

ILLUSTRATION SUPPLEMENT

**To the chapter
"Contributions to geometric visual illusions"
in J. Ninio' web site:
<http://www.lps.ens.fr/~ninio>**

Convexity rules:
 $m(a+b) > m(a) + m(b)$
 if $b > a$ then $m(b)/m(a) > b/a$

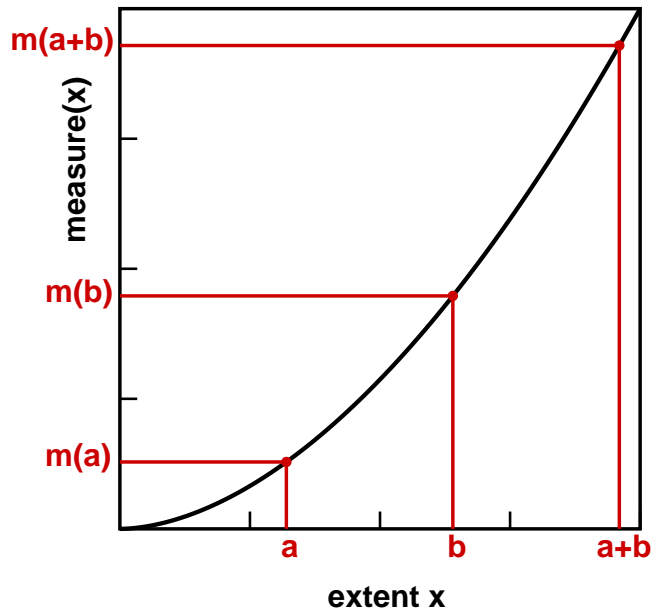


Fig. 1. Convexity effects. The measure of x $y = m(x)$ in ordinate, increases more than linearly with x . Taking two values of x , a and b such that $b > a$, the existence of a convex relationship between $m(x)$ and x implies that $m(b)/m(a) > b/a$.

It also implies that $m(a+b) > m(b) + m(a)$.

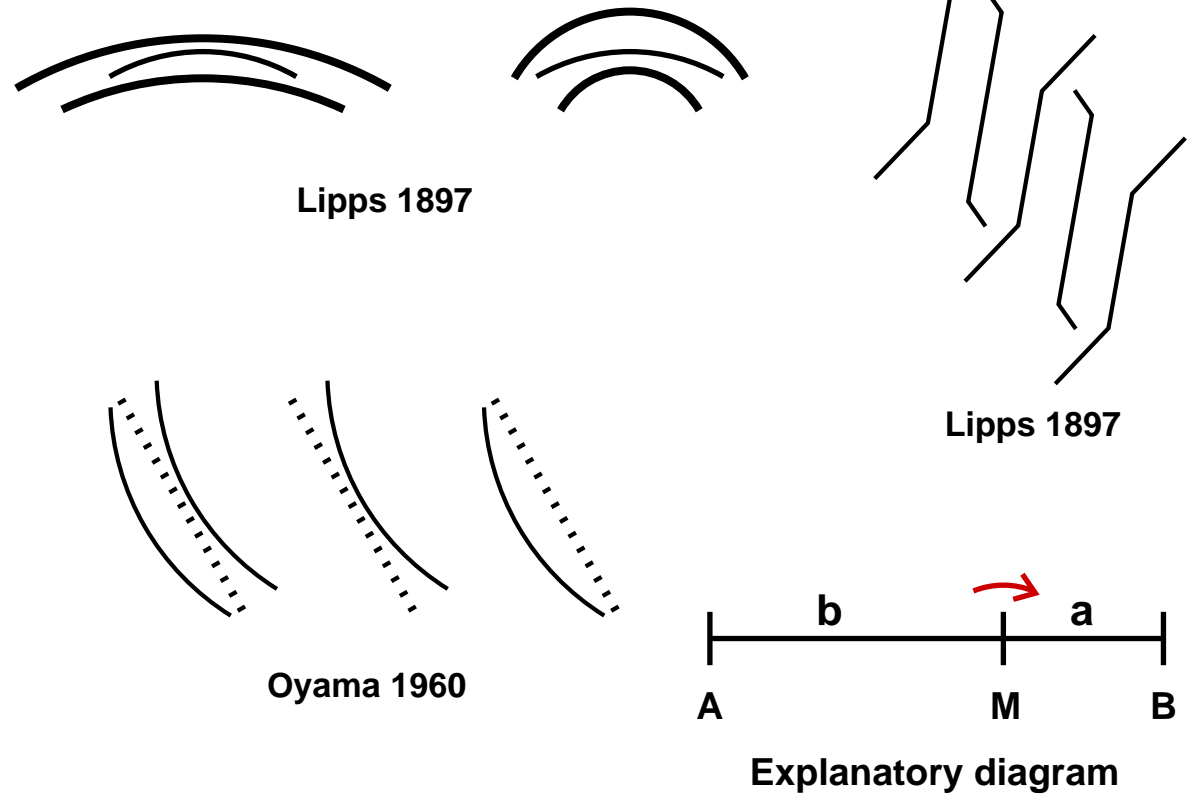
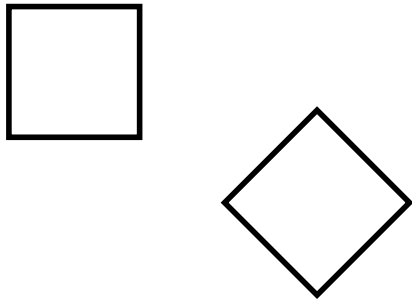
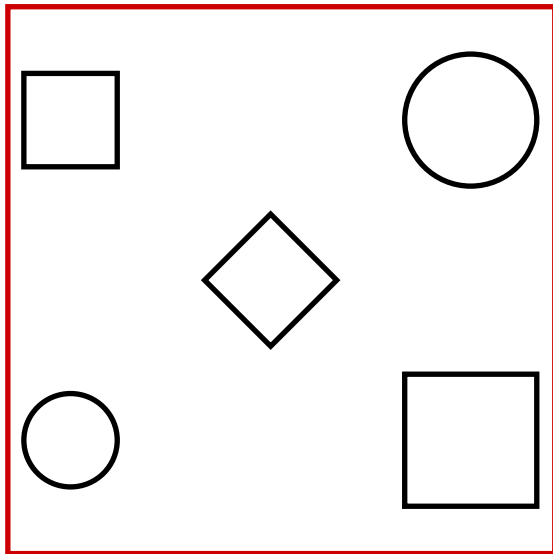


Fig. 2. Illusions explained by $m(b)/m(a) > b/a$
 Consider the endpoints of the intermediate arcs, or the endpoints of the small zigzags in the Lipps figures. According to the explanatory diagram an endpoint M is perceptually displaced towards the nearest neighbouring line.



The square-diamond illusion



Ninio "The science of illusions"
1998-2004

Fig. 3. The side of the big square, bottom right is equal to the diagonal of the central square yet it appears larger, in agreement with the convexity rule $m(b)/m(a) > b/a$ when $b > a$. The opposite would have been predicted, on the basis of the square-diamond illusion (top).

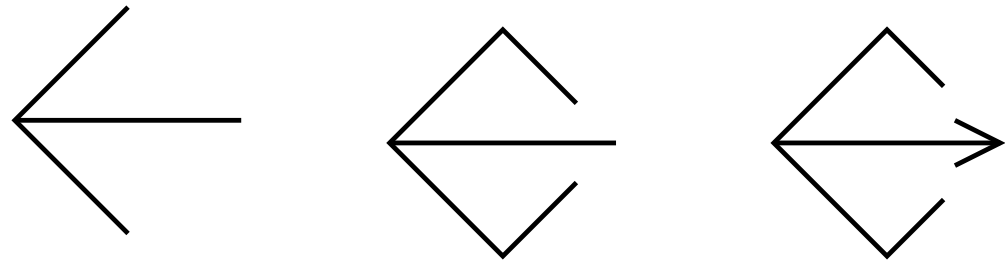
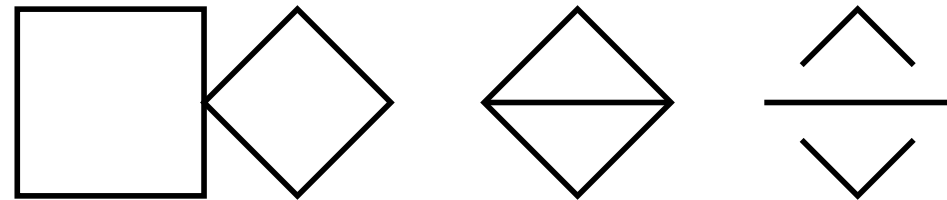
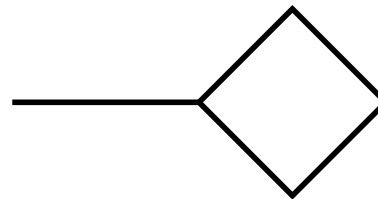


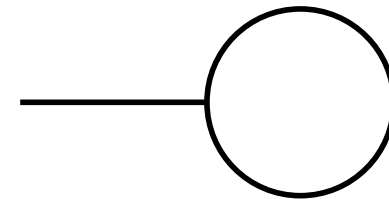
Fig. 4. Pinna's diagonal illusion (2003) Again, an illustration of the convexity rule. When the diagonals are free to expand, they are perceptually enlarged with respect to the sides.



Ninio, Aalen talk 2009



Schumann (1900)



Titchener (1901)

Fig. 5. Further variations on the theme of sides and diagonals.

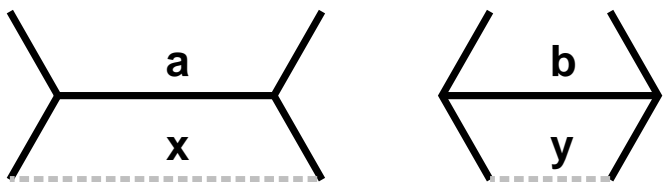


Fig. 6. Muller-Lyer illusion. According to the convexity principle, the "real" illusion is in the perceptual enlargement of x with respect to y : $m(x)/m(y) > x/y$. By construction, $a = b$ but a looks larger than b as a side-effect of consistency, which works in the direction of "assimilation".

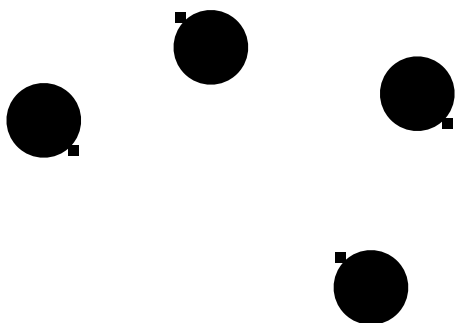


Fig. 7. The gravity lens illusion, by Naito and Cole, 1994. The 4 small squares form the the apexes of a parallelogram. It seems to be a variation on the theme of the Muller-Lyer illusion: equal segments shrink or expand according to the proximity of smaller or larger neighbouring segments.

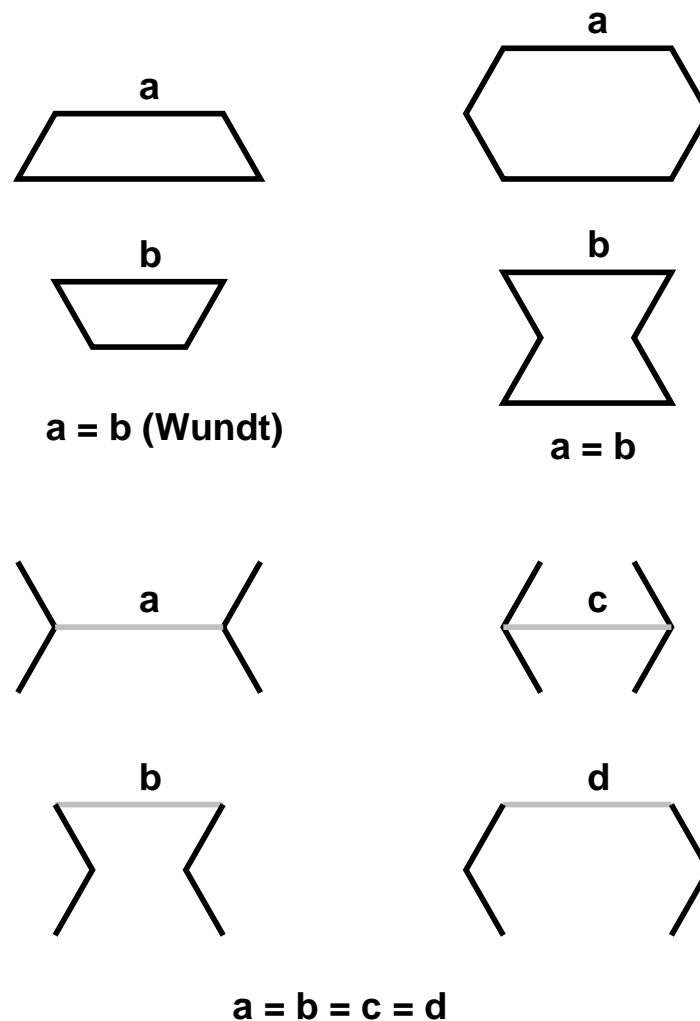
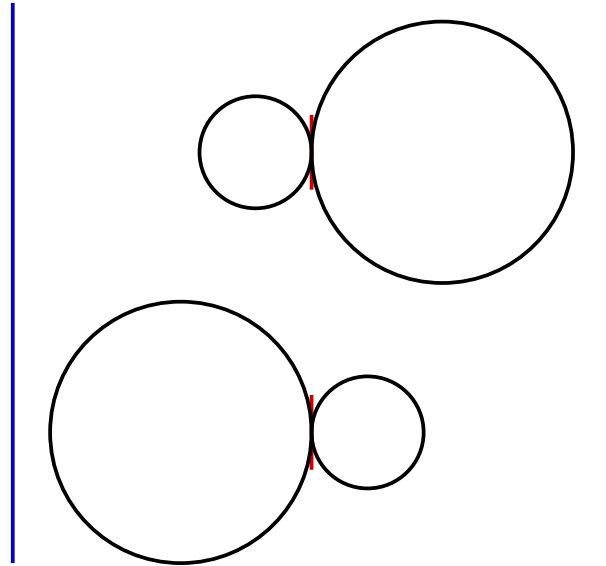


Fig. 8. More variations on the Muller-Lyer theme



Ninio, 1979

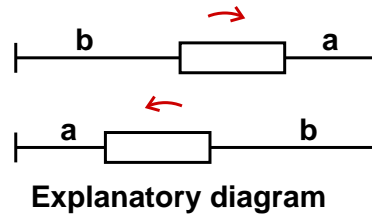
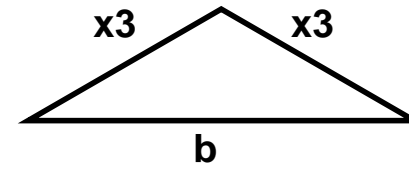
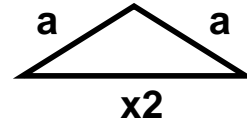
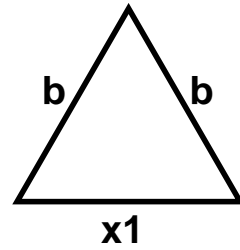


Fig. 9. Attraction to the borders effect.
 The rectangles in the explanatory diagram represent two adjoining circles. The perceptual displacements to the left or to the right are predicted by a convexity rule, $m(b)/m(a) > b/a$, if measurements are made with respect to virtual borders.

Fig. 10. Illusions with triangles.
 There are two prototypical cases, shown on the right.

in both examples $b > a$
 therefore $m(b)/m(a) > b/a$
 then x_2 is perceived as $< x_1$
 in the first case
 and x_2 is perceived as $< x_3$
 in the second case.

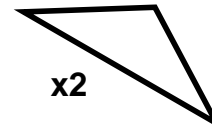
Sander's parallelogram (below)
 can be explained by a triangle
 illusion of the first type.
 (Follow the sequence A, B, C, D.)
 All grey segments are equal.



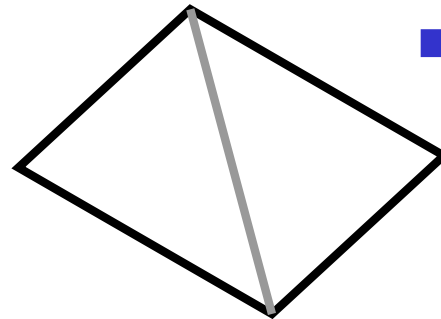
Second case ($x_2 = x_3$)

First case ($x_1 = x_2$)

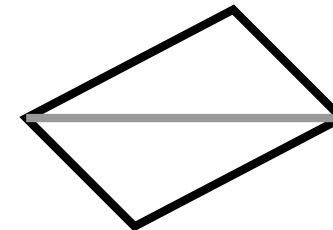
The 1st illusion is more salient when the compared segments are not parallel



A

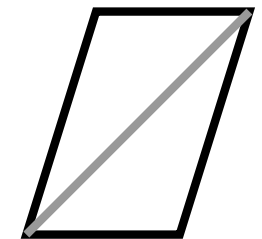
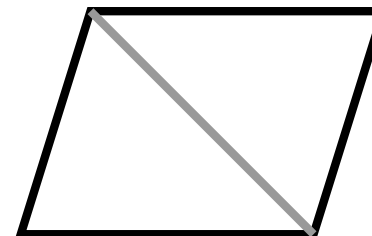
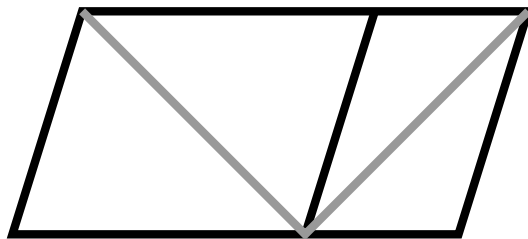


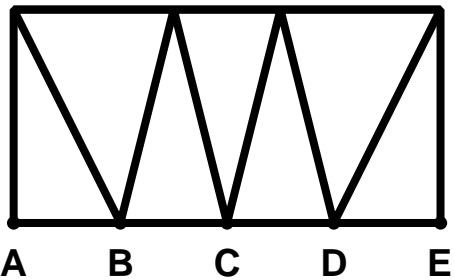
B



C

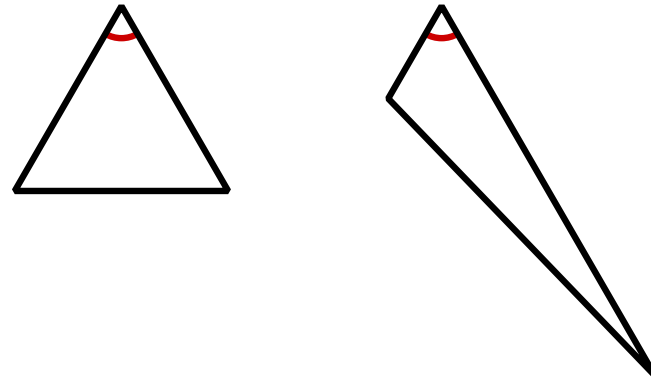
D



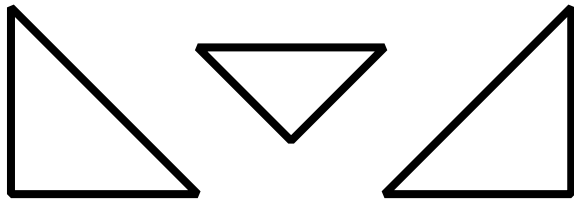


$AB = BC = CD = DE$

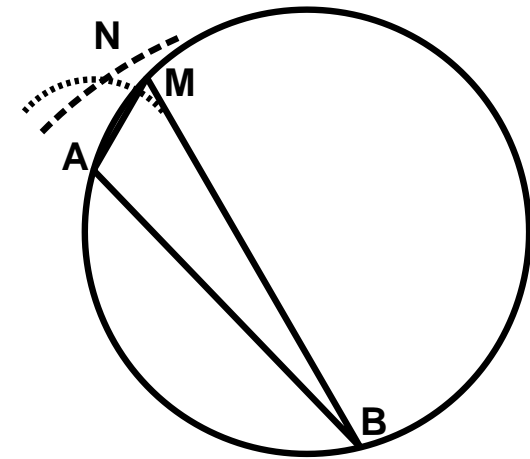
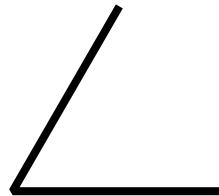
Dr Fee, 1888, in "The science of illusions"



Kennedy, Orbach & Loffler, 2008
The left angle appears larger than the right one.

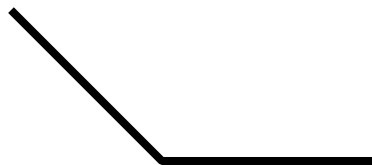


Simplified pattern, Aalen 2009 talk
The 3 horizontal segments are equal

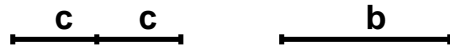


Explanation of the K-O-L illusion.
If the short side is made shorter and the long side made longer M should be replaced by N, and the angle ANB is smaller than AMB (classical geometry).

Fig. 11. Further illusions with triangles.



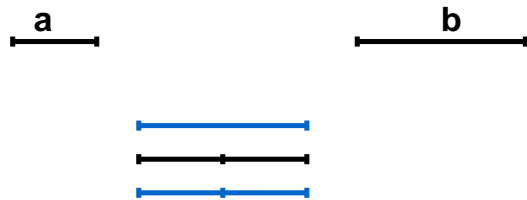
Laska 1890
The sides of the obtuse angle appear larger than those of the acute angle.



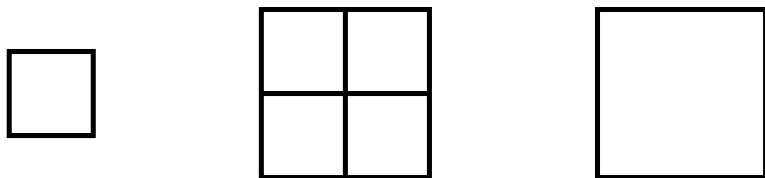
Standard bisection illusion.
 $b = 2c$ but the divided segment
 appears smaller than the undivided one.



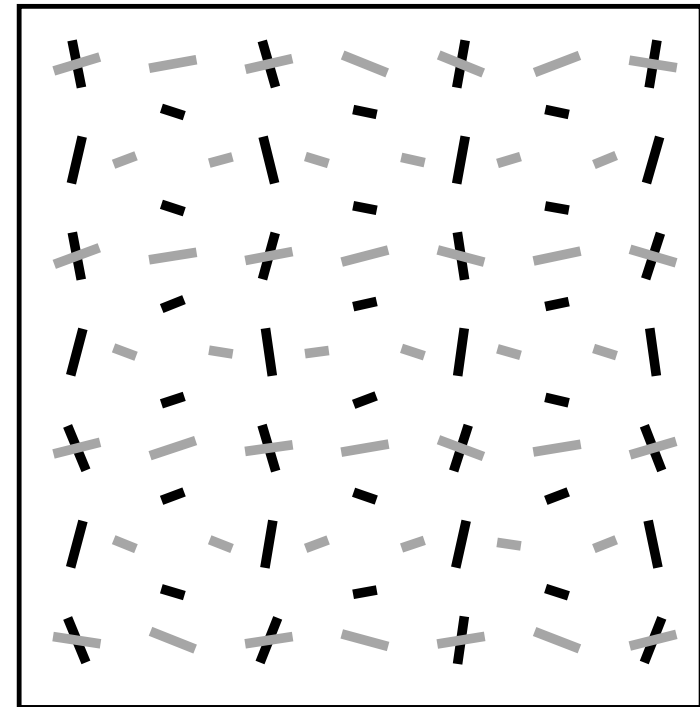
"inverse" bisection illusion
 $a = c$ but a appears smaller than c



"Real" bisection illusion
 $b = 2a$, but $m(b)/m(a) > 2$



The central subdivided square appears larger
 than the square on the right, and each of its
 4 components squares appear smaller than the
 square on the left (Aalen 2009 talk).



Ninio, "The science of illusions"
 (modified)

Fig. 12. Bisection effects. According to the
 convexity principle there is a single "real"
 bisection illusion. It is produced by the
 perceptual enlargement of the undivided
 segment b with respect to a : $m(b)/m(a) > b/a$.
 The standard and the "inverted" variants
 would have the status of pedagogical displays.

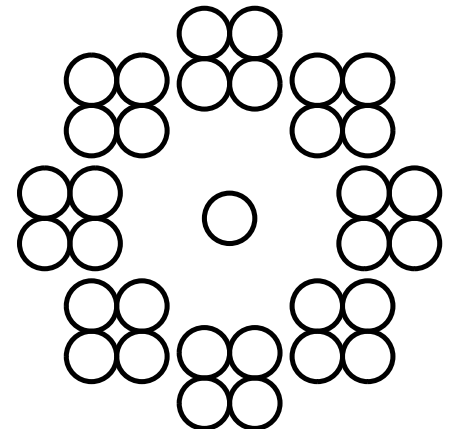
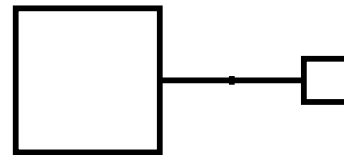
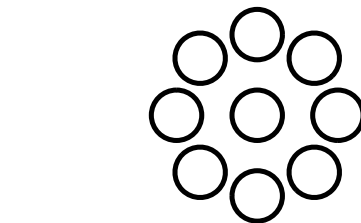
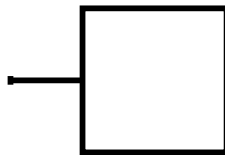
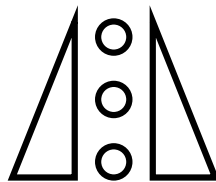
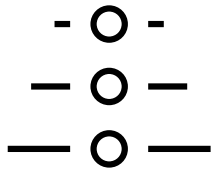
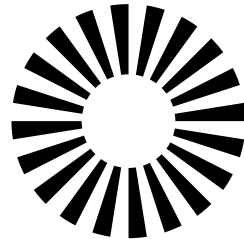
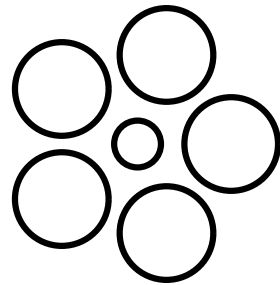
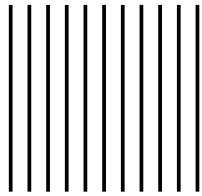
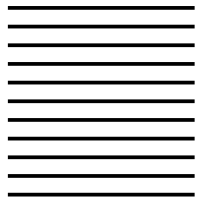


Fig. 13. Space occupation rule.

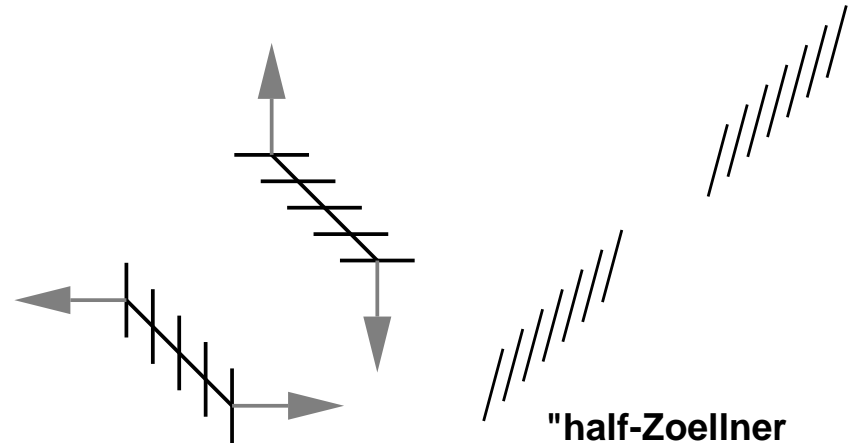
According to this rule, a normalizing factor is applied to whole figures. The large ones are perceptually reduced, and the small ones are perceptually enlarged.

Thus, in the left part of the Ebbinghaus pattern (top left) ALL the circles would be reduced, and in the right part, ALL the circles would be enlarged. The effect is expected to work in all directions. It seems that the illusion works in the bottom right pattern with circles, in which all the circles have the same size. Furthermore, it goes against the classical explanation by a contrast effect.

Baldwin, 1895

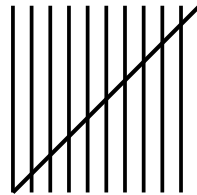
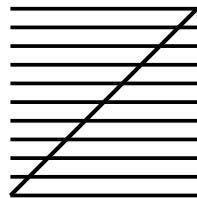
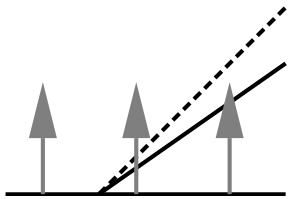


standard expansion effect
in subdivided figures



Orthogonal expansion
and the Zoellner
illusion

"half-Zoellner
illusion, Ninio
& O'Regan, 1996



"Regression to right angles" effects

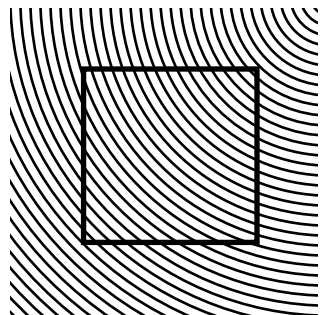
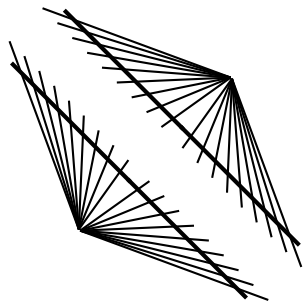
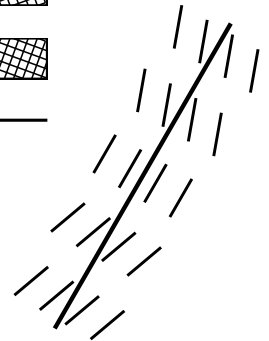
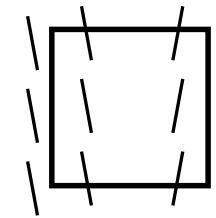
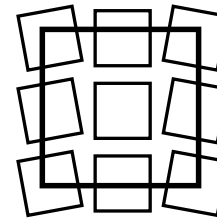
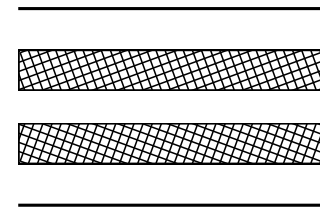
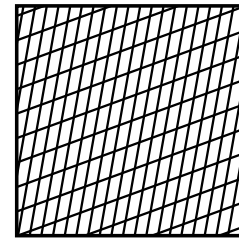
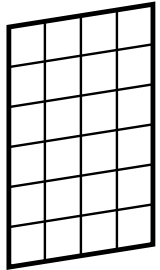
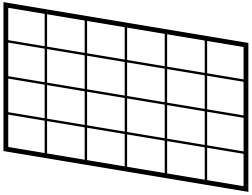


Fig. 14. Orthogonal expansion.
Many classes of illusions can be
described by a principle of
expansion at right angles to a
set of parallel or nearly
parallel lines.

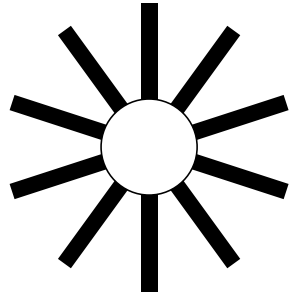


Five patterns from Ninio & Pinna, 2006

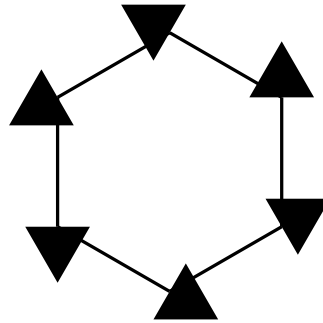
SOME ILLUSIONS THAT I DO NOT UNDERSTAND



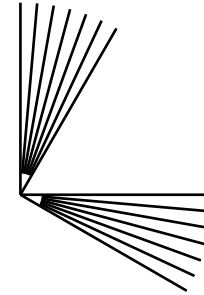
Shepard's tables



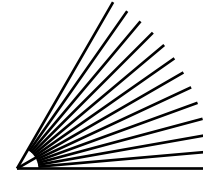
Angularities illusion
Pinna, 1991



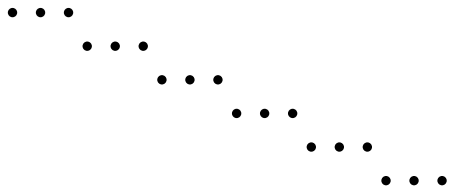
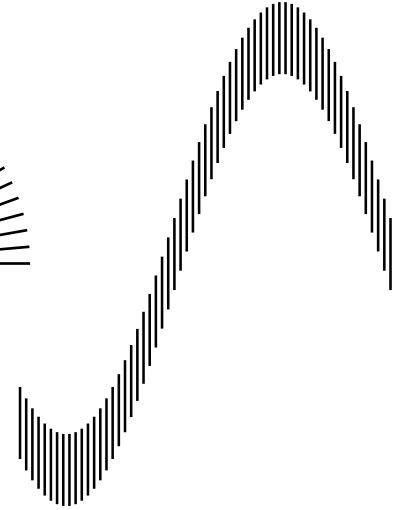
Gerbino



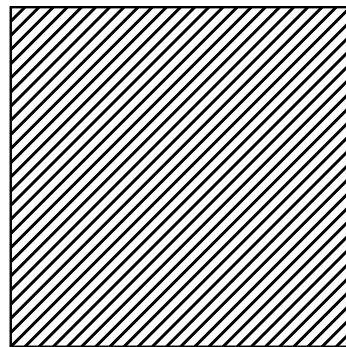
Tolanski



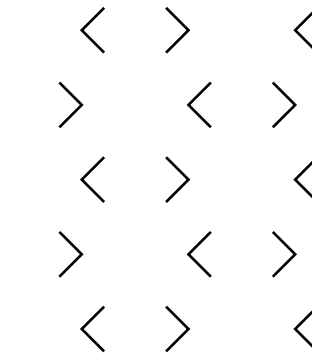
Botti illusion, 1909
Day and Stecher's
pattern, 1991



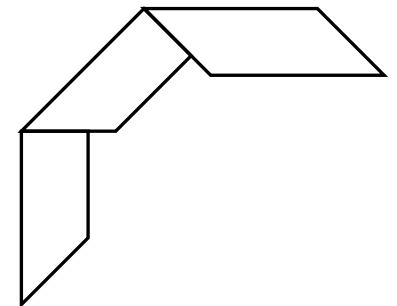
Sloping steps illusion
Vicario, 1978



Rarefaction illusion
Vicario



Displacement illusion
Morinaga, 1954



Bressanelli and
Massironi, 2006