ILLUSTRATION SUPPLEMENT

To the chapter
"Contributions to geometric visual illusions"
in J. Ninio' web site:
http://www.lps.ens.fr/~ninio

[document to be assembled in the "portrait"

orientation, with stapples on the long sides]

Convexity rules:
m(a+b) > m(a) + m(b)
if b > a then m(b)/m(a) > b/a

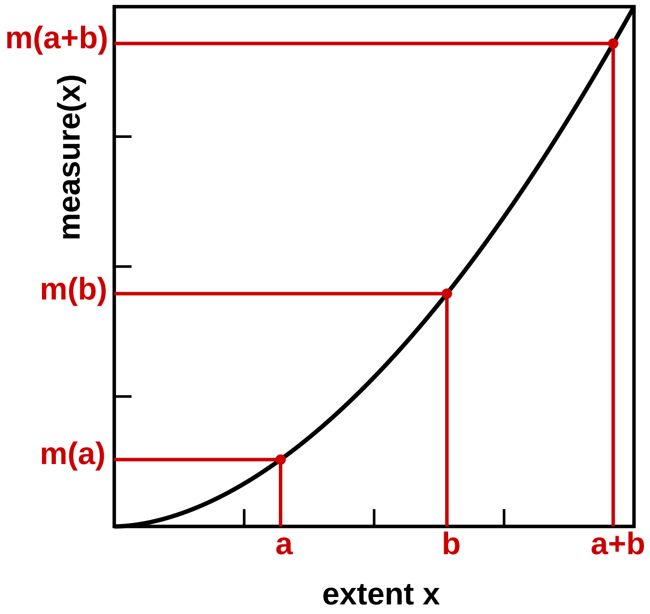


Fig. 1. Convexity effects. The measure of x y = m(x) in ordinate, increases more than linearly with x. Taking two values of x, a and b such that b > a, the existence of a convex relationship between m(x) and x implies that m(b)/m(a) > b/a.

It also implies that m(a+b) > m(b) + m(a).

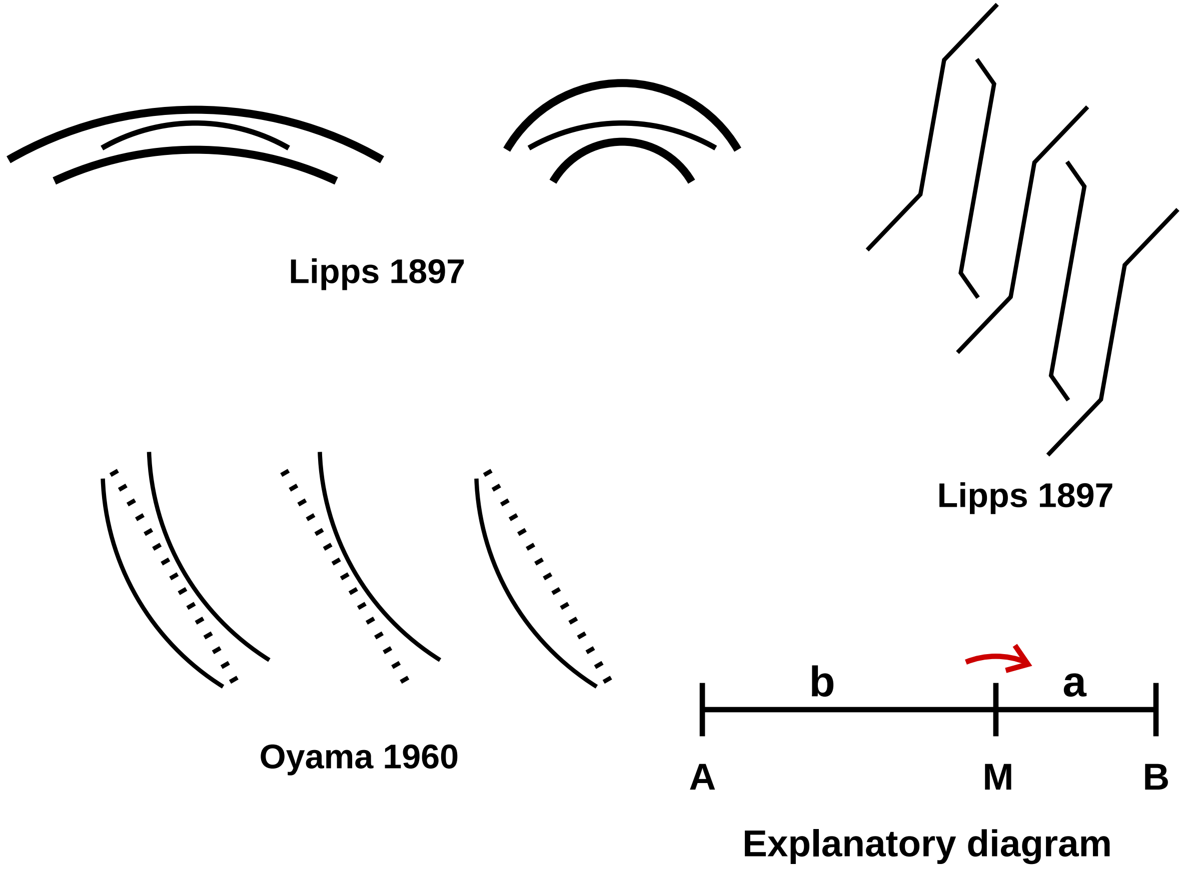
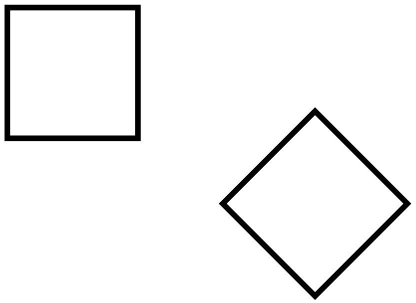
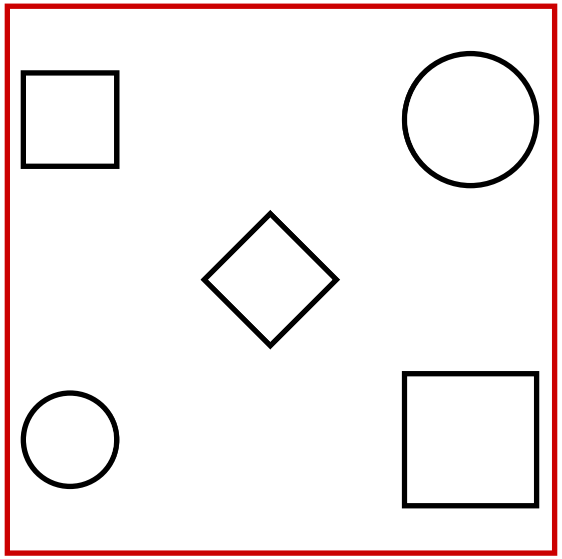


Fig. 2. Illusions explained by m(b)/m(a) > b/a
Consider the endpoints of the intermediate arcs, or the endpoints of the small zigzags in the Lipps figures. According to the explanatory diagram an endpoint M is perceptually displaced towards the nearest neighbouring line.



The square-diamond illusion



Ninio "The science of illusions"
1998-2004

Fig. 3. The side of the big square, bottom right is equal to the diagonal of the central square yet it appears larger, in agreement with the convexity rule m(b)/m(a) > b/a when b>a. The opposite would have been predicted, on the basis of the square-diamond illusion (top).

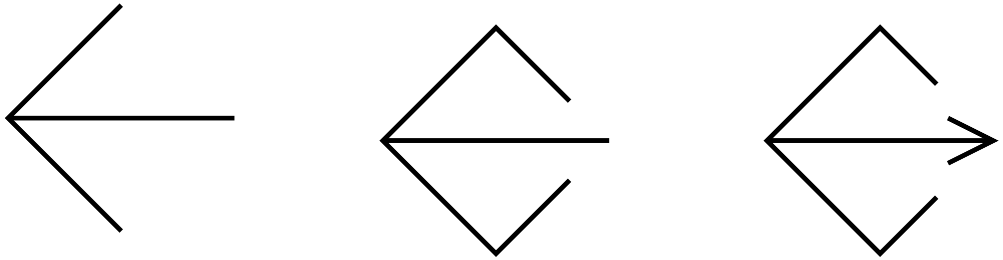
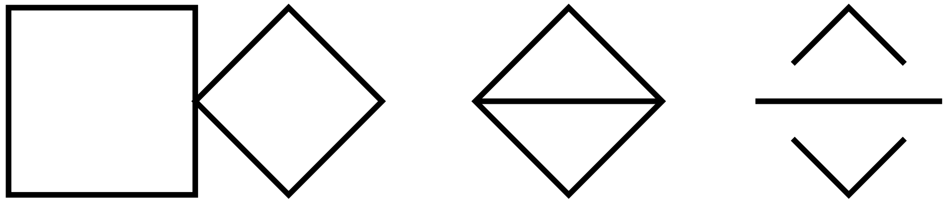
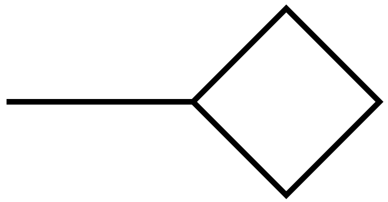


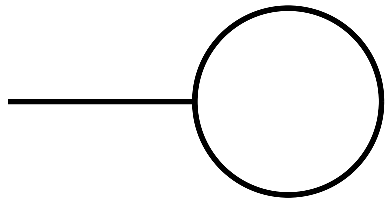
Fig. 4. Pinna's diagonal illusion (2003) Again, an illustration of the convexity rule. When the diagonals are free to expand, they are perceptually enlarged with respect to the sides.



Ninio, Aalen talk 2009



Schumann (1900)



Titchener (1901)

Fig. 5. Further variations on the theme of sides and diagonals.

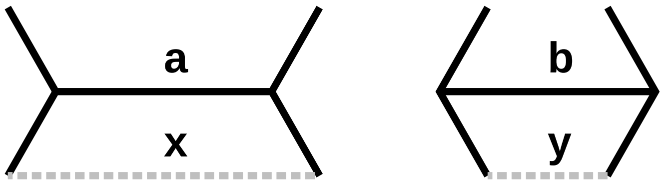


Fig. 6. Muller-Lyer illusion. According to the convexity principle, the "real" illusion is in the perceptual enlargement of x with respect to y: m(x)/m(y) > x/y. By construction, a = b but a looks larger than b as a side-effect of consistency, which works in the direction of "assimilation".

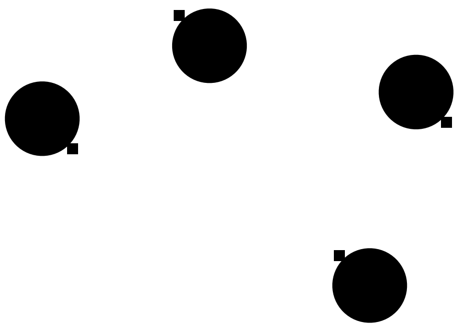


Fig. 7. The gravity lens illusion, by Naito and Cole, 1994. The 4 small squares form the the apexes of a parallelogram. It seems to be a variation on the theme of the Muller-Lyer illusion: equal segments shink or expand according to the proximity of smaller or larger neighbouring segments.

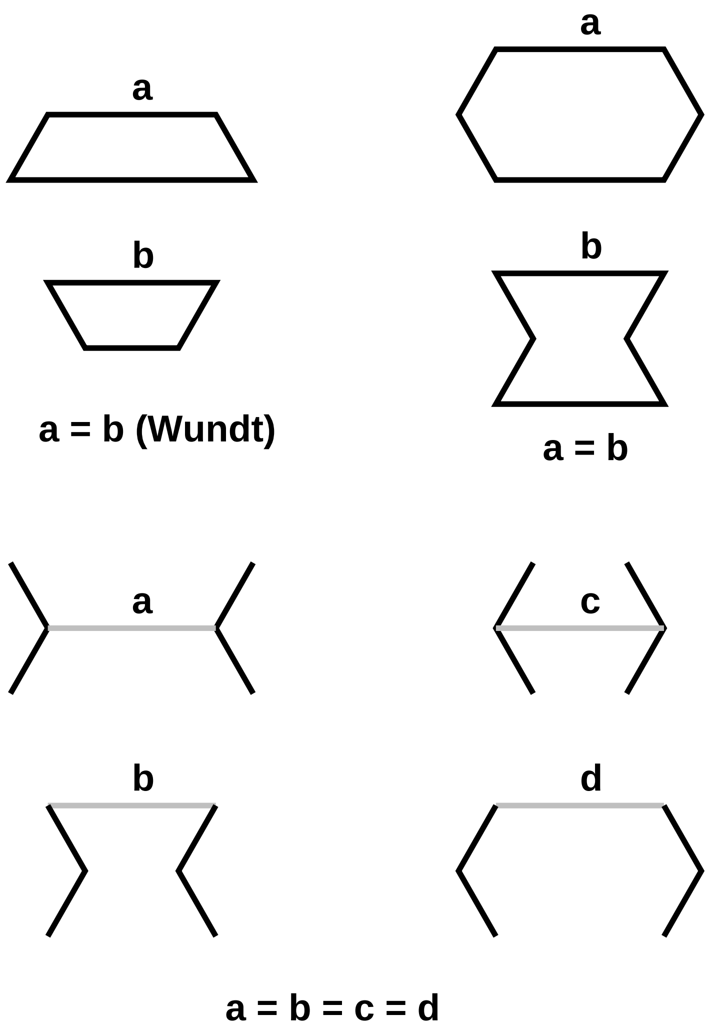
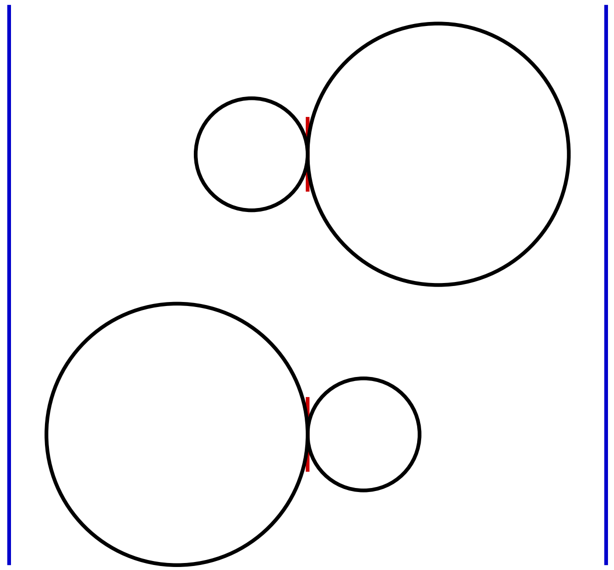
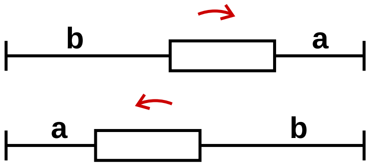


Fig. 8. More variations on the Muller-Lyer theme



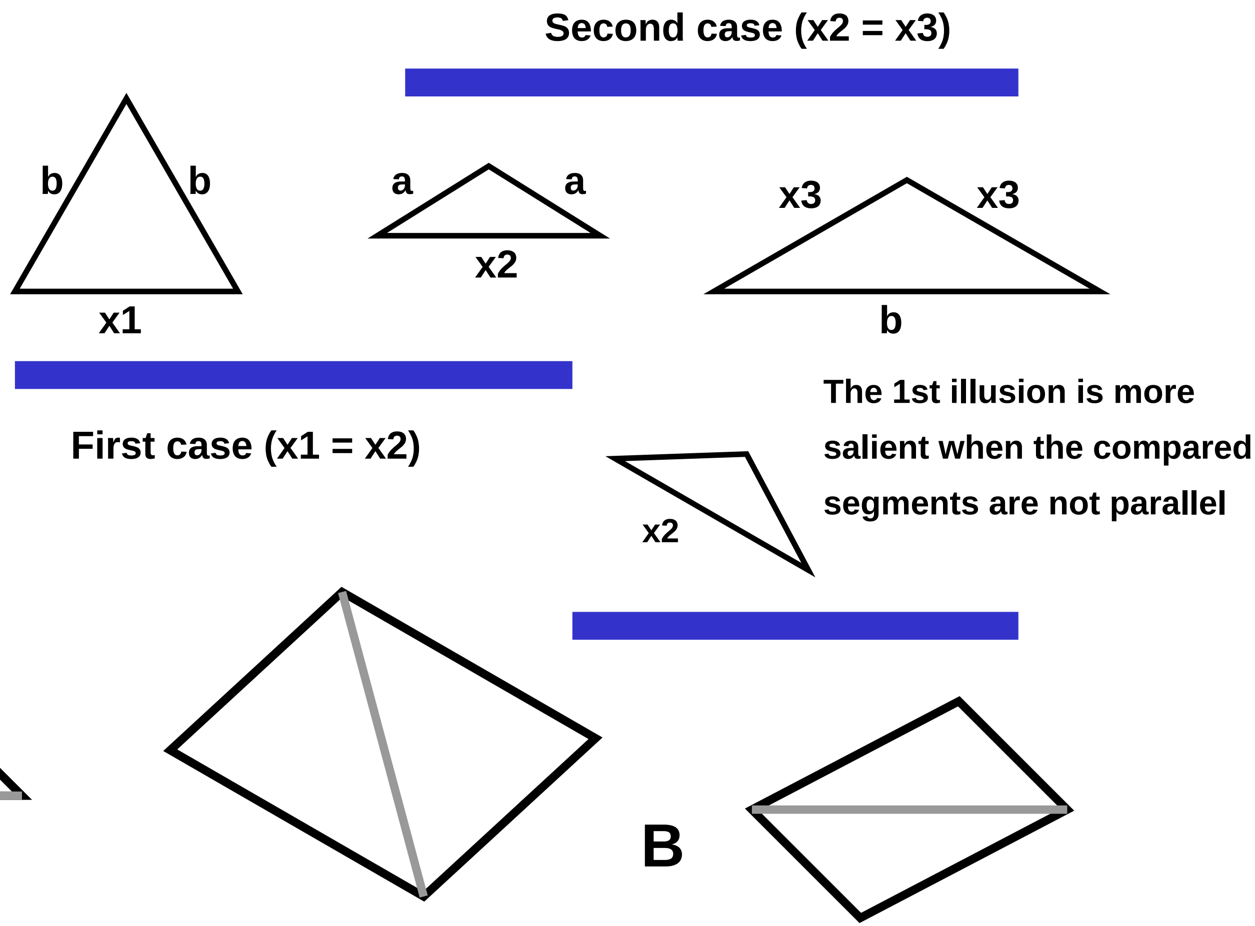
Ninio, 1979

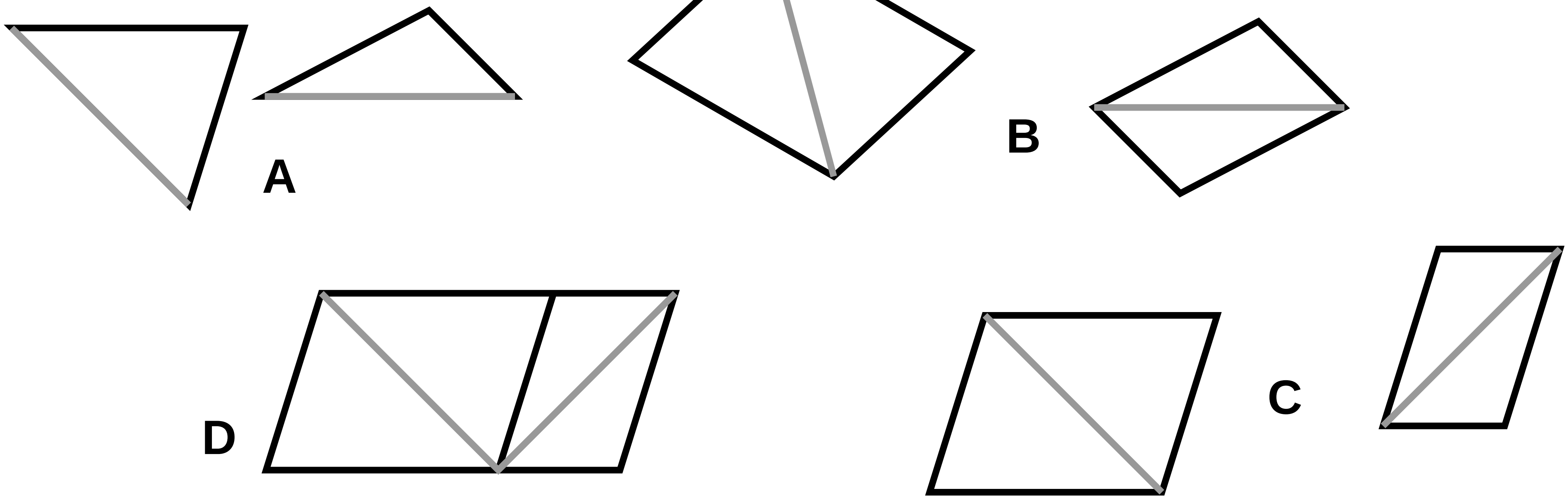


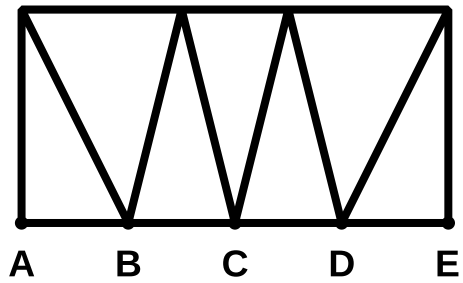
Explanatory diagram

Fig. 9. Attraction to the borders effect. The rectangles in the explanatory diagram represent two adjoining circles. The perceptual displacements to the left or to the right are predicted by a convexity rule, m(b)/m(a) > b/a, if measurements are made with respect to virtual borders.

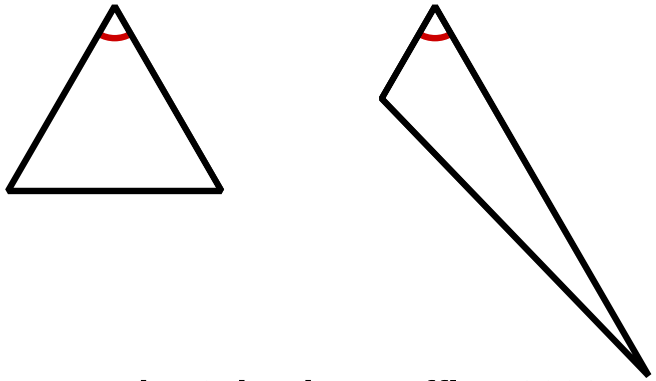
Fig. 10. Illusions with triangles.
There are two prototypical cases, shown on the right.
in both examples b > a
therefore m(b)/m(a) > b/a
then x2 is perceived as < x1
in the first case
and x2 is perceived as < x3
in the second case.
Sander's parallelogram (below) can be explained by a triangle illusion of the first type.
(Follow the sequence A, B, C, D.)
All grey segments are equal.



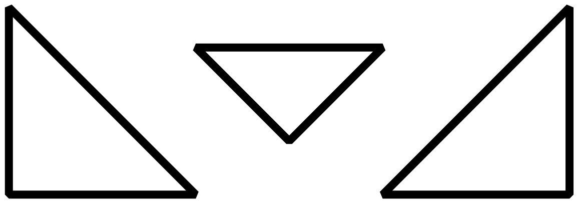




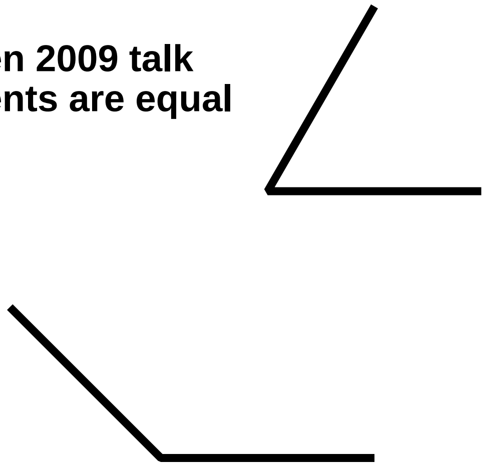
AB = BC = CD = DE
Dr Fee, 1888, in "The science of illusions"



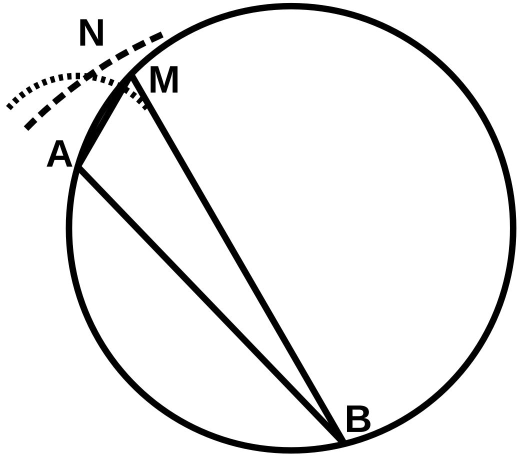
Kennedy, Orbach & Loffler, 2008
The left angle appears larger
than the right one.



Simplified pattern, Aalen 2009 talk
The 3 horizontal segments are equal



Laska 1890
The sides of the obtuse
angle appear larger
than those of the acute angle.

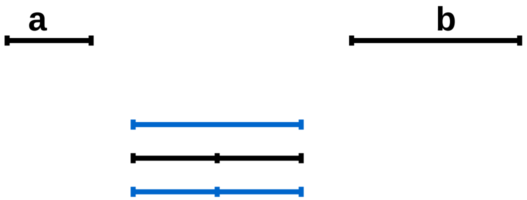


Explanation of the K-O-L illusion.
If the short side is made shorter
and the long side made longer
M should be replaced by N, and
the angle ANB is smaller than AMB
(classical geometry).

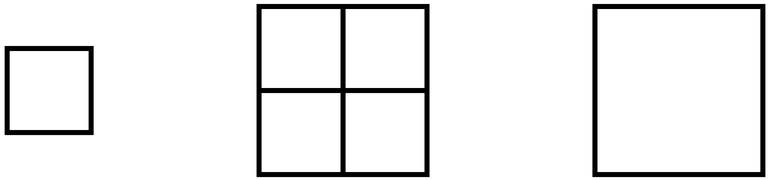
Fig. 11. Further illusions with triangles.

Standard bisection illusion.
b = 2c but the divided segment
appears smaller than the undivided one.

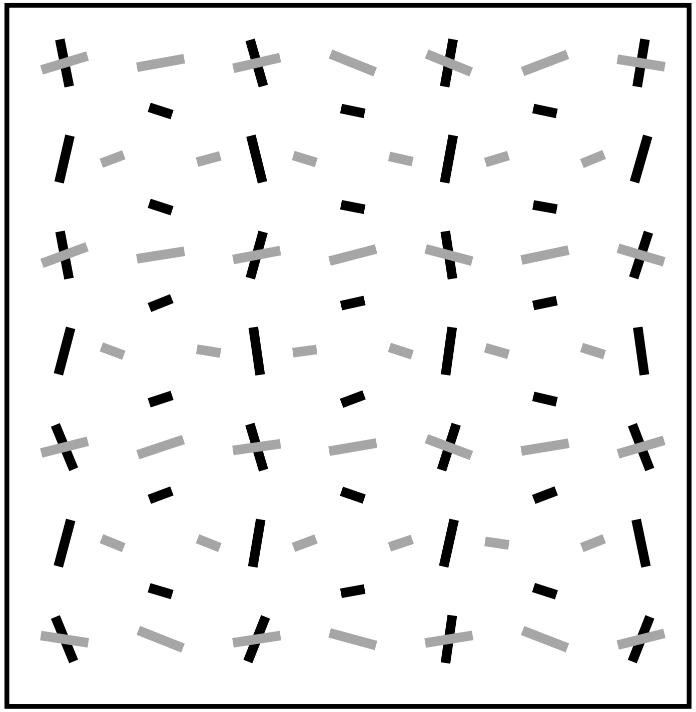
"inverse" bisection illusion
a = c but a appears smaller than c



"Real" bisection illusion
b = 2a, but m(b)/m(a) > 2



The central subdivided square appears larger than the square on the right, and each of its 4 components squares appear smaller than the square on the left (Aalen 2009 talk).



Ninio, "The science of illusions"
(modified)

Fig. 12. Bisection effects. According to the convexity principle there is a single "real" bisection illusion. It is produced by the perceptual enlargement of the undivided segment b with respect to a: m(b)/m(a) > b/a. The standard and the "inverted" variants would have the status of pedagogical displays.

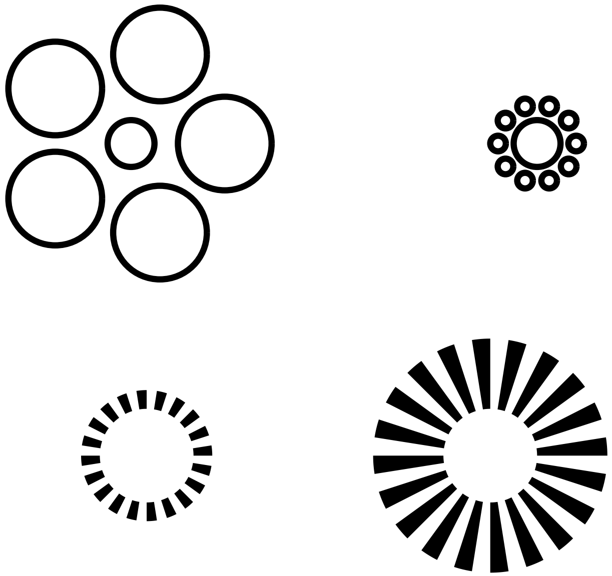
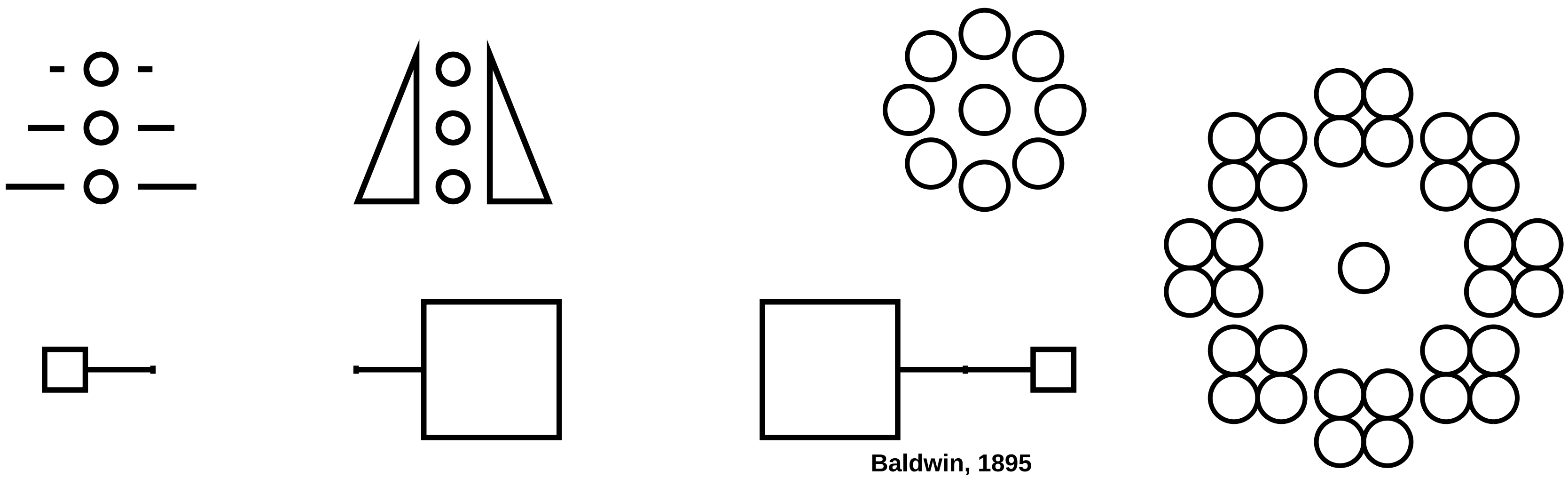


Fig. 13. Space occupation rule.
According to this rule, a normalizing factor is applied to whole figures. The large ones are perceptually reduced, and the small ones are perceptually enlarged.
Thus, in the left part of the Ebbinghaus pattern (top left) ALL the circles would be reduced, and and in the right part, ALL the circles would be enlarged. The effect is expected to work in all directions. It seems that the illusion works in the bottom right pattern with circles, in which all the circles have the same size. Furthermore, it goes against the classical explanation by a contrast effect.



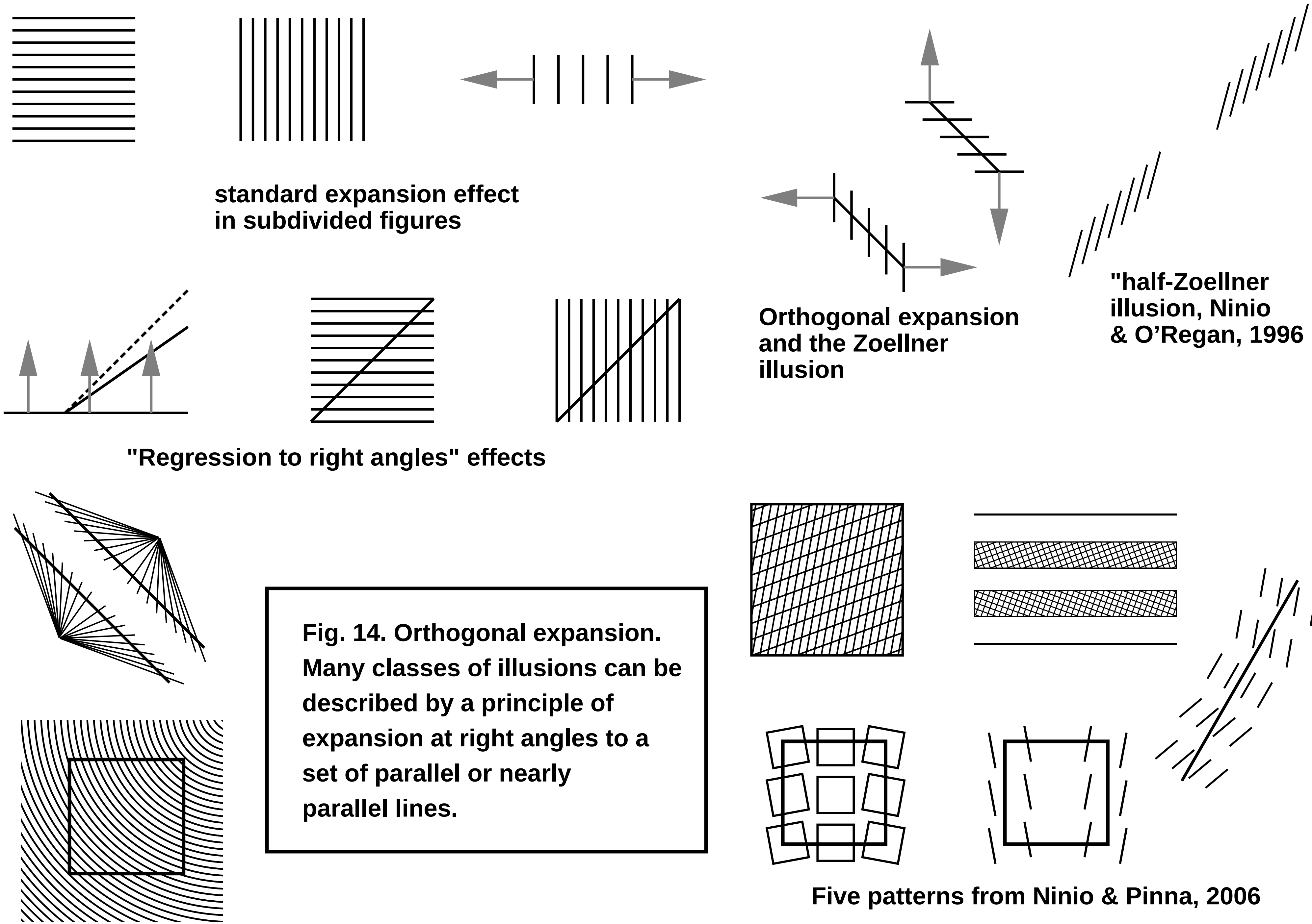
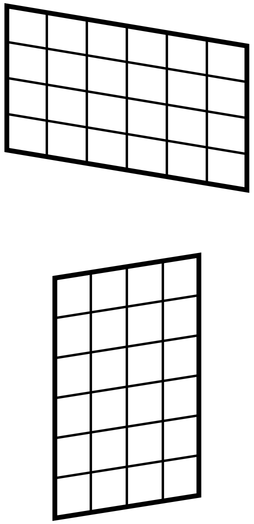
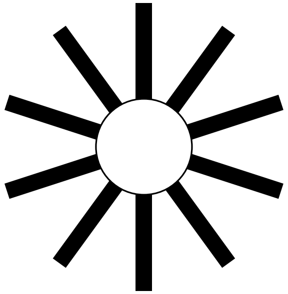


Fig. 14. Orthogonal expansion. Many classes of illusions can be described by a principle of expansion at right angles to a set of parallel or nearly parallel lines.

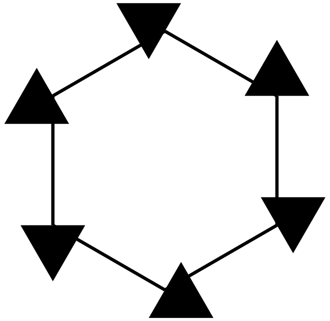
Figure 15 : Some illusions that I do not understand



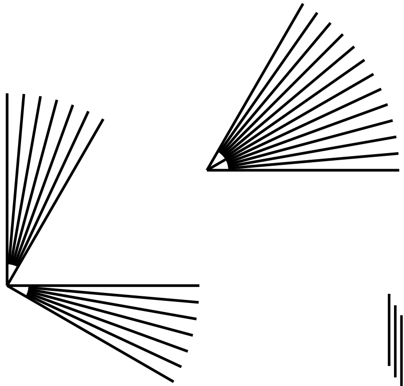
Shepard's tables



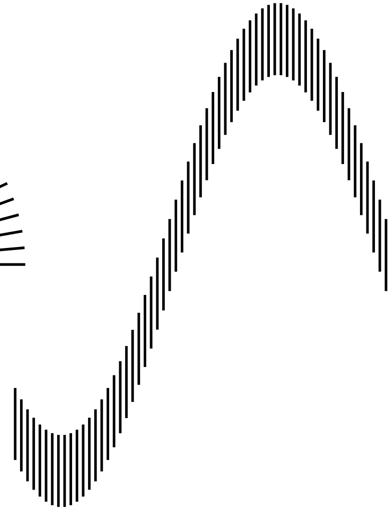
Angularity illusion
Pinna, 1991



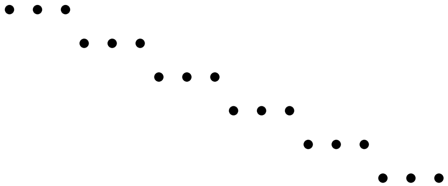
Gerbino



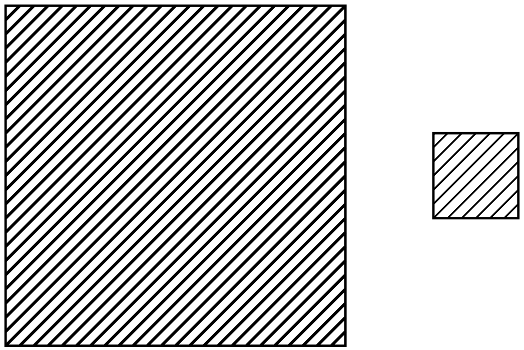
Tolanski



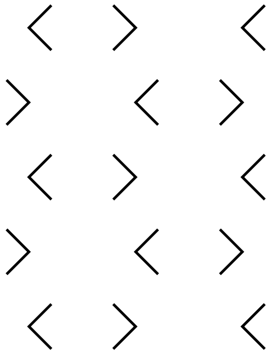
Botti illusion, 1909
Day and Stecher's pattern, 1991



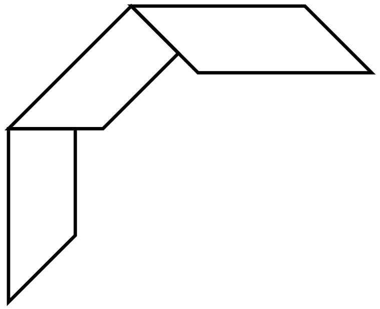
Sloping steps illusion
Vicario, 1978



Rarefaction illusion
Vicario

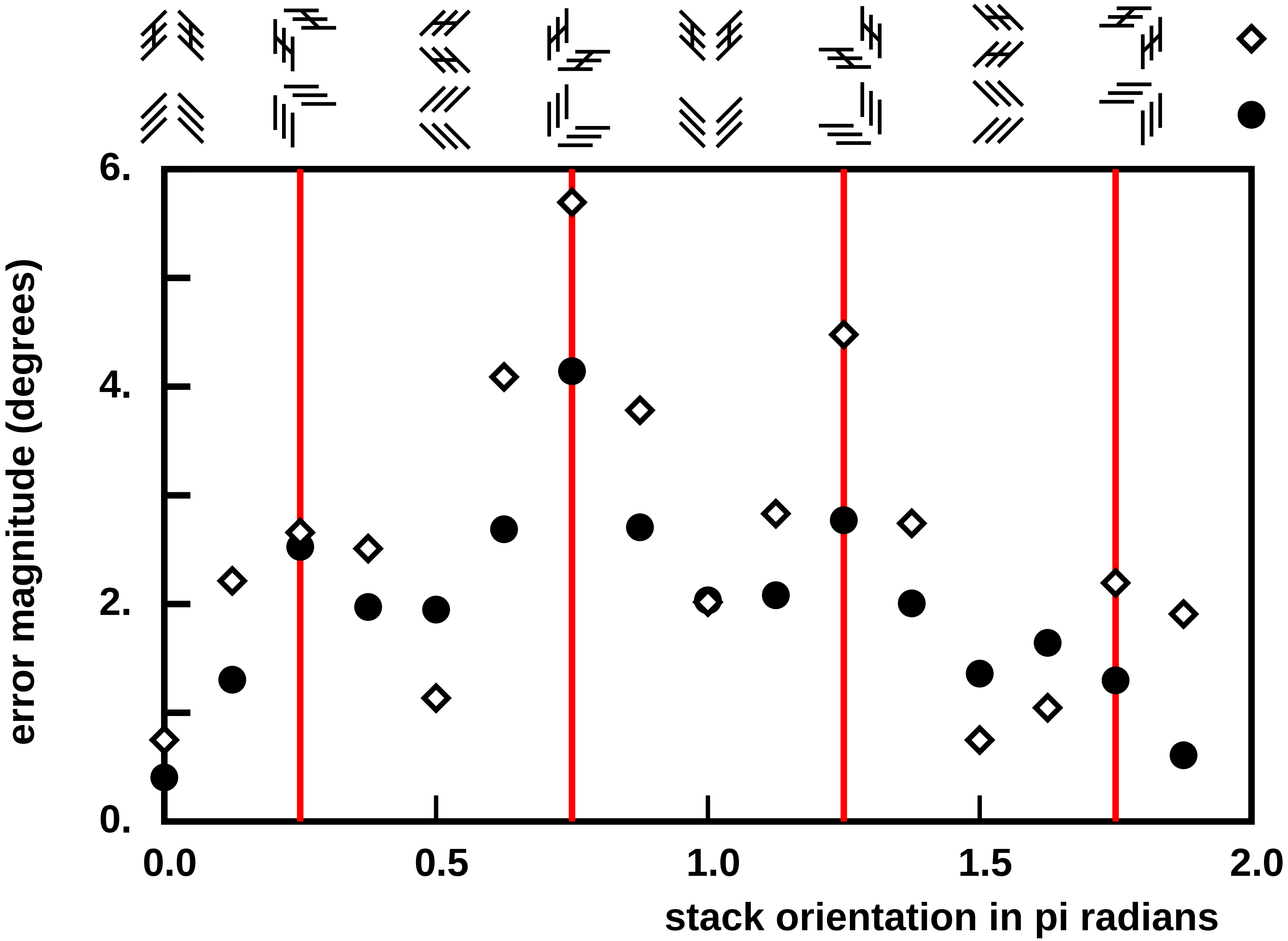


Displacement illusion
Morinaga, 1954



Bressanelli and
Massironi, 2006

Figure 16 : Orientation profiles in the Zoellner illusion with or without axes (with 5 bars per stack)



17 subjects / weighted average / 170 measures per data point

Figure 17 : Orientation profiles in the square-diamond illusions , with or without symmetry

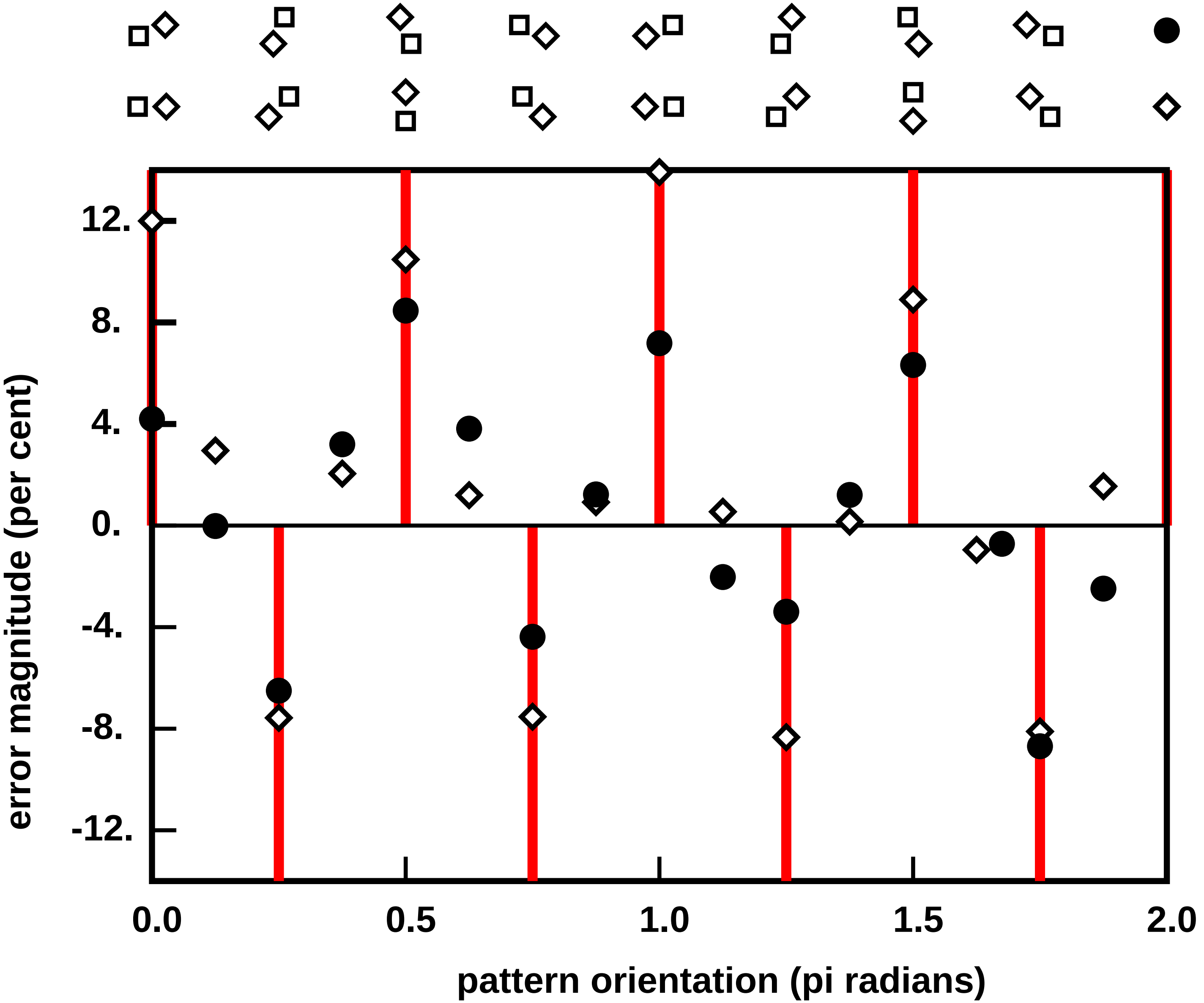


Figure 18 :
Orientation profiles in the square-diamond illusions
square versus diamond within a larger square

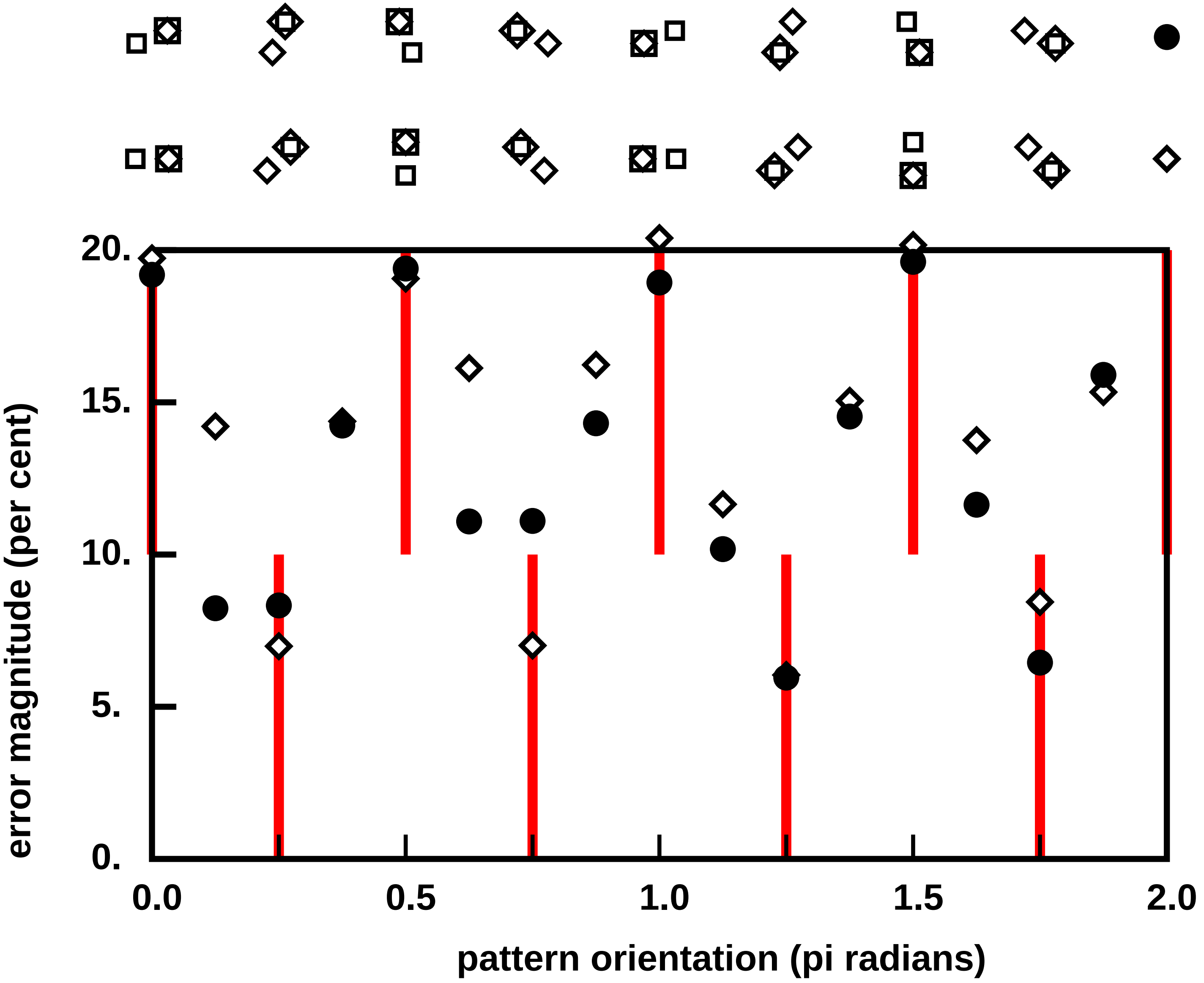


Figure 19 :
Orientation profiles in the square-diamond illusions
diagonal of small square versus side of larger square

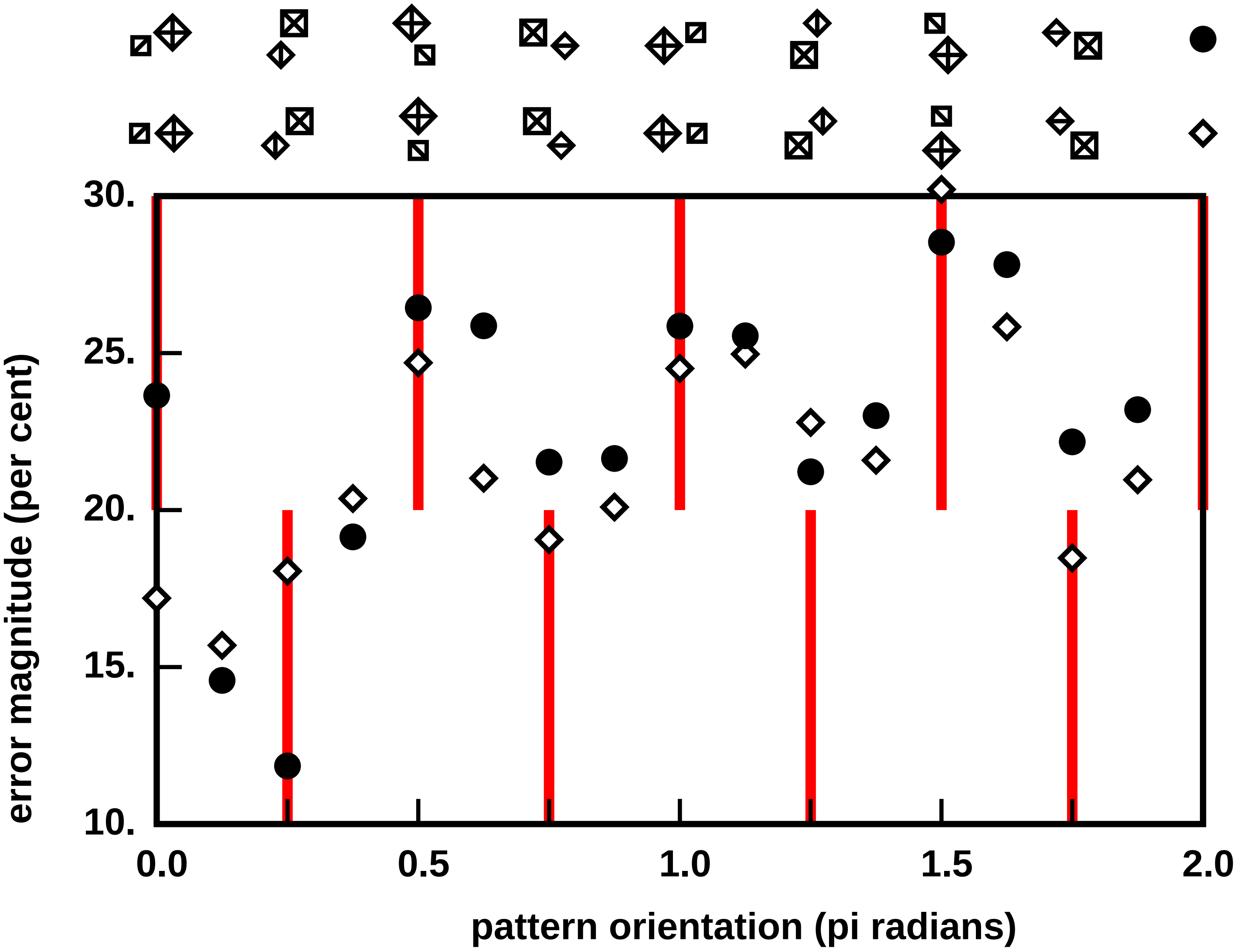


Figure 20 :
Orientation profiles in the trapezium illusions
1-apparent inequality of the large bases

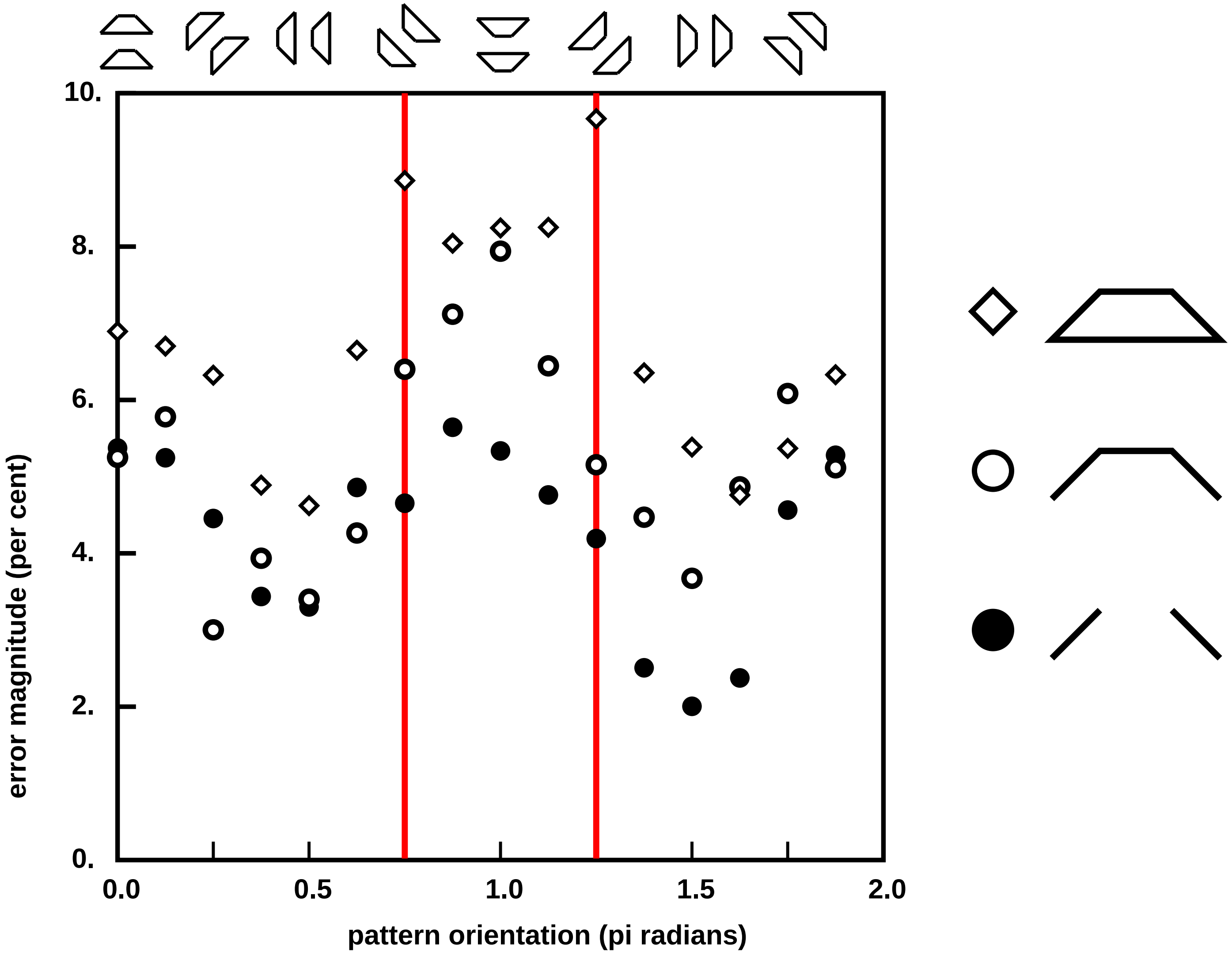


Figure 21 :
Orientation profiles in the trapezium illusions
2--apparent inequality of sides or heights

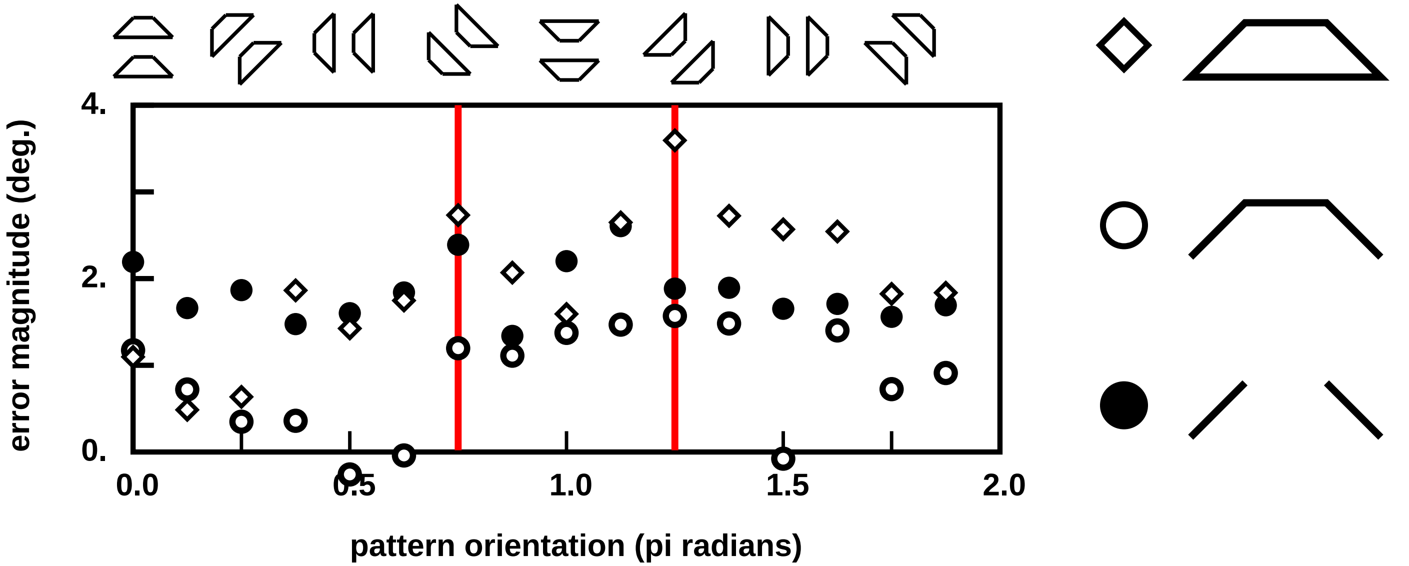


Figure 22 :
Orientation profiles in the trapezium illusions
3-a study of the configuration without sides

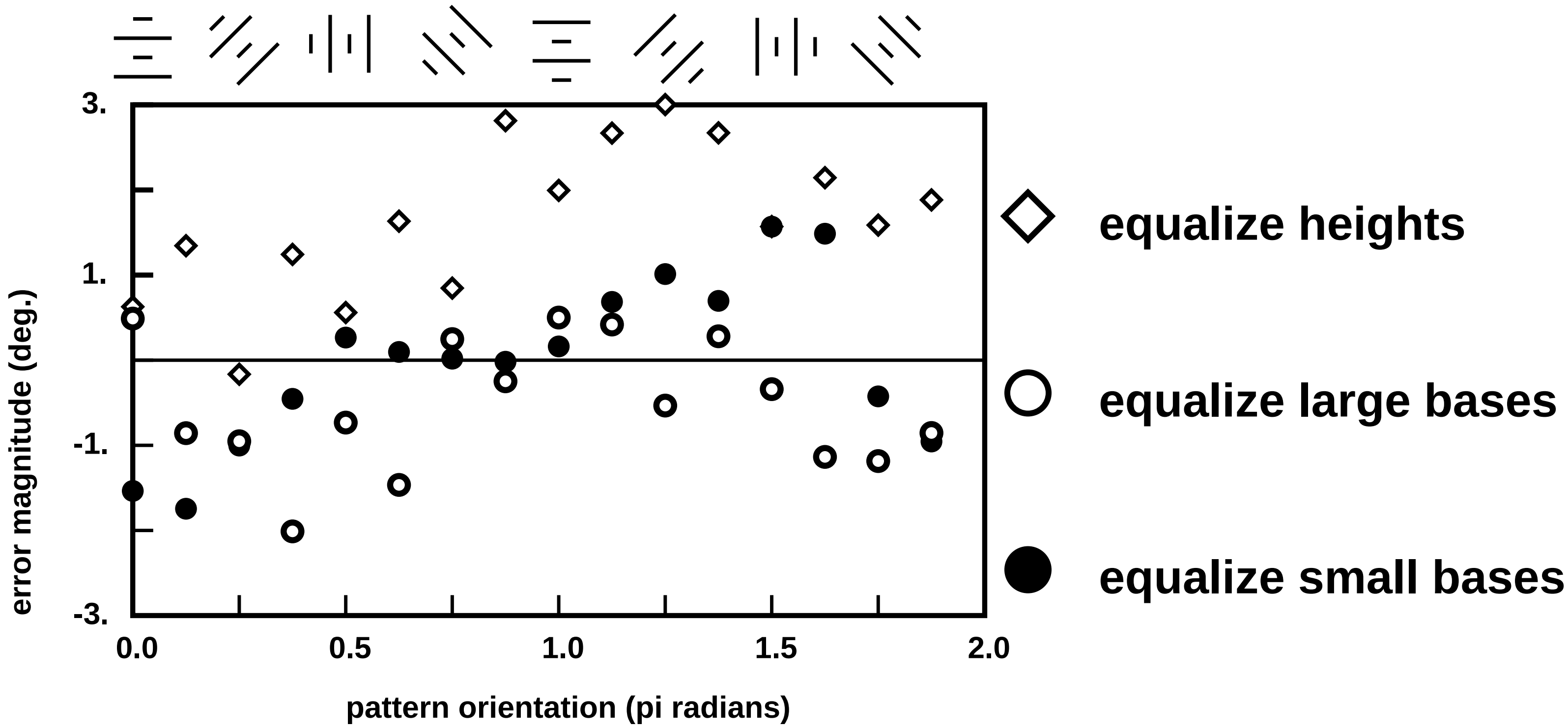


Figure 23 Hybrid Zoellner-Poggendorff patterns :
After subtraction of the Zehender component