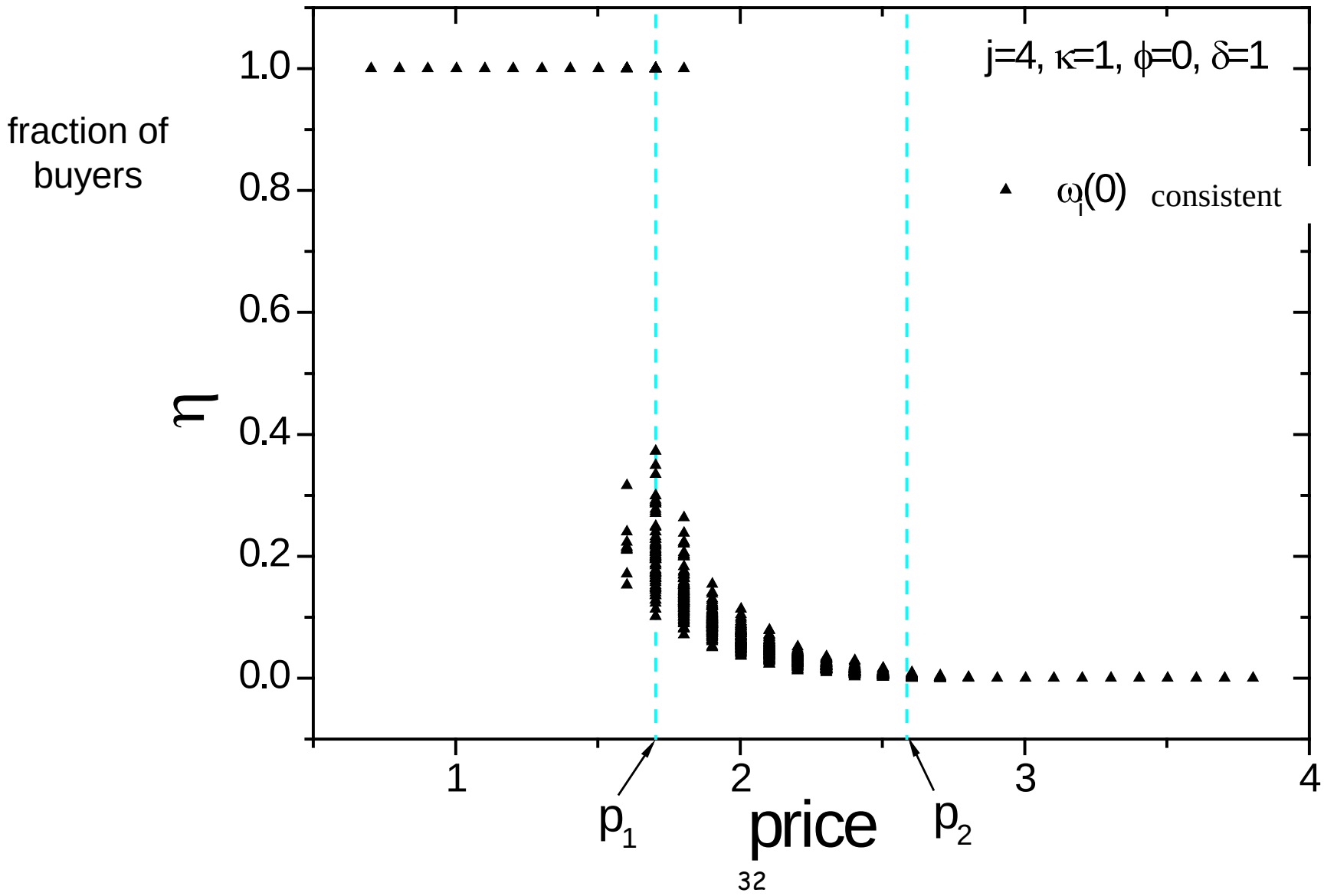
myopic best response



myopic best response



myopic best response



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Use of the Ising framework in socio-economic modelling by physicists… and economists

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