Dynamique des interactions sociales :
du choix individuel au comportement collectif

Dynamics of social interactions:
from individual choice to collective behavior

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• Part 1:
« Qui se ressemble s’assemble » / « Birds of a feather flock together »
Analogy with a model of associative memory (Hopfield), and with Ising spin models

• Part 2: **mimetism**
  - informational influences (A. Orléan)
  - « the dying seminar » (T. C. Schelling and variants)

• Part 3: Dynamics & Equilibria
  (dynamical systems, physics, game theory)
Mimetism

Different types of mimetism (\textit{A. Orléan})

- \textbf{Conformity, Social norms} - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

- \textbf{Informational influences} - private and public informations
  \rightarrow herd behavior, information cascades

- \textbf{Preferential imitation} - externalities in agent's utility function
  Schelling, Granovetter
  « the dying seminar », formation of riots
**Mimetism**

Different types of mimetism (*A. Orléan*)

- **Conformity, Social norms** - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

  « *in Roma do as the romans do* »

  « *politically correct* »

  « *Penser contre son temps c'est de l'héroïsme. Mais le dire, c'est de la folie.* » Eugène Ionesco
Mimetism

• Conformity, Social norms - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

Social psychology: Ash conformity experiment (1951, 1955)

When individual judgment conflicts with a group, the individual will often conform his judgment to that of the group
Mimetism

- **Conformity, Social norms** - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

Social psychology: *Ash conformity experiment* (1951, 1955)

*When individual judgment conflicts with a group, the individual will often conform his judgment to that of the group*

**Neurobiological correlates** of social conformity (*Berns et al 2005*)

http://www.ccnl.emory.edu/greg/

- brain regions classically associated with perception can be altered by social influences

- **independence** (non conformity) is found to be associated with subcortical activity changes indicative of emotional salience (amygdala activation ↔ emotional load associated with standing up for one’s belief)
Mimetism

Different types of mimetism (A. Orléan)

• Conformity, Social norms - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

   endogeneous norm: autoreferential mimetism:
   conformity to the emerging majority

   stock markets

   « C'est bien la pire folie que de vouloir être sage dans un monde de fous » (D. Erasme)

   « C'est une grande folie de vouloir être sage tout seul » (La Rochefoucauld)
Different types of mimetism (A. Orléan)

- **Conformity, Social norms** - peer group pressure; conforming to the prevailing conventions; tradition, culture transmission

- **Informational influences** - private and public informations
  → herd behavior, information cascades
  (Banerjee 1992, Orléan 1995)

- Preferential imitation
Mimetism

Informational influences

Sequential rational decisions → information cascades
   (Banerjee 1992; Bikhchandani, Hirshleifer & Welch 1992)

Simple version
Issue: choosing between restaurant A and restaurant B

Private information
   • « A better than B » or « B better than A »

Public information
   • every arriving agent can see the number of customers in each restaurant

Common knowledge
   • reliability of private information = 70%
   • prior knowledge on the quality of A % B
     (e. g. likeliness of « A better than B »)
Mimetism

information cascade:

Common knowledge:
- reliability of private information = 70%
- 55% chance that B is better than A

Hypothesis: actually A better than B (unknown true state of nature)

First agent: let private information = B  
→ he goes to B

Second agent: private information = A  
public information: one customer in B.  
rational reasoning:  
the customer must have entered because he got ‘B’  
his private information is as good as mine  
→ I go to B

All the agents go to B, eventhough 70% will have received the correct private signal
Mimetism

Informational influences

Case of simultaneous decisions (Orléan 1995)

Issue: choosing between two possible behaviors / hypothesis
Exple: stock market, hypothesis « High » vs. « Low »

Private information:

each agent i receives a private signal \( \sigma(i) \) = +1 or -1

\[ \sigma = +1 \quad \sigma = -1 \]

« H more likely than L » « L more likely than H »

Common knowledge & public information:

- prior knowledge on H % L (e.g. H and L equally likely)
- reliability of private information \( p \) (e.g. \( p = 70\% \))
- at every time step, every agent observes the total number of agents making the choice H
Mimetism - Informational influences

Common knowledge: reliability of private information = \( p > \frac{1}{2} \)

prior distribution:
\[
\rho(H) = \rho(L) = \frac{1}{2}
\]

Optimal (rational) decision rule for a given agent \( i \)?

If private information alone (no public information):

Intuitively:

\( i \) should decide 'H' if signal \( \sigma(i) \) is +1, and 'L' if signal \( \sigma(i) \) is -1

More formally:

Bayesian inference:

Known:
\[
P(\sigma(i) | H), \quad P(\sigma(i) | L)
\]

Wanted:
\[
P(H | \sigma(i))
\]
\[
P(L | \sigma(i)) = 1 - P(H | \sigma(i))
\]
Mimetism - Informational influences

Common knowledge: reliability of private information = \( p > \frac{1}{2} \)

prior distribution: \( \rho(H) = \rho(L) = \frac{1}{2} \)

If private information alone:

**Wanted:** \( P(H | \sigma) \)

**Known:** if true (unknown) state ('state of nature') is \( H \)

\[
P(\sigma = +1 | H) = p \quad \text{= probability to receive the correct information} \\
P(\sigma = -1 | H) = 1-p \quad \text{= probability to receive the erroneous information}
\]

Bayes rule:

\[
P(H | \sigma = +1) = P(\sigma = +1 | H) \rho(H) / p(\sigma = +1) \\
P(H | \sigma = -1) = P(\sigma = -1 | H) \rho(H) / p(\sigma = -1)
\]

where

\[
p(\sigma = +1) = P(\sigma = +1 | H) \rho(H) + P(\sigma = +1 | L) \rho(L) = \frac{1}{2} p + \frac{1}{2} (1-p) = \frac{1}{2} \\
p(\sigma = -1) = P(\sigma = -1 | H) \rho(H) + P(\sigma = -1 | L) \rho(L) = \frac{1}{2} (1-p) + \frac{1}{2} p = \frac{1}{2}
\]
Mimetism - Informational influences

Common knowledge: reliability of private information = \( p > \frac{1}{2} \)

prior distribution \( \rho(H) = \rho(L) = \frac{1}{2} \)

\[
\begin{align*}
P(H | \sigma(i) = 1) &= p > \frac{1}{2} & P(H | \sigma(i) = -1) &= 1-p < \frac{1}{2} \\
P(L | \sigma(i) = 1) &= 1-p < \frac{1}{2} & P(L | \sigma(i) = -1) &= p > \frac{1}{2}
\end{align*}
\]

Thus, if only private information is available:

\( \rightarrow \) Decide 'H' if signal is 1, decide 'L' if signal is -1

(just as intuition told us)

Probability of mistake = 1-p

However, do it again for \( \rho(H) \neq \rho(L) \): condition on \( \rho(H) \) and \( p \) for having the same result?
**Mimetism - Informational influences**

Assume true (unknown) state (‘state of nature’) is $H$

Common knowledge: reliability of private information $= p > \frac{1}{2}$

prior distribution: $\rho(H) = \rho(L) = \frac{1}{2}$

If private information alone: decide ‘$H$’ if signal is 1, decide ‘$L$’ if signal is -1

For $N$ agents, probability that $M$ agents receive the correct information $= C^M_N \cdot p^M \cdot (1-p)^{N-M}$

where

$$C^M_N = \frac{N!}{M! \cdot (N-M)!}$$

For large $N$, $f \equiv \frac{M}{N}$

$= p +$ fluctuations of order $1/\sqrt{N}$

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Graph showing the distribution of $f = \frac{M}{N}$ with $N = 100$ and $p = 0.7$. The graph displays a normal distribution centered around $p = 0.7$ with a standard deviation of $\frac{2}{\sqrt{N}}$. The x-axis represents the probability $f = \frac{M}{N}$ ranging from 0 to 1, while the y-axis shows the probability density from 0 to 0.1. The peak of the distribution is at $f = \frac{M}{N} = 0.7$, with the standard deviation indicated as $2/\sqrt{N}$.
Mimetism - Informational influences

Hence, if one observes $f = M/N$, one can make the correct choice with a very small probability of error:

- if $f > \frac{1}{2}$, decide 'H',
- if $f < \frac{1}{2}$, decide 'L'

But if everyone does the same, instead of using his private information, there is no more any information in the collective behavior (in the value of $f$) → paradox (Grossman & Stiglitz 1980)

For large $N$,

- $f \equiv M/N$
- $= p +$ fluctuations of order $1/\sqrt{N}$
Model (Orléan 1995): propensity to imitate $\mu$

As before: private signal $\sigma(i)$
public information: $f = \text{fraction of agents making choice 'H'}$

Behavioral rule:

If private and public information are in favor of the same hypothesis
$(\sigma(i) = +1 \text{ and } f > \frac{1}{2}, \text{ or } \sigma(i) = -1 \text{ and } f < \frac{1}{2})$
decide accordingly
(decide 'H' or decide 'L')

Otherwise:

$\mu = \text{probability to follow the majority}$

$\sigma(i) = -1 \text{ and } f > \frac{1}{2}, \ P(\text{make the choice 'H'}) = \mu, \ P(\text{make the choice 'L'}) = 1-\mu$
$\sigma(i) = +1 \text{ and } f < \frac{1}{2}, \ P(\text{make the choice 'L'}) = \mu, \ P(\text{make the choice 'H'}) = 1-\mu$
Mimetism - Informational influences

Dynamics

At each time \( t \):
- new private signal \( \sigma(i, t) \)
- public information: \( f(t) = \) fraction of agents having made the choice 'H'

Behavioral rule: applied by all the agents in parallel

For large \( N \) (assuming that 'H' is the true, unknown value)

if \( f(t) > \frac{1}{2} \), then \( f(t+1) = p + (1-p) \mu \)
if \( f(t) < \frac{1}{2} \), then \( f(t+1) = p (1- \mu) \)
Mimetism - Informational influences

Given:\n
\[ f(t+1) = \begin{cases} 
  p + (1-p) \mu & \text{if } f(t) > \frac{1}{2}, \\
  p (1-\mu) & \text{if } f(t) < \frac{1}{2}, 
\end{cases} \]

Fixed point?\n
\[ \mu < \mu^* \equiv 1 - 1/2p, \quad \text{a unique fixed point, } f = p + (1-p) \mu \]
Mimetism - Informational influences

Fixed point
\[ \mu < \mu^* \equiv 1 - 1/2p, \quad \text{a unique fixed point, } f = p + (1-p) \mu \]

if \( f(t) > \frac{1}{2} \),
then \( f(t+1) = p + (1-p) \mu \)

if \( f(t) < \frac{1}{2} \),
then \( f(t+1) = p (1- \mu) \)
Mimetism - Informational influences

if $f(t) > \frac{1}{2}$, then $f(t+1) = p + (1-p) \mu$

if $f(t) < \frac{1}{2}$, then $f(t+1) = p (1- \mu)$

Fixed point?

$\mu > \mu^* = 1 - \frac{1}{2}p$

two fixed points
dynamics

Figure 5. Évolution de $f$ au cours du temps ($\mu = 0.2$ et $p = 0.7$).

$\mu < \mu^*$
Figure 6. Évolution de $f$ au cours du temps ($\mu = 0.8$ et $p = 0.7$).

$dynamics$
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*(not discussed in Class)*