

Statistical physics of social interactions:

Multiple equilibria in a market with heterogeneous agents



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joint work with: Mirta B. Gordon, Denis Phan, Jean Vannimenus

Economics		Statistical Physics
Discrete Choice Theory		Ising and Potts models
Random Utility Models (RUM)		Random Field Ising Models (RFIM)
agents (customers)	N	Ising spins
binary choice: buy/not buy one unit of a good	$\omega_i = 0, 1$	binary state $S_i = \pm 1$
posted price	P	global external field
individual willingness-to-pay (IWP) (reservation price)	h_i	local external field
social influence - positive externality $J_{ij} = J / (\text{nber of neighbors})$	J_{ij} $J > 0$	interactions ferromagnetic coupling
rational agents: i is happy to buy if $V_i \equiv h_i - P + \sum_j J_{ij} \omega_j > 0$	$T = 0$	ground state

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Two different points of view considered as equivalent in economics
(standard case: no interaction)

'economic approach' (McFadden, Manski)	'psychological approach' (Thurstone)
<p>heterogeneous, time independent reservation prices $\{h_i, i = 1, \dots, N\}$</p>	<p>(possibly) homogeneous reservation price h</p>
<p>deterministic choice agent i buys if $V_i > 0$ $V_i = h_i - P$</p>	<p>stochastic choice agent i buys with probability $1/[1 + \exp(-\beta V)]$ $V = h - P$</p>
<p>an observer has not access to the individual h_i only to the fraction of buyers</p>	<p>an observer can only see the fraction of buyers</p>
<p>RFIM, $T = 0$ quenched disorder</p>	<p>standard Ising model, $T > 0$ annealed disorder</p>

Use of the Ising framework in socio-economic modelling by physicists... and economists

- Galam S., Gefen Y., Shapir Y. (1982) "Sociophysics: A Mean Behavior Model for the Process of Strike", Mathematical Journal of Sociology
- Orléan A., (1995) "Bayesian Interactions and Collective Dynamics of Opinion: Herd Behaviour and Mimetic Contagion", Journal of Economic Behavior and Organization
- Durlauf S.N., (1997) "Statistical Mechanics Approaches to Socioeconomic Behavior"
- Weisbuch G., Stauffer D. (2003) "Adjustment and social choice", Physica A
- ...

Case of RFIM, $T = 0$, mean field: $J_{ij} = J/N$, large N limit

Quantities of interest

- equilibrium at a given posted price P

demand curve: $\eta(P) = \frac{1}{N} \sum_j \omega_j$

(\sim magnetization in an Ising model with uniform external field)

- supply side: case of a unique seller (monopoly market)

optimization of the profit by the seller

$$\max_P (P - C) N \eta(P)$$

$C \equiv$ production cost for one unit of the good

Quenched disorder:

generic properties for a smooth pdf $\rho(h_i)$ on $[-\infty, \infty]$, with a single maximum

Parameters: $h \equiv \langle h_i \rangle$ $\sigma^2 \equiv \langle h^2 \rangle_c$

Exple: most commonly used distribution in economics: logistic distribution ("logit")

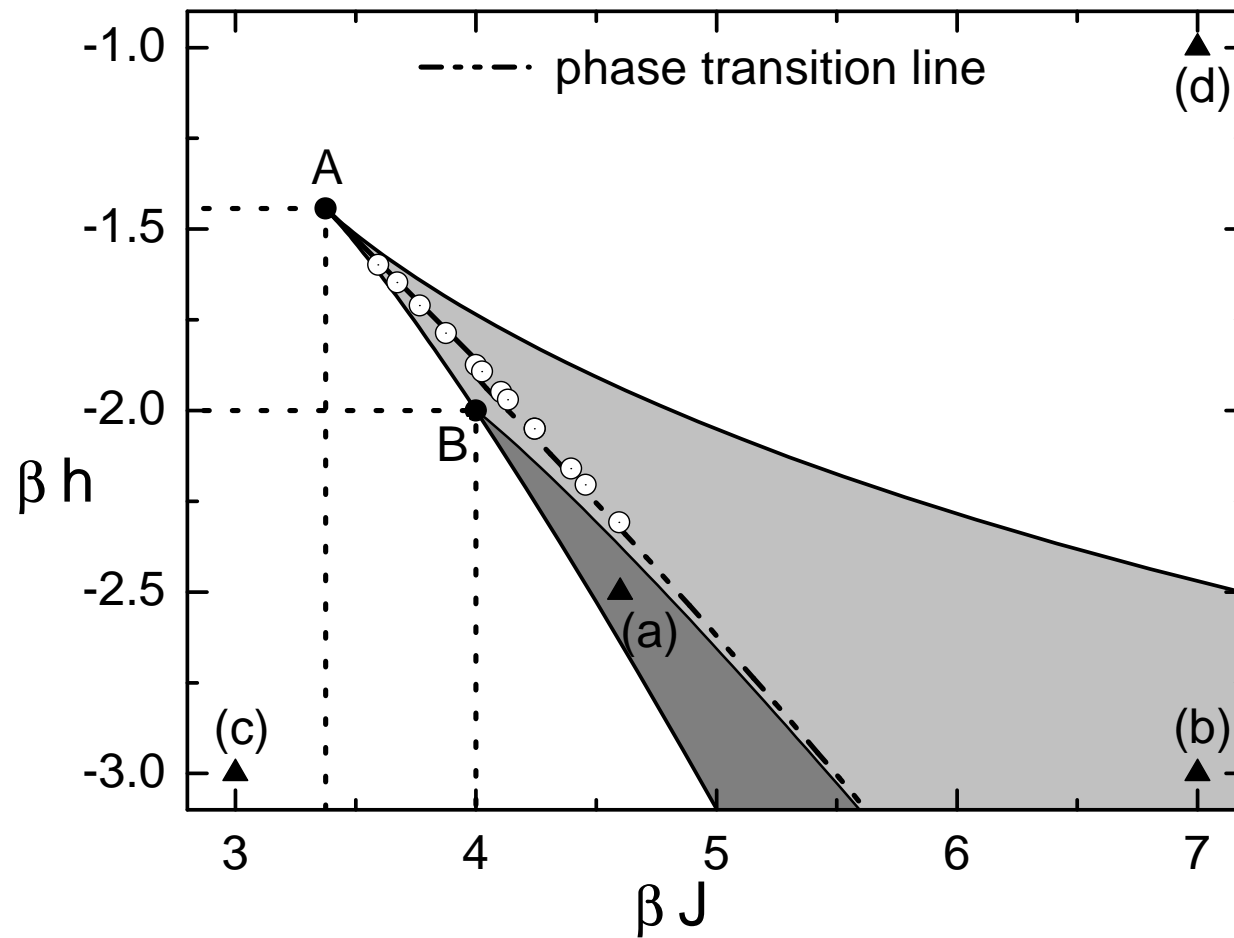
$$\text{Proba}(h_i - h \leq z) = \frac{1}{1 + \exp(-\beta z)} \quad \sigma = \pi / (\beta \sqrt{3})$$

reduced parameters: $\tilde{J} = \beta J$, $\tilde{h} = \beta(h - C)$, $\tilde{P} = \beta(P - C)$

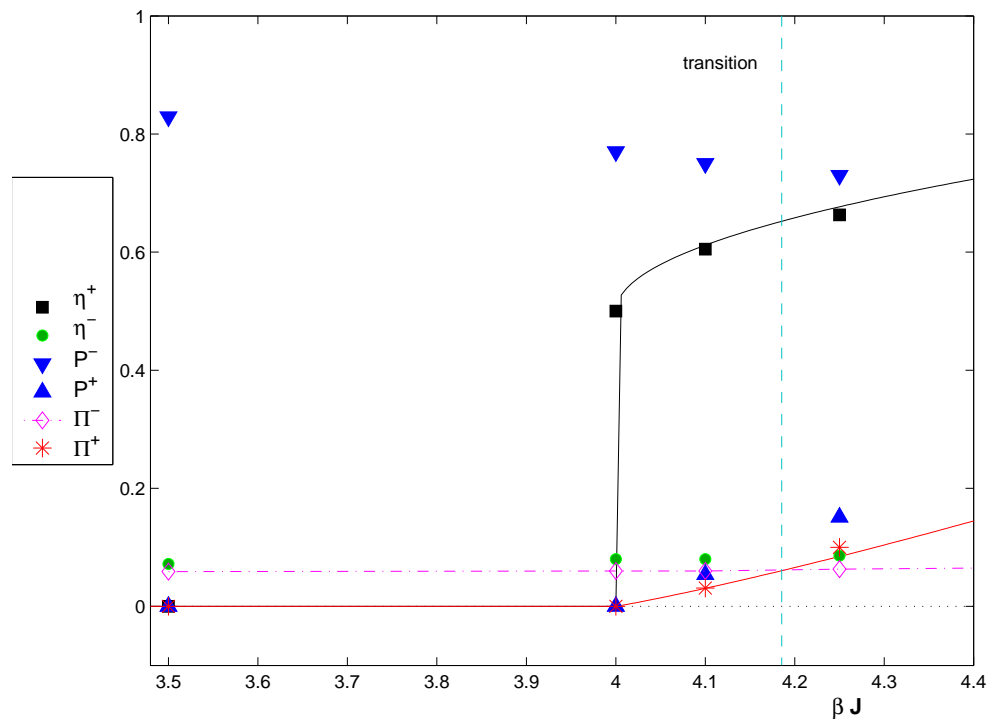
$(\tilde{\beta} = 1, \tilde{C} = 0)$



Monopolist's **phase diagram**:
supply/demand equilibrium (Nash equilibrium)



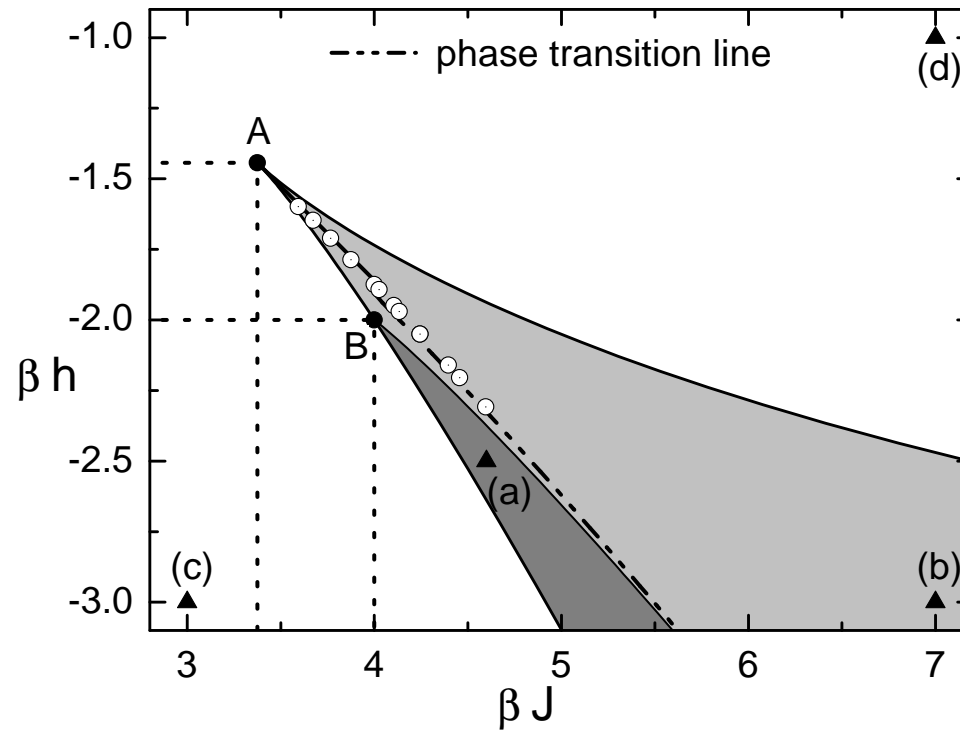
First order phase transition



For $J > J_B$, at a given price P multiple equilibria for the demand $\eta(P)$

For $J_A < J < J_B$, single solution $\eta(P)$

but still multiple equilibria for the optimal monopolist's strategy



Current and future work

- dynamics (hysteresis) (related work: Weisbuch Stauffer Physica A, 2003)
- learning by agents
(actual fraction of buyers known only a posteriori: true surplus not known in advance)
- oligopoly market
- two period model: selling to the few at high price first, then to the many at low price?
- application to other discrete choice problems (e. g. diffusion of criminality)

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