Dynamique des interactions sociales :
du choix individuel au comportement collectif

Dynamics of social interactions:
from individual choice to collective behavior

Jean-Pierre Nadal

Laboratoire de Physique Statistique de l’ENS
&
Centre d’Analyse et de Mathématique Sociales, EHESS

natal@lps.ens.fr
From individual to collective behavior

- From a « microscopic » level (description of agents and their interactions), to a « macroscopic » level (collective behavior).
- Applications: (theoretical) economics, computational neuroscience, statistical physics

<table>
<thead>
<tr>
<th>Elementary units</th>
<th>Interactions</th>
<th>Collective level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agents' preferences</td>
<td>social influences</td>
<td>market: equilibrium price</td>
</tr>
<tr>
<td></td>
<td>(externalities)</td>
<td></td>
</tr>
<tr>
<td>activation rule of neurons</td>
<td>synaptic weights</td>
<td>psychophysics: associative memory</td>
</tr>
<tr>
<td>spins</td>
<td>interactions</td>
<td>thermodynamics: ferromagnetism</td>
</tr>
<tr>
<td>(magnetic moments)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Modelling

• « model »:
  « a precise and economical statement of a set of relationships that are sufficient to produce the phenomenon in question, or an actual biological, or mechanical, or social system that embodies the relationships in an especially transparent way »

• « Most of the models used in the social sciences are families rather than individual models »

• « ... a family of related models that differ in some characteristics but share some essential features »

T. C. Schelling (Prof. of Political Economy, Harvard Univ.)
Part 1:

« Qui se ressemble s’assemble »
« Birds of a feather flock together »

Analogies: associative memory model (J.J. Hopfield); Ising spin models

Part 2:

Mimetism

- Different types of mimetism
- Informational influences (A. Orléan)
- « the dying seminar » (T. C. Schelling and variants)
Part 1:

« Qui se ressemble s’assemble »
« Birds of a feather flock together »

Segregation (T. C. Schelling). Coalitions formation (R. Axelrod). Analogies: associative memory model (J.J. Hopfield); Ising spin models

Part 2:

Mimetism

- Different types of mimetism
- Informational influences (A. Orléan)
- « the dying seminar » (T. C. Schelling and variants)
• (typically large) number of interacting units or ‘agents’ (economy & computer science) (individuals, countrys, firms, neurons): everyone choice is influenced by those of his/her neighbors

• (social) network: neighborhoods – who interact with whom, and how

• « Qui se ressemble s'assemble »/« Birds of a feather flock together » in this lecture, cases where interactions favor the gathering (or common behavior) of agents having similar characteristics.

Three simple models formally related: segregation (Schelling), coalitions formation (Axelrod), associative memory (Hopfield).
A self-forming neighborhood model  Thomas C. Schelling, 1971

- segregation even with weak preferences
  example studied by Schelling: Black/White in the USA

Hypothesis:
every agent accepts a neighborhood where the majority is different from himself, provided the minority is not too small.
Otherwise, he/she move to another place

Test of a specific model with agents on a regular 2D lattice
The prehistory of multi-agents simulations...

A self-forming neighborhood model

Thomas C. Schelling, 1971

« Some vivid dynamics can be generated by any reader with a half-hour to spare, a roll of pennies and a roll of dimes, a tabletop, a large sheet of paper, a spirit of scientific inquiry, or, lacking that spirit, a fondness for games. »

T C Schelling, in From micromotives to macrobehavior (Norton & Cy, 1978)
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has

- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Specific version: two types of agents, blue & red agents on a checkerboard, every location has at most 8 neighbors (8 neighboring locations, some might be non occupied)

Behavioral rules - every agent considers that:
if less than one third of his neighbors is like him, he moves to another location

more exactly: if at a given location, an agent has
- 6 to 8 neighbors, he/she stays if there is at least 3 neighbors of his color
- 3 to 5 neighbors, he/she stays if there is at least 2 neighbors of his color
- 1 or 2 neighbors, he/she stays if there is at least 1 neighbor of his color
- Otherwise, he moves to a randomly chosen empty place
Segregation

Schelling’s model of segregation (1971)

Initial state
stationary state

(time
(snapsots at different times)

Simulations: L. Gauvin, 2009
Segregation

• Generalization: tolerance parameter $T$

  an agent is **satisfied** if the proportion of neighbors different from himself is at most $T$

  (Schelling’s original version: $T=2/3$)

  agent’s utility = 1  if agent is satisfied,
  0  otherwise

• Density of **vacancies**: $p$ (faction of empty sites)
Segregation

Fig. 2. (Color online) Configurations obtained at large times for selected values of $\rho$ and $T$. 

L. Gauvin, 2009
Segregation

• « Phase diagram » (L. Gauvin et al, EPJB 2009)
Segregation

• Variants: different types of utility functions

$s = \text{fraction of neighbors of the same color as the agent under consideration}$

![Graphs showing utility functions](image)

Figure 1.2: Different examples of utility functions presented in the literature. a. Schelling’s utility function is a stair-like function for which $u(s) = 0$ for $s < s_{thr}$ and $u(s) = 1$ for $s \geq s_{thr}$; b. the “linear” function corresponds to $u(s) = s$ for $0 < s < 1$; c. the “asymmetrically peaked” functions correspond to $u(s) = 2s$ for $s < 0.5$ and $u(s) = m + 2(1 - m)s$ for $s \geq 0.5$. They present a maximum for $s = 0.5$ but contain an asymmetry in favor of similar neighborhood, this asymmetry being controlled by the parameter $m = u(1)$. For $m = 0$, the utility function is symmetrically peaked.

(from Goffette-Nagot et al., 2009)

Critical value of $m$:
if $m > m_c$, segregation as in the original Schelling model
Segregation

- Variant : « bounded-neighborhood model » (Schelling)
  - two types of agents, blue and red ;
  - a unique global neighborhood (e. g. a city, a club,...)
    → « mean-field » model
  - each individual has his own threshold:
    agent i accepts (or desires) to leave in this particular neighborhood
    provided the fraction of agents of a different color is at most equal to
    his tolerance threshold $x_i$.

- Main result :
  depending on the distributions of thresholds in the red and blue
  populations:
  - either 2 ‘pure’ fixed points: convergence towards a full blue or full red
    neighborhood
  - or it exists a third fixed point with a mixed population
Schelling

Example of preferences

maximum ratio $B/R$
accepted by agents of type $R$

most tolerant agent

least tolerant agent
Schelling

R (group size)

maximum number B of blue agents accepted by the R most tolerant red agents

(group composed of the R most tolerant red agents)
maximum number $R$ of red agents accepted by the $R$ most tolerant red agents

maximum number $B$ of blue agents accepted by the $B$ most tolerant blue agents

(group composed of the $R$ most tolerant red agents)
Schelling

Other example of preference distributions

- Maximum number $R$ of red agents accepted by the $B$ most tolerant blue agents.
- Maximum number $B$ of blue agents accepted by the $R$ most tolerant red agents.

Diagram showing the relationship between $B$ and $R$.
maximum number $R$ of red agents accepted by the $B$ most tolerant blue agents

maximum number $B$ of blue agents accepted by the $R$ most tolerant red agents

maximum number $R$ of red agents allowed to come
Segregation

• « bounded-neighborhood model » (Schelling)

• Main results:
  depending on the distributions of thresholds in the red and blue populations:
  - either 2 ‘pure’ fixed points: convergence towards a full blue or full red neighborhood
  - or it exists a third (stable) fixed point with a mixed population

Robustness:
In general, small changes in the agents preferences will not affect the number and types of equilibria.

Exceptions:
limiting cases corresponding to locus of « Phase transitions ».
« Phase transition ». Critical case: two examples of (blue) agents preferences, very close from one another. But (given the red preferences), one with, and one without, a mixed fixed point.

maximum number $R$ of red agents accepted by the $B$ most tolerant blue agents

maximum number $B$ of blue agents accepted by the $R$ most tolerant red agents

(group composed of the $R$ most tolerant red agents)
« Phase transition ». Critical case: two examples of (red) agents preferences, very close from one another. But (given the blue preferences), one with, and one without, a mixed fixed point.
Symmetric example, exchanging the role of the red and blue agents: Critical case: two examples of (red) agents preferences, very close from one another, but (given the blue preferences), one with and one without a mixed fixed point.

maximum number $R$ of red agents accepted by the $B$ most tolerant blue agents

maximum number $B$ of blue agents accepted by the $R$ most tolerant red agents

(group composed of the $R$ most tolerant red agents)
Social segregation / coalition formation

$J_{ik} = \text{affinity of agent } i \text{ for agent } k, \text{ either positive or negative}$
Social segregation / coalition formation

- Task / game:

  split into 2 groups, A and B

  every agent wants to be in the group which maximises its mean affinity with the other agents in the group:

  dynamics with myopic best response:

  \[
  \text{if} \quad (\text{affinity with } A) > (\text{affinity with } B)
  \]

  then \quad \text{join group } A

  otherwise \quad \text{join group } B

  iterate...

Affinity of \(i\) with group \(A\) = \(\sum_{k \in A} J_{ik}\)
Case of symmetric affinities
\((J_{ik} = J_{ki})\)

⇒ ‘energy’ (or ‘Lyapunov function’):

Convergence towards one of the (local) minima of the energy

\[ E\text{ (configuration)} = \ - \frac{1}{2} \sum_i \sum_k (+1 \text{ if } i \text{ and } k \text{ are in the same group, } -1 \text{ otherwise}) \ J_{ik} \]

= - (intra - inter) groups affinities

= - total affinity within group A
- total affinity within group B
+ sum of affinities between A and B

Heterogeneous affinities, with some > 0, others < 0,

⇒ in general several minima.

The larger \(N\), the larger the number of minima (\(\Leftrightarrow\) frustration)
Social segregation / coalition formation
Specific model for the affinities

\[ J_{ik} = \text{affinity of agent } i \text{ for agent } k \]

utility of agent \( i \) in group \( A \) = \[ \sum_{k \in A} J_{ik} \]

**Simplest case**: affinity based on a single criterion (e.g. jazz/opera).
Then \( J_{ik} = +1 \) if shared interest, -1 otherwise

In such case, no frustration!
Social segregation / coalition formation
Specific model: affinities based on several criteria

- R Axelrod (1984)
- and also more recent works: S Galam; R Cont and Lowe...

\[ N \text{ agents (or countries, firms,...) } \quad i = 1, \ldots, N \]

affinities: agent \( i \) likes/dislikes agent \( k \) with affinity \( J_{ik} \)

Hyp.: affinities are based on \( p \) criteria

\[ \tau = 1, \ldots, p: \]

\( \tau = 1 \) smoker/non smoker
\( \tau = 2 \) jazz /opera
\( \tau = 3 \) ecotax: for/against
\( \tau = 4 \) ...

\[ J_{ik} = (\text{number of identical criteria}) - (\text{number of different criteria}) \]

\[ = (\text{number of } 👍 ) - (\text{number of } 👎 ) \]
Social segregation / coalition formation

- Task / game: split into 2 groups, A and B

  every agent wants to be in the group which maximises its mean affinity with the other agents in the group.

  dynamics with myopic best response:

  \[
  \text{if} \quad (\text{affinity with A}) > (\text{affinity with B})
  \]

  \[
  \text{then} \quad \text{join group A}
  \]

  \[
  \text{otherwise} \quad \text{join group B}
  \]

  iterate...

  Affinity of agent \( i \) with group \( A \) = \( \sum_{k \in A} J_{ik} \)

  Affinity of agent \( i \) with group \( B \) = \( \sum_{k \in B} J_{ik} \)
Social segregation / neural network

- coalition formation (Axelrod, 1984) / neural network (Hopfield, 1982)

**affinity** \( J_{ik} \) **synaptic efficacy**

- join group A \( S_i = 1 \) send a spike
- join group B \( S_i = 0 \) neuron at rest

**p** features (criteria)
**state of agent** \( i \) **p** patterns to be learned
for criterion \( \tau \) \((\tau = 1, \ldots, p)\)

**affinities in term of the criteria**
(affinity with A) - (affinity with B)
\[
= \sum_{\{k\}} J_{ik} (2 S_k(t) - 1)
\]

Hebb learning rule
\[
\Delta \text{post-synaptic potential} = \sum_{\{k\}} J_{ik} S_k(t) - \theta_i
\]
Social segregation / coalition formation

• Task / game:

split into 2 groups, A and B

every agent wants to be in the group which maximises its mean affinity
with the other agents in the group:

dynamics with myopic best response:

if $(\text{affinity with A}) - (\text{affinity with B}) > 0$ \( \sum_{k} J_{ik} S_k(t) - \theta_i > 0 \)

then join group A \( S_i(t+1) = +1 \)

otherwise join group B \( S_i(t+1) = 0 \)

iterate...

Recall: affinity of \( i \) with group A = \( \sum_{k \in A} J_{ik} \)

\( J_{ik} = \) (number of identical criteria) $-$ (number of different criteria)
• convergence towards a local minimum of the « energy »

• for large \( N \) and \( p = \alpha N \):

\( \alpha < \alpha_c \), equilibria correlated with one of the criteria (self-organised focus on one of the criteria) (e.g. non smokers go together)

\( \alpha > \alpha_c \), no correlation with the criteria (tolerance)

• non deterministic dynamics

  'noise' level \( T \)

  \( \alpha_c (T) \)

\( \alpha_c (T) \) smaller as \( T \) increases

( deterministic limit: \( T = 0 \) )
Coalitions formation \( R \) Axelrod (1984)

\( N \) countries, firms,... \( i = 1, \ldots, N \) \( (N \) not necessarily large) 

**affinities:** agent \( i \) likes/dislikes agent \( k \) with affinity \( J_{ik} \)

Hyp.: affinities are based on \( p \) criteria \( \tau = 1, \ldots, p \):

\( \tau = 1 \) economics interactions (high/low)

\( \tau = 2 \) main religion (same/different)

\( \tau = 3 \) border conflicts (no or rare/frequent)

\( \tau = 4 \) ...

\( \tau = 4 \) ...

\( J_{ik} = (\text{positive terms: favor } i \text{ and } k \text{ in the same coalition}) + 
\) 
(\text{negative terms: favor } i \text{ and } k \text{ in different coalitions})

**Stable coalitions:** (local) minima of the 'energy' \( E \)
Application: formation of coalitions (Axelrod)
(here N small)

• WWII

**TABLE 4-1**
The Two Configurations Predicted for the Second World War in Europe

<table>
<thead>
<tr>
<th>Configuration 2</th>
<th>Alignment 1</th>
<th>Alignment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alignment 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Britain (7.45)</td>
<td>Germany (11.49)</td>
<td></td>
</tr>
<tr>
<td>France (5.32)</td>
<td>Italy (4.03)</td>
<td></td>
</tr>
<tr>
<td>Czechoslovakia (1.15)</td>
<td>Poland (1.83)</td>
<td></td>
</tr>
<tr>
<td>Denmark (0.20)</td>
<td>Romania (0.78)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hungary (0.45)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Portugal (0.27)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finland (0.19)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Latvia (0.13)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lithuania (0.10)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estonia (0.06)</td>
<td></td>
</tr>
<tr>
<td>Alignment 2</td>
<td>Soviet Union (15.01)</td>
<td></td>
</tr>
<tr>
<td>Yugoslavia (0.59)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greece (0.35)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Coalitions**

- **Alliances**

---

**TABLE 5-1**

The Two Nash Equilibrium Configurations

<table>
<thead>
<tr>
<th>Configuration 2</th>
<th>Configuration 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alliance 1</td>
<td>Alliance 2</td>
</tr>
<tr>
<td>Sun (27.2, S)</td>
<td>DEC (18.9, G)</td>
</tr>
<tr>
<td>AT&amp;T (28.5, G)</td>
<td>HP (14.35, G)</td>
</tr>
<tr>
<td>Prime (1.0, G)</td>
<td>Apollo (19.9, S)</td>
</tr>
<tr>
<td>IBM (3.8, G)</td>
<td>Intergraph (4.4, S)</td>
</tr>
<tr>
<td>Nearest empirical match</td>
<td>Open Software Foundation</td>
</tr>
</tbody>
</table>

*Note: Size is shown in parentheses, along with whether the firm was a computer generalist (G) or technical workstation specialist (S).*

*aIn Configuration 1, only the IBM prediction is wrong.*
Emergence: order from disorder

- Case of a large number of units/agents
- Noisy dynamics
  (stochastic/probabilistic behavior) at the agent level
  noise level = $T$

- Critical noise level: $T_c$
- $T < T_c \ll \text{ergodicity breaking}$, ordered state
- $T_c \ll \text{phase transition}$
- $T > T_c$ disordered state (ergodicity)

- 'Order': characterized by a small number of macroscopic (collective) parameters $\ll$ order parameters.
- The appropriate order parameters might be difficult to identify
Generic properties

• **Generic properties** *(stylized facts)*
  = qualitative properties shared by a large class of systems

• More specific: **universal properties**
  = properties quantitatively identical for a large class of systems; typically scaling laws at/near a phase transition

• **Generic properties** may differ depending on:
  - Topology *(network structure: regular lattice, random network, full connectivity, « small world », ...)*
  - Heterogeneity
    (in agents characteristics: idiosynchratic properties)
  - ...


Dynamics

• Dynamics:
• Deterministic (zero noise; « rational agents »)
  or
• Stochastic
  (non zero noise; « trembling hand »; probabilistic choice rule)

• For some dynamical systems:
  existence of a Lyapunov function (or « energy »):
  quantity which decreases with time
  \[ \Rightarrow \] characterization of the equilibria (stationary states) as minima of the energy

• This is the case whenever the interactions are symmetric
• T. C. Schelling
  « Micromotives and Macrobehavior » (Norton & Cy, 1978)
  traduction française : « La tyrannie des petites décisions » (PUF, 1980)

• R. Axelrod
  « The complexity of cooperation » (Princeton Univ. Press, 1977)

• On the model of J. J. Hopfield and his variants :
  D. J. Amit
  « Modeling Brain Function » (Cambridge Univ. Press, 1990)