

Sociophysics Simulations IV: Hierarchies of Bonabeau et al

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Abstract. The model of Bonabeau et al explains social hierarchies as random: People keep a memory of recent fights, and winners have a higher probability to win again. The question of phase transition and the generalization from square lattices to networks is reviewed here.

INTRODUCTION

Why do some speakers at this 8th Granada Seminar give five lectures and expect a million Euro honorarium, while others are allowed only to present a poster and have to pay a registration fee. The elites of all times and regions always had excellent reasons why they should be on top: They were kings by the grace of god, or university professors by their excellent work. Indeed, at the age when Albert Einstein wrote about relativity, diffusivity and quantum photo effect as a technical expert at the patent office in Bern, this author was already paid as university assistant, a position never reached by Einstein. That's why this is this author's fourth contribution to these proceedings.

However, bad people [1] find also other reasons for my position. The Maxwell-Boltzmann statistics for classical ideal gases gives air molecules a probability $\exp(-v^2/2mk_B T)$ to have a velocity vector \mathbf{v} . If one air molecule has a velocity ten times higher than average, then statistical physicists see this as a normal though rare random event and do not associate any wonderful properties with this molecule. And this is how [1] treats us: We got our position in society by accident. These authors are evil because Bonabeau left academia and Deneubourg gave a talk without a tie at a conference where all male speakers were asked months in advance to wear a tie. Nevertheless we now look at their model, following Rasputin's advice that one can reject sin only after having studied it.

STANDARD MODEL

People diffuse on a square lattice filled with density p . Whenever a person wants to move onto a site already occupied by someone else, a fight erupts which is won by the invader with probability q and lost with probability $1 - q$. If the invader wins, the winner moves into the contested site whereas the loser moves into the site left free by the winner; otherwise nobody moves. Each visitor adds $+1$ to a history parameter h , and each loss adds -1 to h . At each iteration, the current h is diminished by ten percent,

so that roughly only the last ten time steps are kept in memory h . The probability q for fighter i to win against fighter k is a Fermi function:

$$q = 1/[1 + \exp((h_k - h_i)\eta)] \quad (1)$$

where the free parameter η could be unity. Initially everybody starts with $h = 0$; then $q = 1/2$ for all fights. After some time, history h accumulates in memory, q differs from $1/2$, and the standard deviation $\sigma(t)$ with

$$\sigma^2 = \langle q^2 \rangle - \langle q \rangle^2 \quad (2)$$

measures the amount of inequalities in society at that time step t and is obtained by averaging over all fights occurring during this iteration t .

This defines the model and the main quantity σ to look at. If instead one looks at the history of one person and integrates its h over time, after sufficiently long times it averages to zero: Who is on top at some time may be away from the top another time. Real Madrid has shown this to the football world: There is always one team winning the Champions League, but it is not necessarily the same team each year. We find similar examples in the political powers dominating Europe during the last two-thousand years.

Bonabeau et al [1] found a phase transition in that for high densities the inequalities are strong, and for densities below some threshold they no longer exist. This effects corresponds to widespread feelings (see the movie *Dances with Wolves*) that social hierarchies developed only with agriculture and cities (but what about the Mongolian empire ?) Unfortunately that effect was based on an assumption which prevented equilibrium and let the $|h|$ go to infinity. When corrected, the phase transition vanished [2]. The transition was restored [3] by a feedback loop: the quantity η in eq.(1) was replaced by the σ as calculated from eq.(2) at the previous time step. (For the first 10 time steps, σ was replaced by one.) Then a (first-order) transition was found again, Fig.1.

PROGRAM

Now follows the Fortran program for this latest version [3].

```

parameter(p=0.3 ,L= 1000 ,Lsq=L*L)
dimension hist(Lsq),latt(Lsq),ipos(Lsq),neighb(0:3)
real*8 q, qsum, qsu2, factor
integer*8 ibm,large
data eta,forget,ibm,maxstep/1.0, .10,1,10000/
1      large/'7FFFFFFFFFFFFFFFFF'X/
print *, p, L, eta, forget, ibm, maxstep
n=p*Lsq
fact=Lsq*1.0d0/large
factor=0.5d0/large
neighb(0)= 1
neighb(1)=-1
neighb(2)= L

```

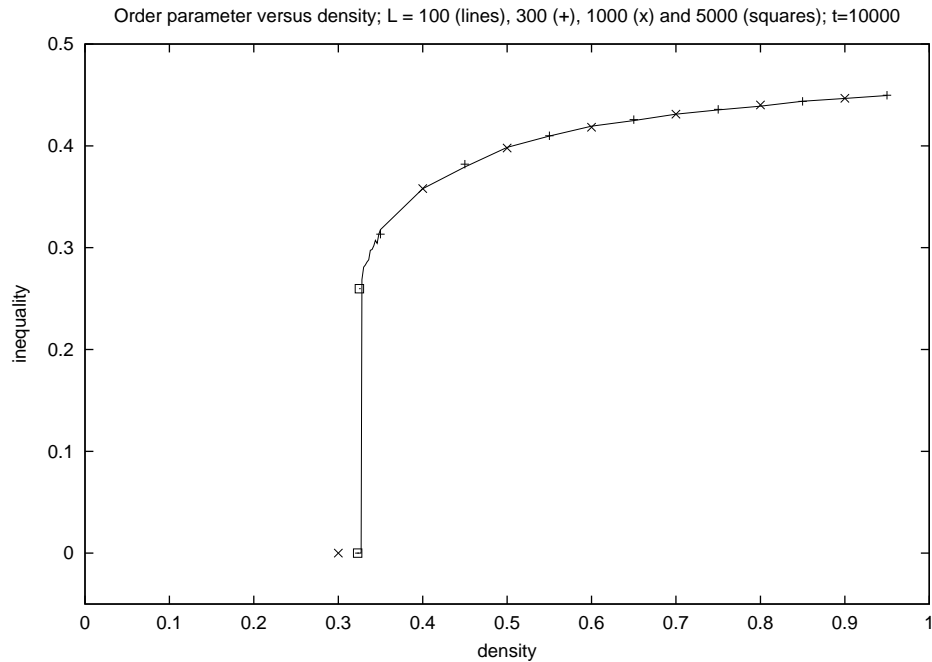


FIGURE 1. Phase transition from egalitarian (low density) to hierarchical (high density) society.

```

neighb(3)=-L
ibm=2*ibm-1
do 1 i=1,n
1   hist(i)=0
do 2 j=1,Lsq
2   latt(j)=0
do 3 i=1,n
4   ibm=ibm*16807
    if(ibm.lt.0) ibm=(ibm+large)+1
    j=1+fact*ibm
c   initially random, no two people on one site
    if(latt(j).ne.0) goto 4
    latt(j)=i
3   ipos(i)=j
c   initialization finished; no dynamics starts
do 5 itime=1,maxstep
icount=0
qsum=0.0d0
qsu2=0.0d0
do 6 i=1,n
    hist(i)=hist(i)*(1.0-forget)
    j=ipos(i)
    ibm=ibm*16807
    jnew=j+neighb(ishft(ibm,-62))

```

```

        if(jnew.gt.Lsq) jnew=jnew-Lsq
        if(jnew.le.0) jnew=jnew+Lsq
        if(latt(jnew).eq.0) then
c          either new site is empty: move there; or it is occupied: fight
            latt(jnew)=i
            latt(j)=0
            ipos(i)=jnew
        else
            k=latt(jnew)
            qq=eta*(hist(k)-hist(i))
            if(itime.gt.10) qq=qq*sigma
            if(abs(qq).lt.10) then
                q=1./(1.0+exp(qq))
            else
                if(qq.lt.0) q=0.9999
                if(qq.gt.0) q=0.0001
            end if
            icount=icount+1
            qsum=qsum+q
            qsu2=qsu2+q*q
            ibm=ibm*65539
            if(0.5+ibm*factor .lt. q) then
c              now i has won over k and moves
                latt(jnew)=i
                latt(j)=k
                ipos(i)=jnew
                ipos(k)=j
                hist(i)=hist(i)+1.0
                hist(k)=hist(k)-1.0
            else
                hist(i)=hist(i)-1.0
                hist(k)=hist(k)+1.0
            endif
        endif
        continue
6      qsum=qsum/icount
        qsu2=qsu2/icount
        sigma=sqrt(qsu2-qsum*qsum)
        if(sigma.lt.0.000001) goto 7
5      print *, itime, sigma, icount
7      continue
        stop
        end

```

This program unfortunately violates the Gerling criterion that nobody should publish more program lines than (s)he has years in life. Thus I start with the core, after the com-

ment line 41: If the site j_{new} to which agent i wants to move is empty, $\text{latt}(j_{\text{new}}) = 0$, then the move is made: the position of the agent is now j_{new} , and the occupation variables of the sites j, j_{new} are interchanged.

Otherwise a fight starts between agent i and the present inhabitant k . The probability q from eq.(1) is calculated (with an escape if the argument of the exponential function is too large) and taken into account in the averages for σ , eq.(2). A random integer i_{bm} , obtained by multiplication with 65539 (or better 16807 as earlier), is compared after normalization with the probability q of the invader i to win; if i wins, again the occupation variables are interchanged, and so are the position variables i_{pos} ; moreover, the history variables h are changes by ± 1 . If the invader loses, nobody moves, and only the history variables are changed in the opposite sense. Then the loop over all n agents ends, σ is evaluated and printed out. If $\sigma < 10^{-6}$ or if the maximum number maxstep of iterations is reached, the simulation ends.

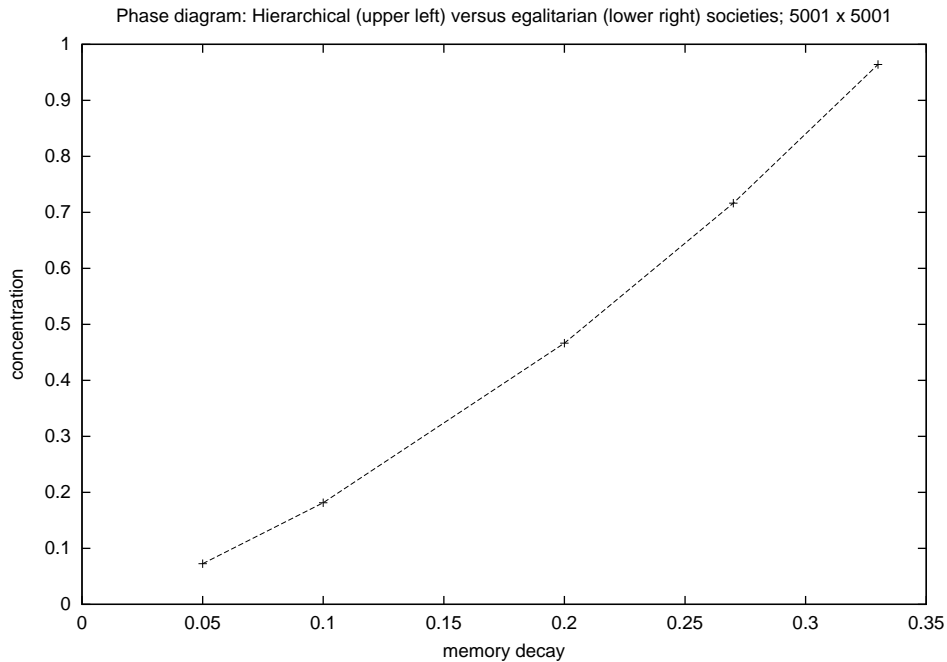


FIGURE 2. Critical oncentration versus fraction of history forgotten at each iteration; from [4] for a case when not all are equal initially.

MODIFICATIONS

One may assume for this last model that some people really are superior to others, and not just more lucky [4]. Fig.2 shows one of the resulting phase diagrams for the still existing phase transition.

If this model [3] leads to hierarchies, then they are symmetric: There as many people on top as they are on bottom. Reality is different: There are few leaders only. This asymmetry was partially reproduced by reducing the history counter h by F points, with

for example $F = 2$, in the case of a loss, while a victory still increases h by only one point [5].

Sá Martins in that paper [5] also looked at scale-free networks of Barabási-Albert type [6]. This aspect was studied more thoroughly by Gallos [7] and Sousa [8]. It means we no longer fight about territory against whoever sits on the lattice site to which we want to move. Instead we fight for power with our acquaintances. And the social network of acquaintances may be described by scale-free networks, where the number k of neighbours for each site follows a probability distribution $\propto 1/k^3$ instead of having $k = 4$ on the square lattice. Details of the simulations differ, and so do their results [7, 5], but the sharp phase transition was recovered. Gallos finds it at a very low concentration < 0.1 , which moreover may decrease towards zero for increasing network size.

A simpler network allows everybody to contact everybody, and also here abrupt changes in the amount σ of hierarchies were seen [9]. Other cases studied were Erdős-Rényi random graphs, Watts-Strogatz small-world networks, and triads where friends of my friends are likely also my own friends [8].

SUMMARY

Even though the model was already published in 1995, it seems to become fashionable only now with three independent papers in the first few months of 2005 [9, 7, 8]. Some crayfish [10] followed Bonabeau et al earlier as we physicists.

ACKNOWLEDGMENTS

We thank A.O. Sousa for a critical reading of the manuscript.

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