Cogmaster - CS1

http://www.lps.ens.fr/~risc/CS/

Dynamique des interactions sociales : du choix individuel au comportement collectif

Dynamics of social interactions: from individual choice to collective behavior

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From individual to collective behavior

- From a « microscopic » level (description of agents and their interactions), to a « macroscopic » level (collective behavior).
- Applications: (theoretical) economics, computational neuroscience, statistical physics

Elementary units	Interactions	Collective level
Agents' preferences	social influences (externalities)	market: equilibrium price
activation rule of neurons	synaptic weights	psychophysics: associative memory
spins (magnetic moments)	interactions	thermodynamics: ferromagnetism

Modelling

- « model »:

  « a precise and economical statement of a set of relationships that are sufficient to produce the phenomenon in question,

  or

  an actual biological, or mechanical, or social system that embodies the relationships in an especially transparent way »

- « Most of the models used in the social sciences are families rather than individual models »
- « ... a family of related models that differ in some characteristics but share some essential features »

T. C. Schelling (Prof. of Political Economy, Harward Univ.)

Qui se ressemble s'assemble

Birds of a feather flock together

plan

- Introduction

- T. C. Schelling
  - Social segregation emerging from weak preferences
  - Illustration : « multi-agents » simulations

- R. Axelrod
  - Formation of coalitions
  - Application : coalitions before World War II

- J. J. Hopfield
  - model of associative memory
  - link with statistical physics: Ising spins (ferromagnetism)

introduction

- (typically large) number of interacting units or 'agents' (economy & computer science) (individuals, countrys, firms, neurons):
  everyone choice is influenced by those of his/her neighbors

- (social) network: neighborhoods – who interact with whom, and how

- « Qui se ressemble s'assemble »/« Birds of a feather flock together »
  in this lecture, cases where interactions favor the gathering (or common behavior) of agents having similar characteristics.

  Three simple models formally related: segregation (Schelling), coalitions formation (Axelrod), associative memory (Hopfield).

Segregation

A self-forming neighborhood model Thomas C. Schelling, 1971

- segregation even with weak preferences
  example studied by Schelling : Black/White in the USA

  Hypothesis :
  every agent accepts a neighborhood where the majority is different from himself, provided the minority is not too small.
  Otherwise, he/she move to another place

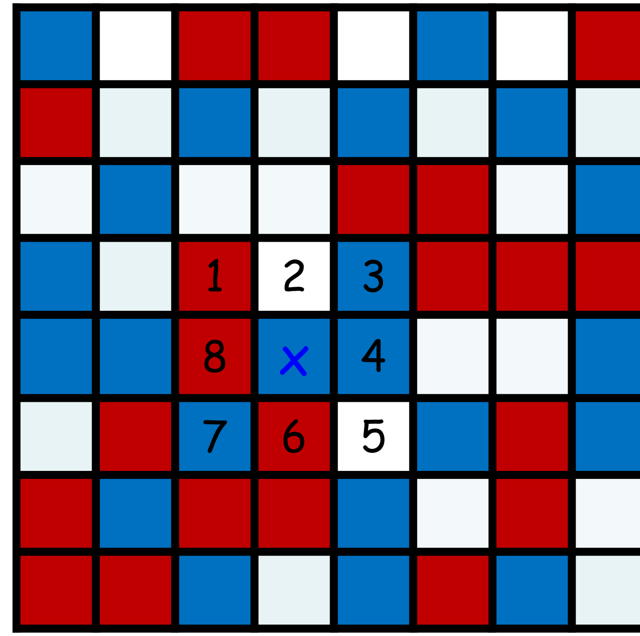
  Test of a specific model with agents on a regular 2D lattice

A self-forming neighborhood model Thomas C. Schelling, 1971

« Some vivid dynamics can be generated by any reader with a half-hour to spare, a roll of pennies and a roll of dimes, a tabletop, a large sheet of paper, a spirit of scientific inquiry, or, lacking that spirit, a fondness for games. »

T C Schelling, in *From micromotives to macrobehavior* (Norton & Cy, 1978)

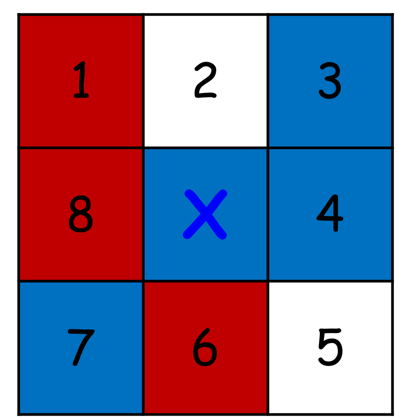
Schelling's model of segregation



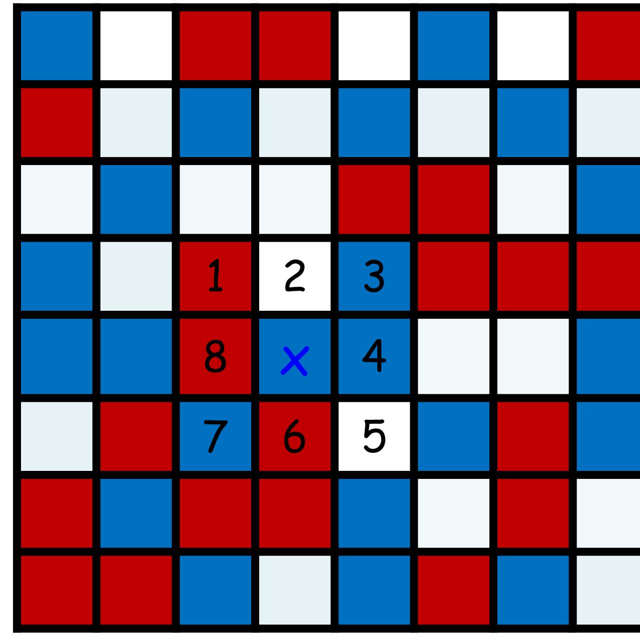
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:

if less than one third of his neighbors is like him, he moves to another location



Schelling's model of segregation



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Behavioral rules - every agent considers that:

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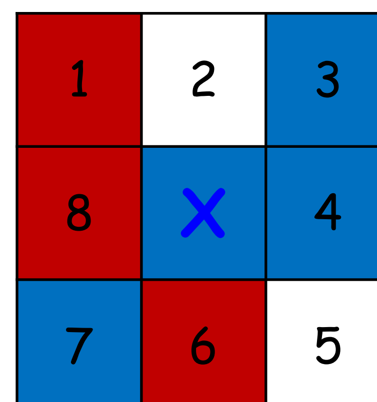
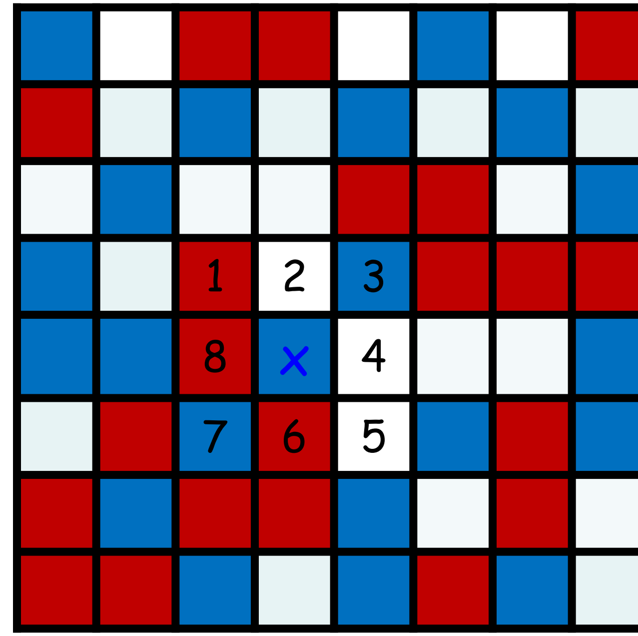
more exactly: if at a given location, an agent has

- **6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color**



Schelling's model of segregation



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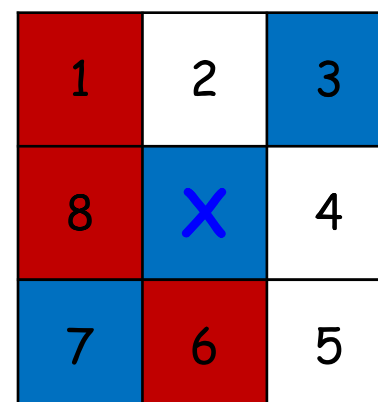
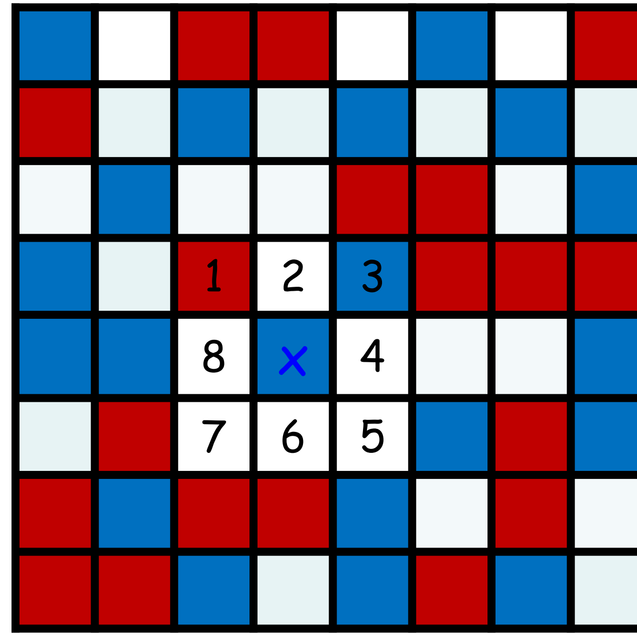
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- **3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color**



Schelling's model of segregation



Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

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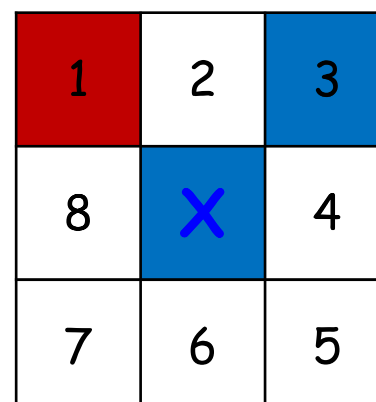
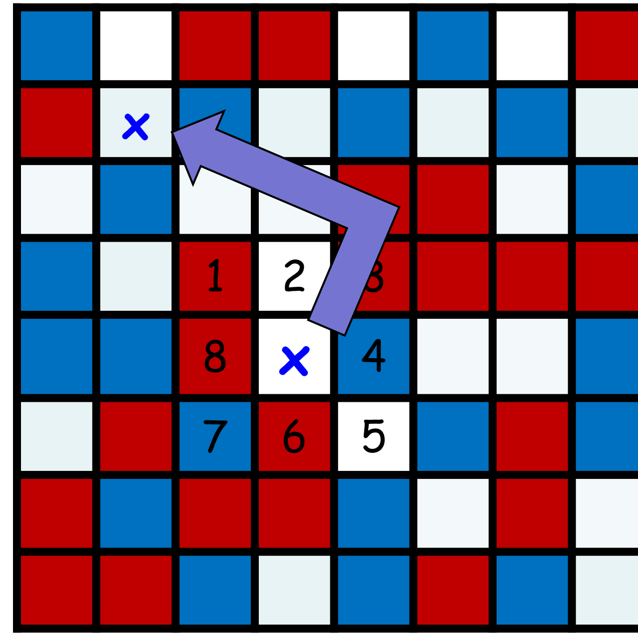
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has

- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- **1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color**



Schelling's model of segregation



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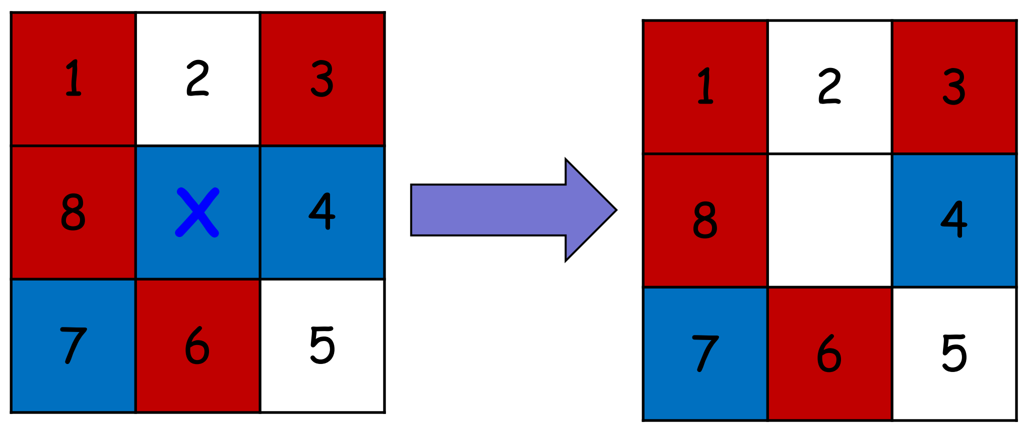
more exactly: if at a given location, an agent has

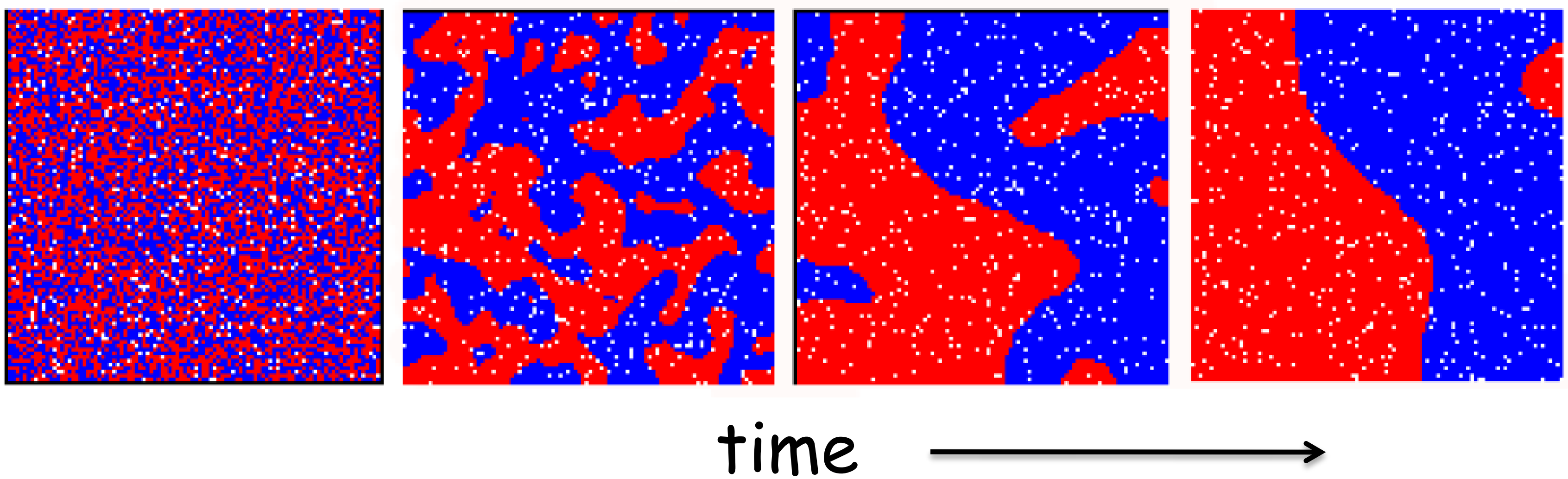
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- **Otherwise, he moves to a randomly chosen empty place**



Some references :
http://gemas.msh-paris.fr/dphan/segregationSchelling.htm

Segregation

Schelling's model of segregation (1971)



Simulations : L. Gauvin, 2009

Segregation

- Generalization: tolerance parameter T

  an agent is satisfied if the proportion of neighbors different from himself is at most T

  (Schelling's original version: T=2/3)

  agent's utility = 1 if agent is satisfied,
0 otherwise

- Density of vacancies: ρ (faction of empty sites)

Segregation

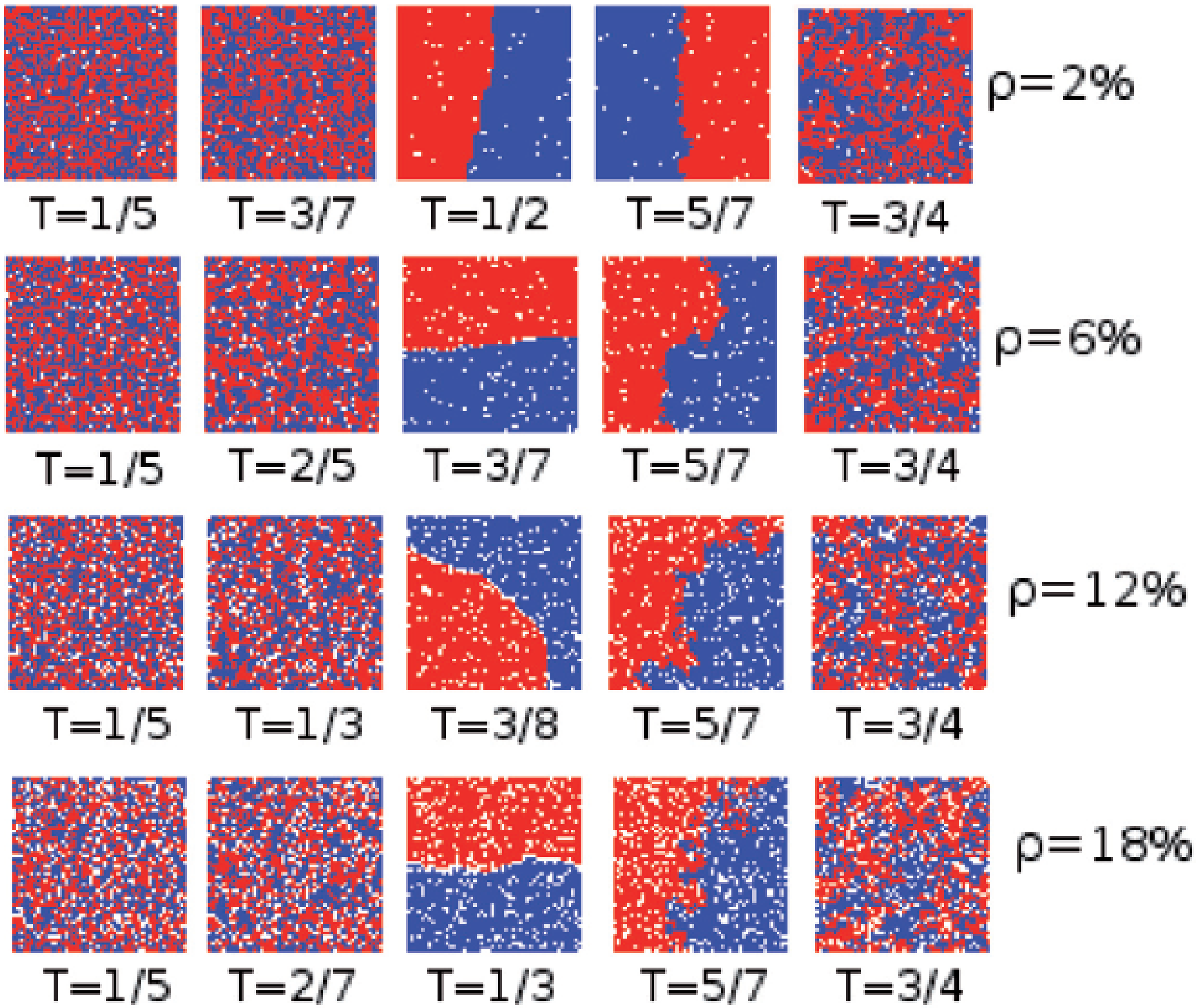
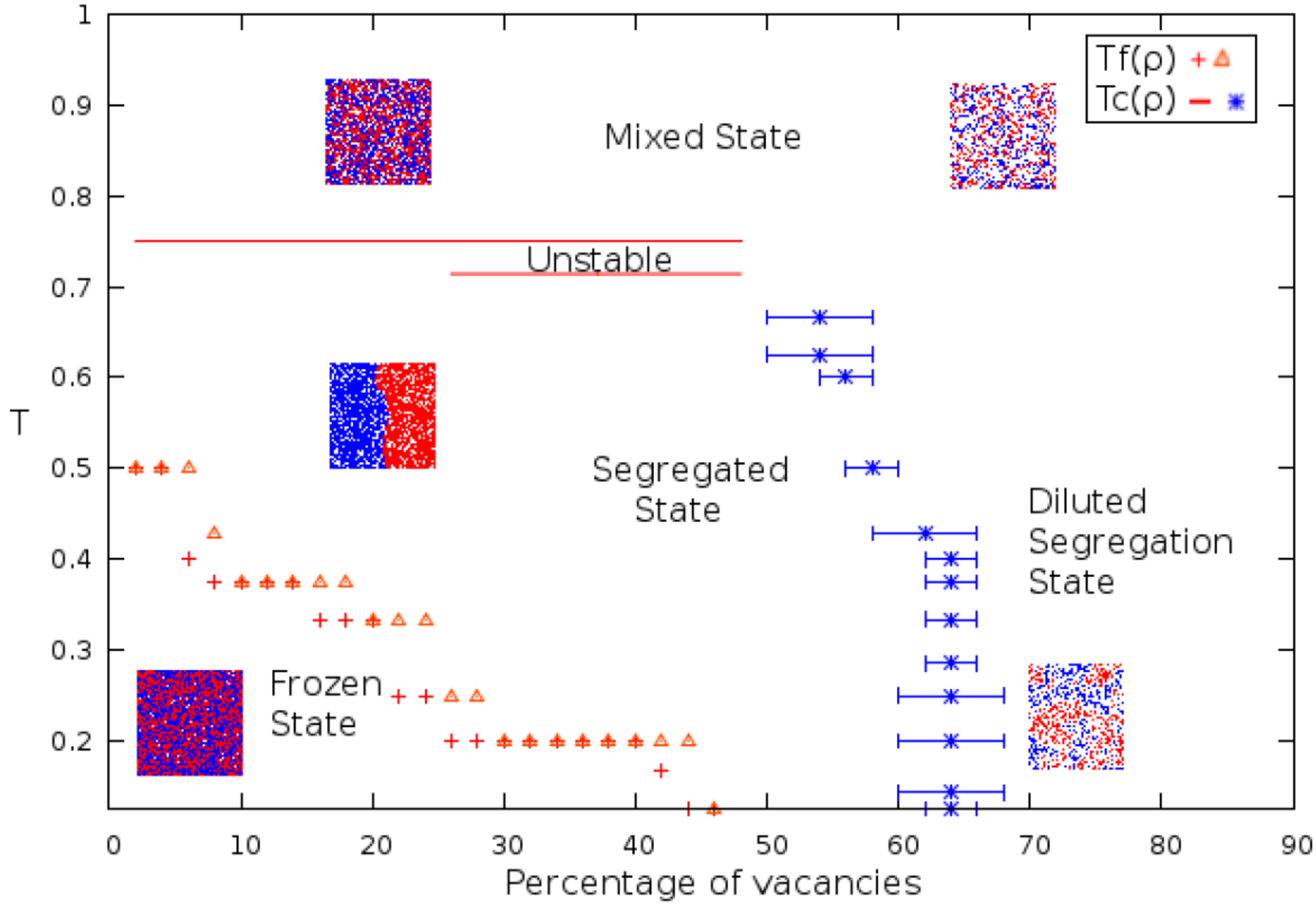


Fig. 2. (Color online) Configurations obtained at large times for selected values of ρ and T.

L. Gauvin, 2009

Segregation

- « Phase diagram » (L. Gauvin *et al*, EPJB 2009)

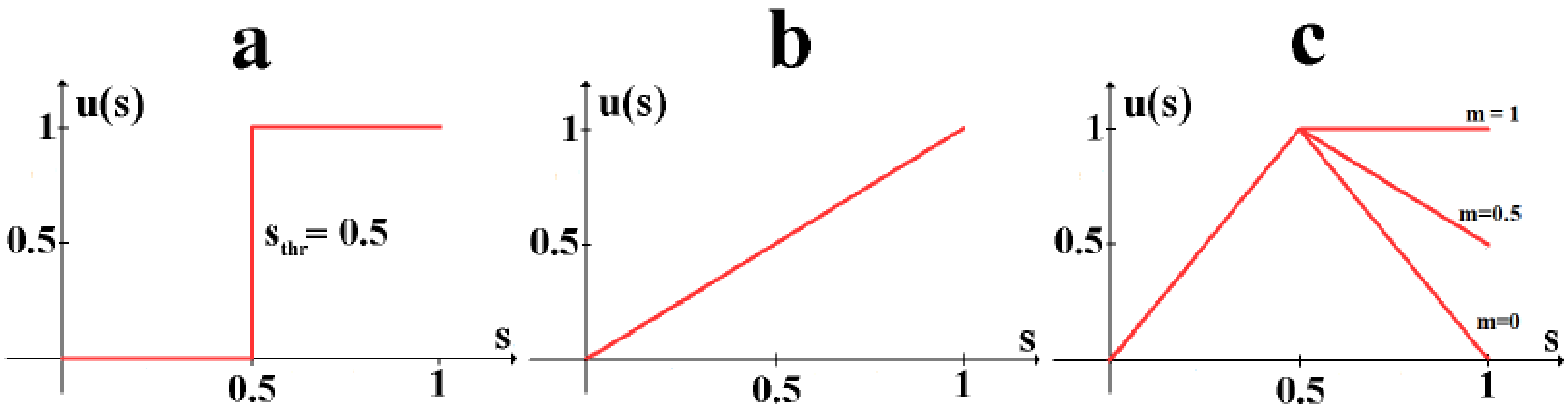


Segregation

- Variants: different types of utility functions

Schelling case (T=1/2)

3 cases with preference for a mixed neighborhood



s = fraction of neighbors of the same color as the agent under consideration

Figure 1.2: Different examples of utility functions presented in the literature. **a.** Schelling's utility function is a stair-like function for which $u(s) = 0$ for $s < s_{thr}$ and $u(s) = 1$ for $s \geq s_{thr}$; **b.** the "linear" function corresponds to $u(s) = s$ for $0 < s < 1$; **c.** the "asymmetrically peaked" functions correspond to $u(s) = 2s$ for $s < 0.5$ and $u(s) = m + 2(1 - m)s$ for $s \geq 0.5$. They present a maximum for $s = 0.5$ but contain an asymmetry in favor of similar neighborhood, this asymmetry being controlled by the parameter $m = u(1)$. For $m = 0$, the utility function is symmetrically peaked.

(from Goffette-Nagot *et al*, 2009)

Critical value of m:

m > mc, segregation occurs as in the original Schelling model

Segregation

- Variant : « bounded-neighborhood model » (Schelling)
- - two types of agents, blue and red ;
- - a unique global neighborhood (e. g. a city, a club,...)

  -> « mean-field » model
- - each individual has his own threshold:

agent i accepts (or desires) to leave in this particular neighborhood provided the fraction of agents of a different color is at most equal to his tolerance threshold x_i.

- Main result :

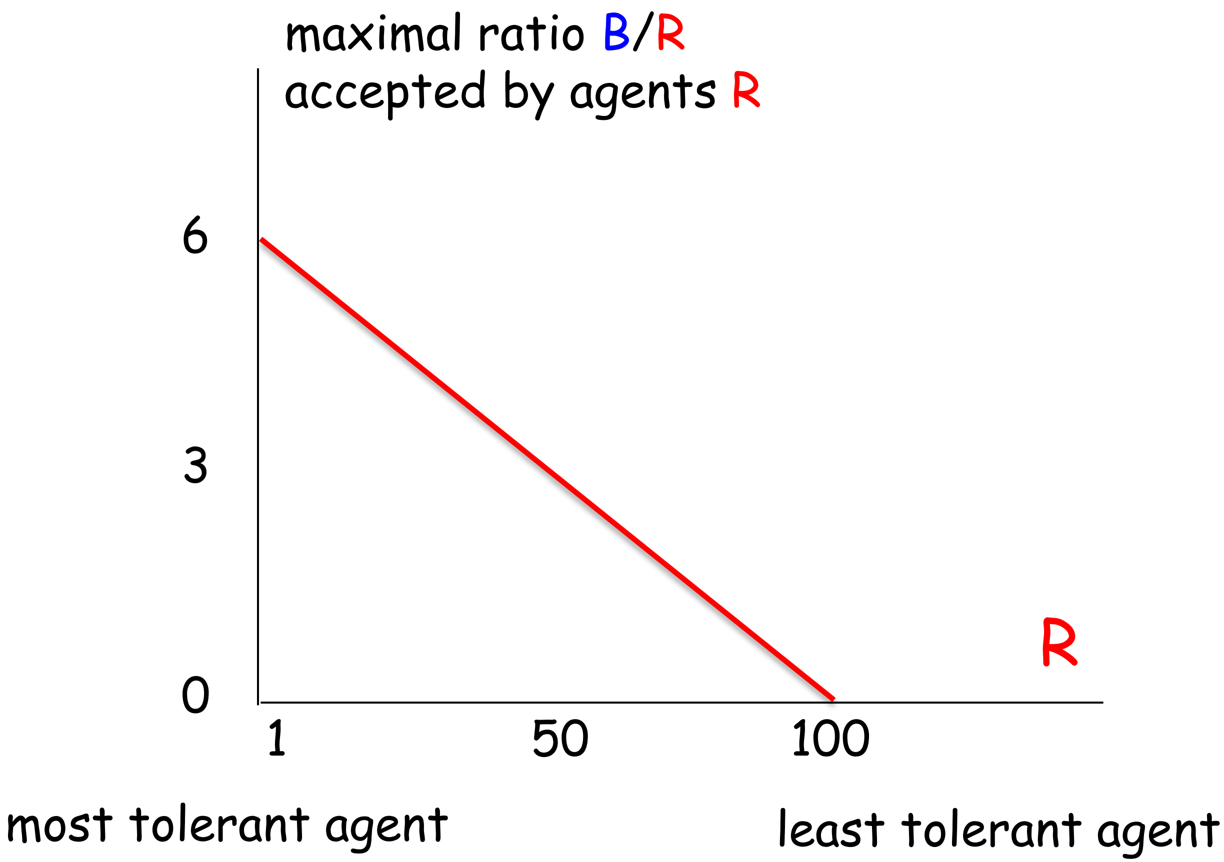
depending on the distribution of thresholds in the two (red and blue) populations:

- either 2 'pure' fixed points: convergence towards a full blue or full red neighborhood

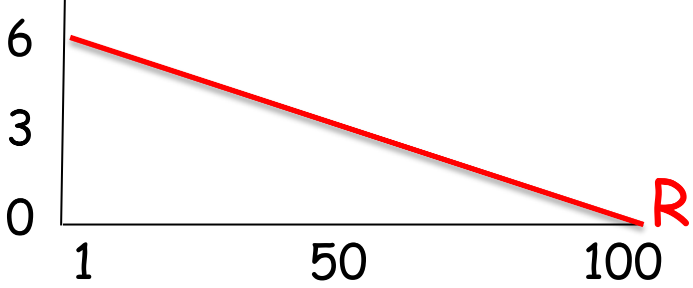
- or it exists a third fixed point with a mixed population

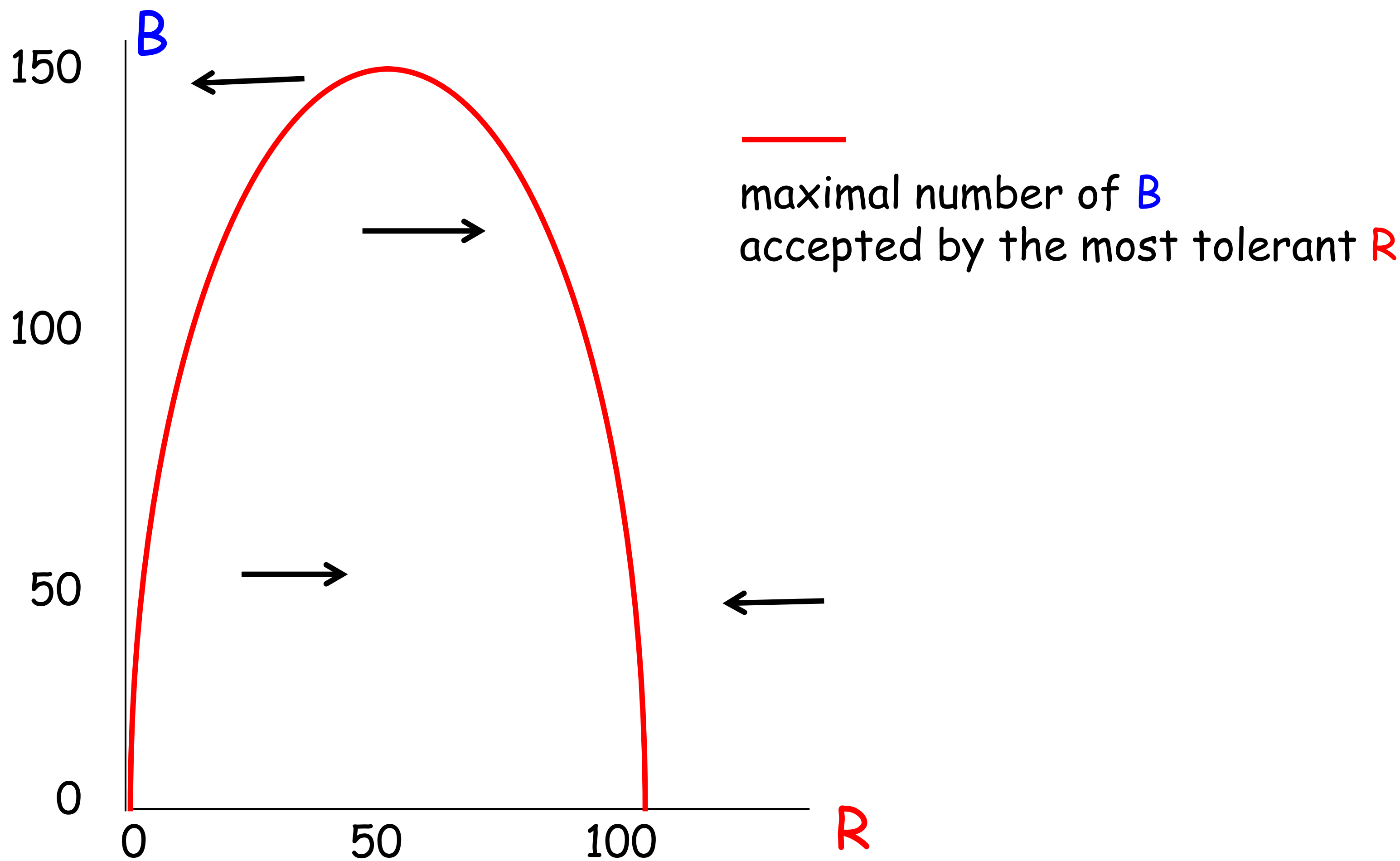
Schelling: bounded-neighborhood model

Example of preferences



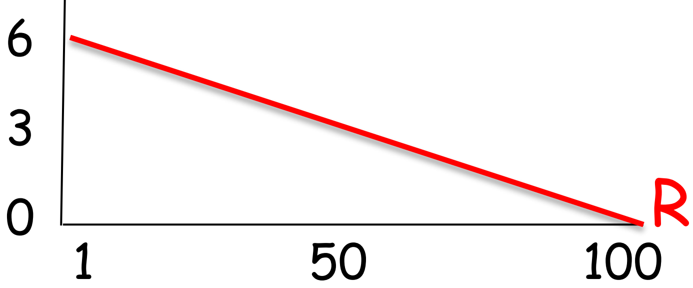
Schelling

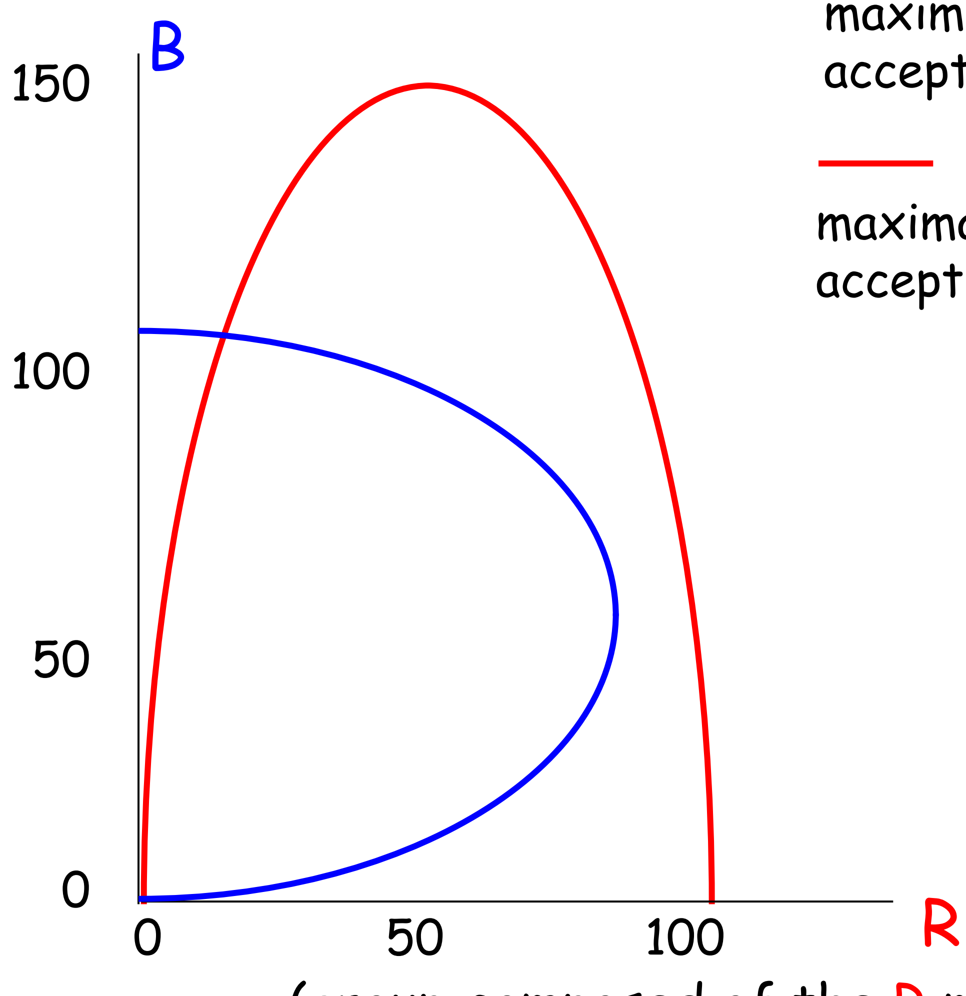




(group composed of the R most tolerant red agents)

Schelling

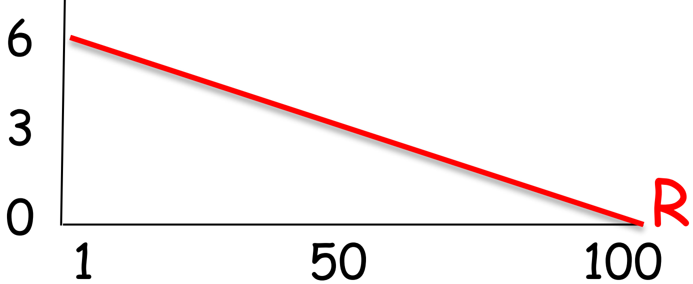


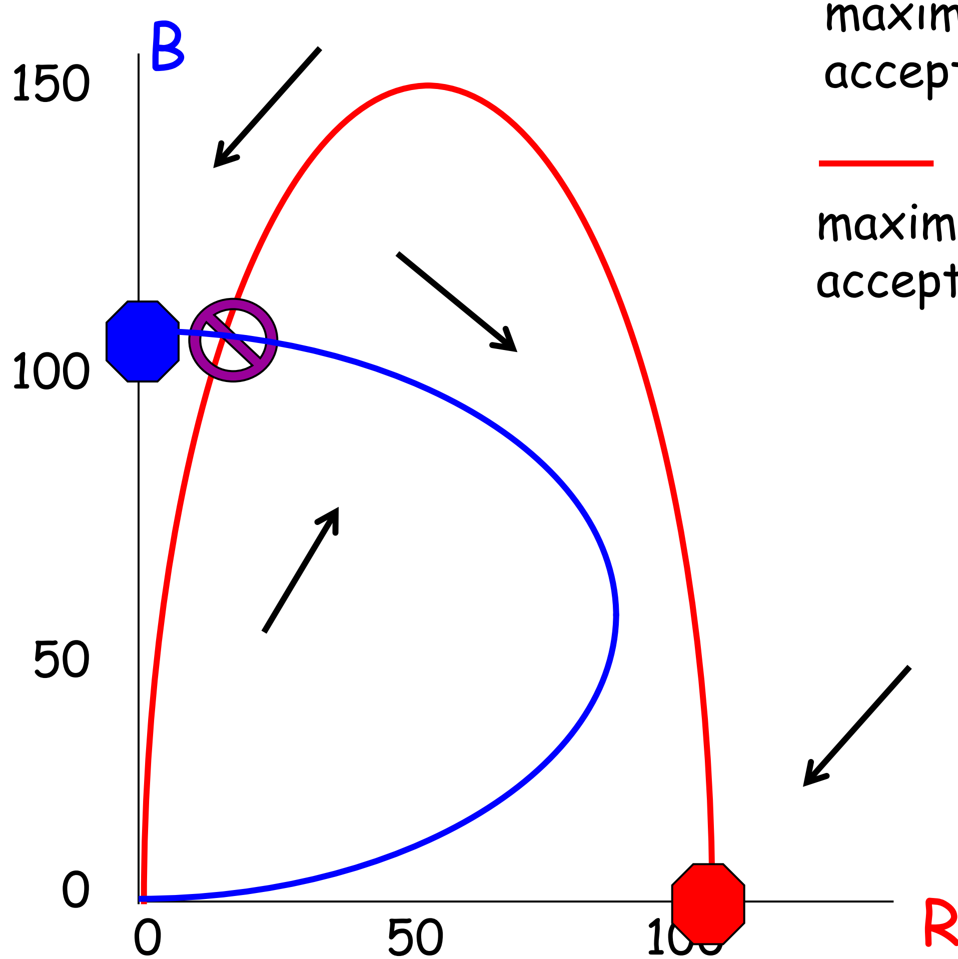


Blue line: maximal number of R accepted by the most tolerant B

Red line: maximal number of B accepted by the most tolerant R

Schelling

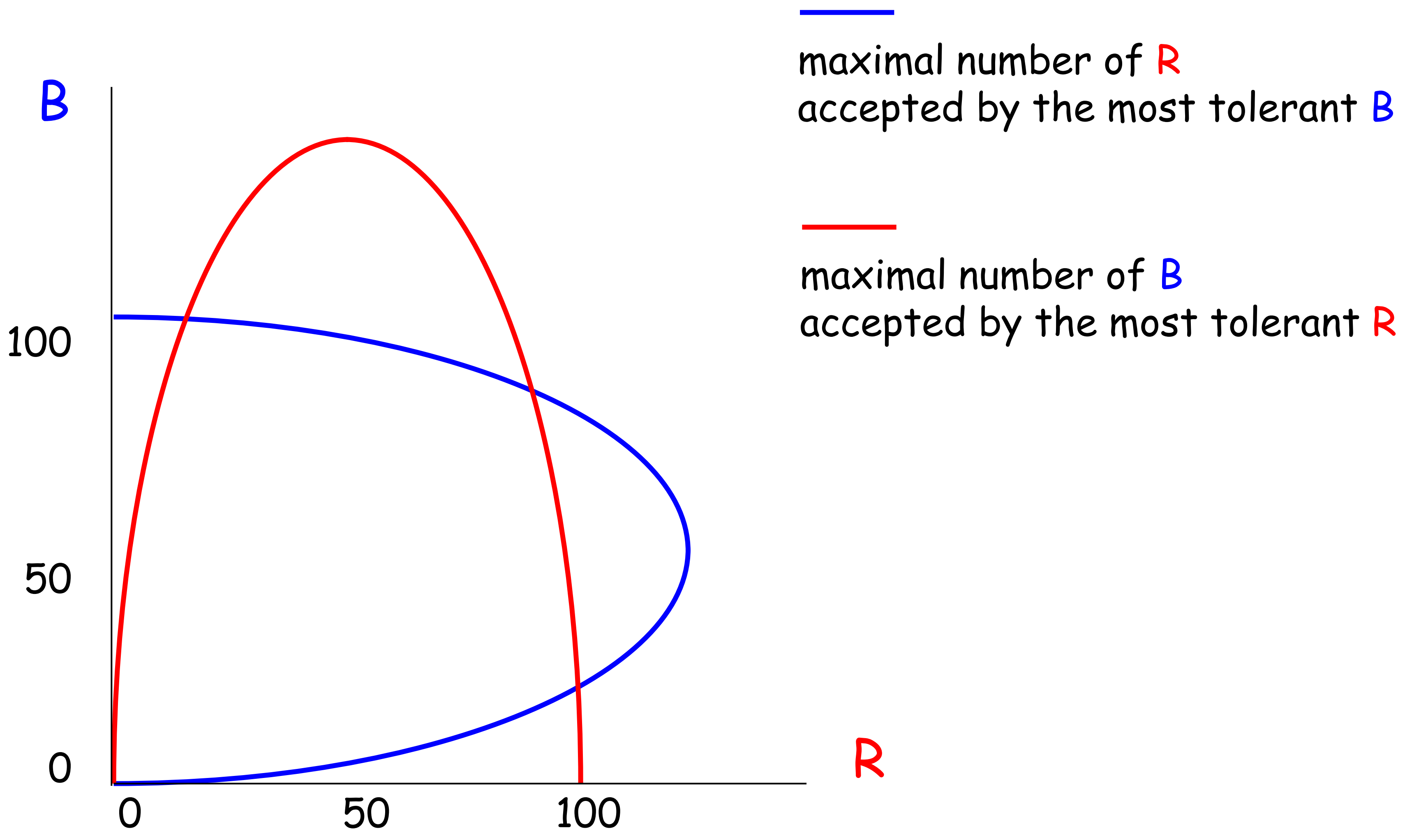




(group composed of the R most tolerant red agents)

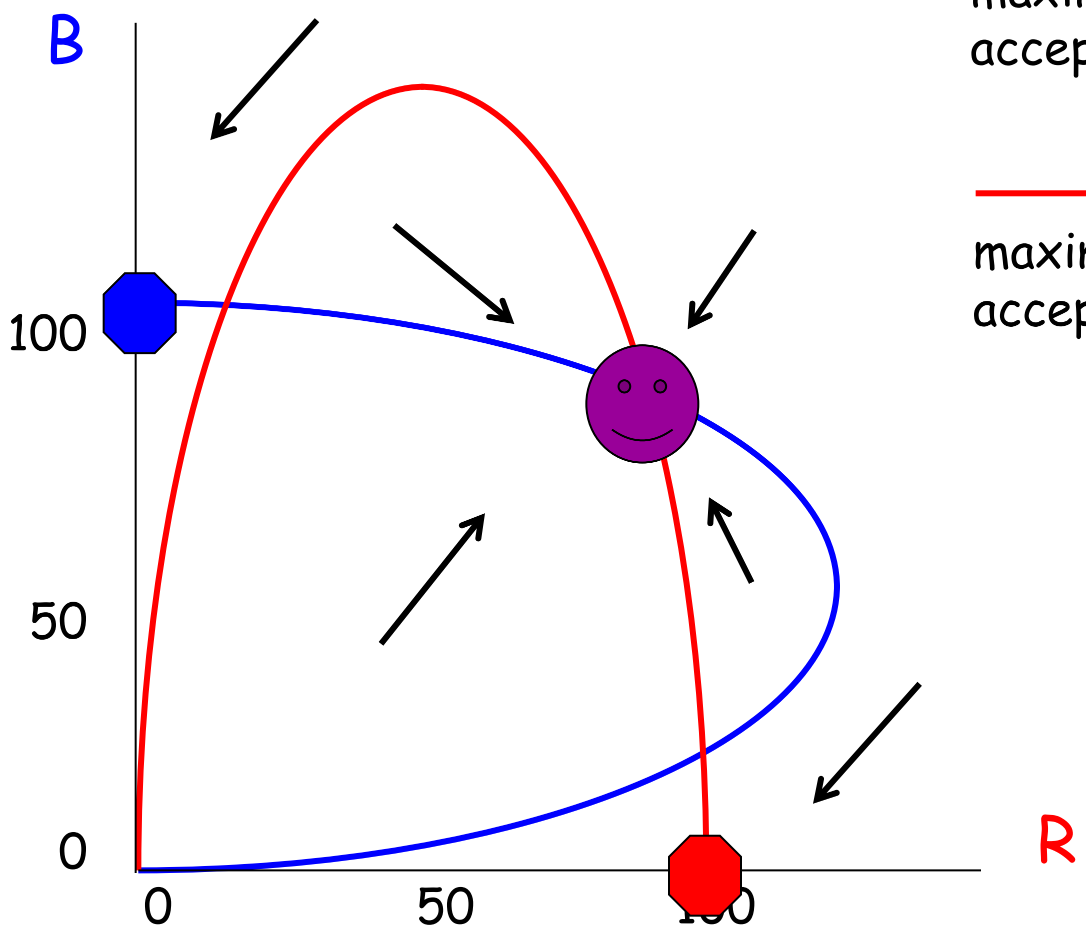
Schelling: bounded-neighborhood model

- Other possibility (different distributions of preferences)



Schelling: bounded-neighborhood model

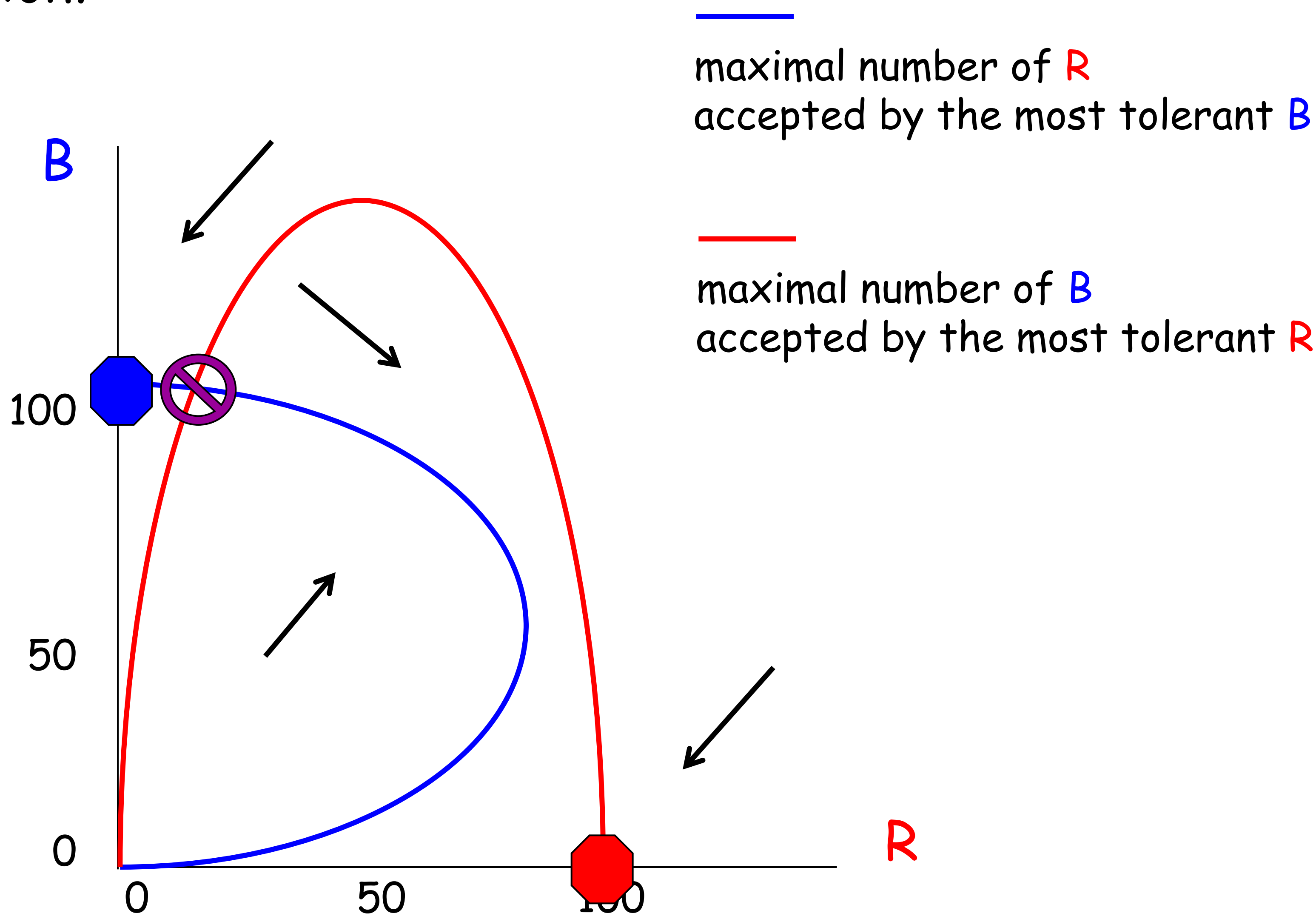
- Other possibility (different distributions of preferences)



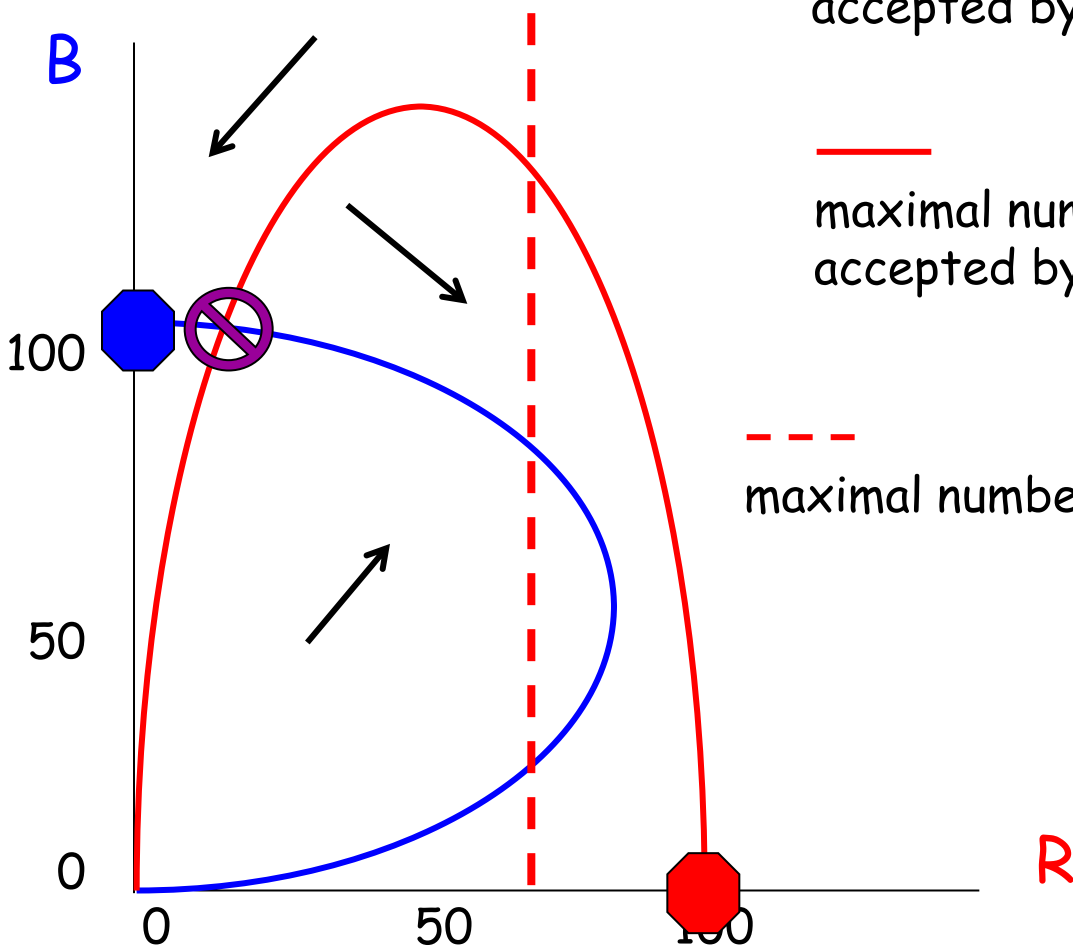
maximal number of R accepted by the most tolerant B

maximal number of B accepted by the most tolerant R

- Regulation?


B
100
50
0
0
50
R
maximal number of R accepted by the most tolerant B
maximal number of B accepted by the most tolerant R

- Regulation?



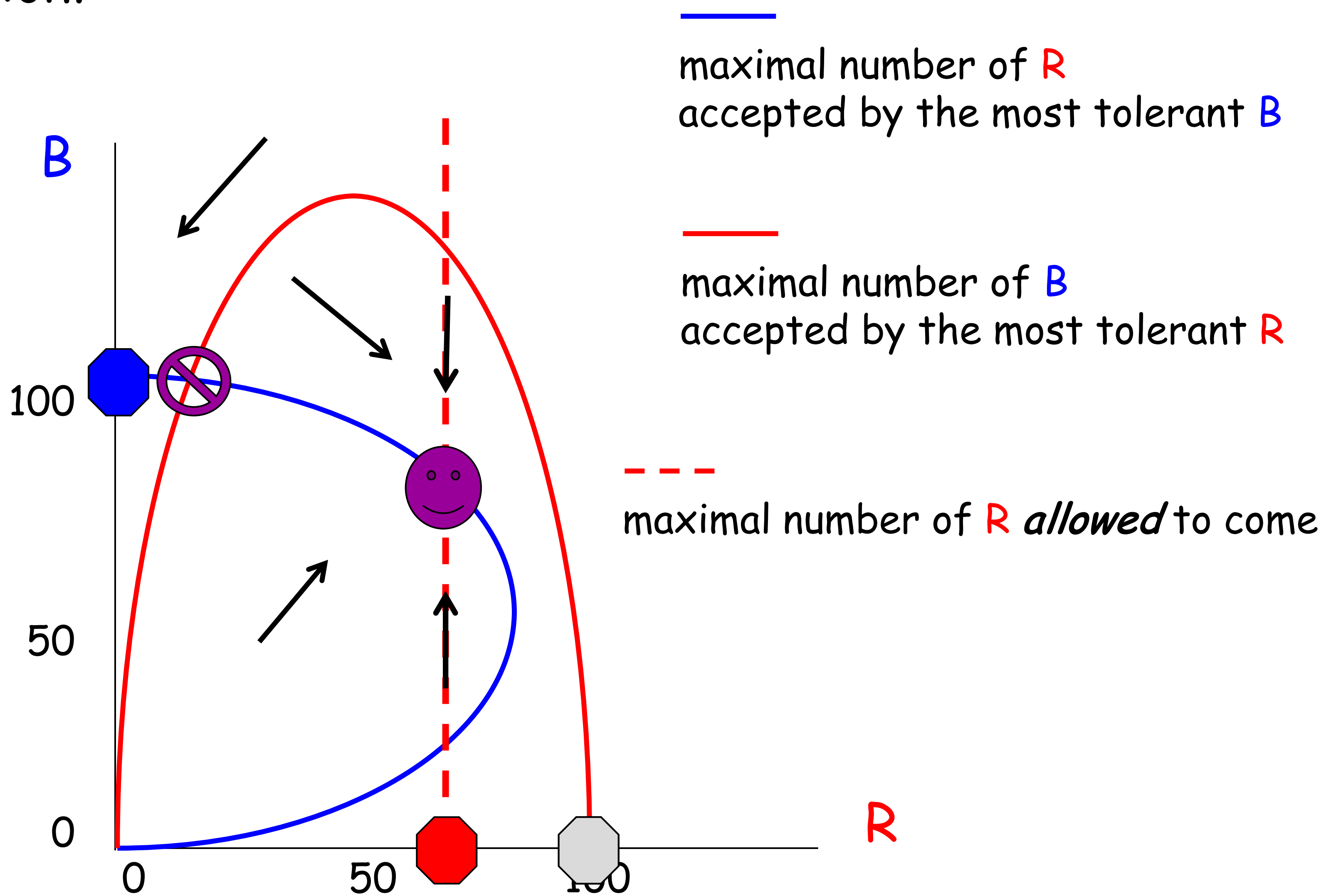
maximal number of R accepted by the most tolerant B

maximal number of B accepted by the most tolerant R

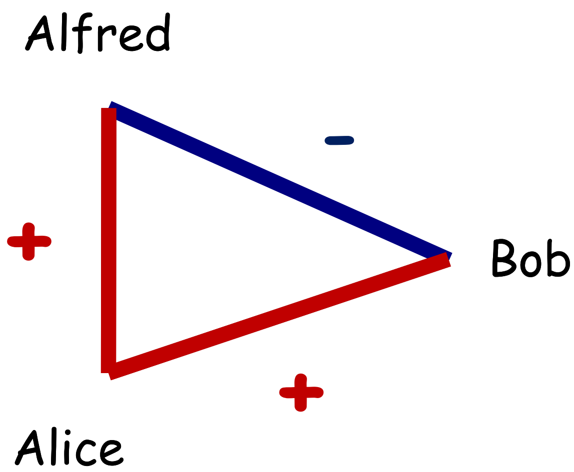
maximal number of R *allowed* to come

Schelling: bounded-neighborhood model

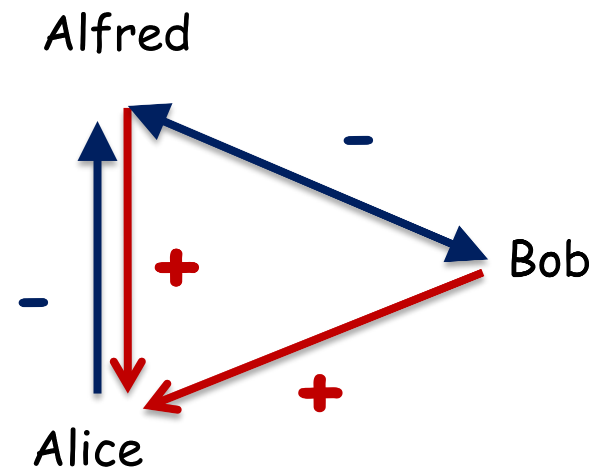
- Regulation?



Social segregation / coalition formation



« frustration »



J_{ik} = affinity of agent i for agent k

utility of agent i in group A = $\Sigma_{\{k \text{ in } A\}} J_{ik}$

Simplest case: affinity based on a single citerion
(e.g. smoker/non-smoker, or junior/senior ...)

Social segregation / coalition formation

- R Axelrod (1984)
- and also more recent works: S Galam, R Cont and Lowe...

N agents (or countries, firms,...) $i = 1,\ldots,N$

affinities: agent i 'likes/dislikes' agent k with affinity J_{ik}

affinities are based on p criteria $\tau = 1,\ldots,p$:

$\tau = 1$ smoker/non smoker

$\tau = 2$ baseball/opera

$\tau = 3$ junior/senior

$\tau = 4$ Bush/Obama

$\tau = \ldots$

J_{ik} = (number of identical criteria) – (number of different criteria)

Social segregation / coalition formation

- Task / game:

make 2 groups, A and B

every agent wants to be in the group which maximises its mean affinity with the other agents:

dynamics with myopic best response:

if: (affinity with A) – (affinity with B) > 0

join group *A*

otherwise: join group B

affinity of i with *A* = $\Sigma_{\{k \text{ in } A\}} J_{ik}$

J_{ik} = (number of identical criteria) – (number of different criteria)

Social segregation / formal neural network

- social segregation (Axelrod) / neural network (Hopfield)

affinity	J_{ik}	synaptic efficacy
join group A	$S_i = 1$	send a spike
join group B	$S_i = 0$	neuron at rest

(affinity with A) – (affinity with B) ~ post-synaptic potential

$= \Sigma_{\{k\}} J_{ik} (2\, S_k(t) - 1)$ $\qquad$ $= 2\, \Sigma_{\{k\}} J_{ik}\, S_k(t) - \theta$

criteria $\qquad$ patterns to be learned

Associative memory

(formally same model) :

Hopfield 1982 : attractor neural networks

N binary neurons S_i = 0 (rest) or 1 (sends a spike), $i = 1,\dots,N$

J_{ik} = synaptic weight (efficacy) between neurons i and k

P objects to memorise : ($\tau = 1,\dots,P$)

$\{\xi_i^\tau , i = 1,\dots,N \}$ = activity of the network when coding for object τ

Network dynamics (simplest case: deterministic dynamics):

- S_i (t=0) = activity imposed by a stimulus
- $S_i(t+1)$ = 1 if: $h_i(t) = \sum_{\{k\}} J_{ik} S_k(t) > \theta$
  = 0 otherwise
- Fixed point (= network's response) = $\{S_i^* , i = 1,\dots,N \}$
- If $\{S_i(t=0) , i = 1,\dots,N \}$ similar to $\{\xi_i^1 , i = 1,\dots,N \}$ (stimulus is ~ object $\tau = 1$)
  and $\{S_i^* , i = 1,\dots,N \}$ (almost) identical to $\{\xi_i^1 , i = 1,\dots,N \}$,
  then object $\tau = 1$ is « recognized » / « memorized », by the network

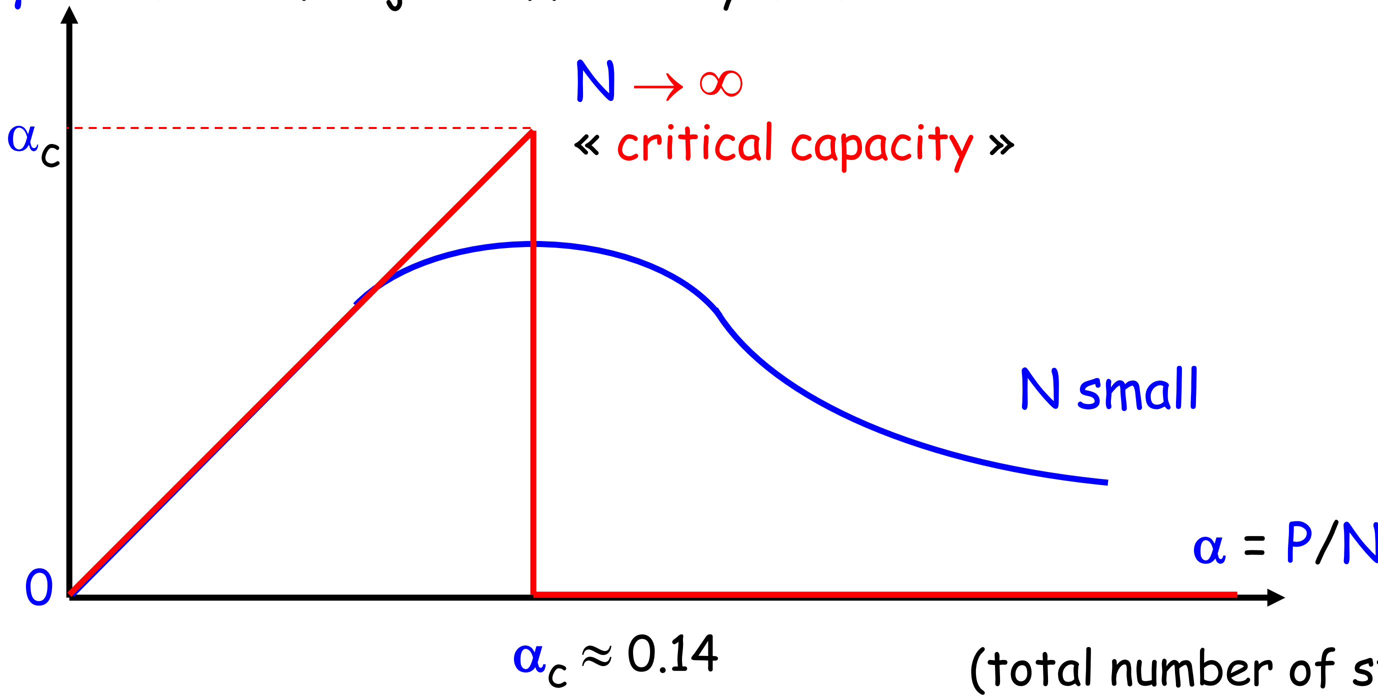
Learning as a reinforcement dynamics

Learning (inspired from Hebb, 1949) = modifying the weights according to the neural activity associated to the objects to be memorised

- $J_{ik}(0) = 0$ (Hopfield model, 1982)
- $J_{ik}(\tau) = J_{ik}(\tau - 1) + 1$ if $\xi_i^{\tau} = \xi_k^{\tau}$ (affinity is reinforced)

  $= J_{ik}(\tau - 1) - 1$ if $\xi_i^{\tau} \neq \xi_k^{\tau}$ (affinity is weakened)

γ = number of object effectively memorised /N



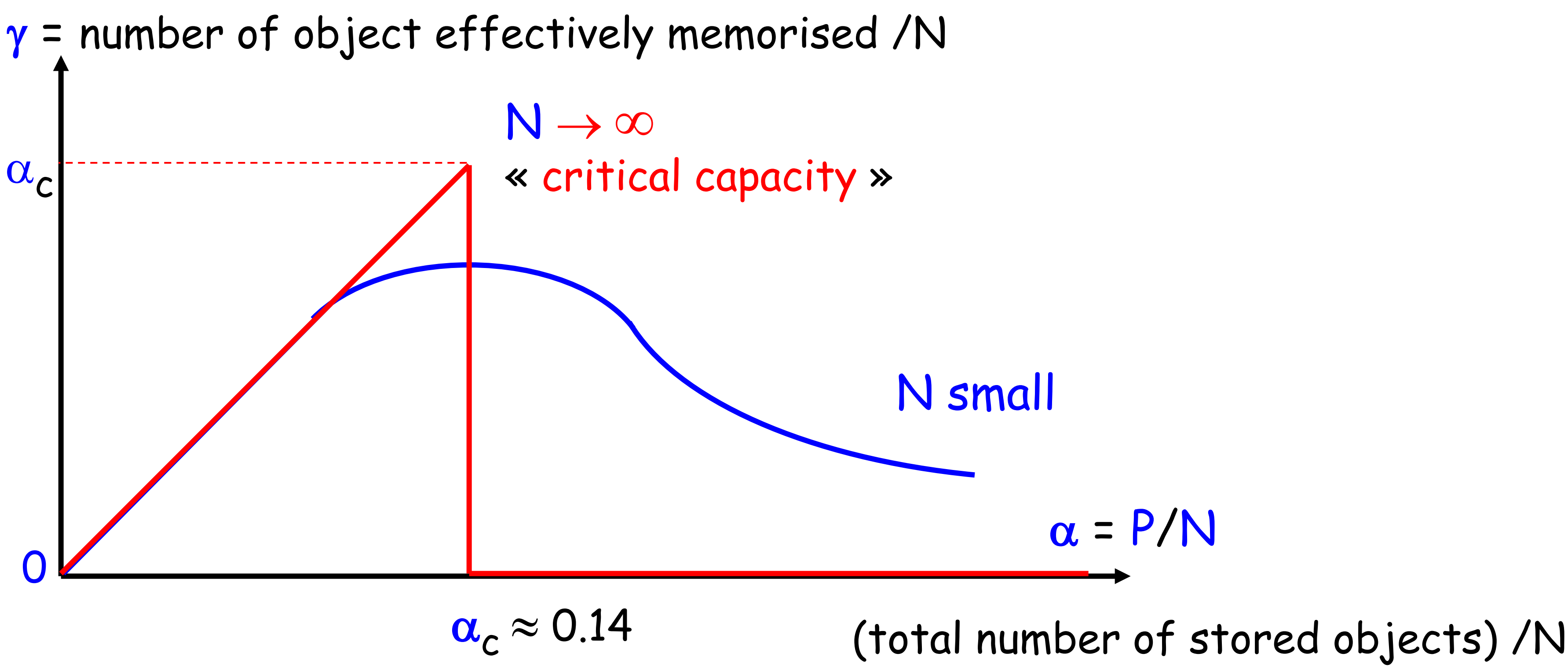
(total number of stored objects) /N

Learning as a reinforcement dynamics

Learning (inspired from Hebb, 1949) = modifying the weights according to the neural activity associated to the objects to be memorised

(Hopfield model, 1982)

J_{ik} = (number of patterns for which the activity is identical)
– (number of patterns fr which the activity is different)



- convergence towards a local minimum of the « energy »

- for large N and $p = \alpha N$:

$\alpha < \alpha_c$, solutions correlated with one of the criteria (self-organised focus on one of the criteria) (e.g. non smokers go together)

$\alpha > \alpha_c$, no correlation with the criteria (tolerance)

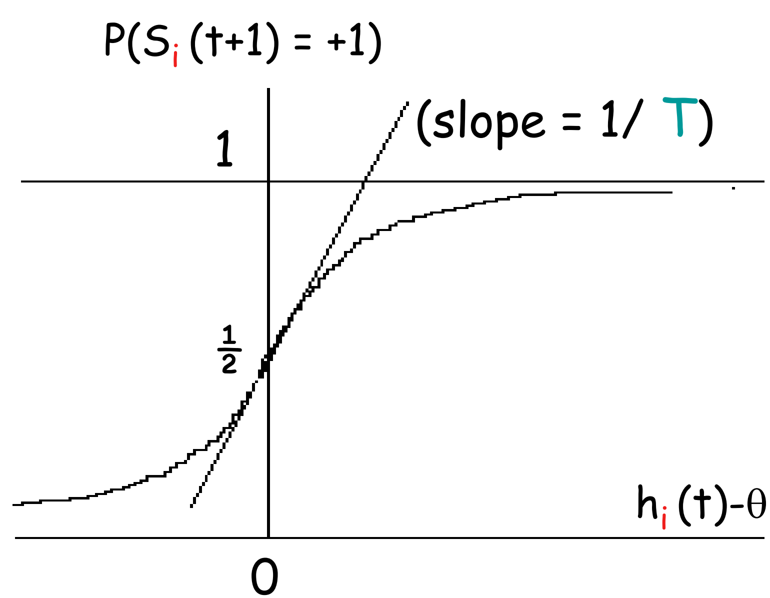
- non deterministic dynamics

'noise' level T

$\alpha_c(T)$

$\alpha_c(T)$ smaller as T increases

(deterministic limit: $T = 0$)



« Landscape theory » (Axelrod)

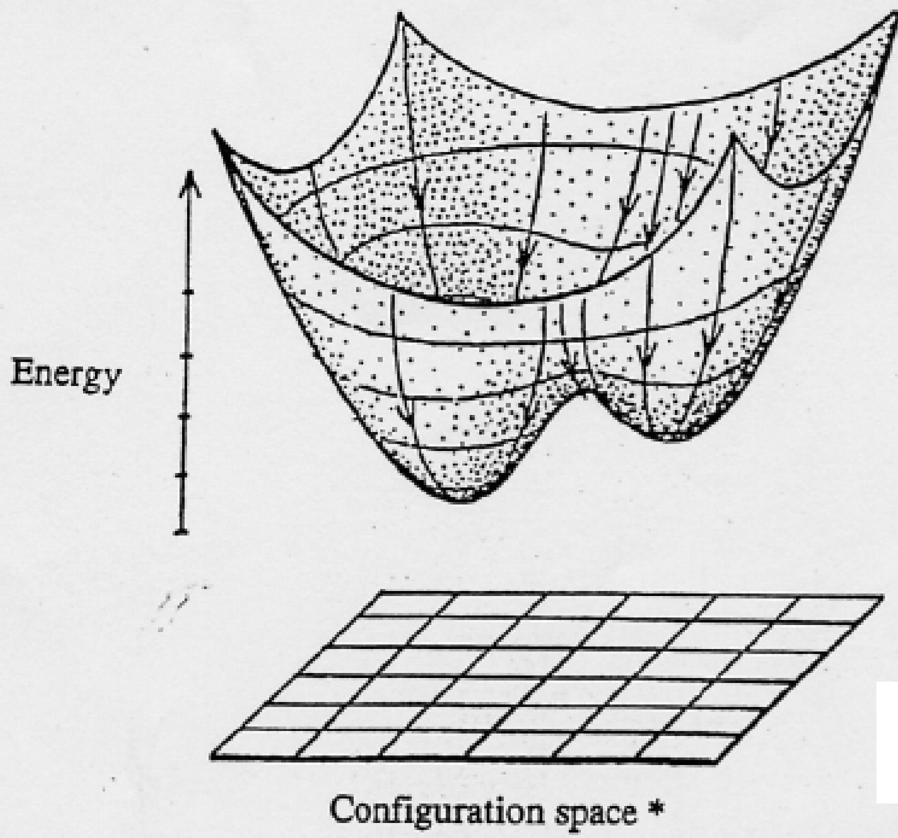


Figure 4-1. A Landscape with Two Local Optima. *Source:* Adapted from Abraham and Shaw, *Dynamics—The Geometry of Behavior.* Used with permission.

* The configuration space is an *n*-dimensional binary hypercube. The hypercube has one dimension for each actor indicating which of two possible alignments that actor is in.

Symmetric interactions

($J_{ik} = J_{ki}$)

⇒ '**energy**' E

Convergence towards one of the (local) minima of E

$$E(\underline{S}) = -\frac{1}{2}\sum_{ik} J_{ik} S_i S_k$$

Heterogeneous affinities
with some > 0 others < 0
⇒ For large N
large number of minima

Application: formation of coalitions (Axelrod)

(N not large)

- WWII

TABLE 4-1
The Two Configurations Predicted for the Second World War in Europe

	Configuration 1	
Configuration 2	*Alignment 1*	*Alignment 2*
Alignment 1	Britain (7.45) France (5.32) Czechoslovakia (1.15) Denmark (0.20)	Germany (11.49) Italy (4.03) Poland (1.83) Romania (0.78) Hungary (0.45) Portugal (0.27) Finland (0.19) Latvia (0.13) Lithuania (0.10) Estonia (0.06)
Alignment 2	Soviet Union (15.01) Yugoslavia (0.59) Greece (0.35)	(None)
Nearest empirical match[a]	Allies (and those invaded by Germany)	Axis (and those invaded by the Soviet Union)

Note: The size is shown in parentheses, in terms of percentage of world capabilities. The predictions are based upon 1936 data.

[a]In Configuration 1, only Poland and Portugal are wrong.

Coalitions

- Alliances

TABLE 5-1
The Two Nash Equilibrium Configurations

	Configuration 1	
Configuration 2	*Alliance 1*	*Alliance 2*
Alliance 1	Sun (27.2, S)	DEC (18.9, G) HP (14.35, G)
Alliance 2	AT&T (28.5, G) Prime (1.0, G) IBM (3.8, G)	Apollo (19.9, S) Intergraph (4.4, S) SGI (4.15, S)
Nearest empirical match[a]	UNIX International	Open Software Foundation

Note: Size is shown in parentheses, along with whether the firm was a computer generalist (G) or technical workstation specialist (S).

[a]In Configuration 1, only the IBM prediction is wrong.

Emergence: order from disorder

- Large number of units/agents
- Noisy dynamics

  (stochastic/probabilistic behavior) at the agent level

  noise level = **T**

- Critical noise level: **Tc**
- T < Tc « ergodicity breaking », ordered state
- Tc « phase transition »
- T > Tc disordered state (ergodicity)

- 'Order': characterized by a small number of macroscopic (collective) parameters « order parameters ».
- The appropriate order parameters might be difficult to identify

Generic properties

- Generic properties (stylized facts)

  = qualitative properties shared by a large class of systems

- More specific: universal properties

  = properties quantitatively identical for a large class of systems ; typically scaling laws at/near a phase transition

- Generic properties may differ depending on:
  - Topology (network structure: regular lattice, random network, full connectivity, « small world », ...)
  - Heterogeneity

    (in agents chracteristics: idiosynchratic properties)
  - ...

Dynamics

- Dynamics:
- Deterministic (zero noise; « rational agents »)

  or
- Stochastic

  (non zero noise; « trembling hand »; probabilistic choice rule)

- For some dynamical systems:

  existence of a Lyapunov function (or « energy »):

  quantity which decreases with time

  → characterization of the equilibria (stationary states) as minima of the energy

- This is the case whenever the interactions are symmetric

Bibliography

http://www.lps.ens.fr/~nadal

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- R. Axelrod
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- On the model of J. J. Hopfield and his variants :
  D. J. Amit
  « Modeling Brain Function » (Cambridge Univ. Press, 1990)