Bad Honnef

Economic Dynamics from the Physics Point of View

Hits and Flops Dynamics

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http:www.lps.ens.fr/˜weisbuch/adoption.ps

http:www.lps.ens.fr/˜weisbuch/socper.ps

http:www.lps.ens.fr/˜weisbuch/simulcomp.ps

1 Motivations

Why does one observe either hits or failures in

- certain markets such as toys, gadgets,
- in the movies industries ,
- but also in the adoption of technological changes,
- of political and economical measures,
- in the political arena

rather than a featureless distribution of partial successes ?

Possible answer: “social choice”

- Information contagion accross a social network
- positive “externalities” (e.g. markets for equipments)

2 Social Percolation Models

http:www.lps.ens.fr/˜weisbuch/socper.ps

Suppose a lattice or a random graph with percolation critical density p_c [9]. On each site there is an agent i ("movie-goer") which can communicate information about the movie and its quality q to its "nearest neighbours"

Agents i informed about the movie will decide to go to it, if and only if the quality q (a real number between zero and one) of the movie is larger than his/her personal preference p_i, i.e. if $q > p_i$.

2.1 "Frozen"preferences

A classical percolation problem.

When a movie happens to have a quality $q < p_c$, after a certain short time its diffusion among the public will stop. The movie will then be a flop.

On the opposite, when $q > p_c$ percolation accross the net occurs: the movie is a hit.

2.2 SOC dynamics

- after opportunities during which they went to the movie, the agents will be more demanding and typically increase their preferences p_i; on the opposite, those who did not go, lower their expectations and preferences.
- after hits (resp. flops) the movie producers will decrease (resp. increase) the quality q of the offered movie(s), in their effort to remain above the threshold while minimizing expenses.

For Moore neighborhood (4 neighbours)

- Adaptive movie quality: for $\delta p = 0,\ \delta q > 0$, q moves to the usual percolation threshold;
- Adaptive customer preferences: for $\delta p > 0,\ \delta q = 0$, the p_i distribution drifts towards a single peak centered on the fixed q value, taken equal to 0.5 (no spanning cluster) or 0.593 (some spanning clusters).
- When both δp and δq are positive, p_i and q drift towards $p_c = 0.593$; the dynamic percolator shows self-organized criticality.

S. Solomon, G. Weisbuch, L. de Arcangelis, N. Jan and D. Stauffer, Physica A 277, 239 (2000)

2.3 Extensions: more choices

E. Ahmed and H.A. Abdusalam, Eur. Phys. J. B, in press

One party first invade the network, then another one appears; conservatism prevails, the first party dominates.

Z.F. Huang, Int. J. Mod. Phys. C 11, No. 2 (2000)

D. Stauffer and GW. Simultaneous Competition (in progress).

http:www.lps.ens.fr/˜weisbuch/simulcomp.ps

Competition between two parties A $q + \epsilon$ and B q.

Von Neumann neighbourhood, $q = p_c = 0.407$.

If site i gets information on one movie, this movie is visited if $q > p_i$; If different neighbours have gone to different movies, then i visits the better movie A.

Results

"Birds of a feather flock together"

AAAAAAAAAAAAAAAABBBBBBBBBBBBBB

Equal probability of percolation for A and B.

For random distribution of A's and B's, A wins with very high probability.

3 From utilities to INCA

http:www.lps.ens.fr/˜weisbuch/adoption.ps

Actors i have private different utilities u_0 and u_1 for choices 0 and 1, and they poll all their neighbors.

$$U_0 = u_0 + Jf_0 \tag{1}$$

$$U_1 = u_1 + Jf_1 \tag{2}$$

$$\Delta U = \Delta u + J(2f_1 - 1) \tag{3}$$

For 8 neighbours v, the condition for adoption $\Delta U > 0$ becomes:

$$\Delta u + \frac{v_1}{4} > 1 \tag{4}$$

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Adoption thresholds:
5    4     3     2     1    0
__|_____|_____|_____|_____|_
  0    0.25  0.5   0.75  1.0
Private utility differences.
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4 Hopeful Monsters

For threshold 2, a seed is made of one early adopter plus another one among its 24 neighbors. After a first adopter has been randomly positionned a new random sampling wil create a "hopeful monster" with probability p:

$$p = 24/N \tag{5}$$

where 24 is the number of cells available to useful neighbours and N is the total number of lattice sites. If n early adopters are randomly generated, the probability $\mathcal{P}$ of not generating any hopeful monster is:

$$\mathcal{P} = (1-p)(1-2p).....(1-(n-1)p) \tag{6}$$

The magnitude of n_c such that $\mathcal{P}$ is $1/e$ is given by:

$$\mathcal{P} = exp(-\frac{p\, n\, (n-1)}{2}) = 1/e \tag{7}$$

$$n_c \simeq \sqrt{\frac{N}{12}} \tag{8}$$

5 INCA: Inhomogeneous cellular automata: Crystal Growth

Probability of growth on any site at the interface is written:

$$\pi = 1 - q. \tag{9}$$

Where q is the probability that the site stays empty The probability $P1$ that at least one site among $s1$ opportunities at the first step is an actual inter-facial seed is then:

$$P1 = 1 - q^{s1}. \tag{10}$$

The probability $P2$ that any site among $s2$ opportunities at the second step is actual inter-facial seed is then:

$$P2 = 1 - q^{s2}. \tag{11}$$

When $\frac{s1}{s2}$ is small, $\frac{P2}{P1}$ can get big: as soon as growth starts at the first layer outside the seed, it has good chances to proceed further.

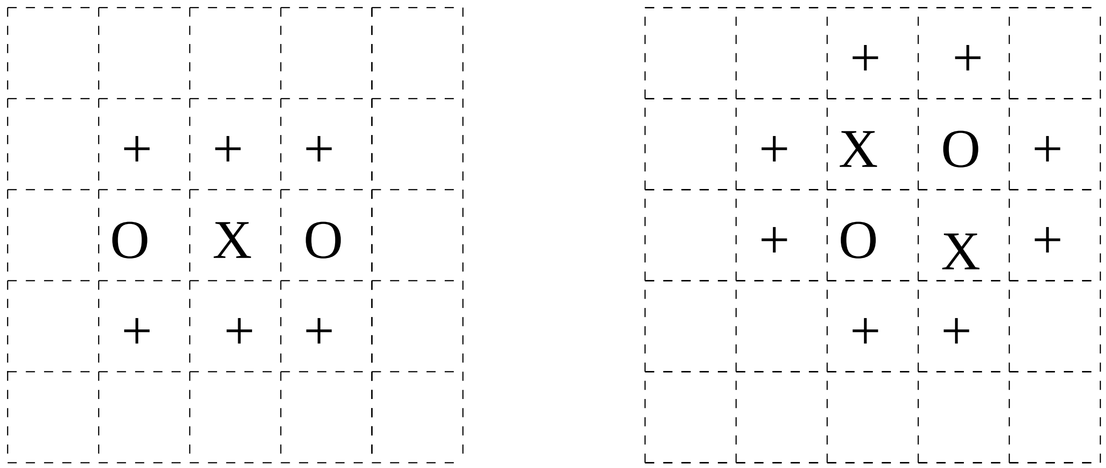

Figure 1: For the left configuration s1=1, s2=6 and for the right configuration s1=2, s2=8.

6 Critical adoption densities in random networks

For threshold 2, the M_t agents who adopt at time t are those with at least 2 ”parents” in S_{t-1} who have adopted before time t. We then obtain the recurrence relation:

$$M_t = \frac{(k-1)(k-2)N}{2N^2}(M_{t-1}^2 + 2M_{t-1}S_{t-2}) \quad (12)$$

$$s_t - s_{t-1} = \frac{(k-1)(k-2)}{2}(s_{t-1}^2 - s_{t-2}^2) \quad (13)$$

If we now suppose the existence of a critical density n_c, s_t should follow an exponential dynamics in the neighbourhood of n_c:

$$s_n = n_c * \alpha^n \quad (14)$$

Using the above ansatz in the previous equation predicts the critical density when $\alpha = 1$:

$$n_c = \frac{1}{(k-1)(k-2)} \quad (15)$$

The same method for threshold 3 gives:

$$n_c = \sqrt{\frac{2}{(k-1)(k-2)(k-3)}} \quad (16)$$

7 Conclusions

The same phenomenon, observation of extreme fraction of choices, close to 0 or close to 1, is observed for binary choices under conditions of “social choice”:

- For regular lattices and random networks.
- for contagion dynamics (threshold 1) and poll dynamics (larger thresholds).
- For different kinds of disorder.

These observations about dynamics can be used not only to describe empirical phenomena, but also to build “selling” strategies for sellers or policy implementers: importance of the notion of seed and interface between adopters and non adopters.